Assignment - 2
Computer vision
By
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Q-1. Yes, with 3 cameras we can get unique point of correspondence.

⇒ As shown in fig. 1 in the question itself. The scenario where single world point M is projected on 3 image planes defined as P_1, P_2, P_3.

⇒ As visible in the figure. 1 by intersecting lines we can determine that the correspondence point can be uniquely defined as point of intersection of epipolar lines in 3 ~~image~~ camera stereogeometry.

⇒ Hence, in case of 3 cameras, if we know the corresponding points in 2 camera planes, then the third correspondence to the points in 2 known camera frames.

Problem (2): (a) Given the definition of epipolar plane as the plane defined by two optical centers and a world point, explain why the geometry for a forward moving camera results in a set of radially oriented intersecting planes.

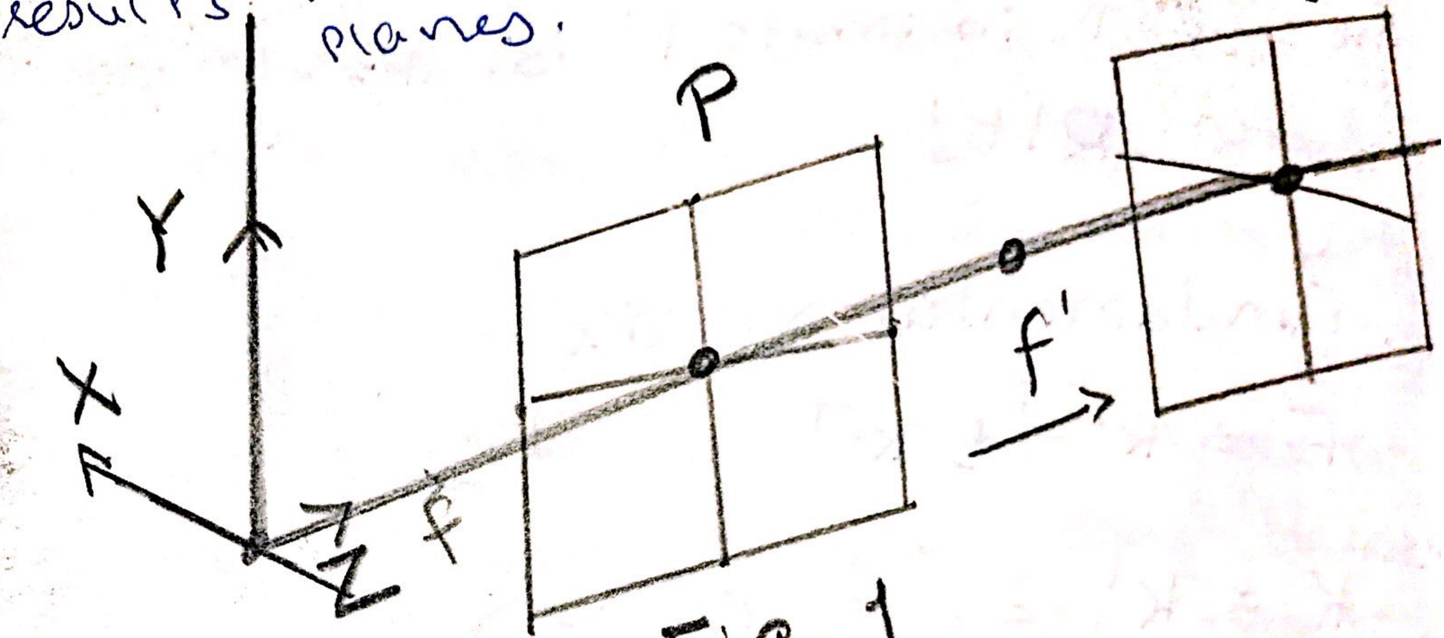

Fig. 1

As shown in Fig. 1. The translation of camera is along z-direction.

$\therefore$ translation $t = [0, 0, t_z]$ and

Rotation $R = I$ (Identity matrix)

$\therefore$ Essential matrix

$$E = [t_x] R$$

$$E = \begin{bmatrix} 0 & -t_z & 0 \\ t_z & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The point P in image P is

$$P = K[I|0]$$

The point p' in image p' is

$$p' = K'[R|t]$$

$\therefore$ Fundamental matrix

$$F = K'^{-T} E K^{-1}$$

Also,

$$K = K' = \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\therefore F = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\therefore$ Epipolar line for point in an image $p(x, y)$ is given as $l = Fx$

$$l = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} -y \\ x \\ 1 \end{pmatrix}$$

Hence, epipolar line $l(x,y) = x - y = 0$.

This line is passing through center (0,0). Hence all epipolar lines passes through the center. We know that by definition of epipole ~~th~~ lines passes through epipole. Here as camera-1 is in front of camera-2 the line joining their centers passes through image plane & epipole lies on that line. Hence all epipolar lines passing through the epipole appears as radially oriented intersecting lines.

(b) Given ~~[illegible]~~ geometric consideration of forward moving camera...

$\boxed{\text{Q-2}}$

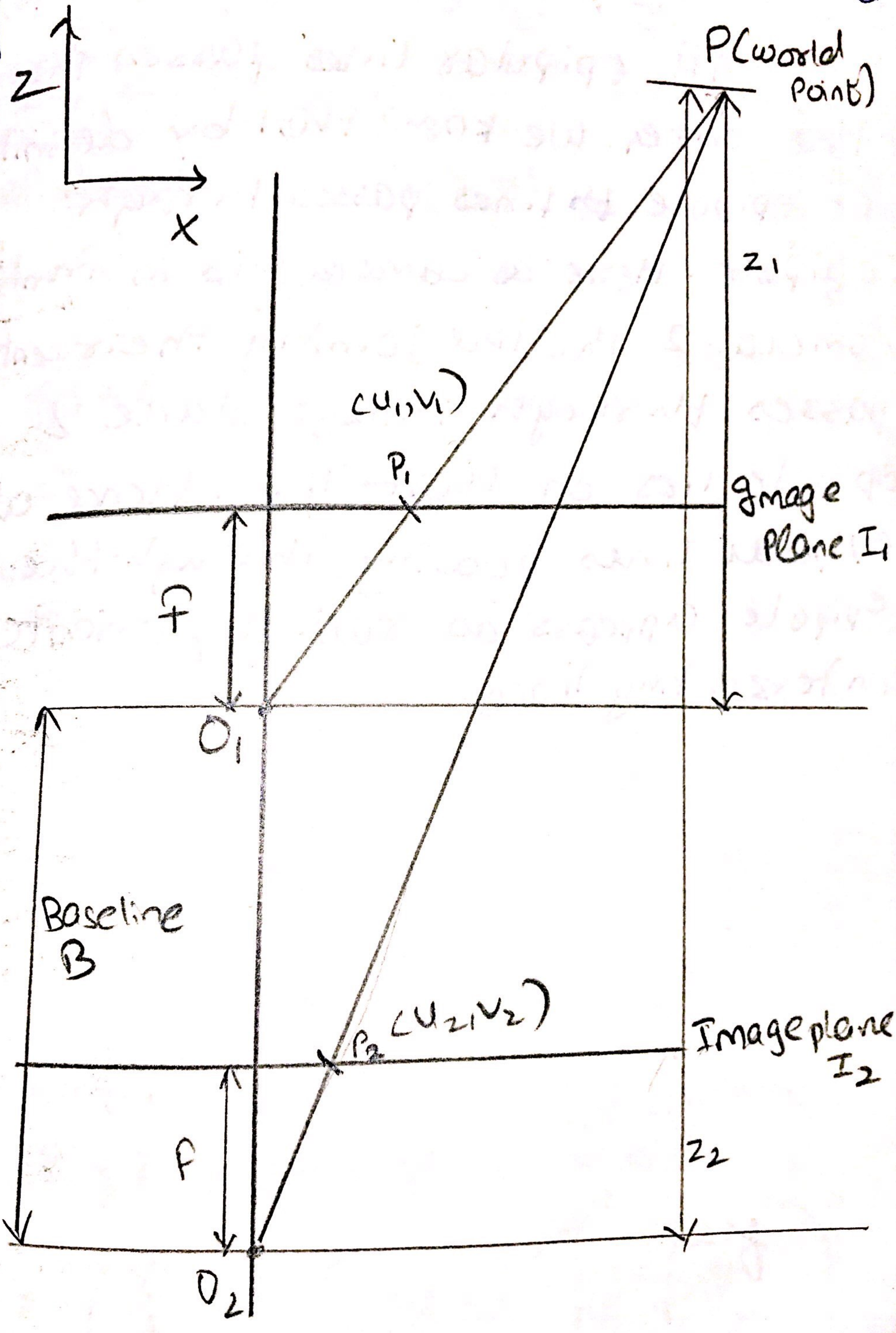

Fig. 2

We know that for a point (x, y) in image 1 the correspondence point lies on epipolar line passing through the center. The corresponding point in 2nd image is (x', y') then $\frac{x}{y} = \frac{x'}{y'}$

In camera coordinates

$P_{11} = [x_1, y_1, z_1]^T$

$P_{22} = [x_2, y_2, z_2]^T$

Assuming we are using a single camera $z_1 = f$ & $z_2 = f$ where f is focal length of camera. so,

$P_{11} = [x_1, y_1, f]^T$ $P_{22} = [x_2, y_2, f]^T$

In world coordinate system the vector joining optical center O_1 & O_2 & point P are given as $P_1 = [X_1, Y_1, Z_1]^T$

$P_2 = [X_2, Y_2, Z_2]^T$

We want to find depth Z_1 & Z_2. We know that

$\frac{x_1}{X_1} = \frac{f}{Z_1}$, $\frac{x_2}{X_2} = \frac{f}{Z_2}$.

Hence, $Z_1 = f\frac{X_1}{x_1}$ $Z_2 = f\frac{X_2}{x_2}$

COMPUTER VISION
ASSIGNMENT 2
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PRACTICAL SOLUTION

4a) Epipolar geometry from F-matrix
4b) 3D object geometry via Triangulation

Aim: The aim of this assignment is to take a stereo pair of images, and explore the epiploar gemeotry and the ability to recover depth from a pair of images.

Experimental Procedure:

Image Capture: In this experiment I captured two pairs of Stereo Images. One is a stereo Image of Checkerboard and other is a stereo image of cube like object. This both stereo pairs are represented as in figures below.

Figure 1: Stereo Image pair 1

As mentioned earlier , I used two different pairs of stereo images in this assignment. So my second pair is as below:

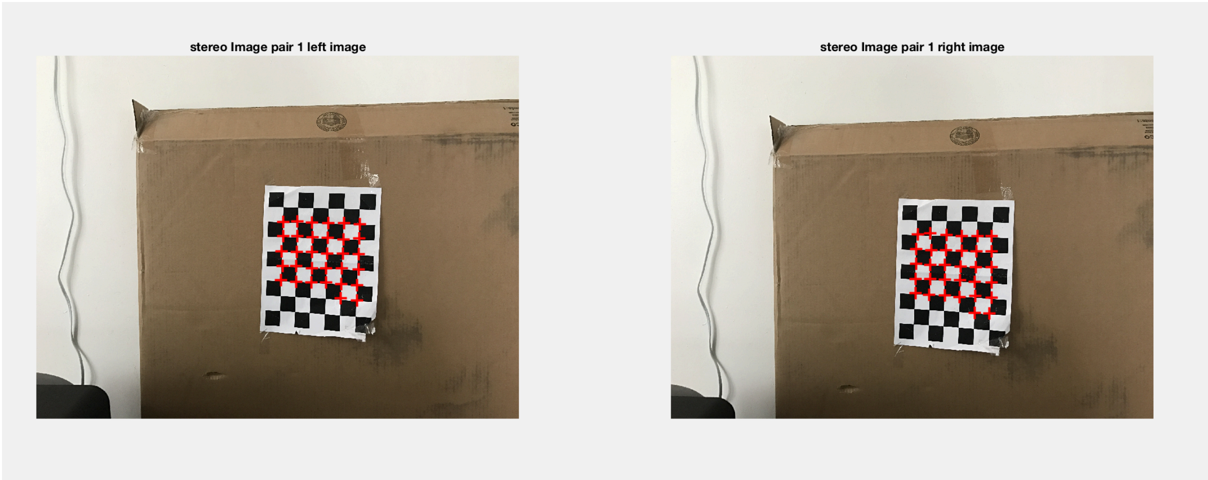

Figure 2: Stereo image pair 2

Once the images are captured as in Figure 1 and Figure 2, the coordinate system is defined in X,Y,Z direction as in the previous experiment performed for image calibration.
After the images are captured, 32 points were selected in both the images using ginput(32) function in Matlab. This coordinates are stores in the file I11.txt and I22.txt for stereo image pair one and I1.txt and I2.txt for Stereo image pair 2.
Once the corresponding points are selected in both the images , the images are visualized as shown in above figure 1 and figure 2. The Matlab code for this is in the project folder as DataConfirm.m . Each of the 32 points selected are the corresponding points in both the images which would be required to construct the epipolar geometry in both the stereo image pairs.

Estimating Fundamental Matrix:
The process of estimating fundamental matrix is based on the lecture discussions and notes to derive F from the coordinates of the points of both the image planes as $u^T F u' = 0$. In estimating this fundamental matrix 8-point algorithm is implemented. Each point correspondence is expressed a pair of linear equation

$$[u\ v\ 1]\ [\ F11\ F12\ F13\ ;\ F21\ F22\ F23\ ;\ F31\ F32\ F33][u'\ v'\ 1]^T = 0$$

Solving this set of equation and then inverting we solve to get the value of F. Using this above algorithm and the implementation. Also normalizing the points to the image scale we are able to determine F.

F =

1.37642042639166e-09	-2.27118064945027e-07	0.000373332769926365
2.22312017834481e-07	7.74289797587877e-10	-0.000547850153526548
-0.000395304659185434	0.000554602660370469	0.0519676518916615

The F is solved using the above equations known as AF = 0 in the Matlab code. As the values of F are used further for the epipolar geometry , I find the values of F to be acceptable.

Epipolar Geometry from F:

Now from the equation $u^T Fu' = 0$ we can find that Fu' = l', where l' is the epiploar line associated with the u. The epipolar lines are calculated using this formula for both the image planes in both the pairs of Stereo images. The result of the epipolar line for the 32 points is given as below:

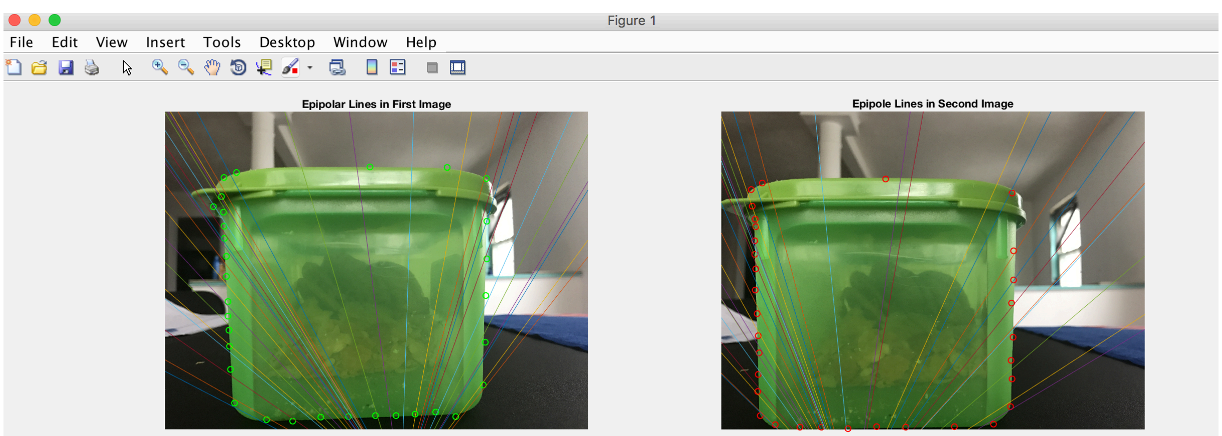

Figure 3: Epipolar lines for the corresponding points for first stereo pair.

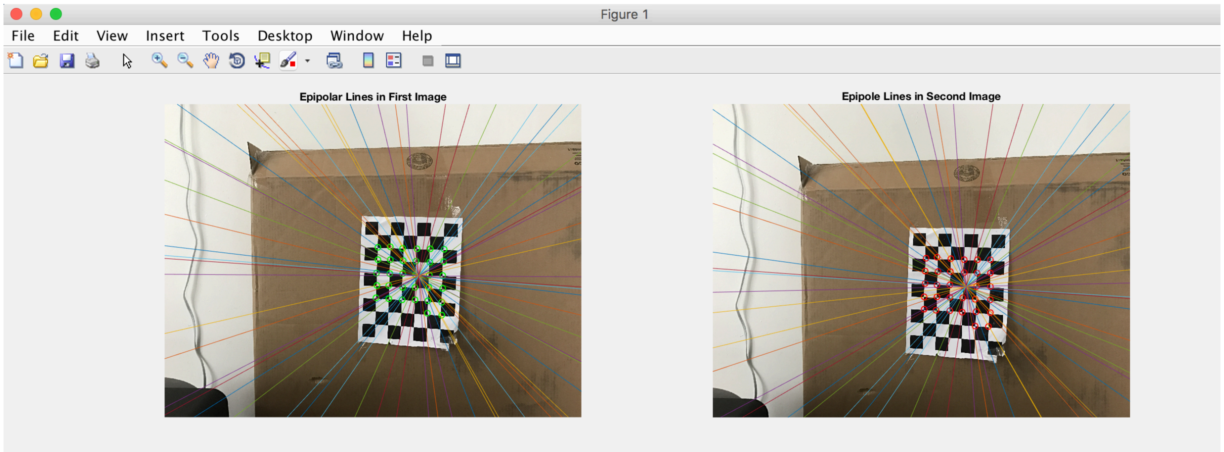

Figure 4: Epipolar lines for the corresponding points for second stereo pair.

As shown in the above figures the epipolar lines are based on the corresponding points of left image on right image plane and vice versa. The epipole of the both the image planes of both stereo pair of images can be determined using the formula $F^*e_L = 0$, where e_L is the episode of left image plane here. From the equation we can derive that $F^*e_L = 0$ vector in the right null space of matrix F. Thus, epipoles can easily be determined by determining nullspace of F. Epipoles determined for the second pair of stereo images are given as :

e1= $[-0.828980044659197 , -0.559278080980350 , -0.000337182108275077]^T$

e2 = $[0.811457077108891, 0.584411928459442, 0.000331487874259874]^T$

Epipoles determined for the first pair of stereo images are given as :

e1 = $[0.526202258324656, 0.850359411453936, 0.000233849839725296]^T$

e2 = $[-0.320385056879802, -0.947287364422721, -0.000254033817484734]^T$

Estimating Essential Matrix:

The Essential matrix is estimated using the formula K'*F*K=0, where K is the intrinsic matrix. The K is derived as the intrinsic matrix from the previous

homework of camera calibration. After using the above relation between K and E , we can derive Essential matrix as:
E =.

4.38695832907520e-05	-0.00088142 4984209718	-0.0285482622336430
0.000786946459371538	4.938864365 39407e-05	0.0231961402497720
0.0236224488325928	-0.01888500 83568301	0.00888556829754911

As per the reconstruction produced later in the report, I would define values of E estimated as to be acceptable and fair.

Computation of camera matrix P and P':
The camera matrix P and P' are used for the 3D reconstruction of the images from stereo pair of images. The camera matrix P is taken as matrix P = K[I|0] . The second camera matrix is calculated using the intrinsic matrix as well as rotation and translation as P' = K'[R|t] . In this experiment we have rotation as Identity matrix but the translation is calculated using the Depth of the image and the disparity is calculated.
P2 =

0.795493955637404	0.140682218725426	-0.589404682606821	0.626458936899639
0.204364149614896	0.853404478038134	0.479516518189562.	0.779364467769892
0.570460043207704	-0.501905678588070	0.650127701997472	0.0118417378929769

Triangulation(3D Reconstruction):
In this part of the project , the triangulation is used on the pair if stereo images to produce the 3D reconstruction. Using the camera matrix and the corresponding points we can determine the triangulation. The triangulation for both the image pairs are as follows:

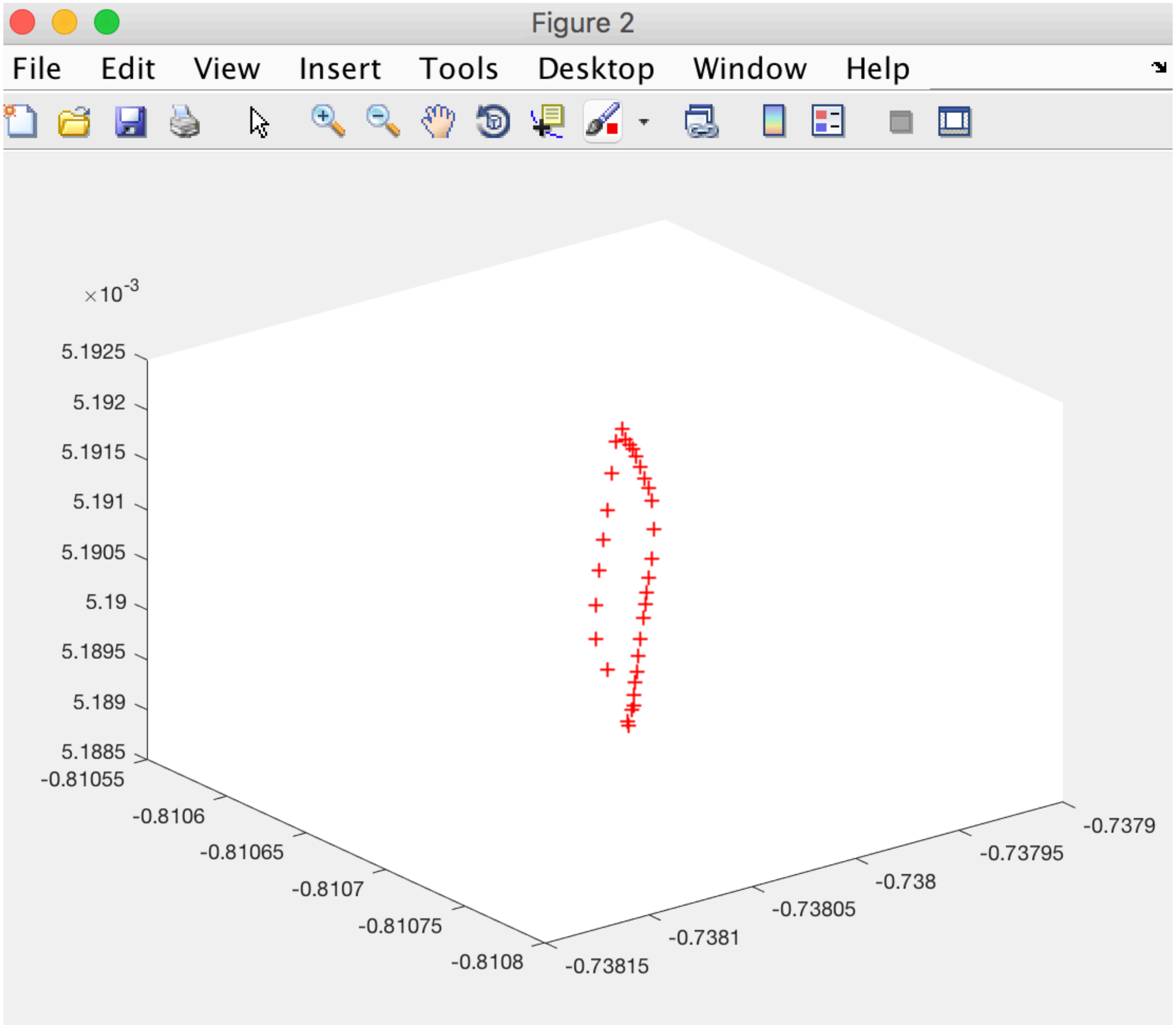

Figure 5: 3D reconstruction using triangulation for stereo image 1

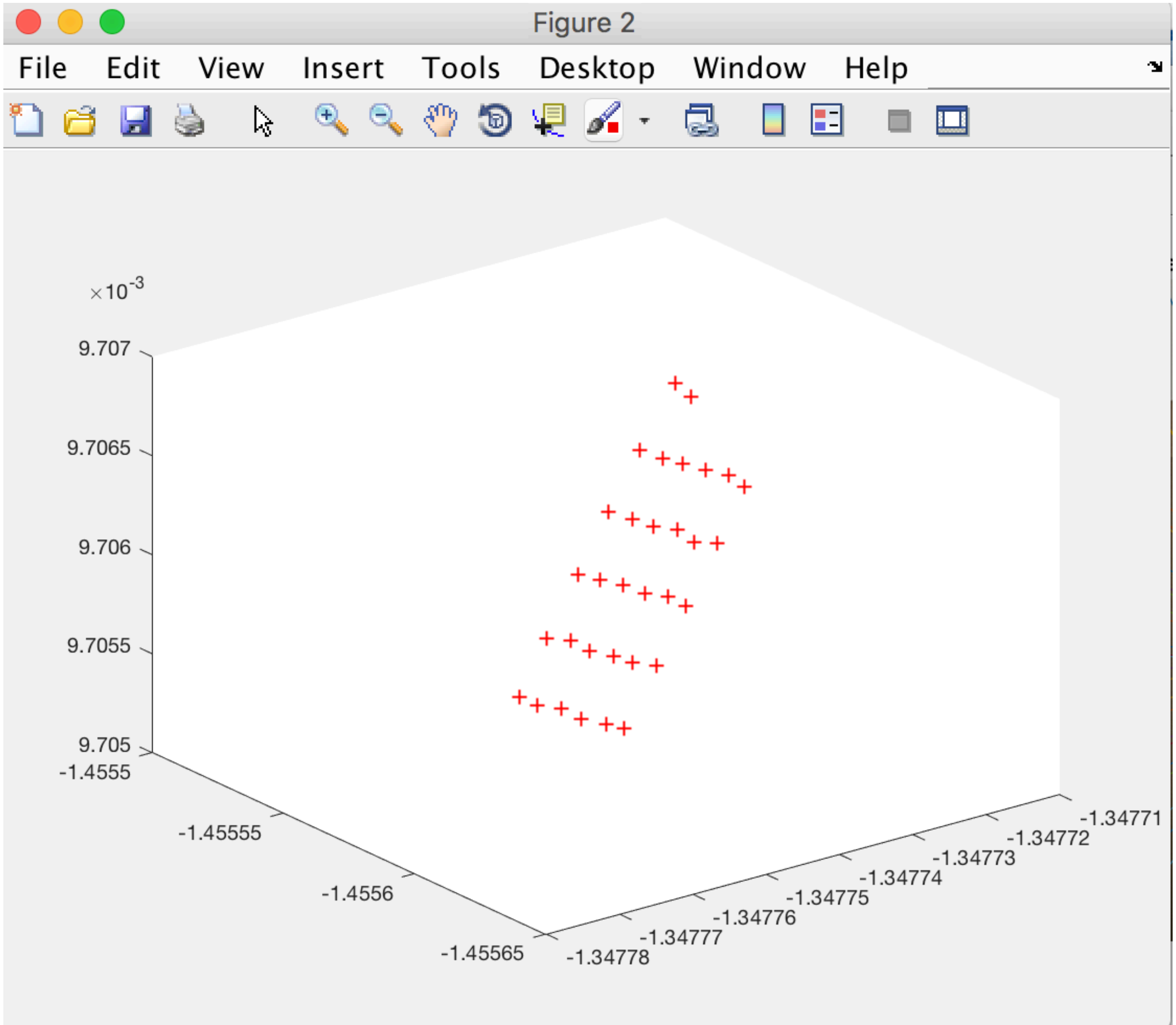

Figure 5: 3D reconstruction using triangulation for stereo image 2

The p = fP/Z is the basis of equation used for the reconstruction. As per the results obtained the algorithm works on a proper level. The results are constructed as per the algorithm and the images are constructed which shows the corresponding points in 2D space to 3D space.

Hence, as per the experiment the fundamental matrix and the 3D reconstruction of the points is derived.