

We will learn how to factorise a number in $O(\log N)$ time complexity . We know that any number can be represented as multiplication of primes . Let us suppose that we want to prime factorise a number N . And we have a array $pf[]$ where $pf[i]$ indicated any one prime factor of i . Now we have a prime factor of N i.e. $pf[N]$. Now we will keep on dividing N by $pf[N]$ until it becomes indivisible by $pf[N]$. After all this division let the number N became some x where ($x < N$) . We will keep on dividing x by $pf[x]$ i.e. repeat the same procedure that we did for N until we are left with N as 1 . Now the only thing that is a problem is how to compute $pf[x]$, right? Well that's easy , We can obtain that from sieve algorithm

(If you have not yet read sieve algo then please read it first) . Here is pseudocode for that.

```
bool prime[100010];          // prime[i] is true if i is a prime number and
                             // otherwise false

int pf[100010];              // pf[i] indicates any one prime number of i

void sieve(){
    fill(prime,prime+100010,true);
    prime[0]=prime[1]=false;

    for(lli i=2;i<100010;i++){
        if(prime[i]){
            for(lli j=i*i;j<100010;j+=i)
                {prime[j]=false; pf[j]=i ; }      // i is the prime number that divides j
        }
    }
}
```

In this way we can precompute the pf array . Here is pseudocode for prime factorisation .

We have a number N whose prime factors we want to find and a vector v in which all prime factors of N will be stored .

```
While(N>1){  
    v.push_back(pf[N]);  
    int y=N;  
    while(y%pf[N]==0)y/=pf[N];  
    N=y;  
}
```

We understood how to factorise a number by this method but how it's complexity came to be $O(\log N)$. Well it's a homework exercise for you . (hint: count how many max different prime factors can a number less than 10^6 have)

There is a limitation of this method that we have to precompute the array $pf[]$ and there are issues with memory limit . But still this will run efficiently for 10^6 . For 10^7 the sieve itself takes $N \log N$ time which may or may not pass the time limit (depends on how many seconds are given).

You can also find all factors of N in $O(\sqrt{N})$. It's quite an easy algorithm that's why we are not explaining it here , (Read it from here:

<http://www.geeksforgeeks.org/print-all-prime-factors-of-a-given-number/>) .