Physics 335 February 16, 2023
Physics 635 **Due**: February 23, 2023

Homework #6

Problem 1

The Bessel function $J_1(x)$ satisfies the equation

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + \left(1 - \frac{1}{x^2}\right) y = 0. \tag{1}$$

Given starting values at $x = 1$, integrate the equation to find J_1 at $x = 30$. Do the integration once using `StepperDopr5` and once using `StepperBS` and compare the two routines. You can call each routine in turn from `Odeint` just by changing the argument in the call to `Odeint` from `StepperDopr5` to `StepperBS`. Specifically:

1. Get the value of J_1 at $x = 1$ from the MatLab routine `besselj(1,x)` and the value of the derivative from the recurrence relation $J_1' = J_0 - J_1/x$.

2. Compare your values of J_1 with that from `besselj` in a plot. Also plot their difference.

3. Put a "counter" in the functor that computes the derivatives to determine how many times it is called. Compare Runge-Kutta with Bulirsch-Stoer

Problem 2

Integrate Eq. (1) with any method you like given J_1 at $x = 0$ to find $J_1(1)$. Don't use `besselj` or the recurrence relation to get initial conditions. Instead use the series approximation for $J_\nu(x)$ for small x. NOTE:

1. Can you start the integrations exactly at $x = 0$?

2. Try integrating the equations as you did in Problem 1. Do the integrations behave as nicely?

3. Instead of using $z(x) \equiv \frac{dy}{dx}$, try instead $w(x) \equiv x\frac{dy}{dx}$. Does this work better? Why?

Problem 3

Implement (your own version of) both the explicit and implicit Euler method to integrate the differential equation

$$y' = -\lambda y \tag{2}$$

from $x = 0$ to $x = 1$ in n equal steps (so $h = 1/n$). Use $\lambda = 100$, and start with the initial condition $y = 1$ at $x = 0$.

1. Do the integration with $n = 1000$, make a graph of $y(x)$ for both integrations, and compare with the analytical result.

2. What is the minimum number of steps n for the explicit Euler method to be stable?

3. Do the integrations again for $n = 49, 50$ and 51. For each n, make a graph of $y(x)$ for both integrations and compare with the analytical result.