Physics 335 February 2, 2023
Physics 635 **Due**: February 9, 2023

Homework #4

Problem 1

Do by Monte Carlo integration the same integral as in Set 3/Problem 2, namely

$$\int\int e^{-x\sin y} dxdy$$

over the interior of an ellipse whose boundary is given by

$$5x^2 - 6xy + 5y^2 = 2.$$

Get an accuracy better than 0.5%, and give both an answer and a (calculated) estimated error. Keep track of how many times you evaluate the integrand in total, and give the total number N. Before you start, think about any tricks you can use on the integrand or the region of integration to keep N as small as possible.

Project #1

Simulate the magnetic properties of a `feromagnetic material` with the help of the `Ising` model.

You will implement a two-dimensional Ising model. You should follow the steps that were outlined in class. Store the interacting spins in an array of size $N \times N$. Ultimately, $N = 32$ or $N = 64$ is probably a good number, but for developing your code you can start with arrays as small as $N = 4$.

It is convenient to store the array as a property of a MatLab class, and define class methods for all the operations needed in this project. Your goal is to find the following properties as functions of temperature:

- Magnetization
$$M = \sum_{i,j} S_{ij} \tag{1}$$
(where $\{i, j\}$ run over all elements of the array)

- Energy
$$E = -J \sum_{i,j} S_{ij}(S_{i+1\ j} + S_{ij+1}) \tag{2}$$

- Specific heat
$$C_v = \frac{1}{k_B T^2} \left[\langle E^2\rangle - \langle E\rangle^2\right] \tag{3}$$

- Magnetic susceptibility
$$\chi = \frac{1}{k_B T} \left[\langle M^2\rangle - \langle M\rangle^2\right] \tag{4}$$

Without loss of generality, you may assume $k_B = J = 1$ in this project.

You will want to start each temperature with a random state of spins, then perform some number of "sweeps" through the grid. During each sweep, apply the `Metropolis` algorithm to N^2 random points in your grid. After performing a sufficient number of sweeps to reach "equilibrium", perform additional sweeps during which you will collect the data needed to construct the desired plots.

The **FIRST** objective of this project is to write the code necessary to compute the data. This will be turned in at the end of one week. You should have a complete working code that is tested for small values of the grid size m and for several values of T.

The **SECOND** objective of this project is to run the code to produce the best data that you can. This will be done in teams. You will first verify that your various codes give consistent results. Then you will need to decide what you have to do (as a team) to produce the best and most reliable results. These *team* results will be delivered at the end of the second week.