Assignment 4 : Optical Flow
Computer Vision

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Practical Problem:

Optical Flow:

Aim : Optical Flow reconstruction from sequence of images. On a set of images capturing the motion of one or multiple objects, we want to be able to reconstruct the displacement field associated to each pixel during the time difference from one frame to an other.

Solution:
1.1 Normal Flow:

A set of two consecutive images were taken from the set of images extracted from the video sequence. Our dataset consists of sequence of images extracted from a video over time. Hence, we have a three dimensional dataset for the experiment. As it is a sequence of images we have two variables for space over each image and an time variable for image.

As explained in the aim, our dataset consists of images which have one or multiple objects whose motion changes between two frames that changes its position from (x,y) to (x+u,y+v). The example of a image sequence pair is as below.

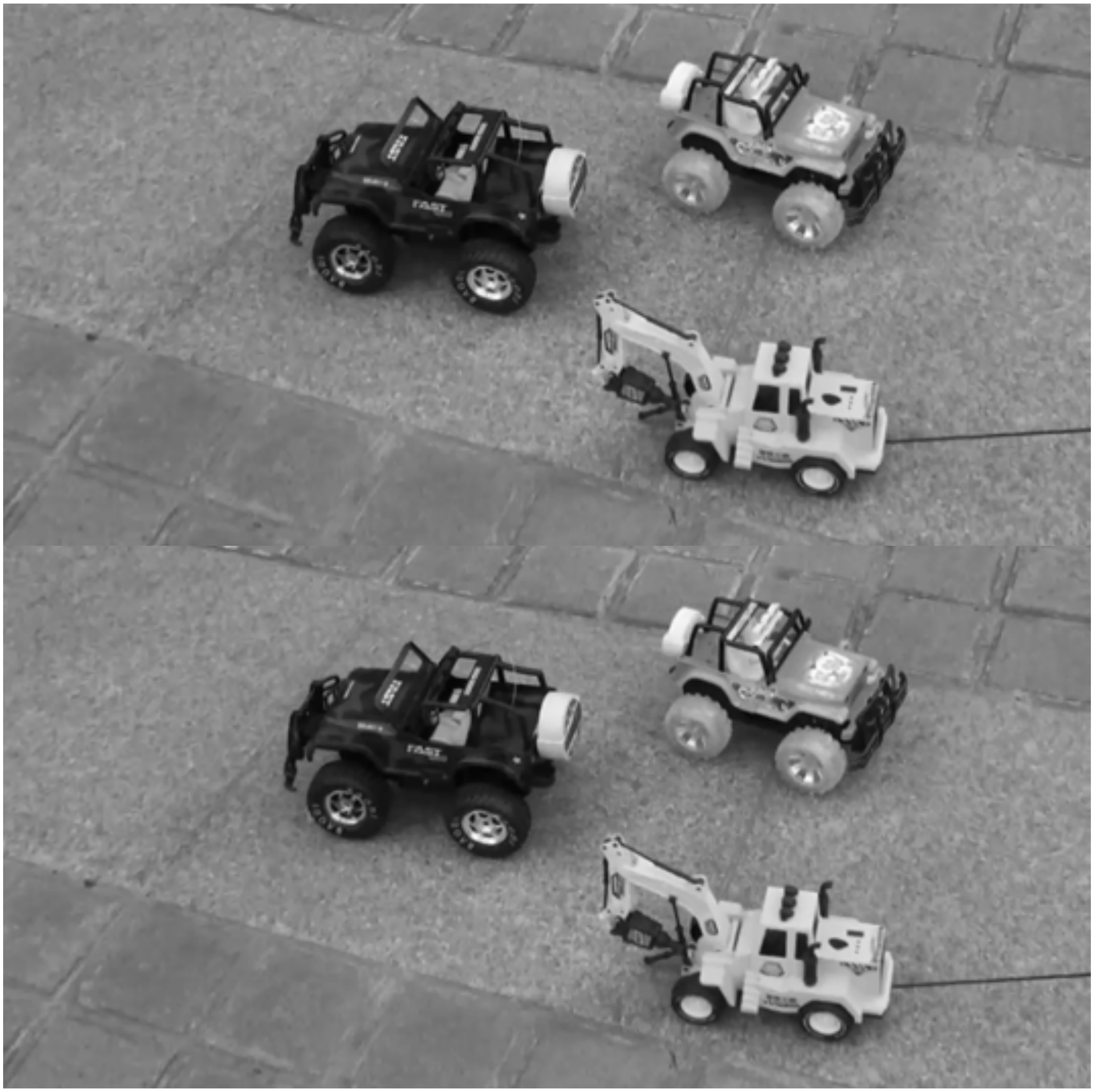
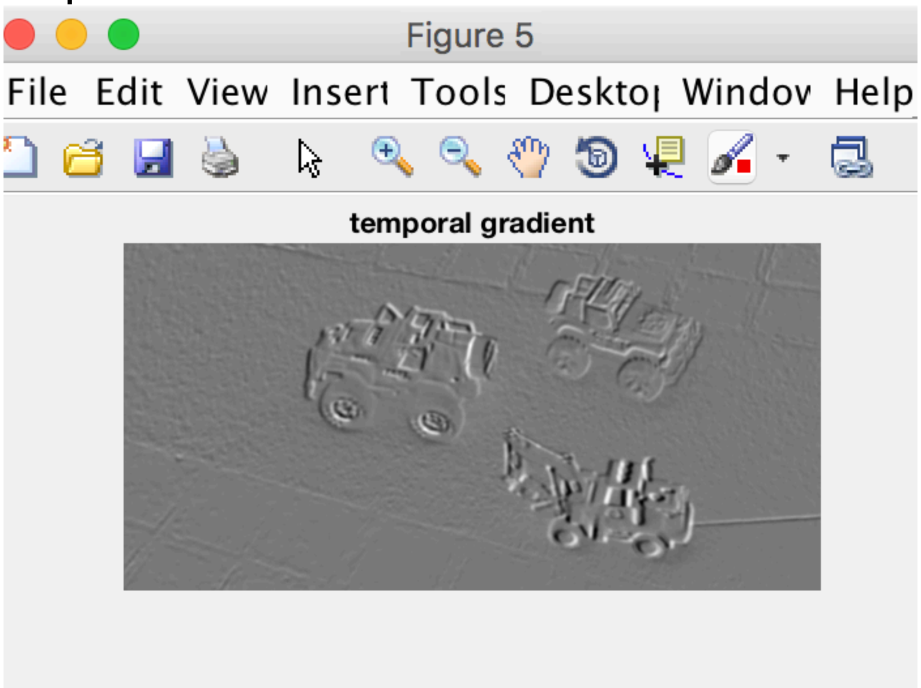


Figure 1. Example of pair of image sequence from video.

Once we have the image sequence pairs we need to smooth the images using the Gaussian filter. As the Image processing part of the Gaussian filter was explained , the direct provided function was used to smooth the image with sigma 1 and sigma 2.

As we have three dimensional dataset we can get the spatial and temporal derivatives of the image. The temporal gradient between the two consecutive images can be computed by the formula $\frac{\partial E}{\partial t}$. This temporal gradient is computed subtracting two frames as I(x,t+1) - I(x,t) . The result of temporal gradient can be given as below for images with smoothing sigma 1 and sigma 2, as well as on the raw image.

Temporal Gradient Results:



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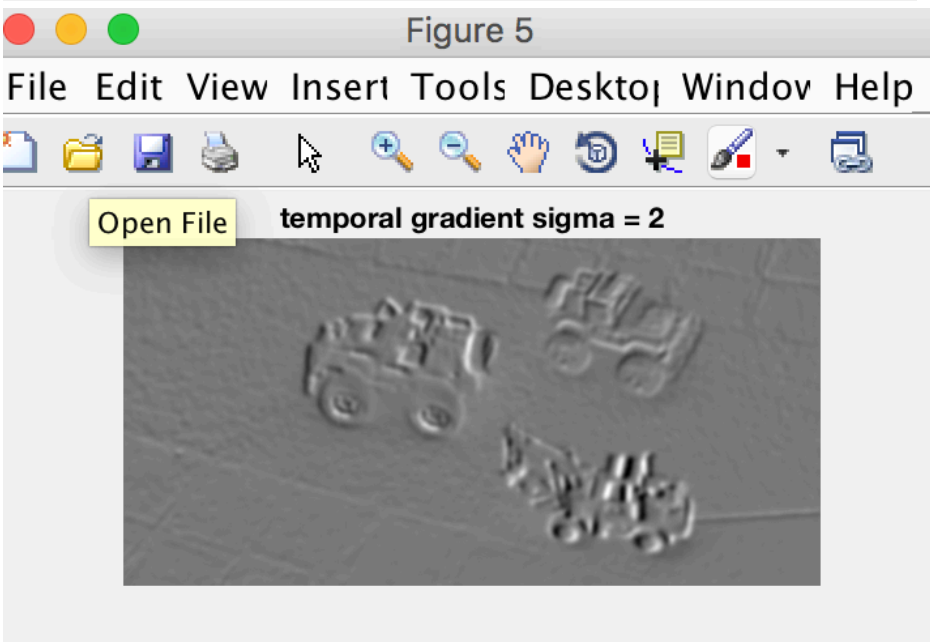
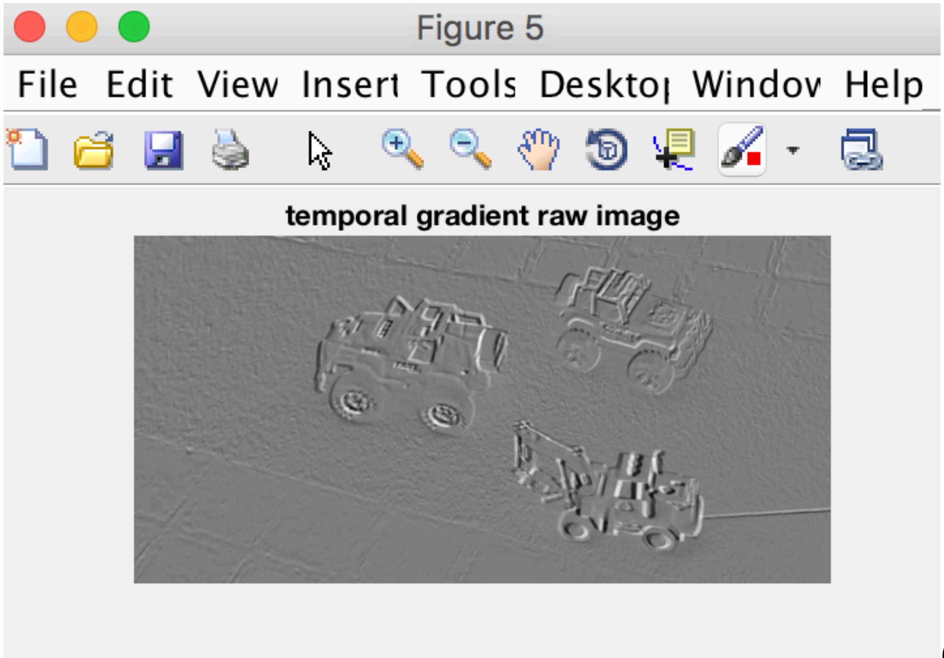


. (B)



(C)

Figure 2: A) Temporal gradient Result for image smoothed with sigma 1, B) Temporal gradient Result for image smoothed with sigma 2, C) Temporal gradient Result for raw image.

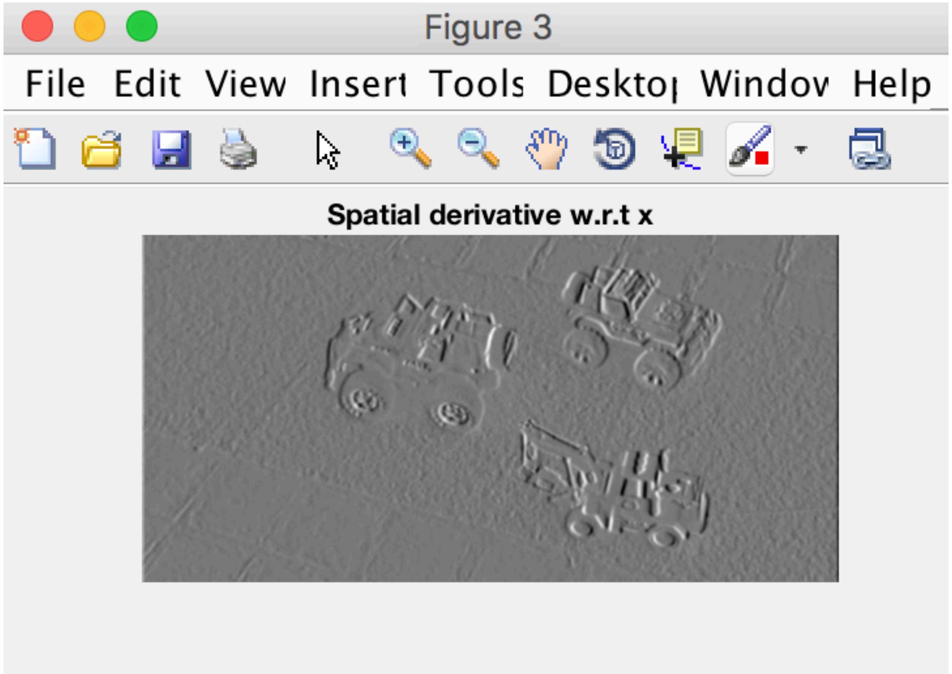
Discussing the results for the temporal gradients which is basically the subtraction of the intensities of the two consecutive images. As we can see we have well defined line features for the raw image, then the results for the smoothed images with sigma 1 and 2 have images with image intensity issues if we have a more sigma has the gradient is not that well defined but instead it shows the path with more intensity as more defined compared to the one with low intensity.

Once we have temporal gradient, next step is to get the Spatial derivates for the images. The spatial derivatives $E_x = \frac{\partial E}{\partial x}$ and $E_y = \frac{\partial E}{\partial y}$ are derived from the image. The spatial derivative with respect to x is calculated using I(x+1,y,t)-I(x,y,t) for $E_x = \frac{\partial E}{\partial x}$ and the spatial derivative with respect to y is calculated using I(x,y+1,t)-I(x,y,t) for $E_y = \frac{\partial E}{\partial y}$.

The results for spatial derivatives for image smoothed with sigma 1 and sigma 2 , also for raw image are given as below:

Spatial Derivative Results:

Images with Smooth Sigma = 1



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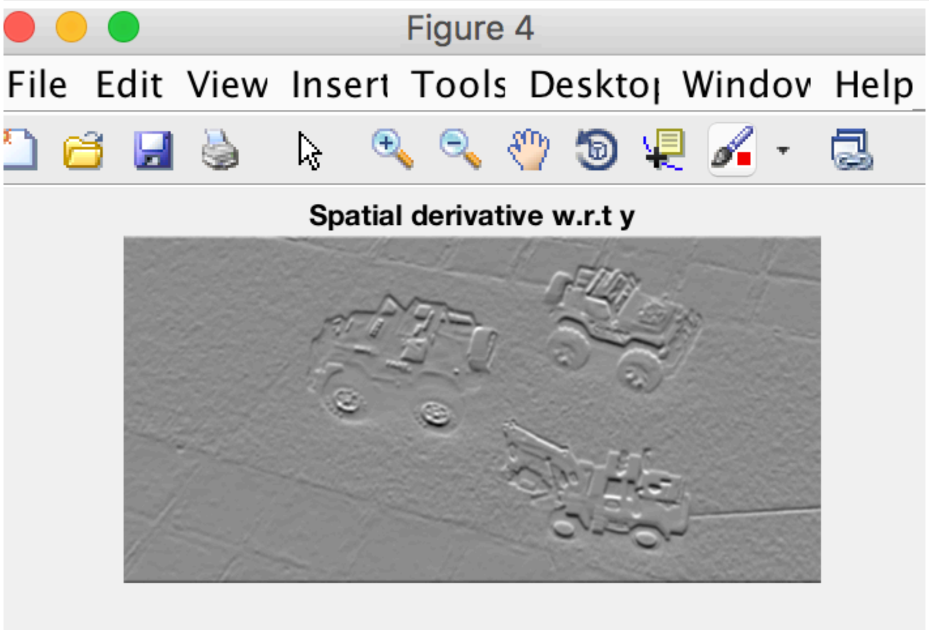

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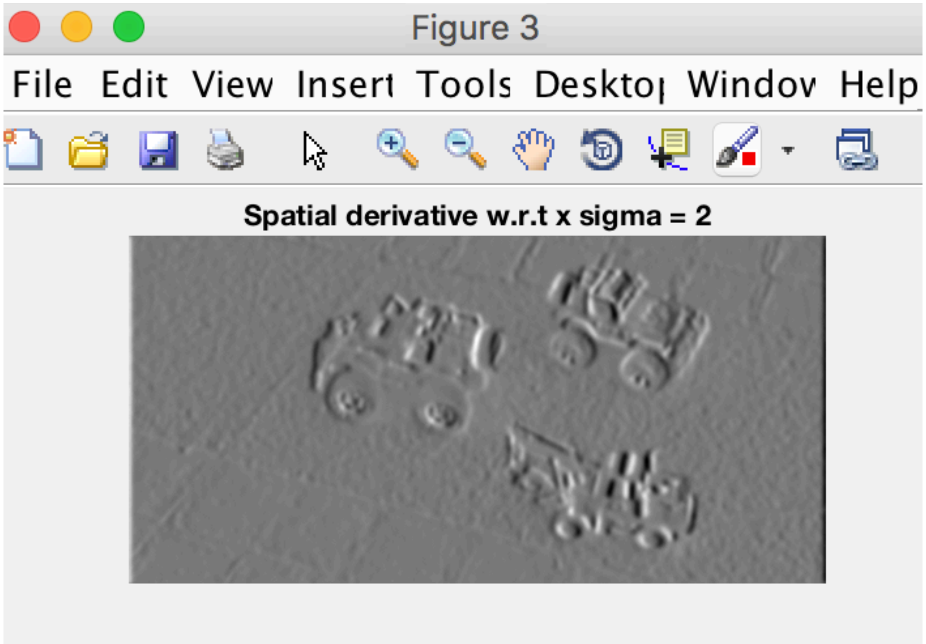
Figure 3: A) Spatial derivative w.r.t x for sigma = 1 , B) Spatial derivative w.r.t y for sigma = 1

Images with Smooth Sigma = 2



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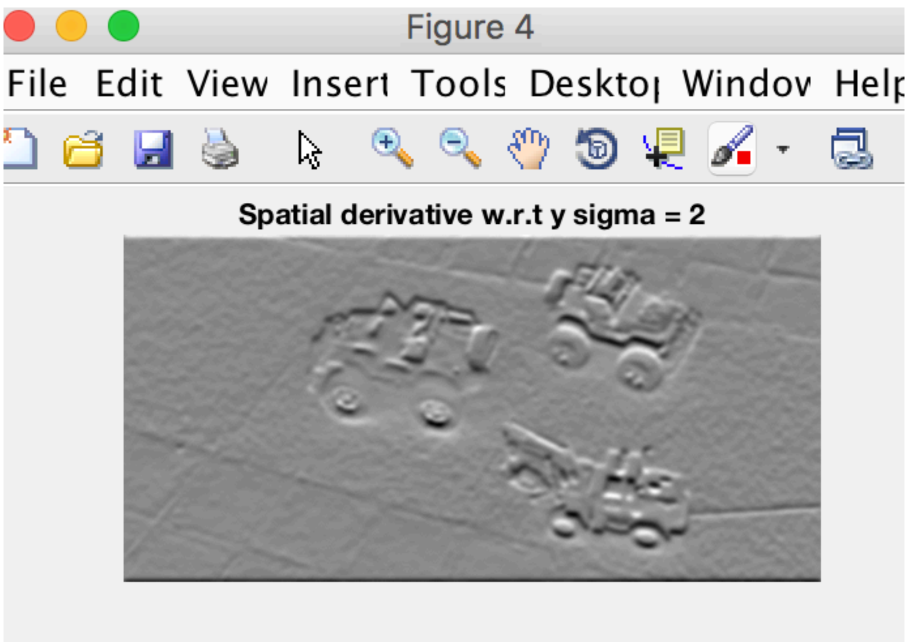
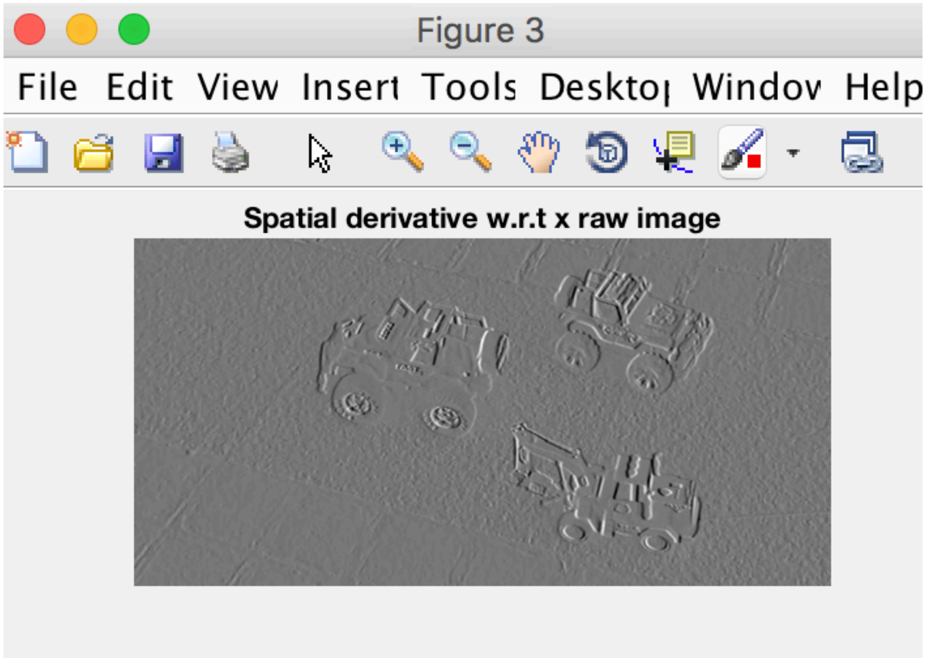


. (B)

Figure 4: A) Spatial derivative w.r.t x for sigma = 2 , B) Spatial derivative w.r.t y for sigma = 2

For raw Images



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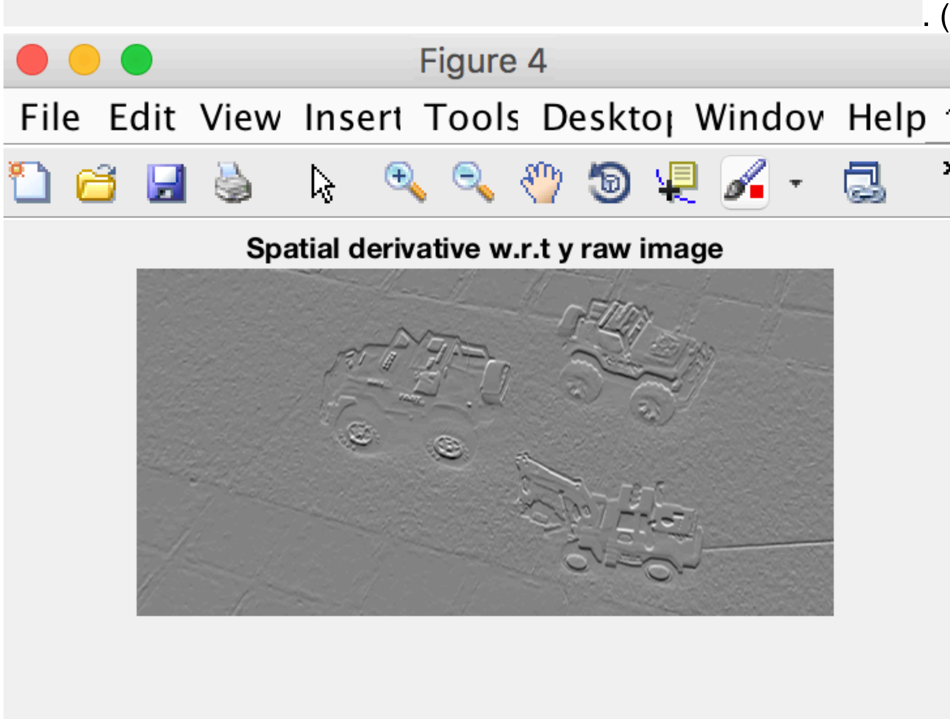

(B)

Figure 5: A) Spatial derivative for raw image, B) Spatial derivative for raw image

Discussing the results for spatial derivatives, as we can see we have well defined edges with the raw image but it contains noise. So we removed noise with the Gaussian filter provided. Once we smooth the image using sigma 1 and sigma 2, the noise from the images is removed. The spatial derivative with sigma 2 has more of the image blurred hence the results are not as expected. Thus, the image smoothed with sigma 1 has perfect mixture of no noise and smoothing to provide reasonable results.Also, we can see the (A) images have clear vertical edges , thats the effect of the gradient on the horizontal direction and the stronger are created by vertical lines. The same thing is to be noticed for (B) images, we have gradient according to the vertical direction where the strongest discontinuities enhanced are horizontal lines.

Normal Flow:

The normal flow for the one dimensional spatial derivative is calculated using the set of formula to solve for v.

Assumptions :

The motion between frames is small

Conservation of pixel brightness over time

Brightness Constancy Assumption:

$$f(t) \equiv I(\underbrace{x(t),t}) = I(x(t+dt),t+dt)$$

$$\frac{\partial f(x)}{\partial t} = 0 \quad \text{Because no change in brightness with time}$$

$$\underbrace{\left.\frac{\partial I}{\partial x}\right|_{t}}_{I_x} \underbrace{\left(\frac{\partial x}{\partial t}\right)}_{v} + \underbrace{\left.\frac{\partial I}{\partial t}\right|_{x(t)}}_{I_t} = 0$$

$$\Rightarrow \quad v = -\frac{I_t}{I_x}$$

As per the above formula we can solve for v by using the spatial derivative for x and the time gradient.

Normal Flow results:

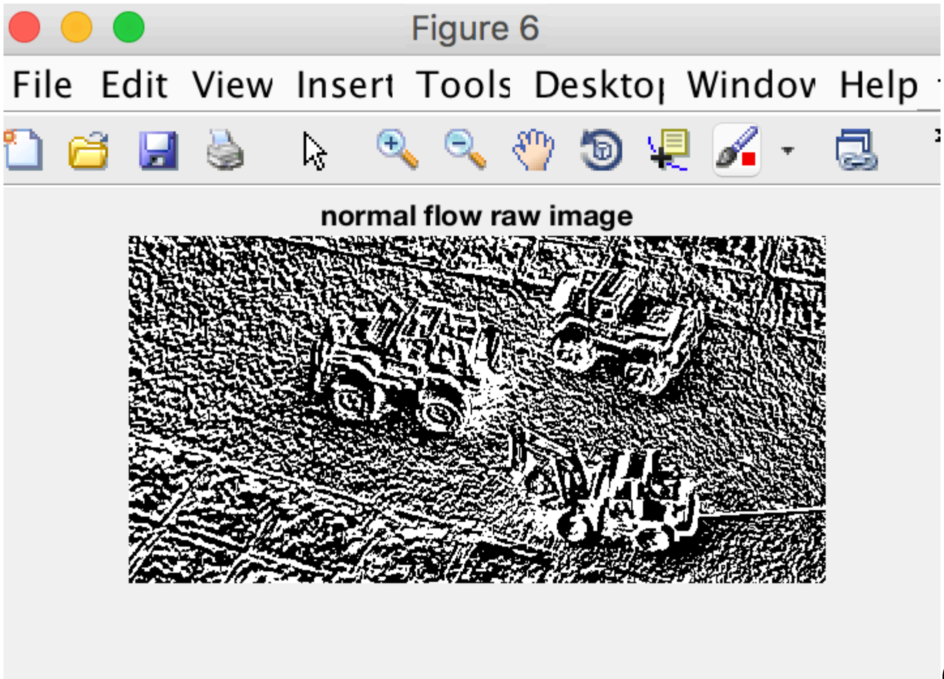
Figure 6
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normal flow raw image

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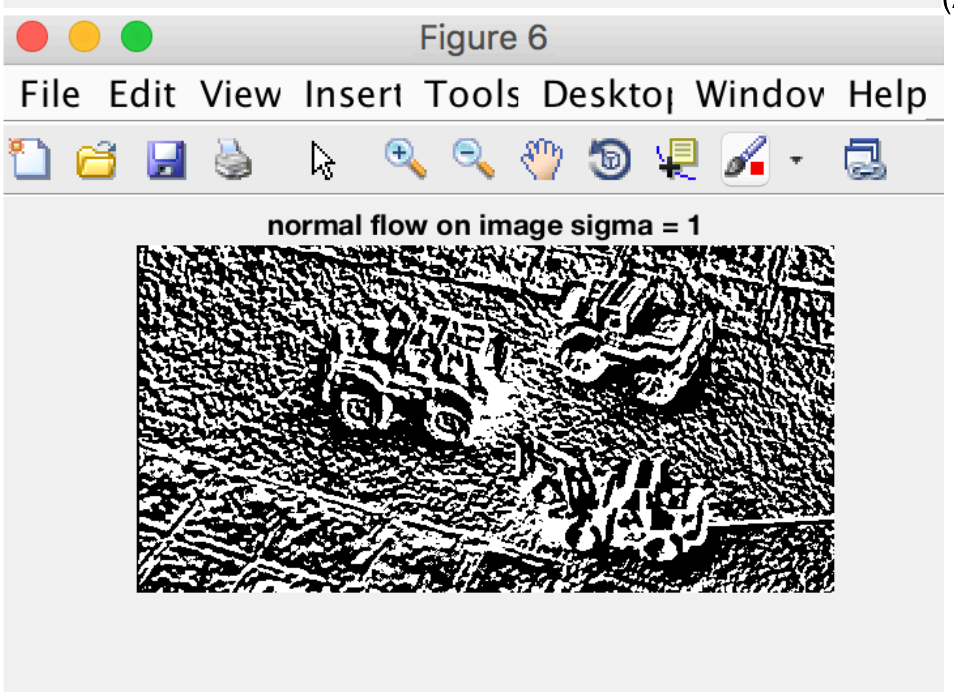
Figure 6
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normal flow on image sigma = 1

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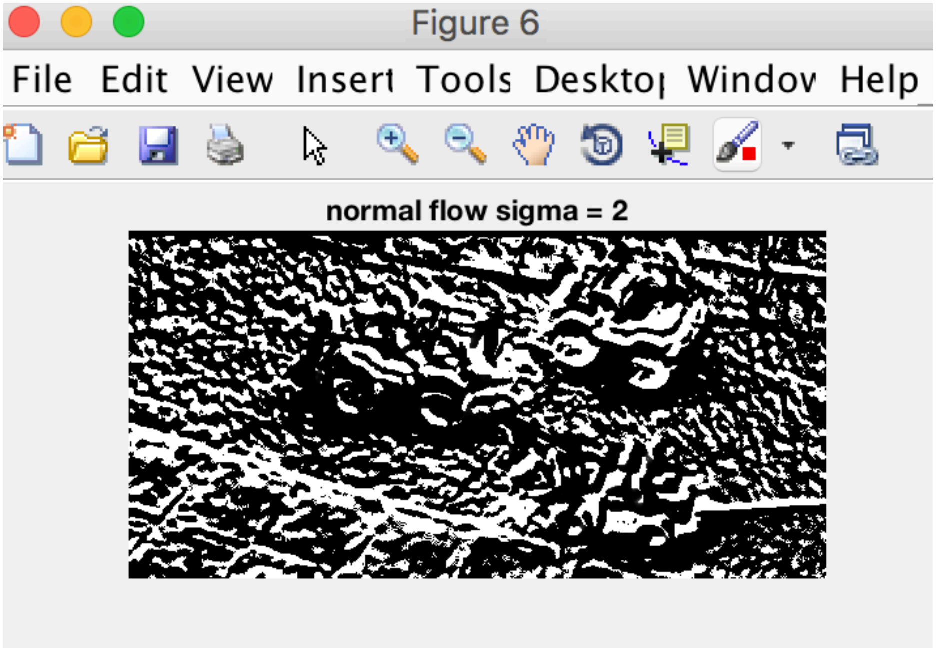

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Figure 6 : A) Normal Flow for raw image, B) Normal Flow for smooth image with sigma = 1, C) Normal Flow for smooth image with sigma = 2

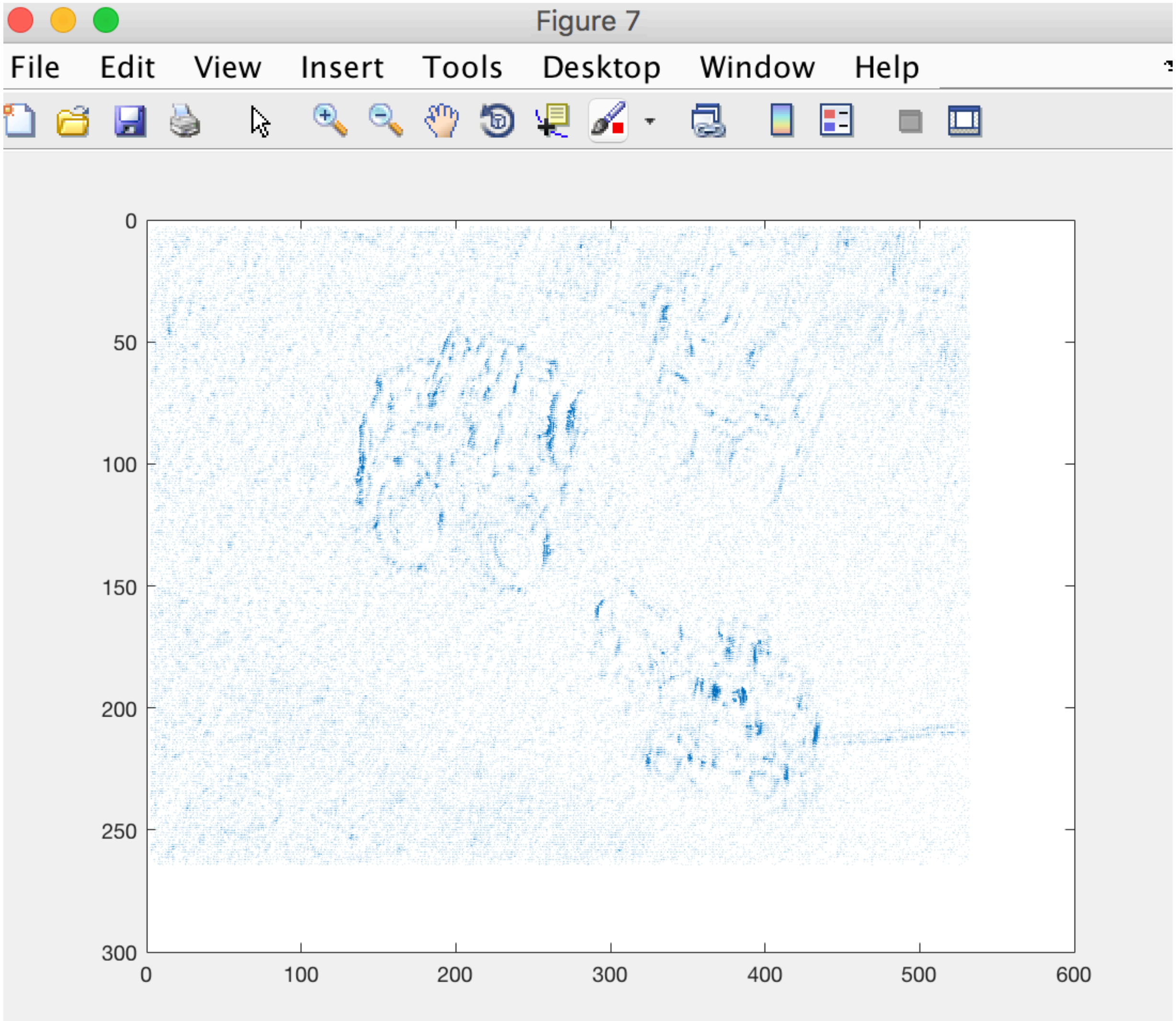
Discussing the results for normal flow, as we can see the results for the normal flow are well established for the image with smoothing with sigma = 1. The images with smoothing sigma = 1 we have balanced intensity distribution over the entire image. While in the raw image the intensity of points for the white points is higher while the intensity for black points is higher for smooth images with sigma = 2.

Flow Calculated over pixel neighborhood:

Once we have computed and smoothed the images , the normal flow can be calculated by using the 2x2 grid pixel from the local velocity estimates and solving the system of equations. As per the inclass discussion , we have a system of equations to solve for v, which can be given as

$$\begin{aligned} Av + b &= 0 \\ A^T A v &= -A^T b \\ v &= -(A^T A)^{-1} A^T b \\ C &= A^T A \end{aligned}$$

Hence , solving this system for 2x2 pixel neighborhood for v, we can get the following results: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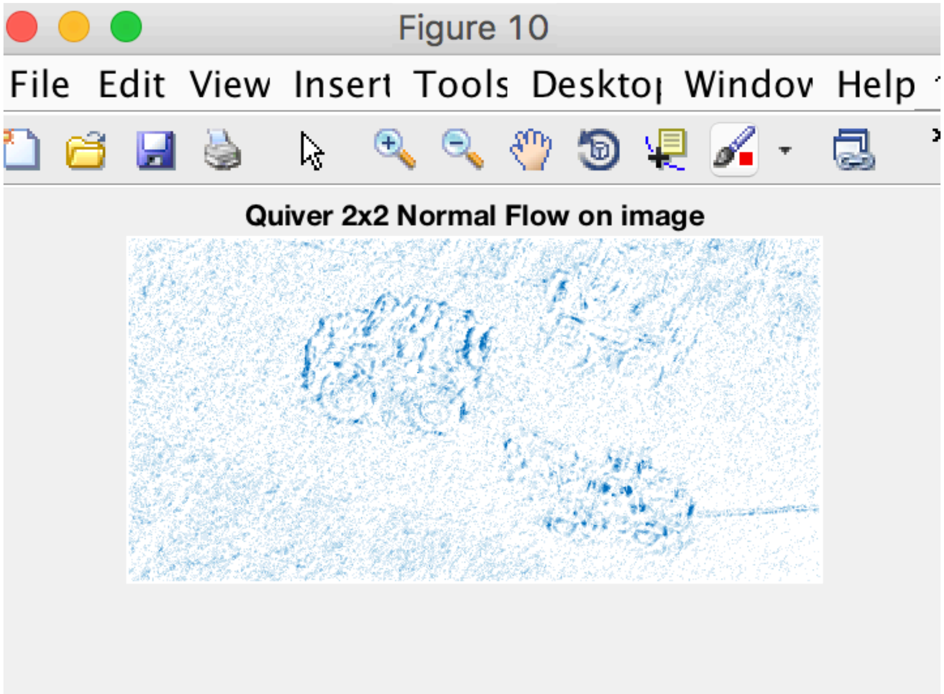
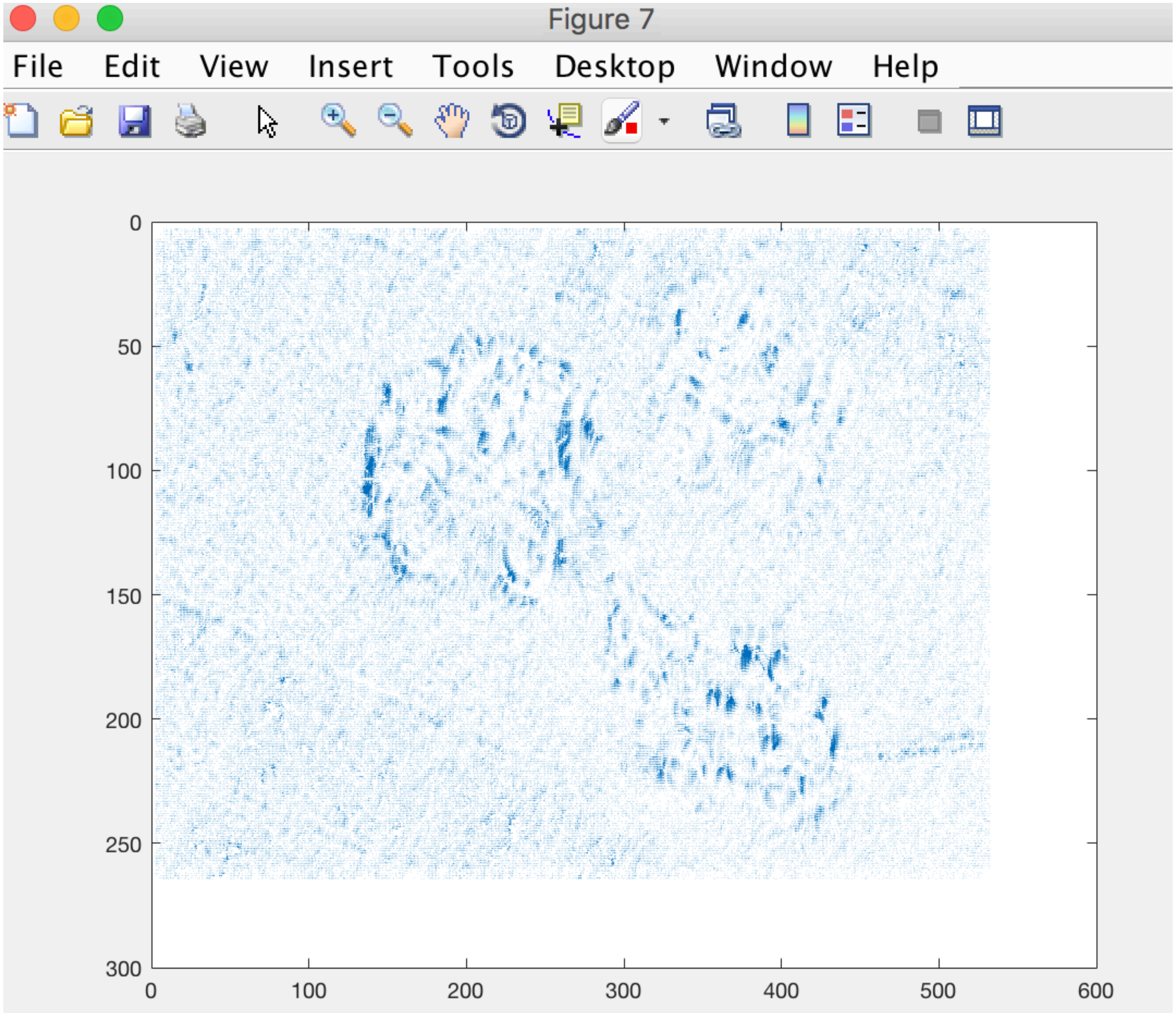
.(A)  (B)

Figure 7: A) Flow calculated over neighborhood pixel with Smooth image sigma =1 , B) Flow calculated over neighborhood pixel with Smooth image sigma =1 superimposed on original image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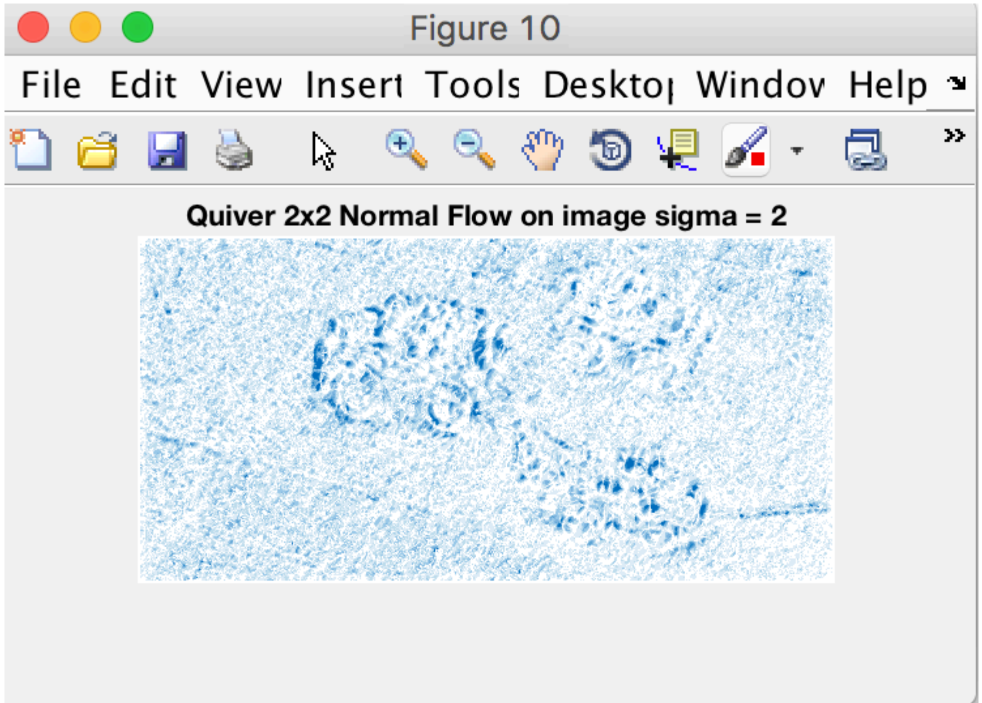
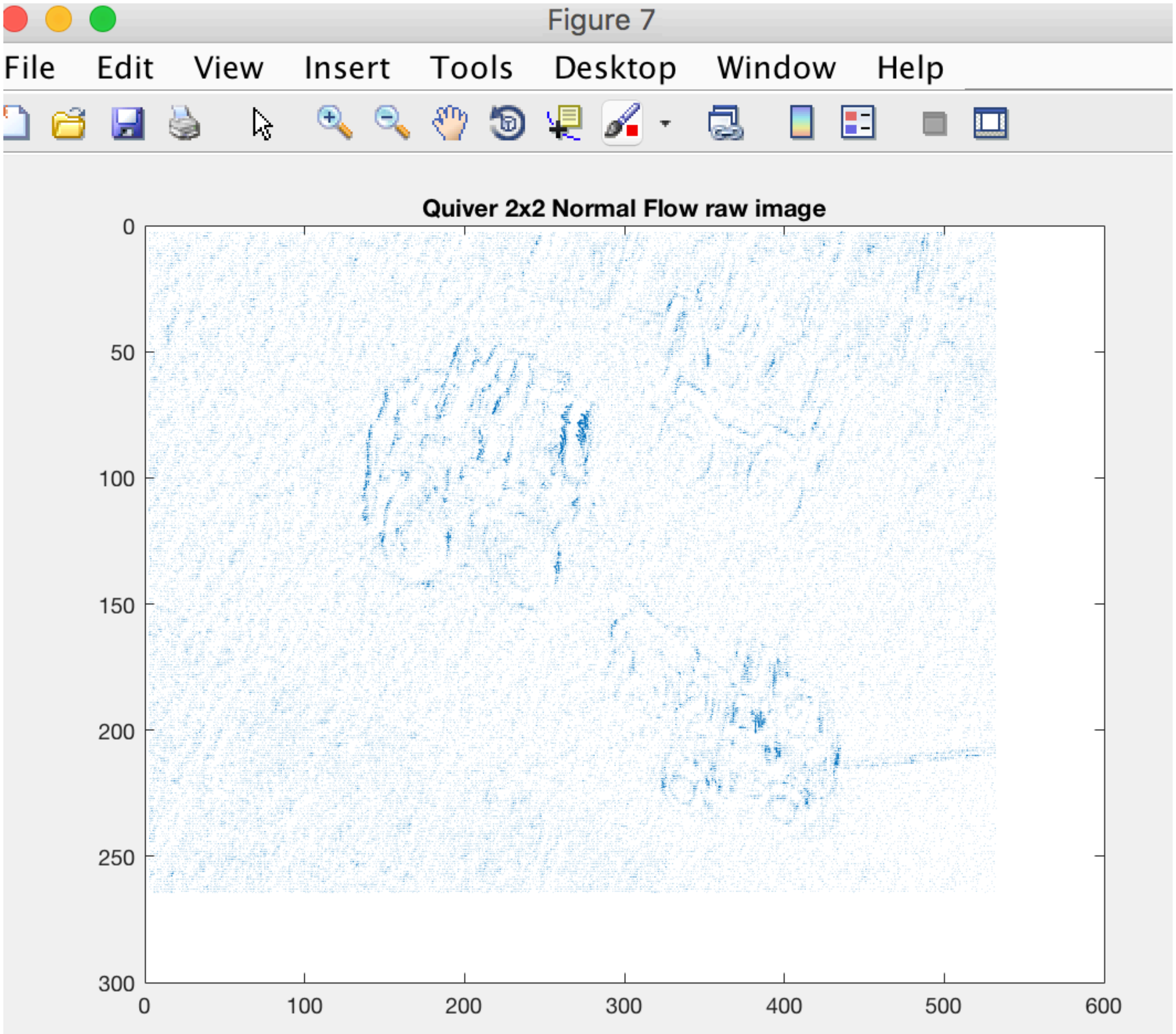


(B)

Figure 8: A) Flow calculated over neighborhood pixel with Smooth image sigma =2 , B) Flow calculated over neighborhood pixel with Smooth image sigma =2 superimposed on original image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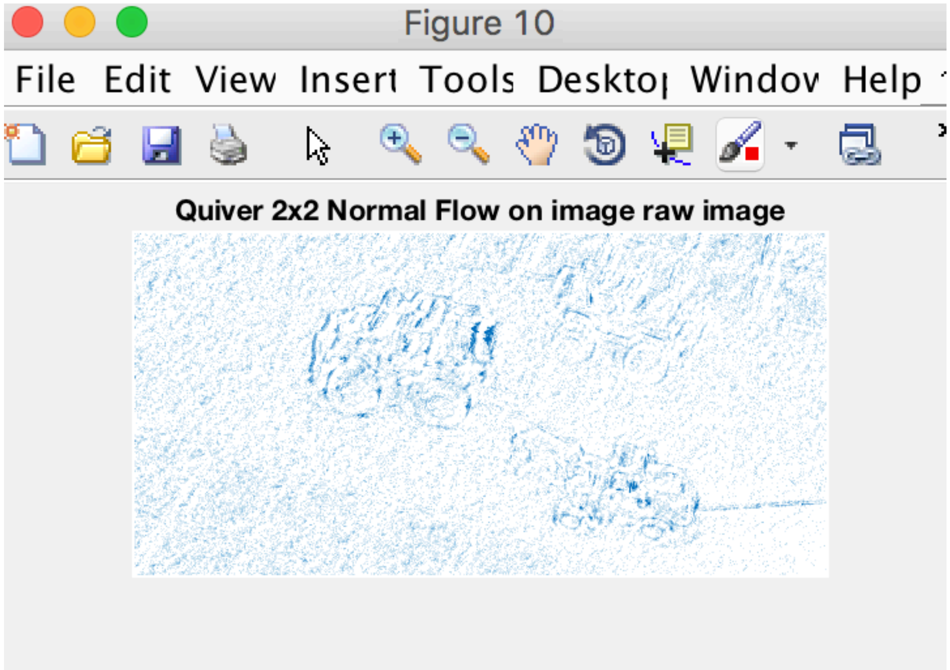
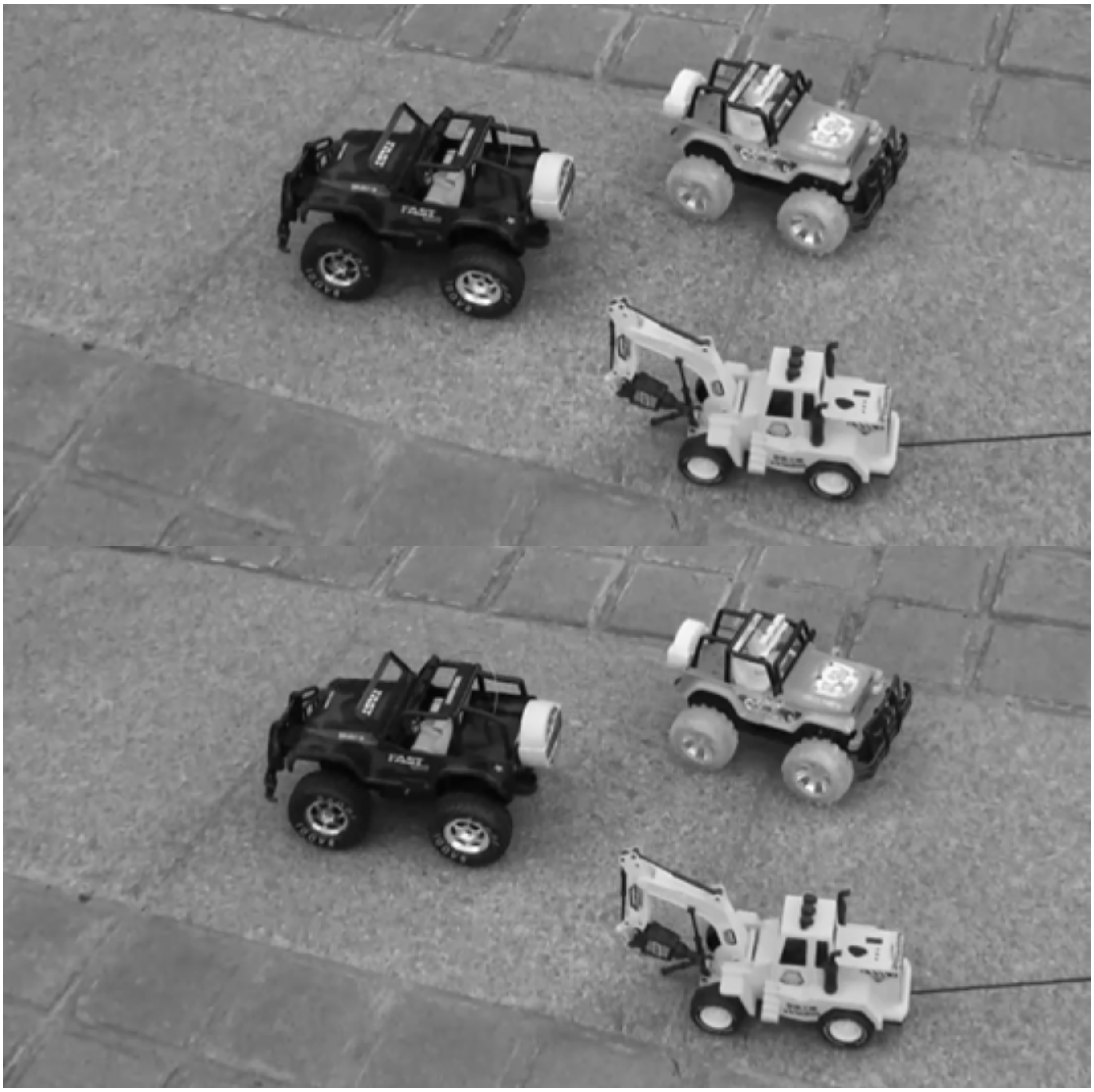
. (A)  (B)

Figure 9: A) Flow calculated over neighborhood pixel with raw image , B) Flow calculated over neighborhood pixel with raw image superimposed on original image

Discussion over the flow calculated using the pixel neighborhood of 2x2 pixel grid, the results with the smooth images with sigma = 1 prove to be better than the results with the raw image and the smooth image with sigma=2. As discussed earlier , the intensities of white points are more with the raw images while intensities for black points is more with sigma = 2 images.

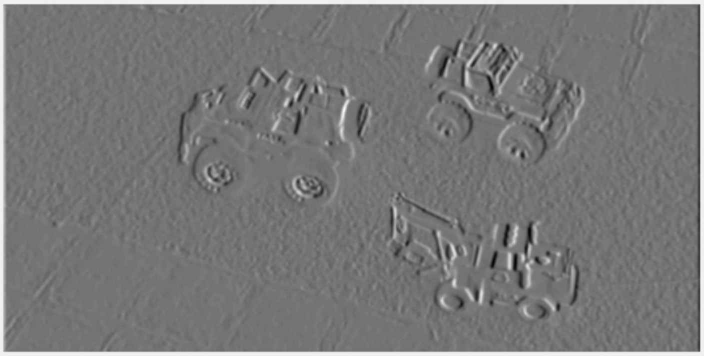
Comparing Results with Another Pair:

The pair of images considered is :

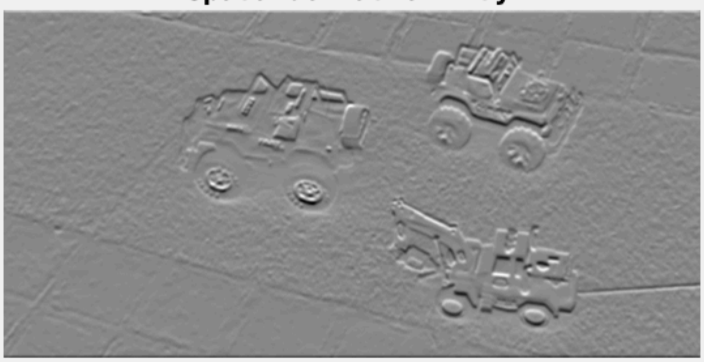


Results: For another (above) pair of images for smoothed images with sigma = 1

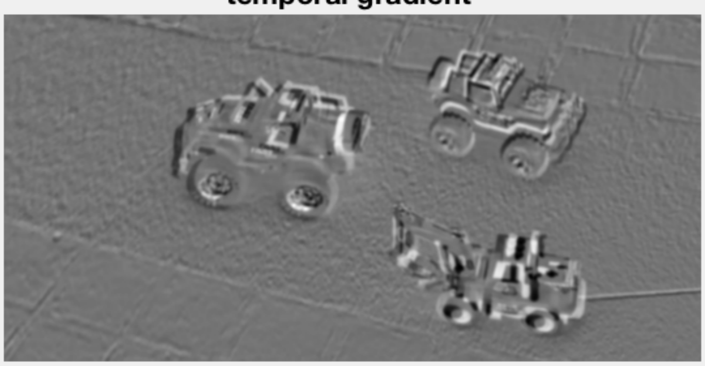
Spatial derivative w.r.t x

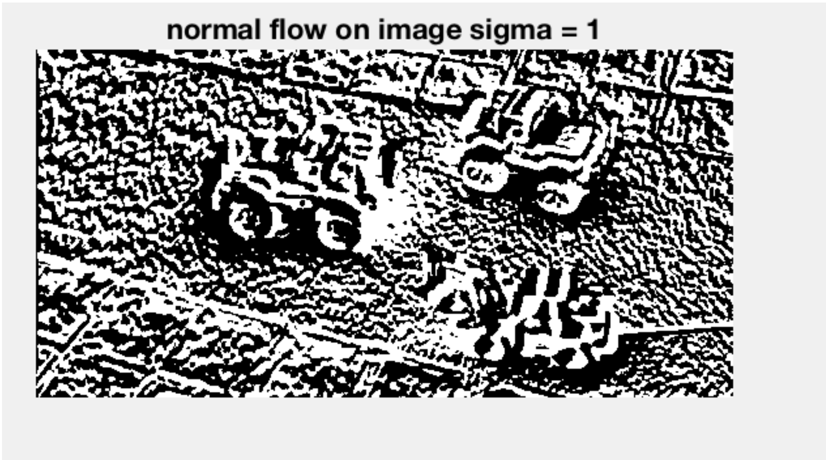


Spatial derivative w.r.t y



temporal gradient





Discussing about the results, we can see there is not much of a difference in the results if we take different pair of image from the video sequence, but as the image pair is chosen from later part of sequence the noise in the images is increased and the results are bit distorted compared to the the first pair we chose.

Bonus HomeWork 1 : Horn and Schunck Algorithm

Horn and Schunck method for displacement field reconstruction. Implementation is based on the following algorithm:

Data: two frames with moving objects
Result: displacement vector field $\overrightarrow{u} = (u, v)$
initialization;
compute spatial gradient I_x and I_y and time gradient I_t;
while *iteration* $n \leq n_0$ **do**

$\bar{u} = \frac{1}{4}$(u(i-1,j)+u(i+1,j)+u(i,j-1)+u(i,j+1)) ;

$\bar{v} = \frac{1}{4}$(v(i-1,j)+v(i+1,j)+v(i,j-1)+v(i,j+1)) ;

$\alpha = \lambda \frac{I_x \bar{u}^{(n)} + I_y \bar{v}^{(n)} + I_t}{1 + \lambda (I_x^2 + I_y^2)}$;

$u^{(n+1)} = \bar{u}^{(n)} - \alpha I_x$;

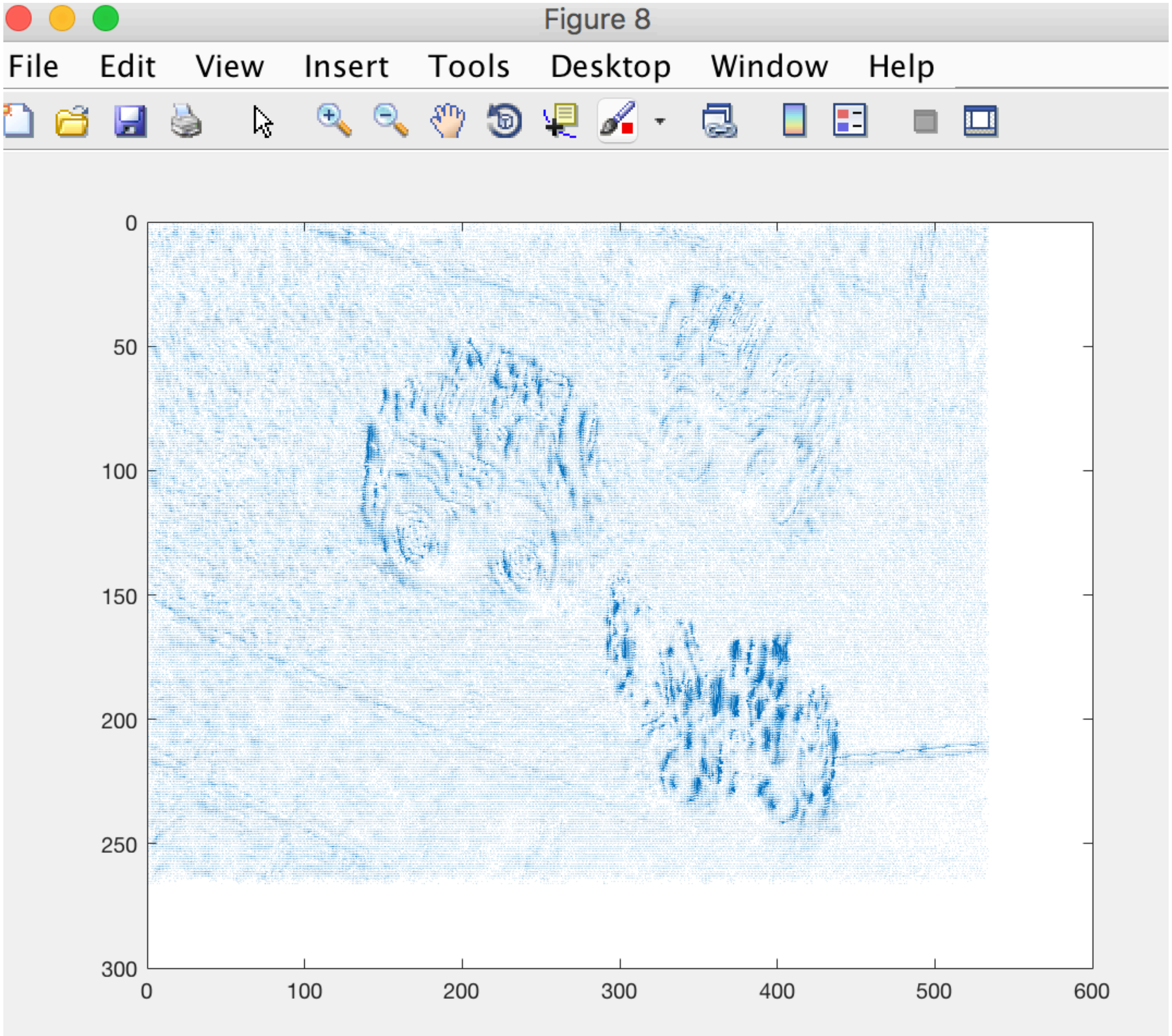
$v^{(n+1)} = \bar{v}^{(n)} - \alpha I_y$;

$n = n + 1$;

end

Algorithm 1: Horn and Shunck Optical Flow method

Results for Horn and Schunck Algorithm: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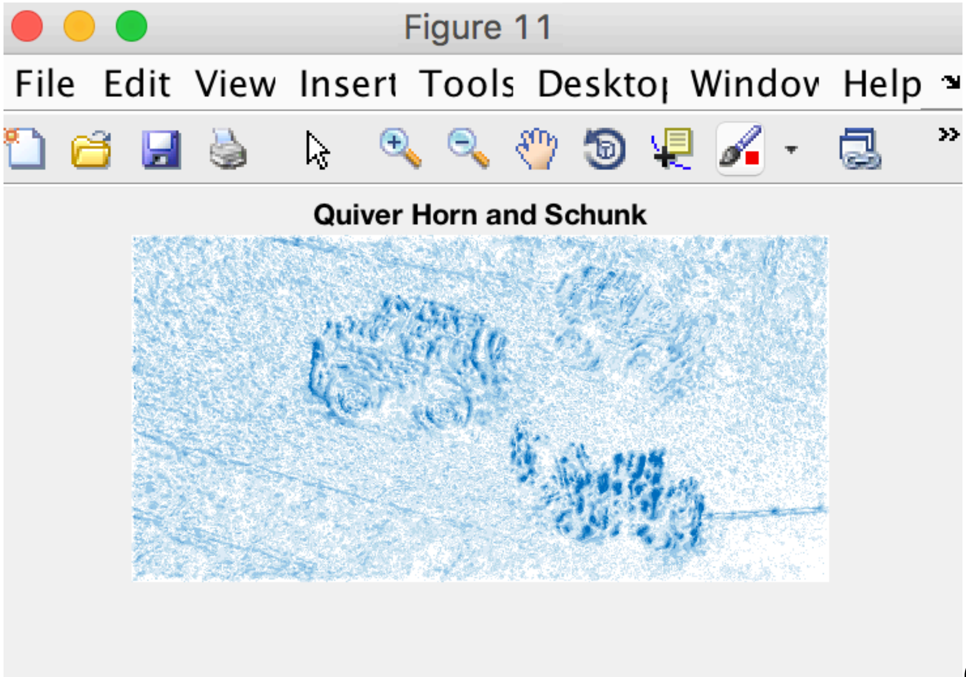
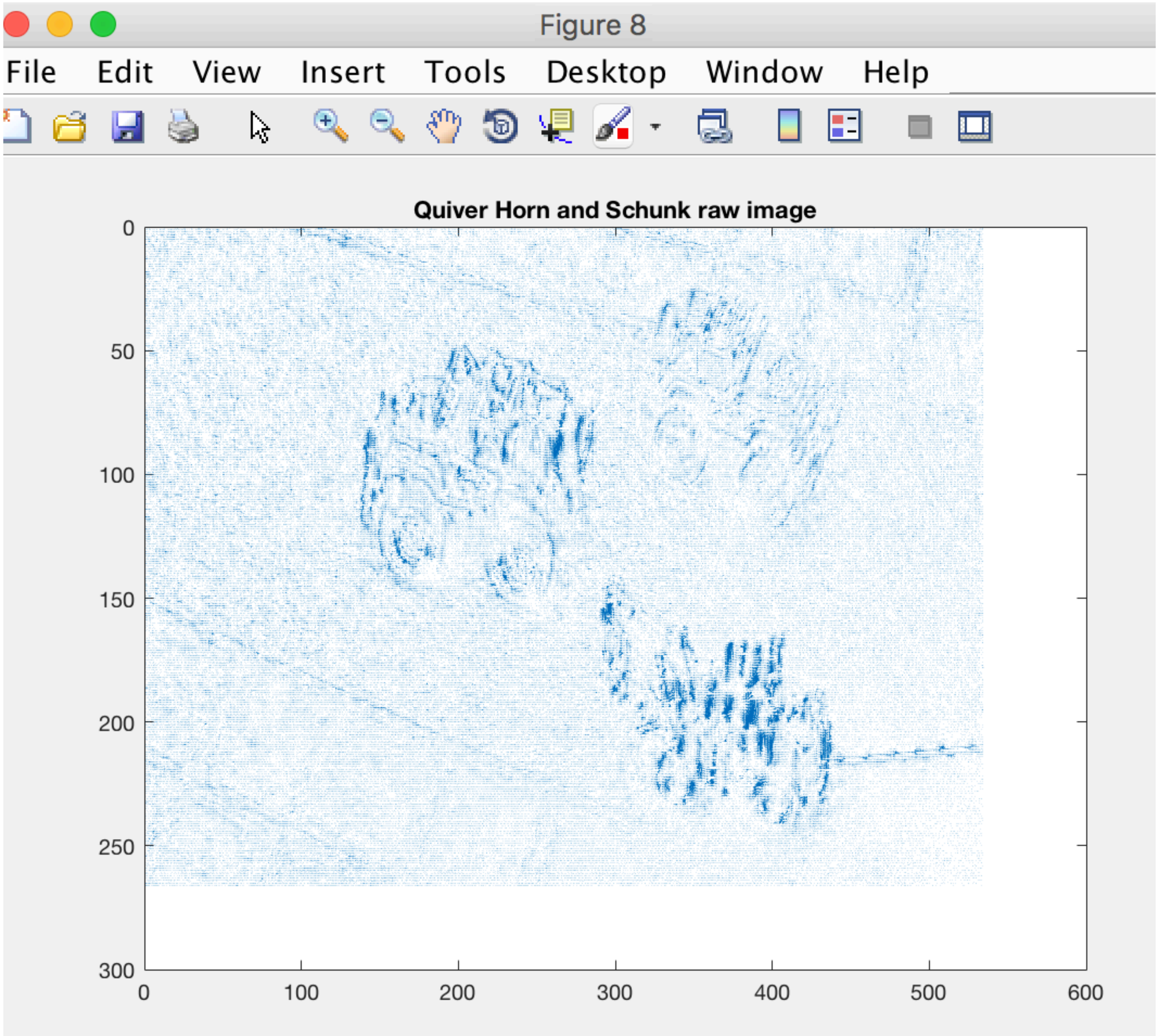
(A) 

(B)

Figure 10: A) Flow calculated using Horn and Schunck with smooth image sigma =1 , B) Flow calculated using Horn and Schunck with smooth image sigma = 1 superimposed on original image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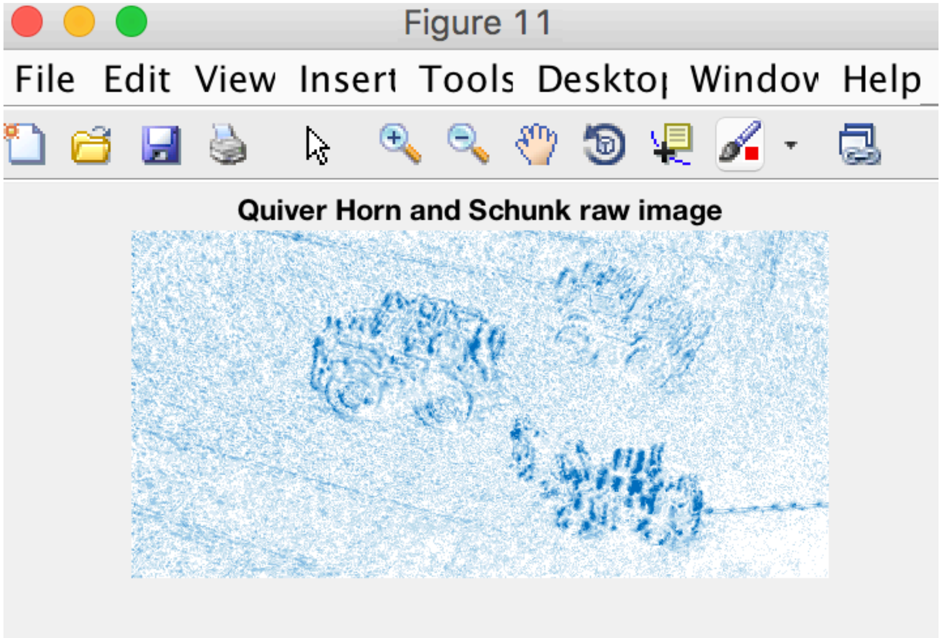
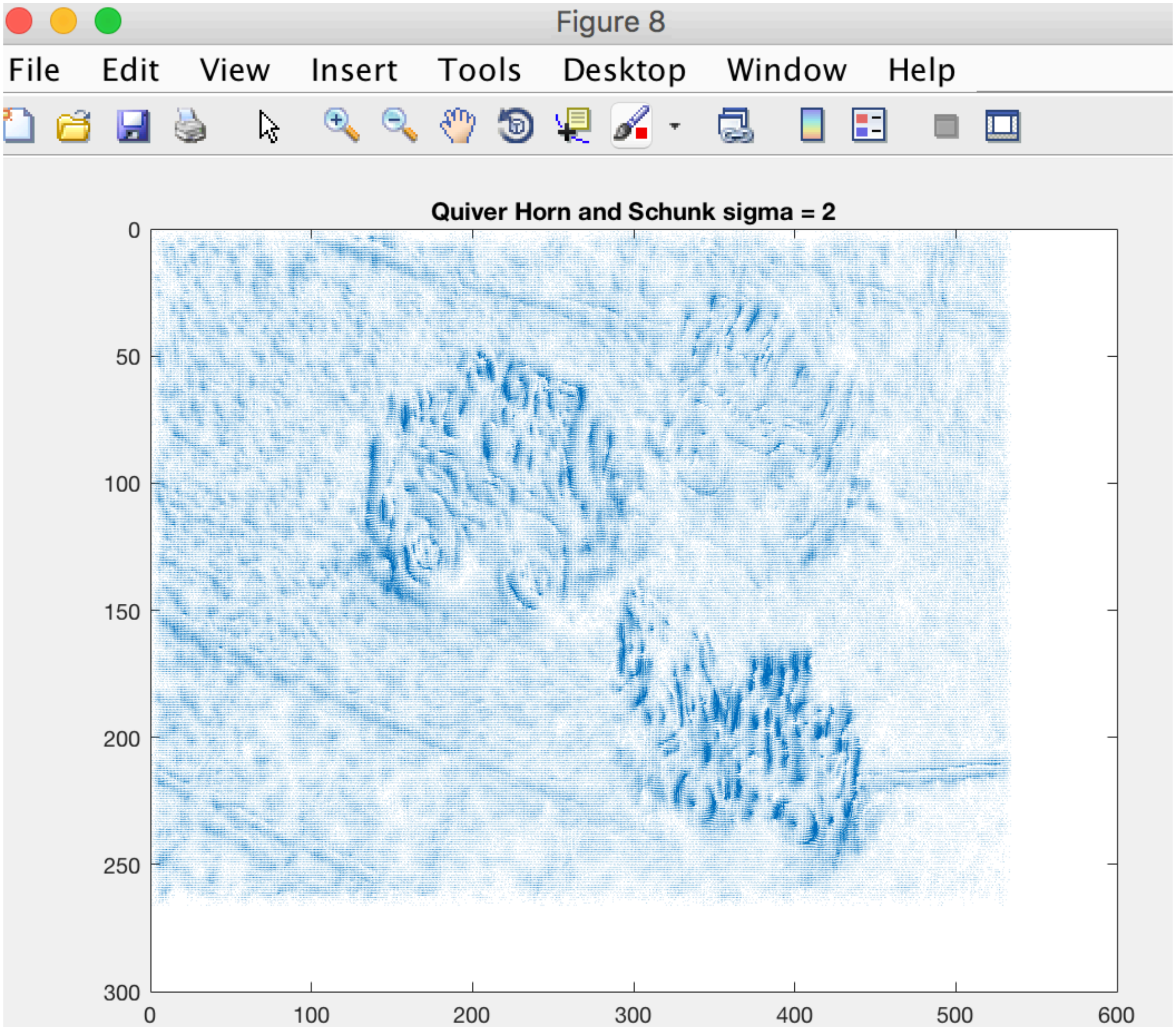
(A)  (B)

Figure 11: A) Flow calculated using Horn and Schunck with raw image , B) Flow calculated using Horn and Schunck with raw image superimposed on original image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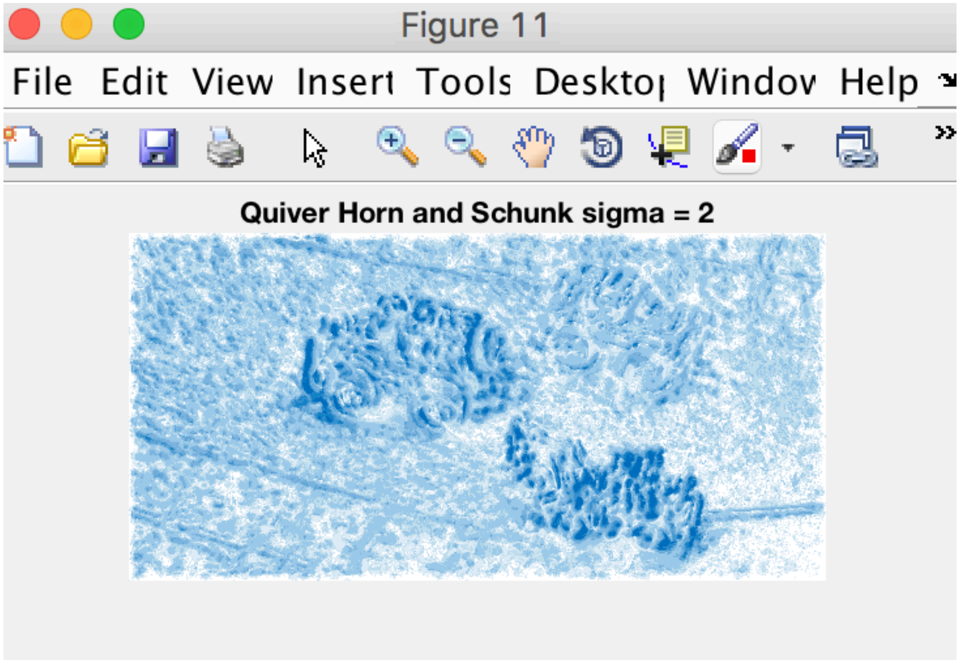
(A) 

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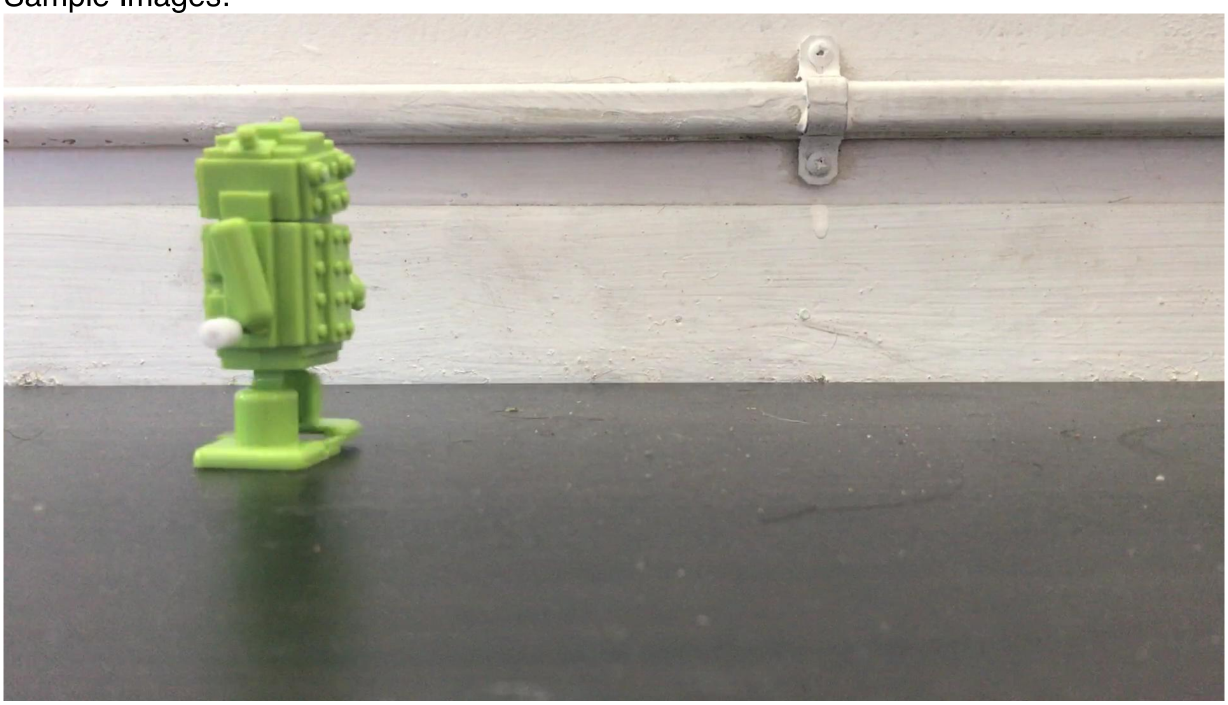
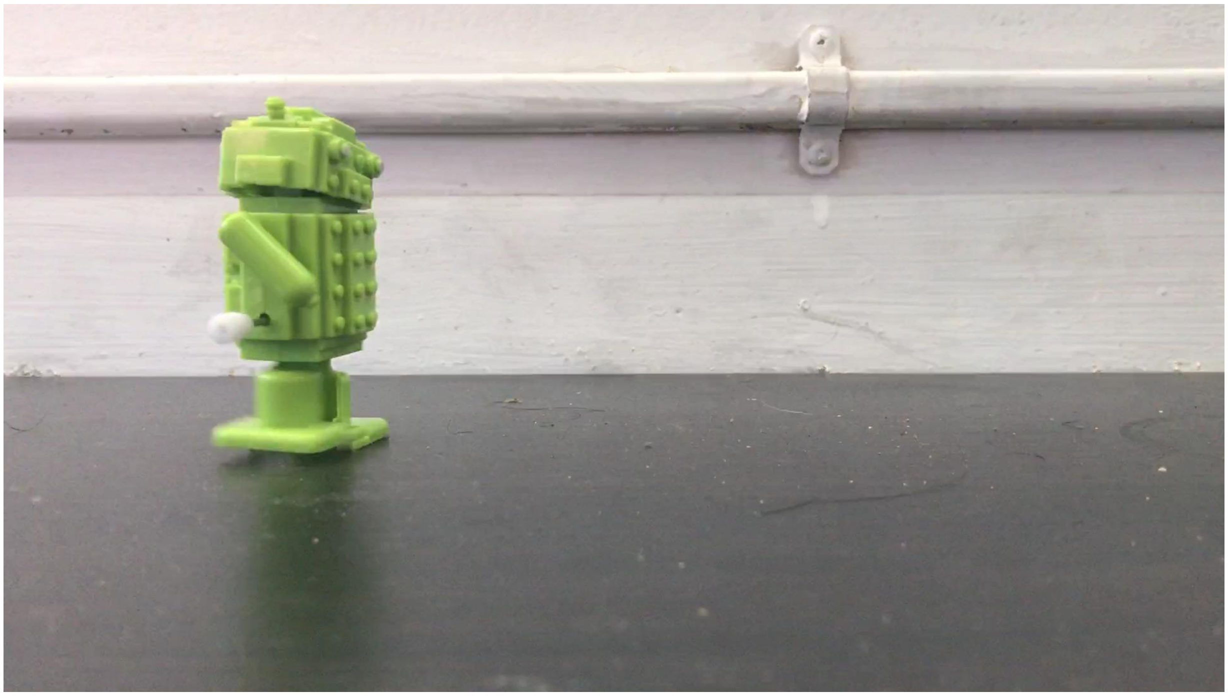
Figure 12: A) Flow calculated using Horn and Schunck with smooth image sigma =2 , B) Flow calculated over neighborhood pixel with smooth image sigma =2 superimposed on original image

Bonus HomeWork 2:

Sample Images:





Results for own video sequence:

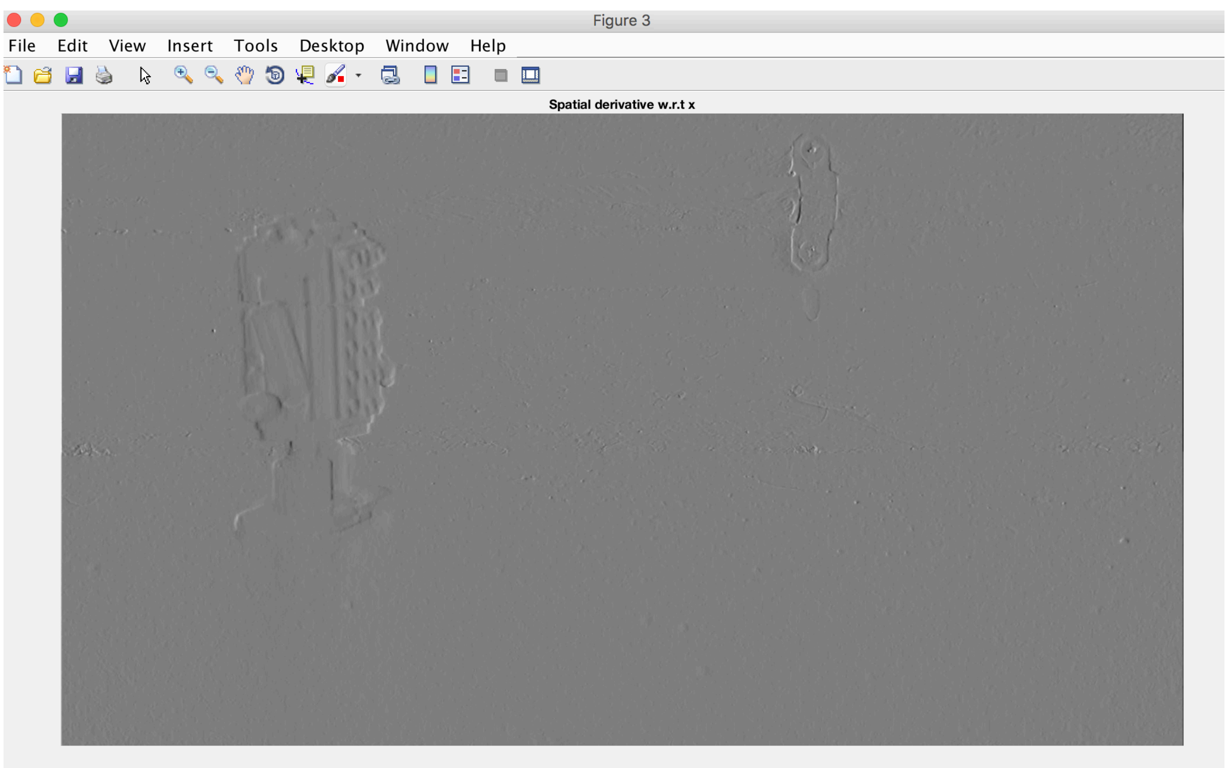


Figure 13: Spatial Derivative w.r.t x

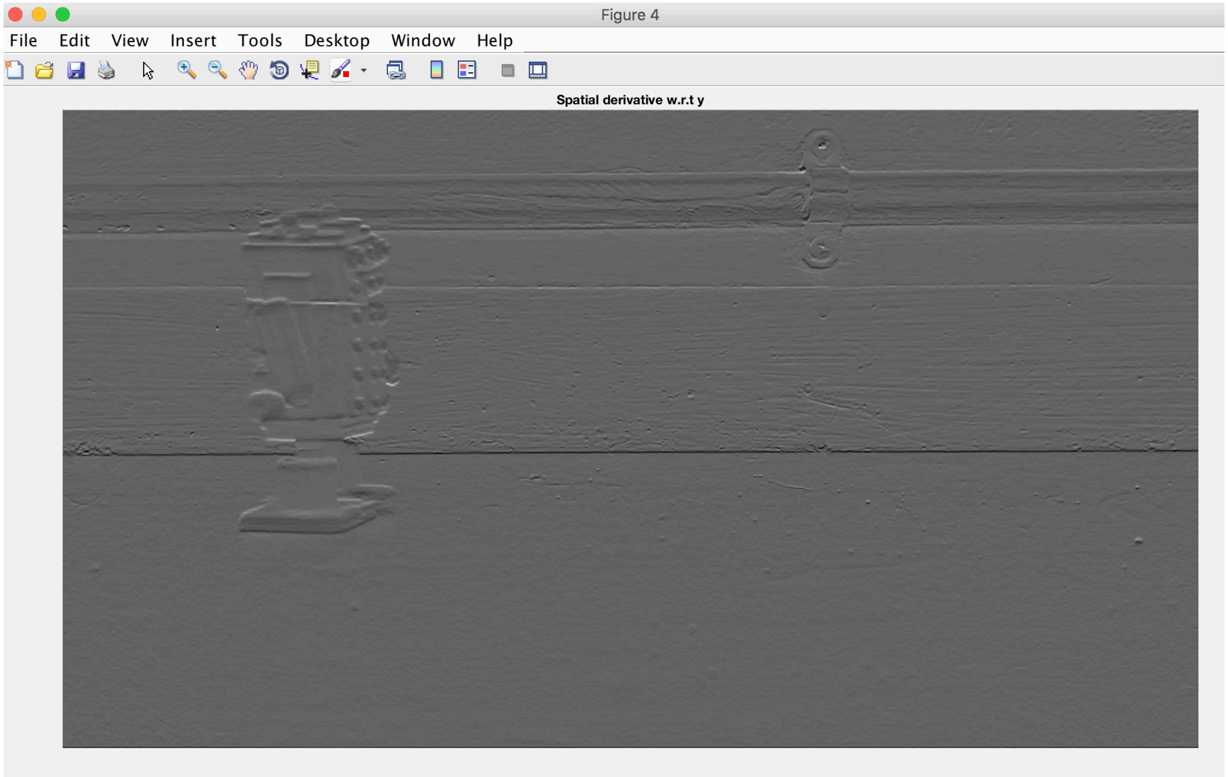


Figure 14:Spatial Derivative w.r.t y

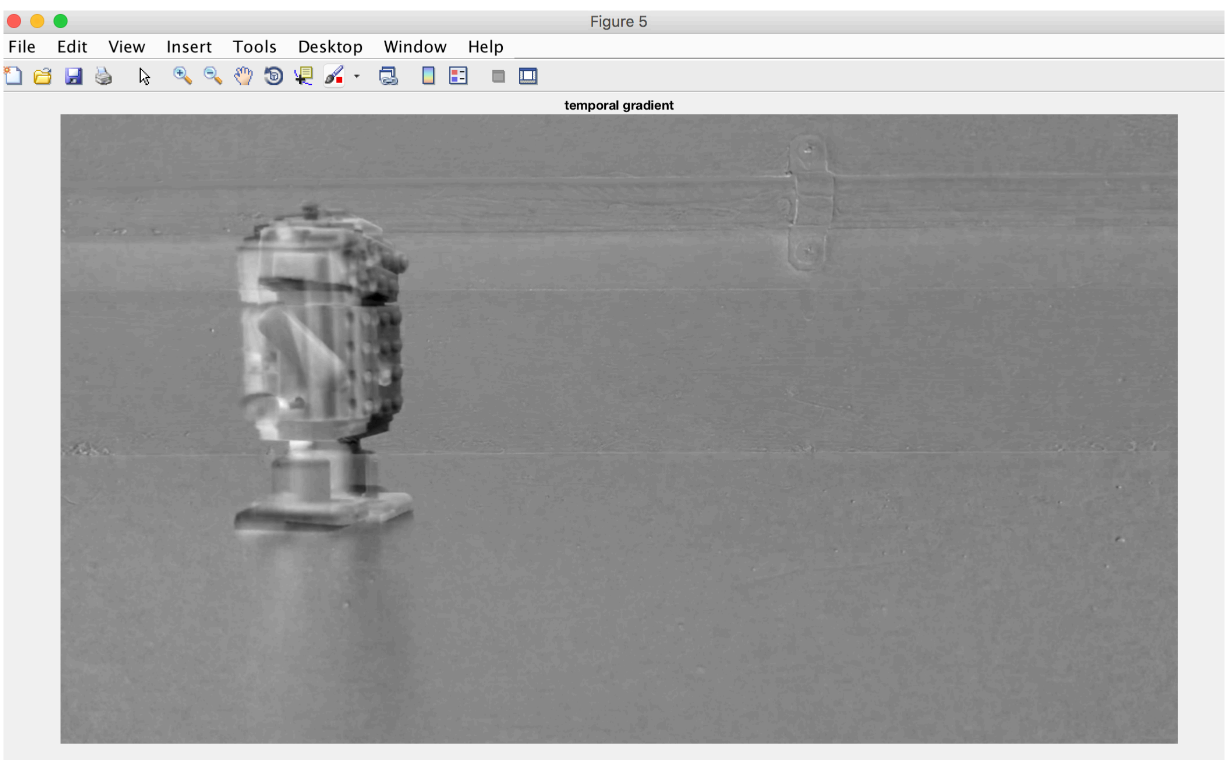


Figure 15: Temporal Gradient

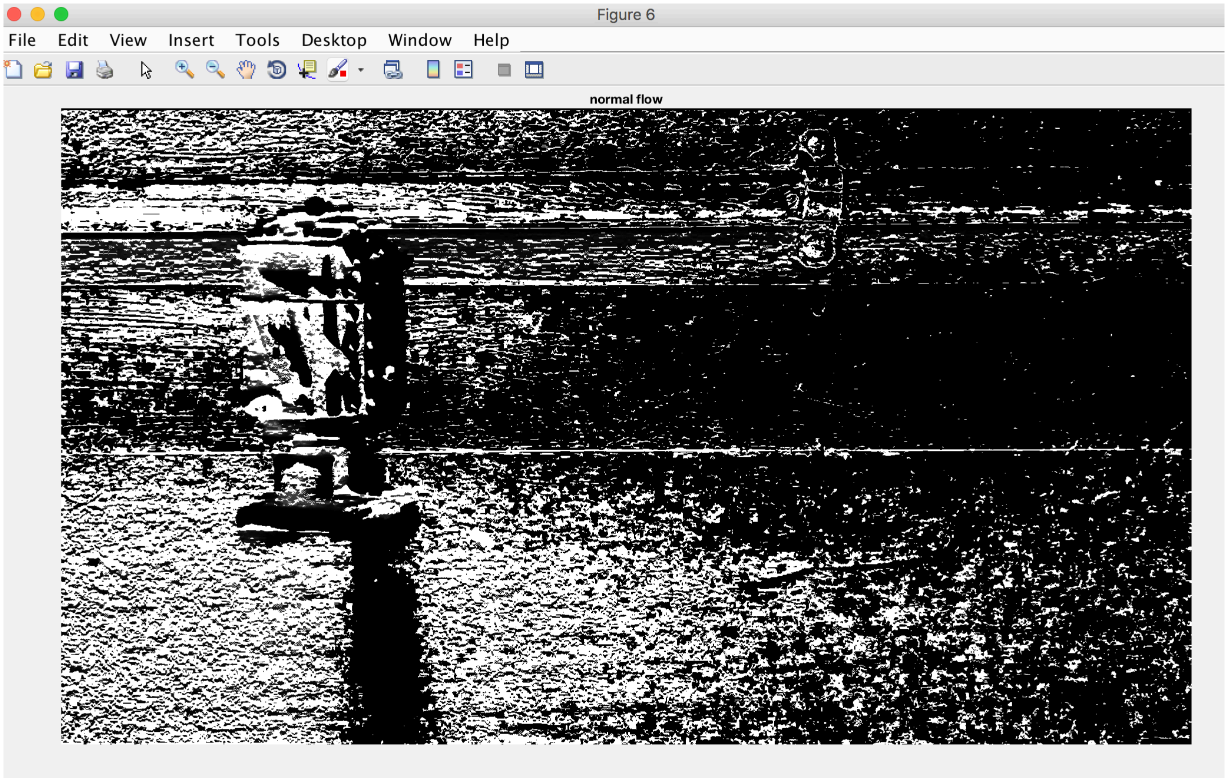
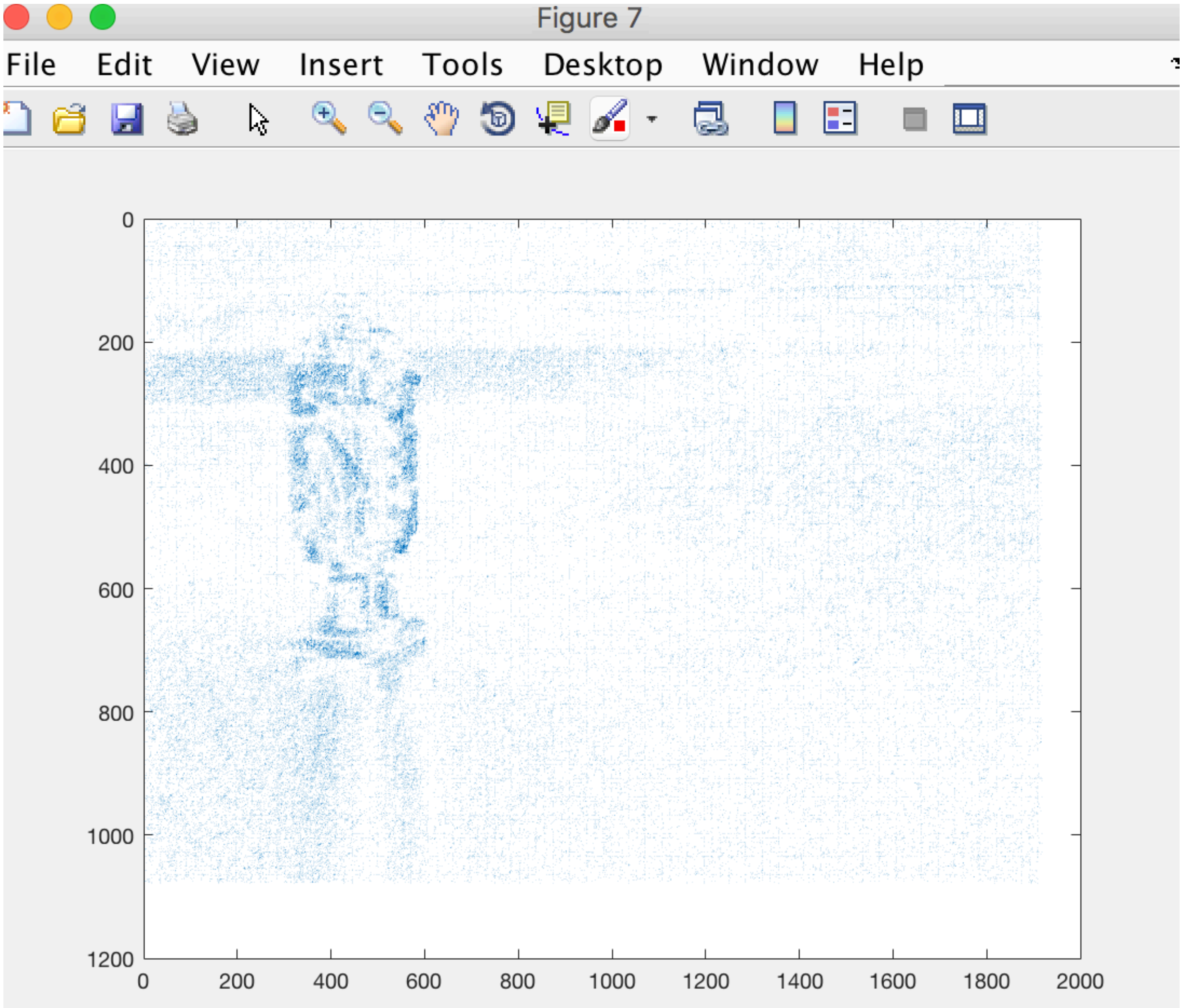


Figure 16: Normal Flow for the calculated image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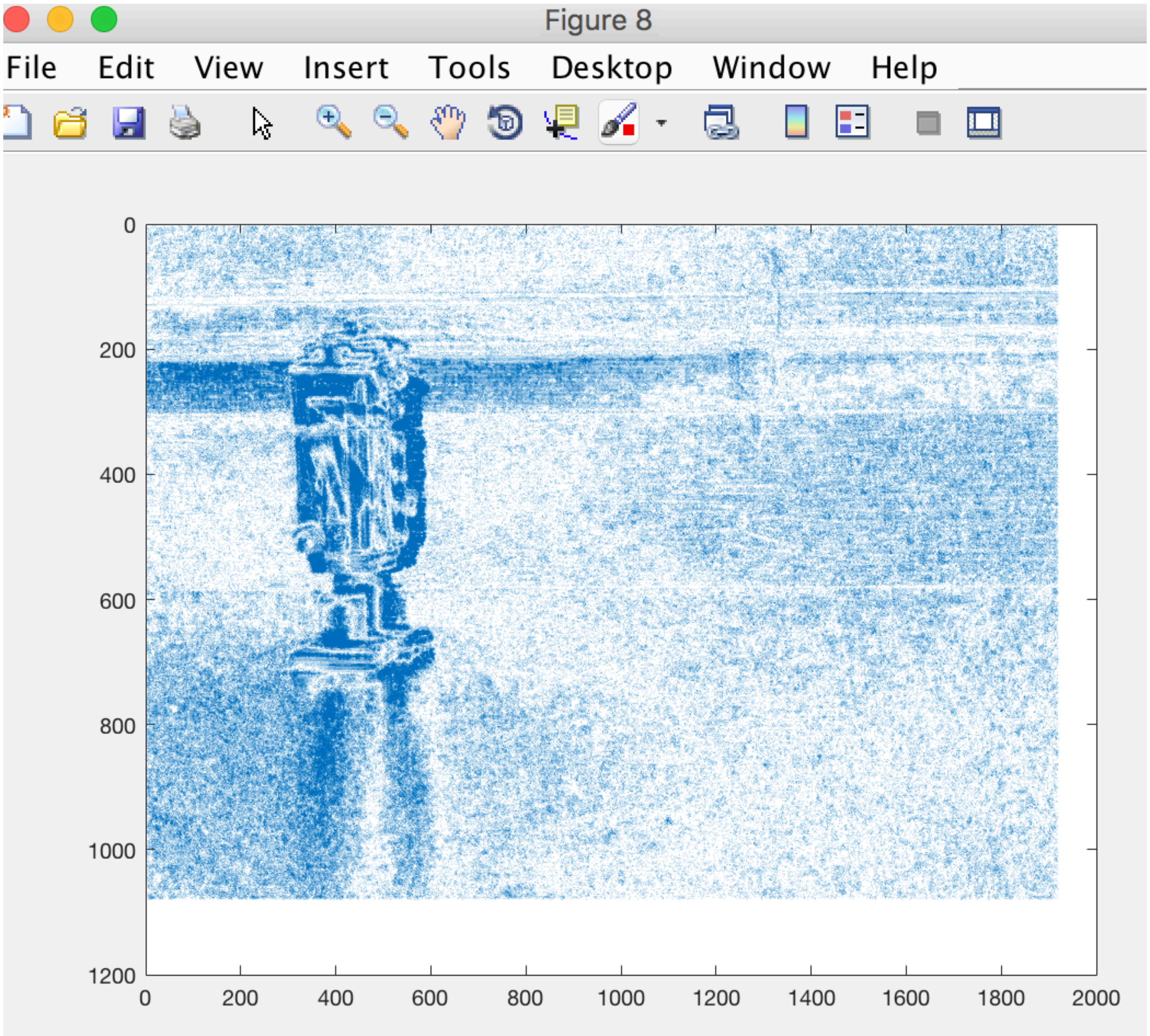

Figure 17: Normal Flow using 2x2 pixel neighborhood
Figure 18: Optical Flow using Horn and Schunck