Deep Reinforcement Learning

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Agenda

Introduction and Overview

Markov Decision Processes

Reinforcement Learning via Black-Box Optimization (CE method)

Policy Gradient Methods

Variance Reduction for Policy Gradients

Trust Region and Natural Gradient Methods

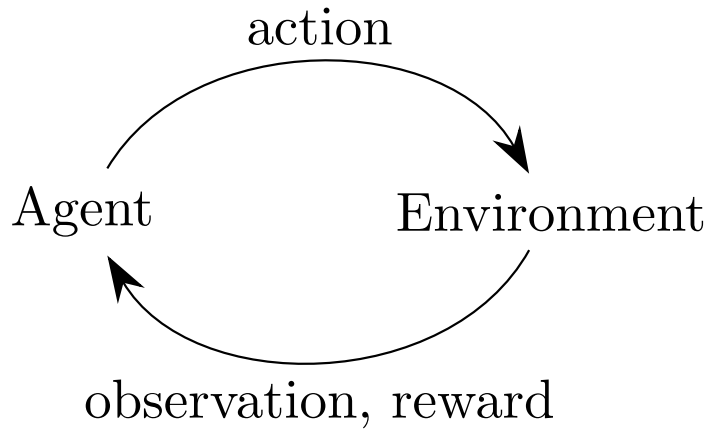
Open Problems

Course materials: goo.gl/5wsgbJ

Introduction and Overview

What is Reinforcement Learning?

- Branch of machine learning concerned with taking sequences of actions
- Usually described in terms of agent interacting with a previously unknown environment, trying to maximize cumulative reward



Motor Control and Robotics

Robotics:

- Observations: camera images, joint angles
- Actions: joint torques
- Rewards: stay balanced, navigate to target locations, serve and protect humans

Business Operations

- Inventory Management
  - Observations: current inventory levels
  - Actions: number of units of each item to purchase
  - Rewards: profit
- Resource allocation: who to provide customer service to first
- Routing problems: in management of shipping fleet, which trucks / truckers to assign to which cargo

Games

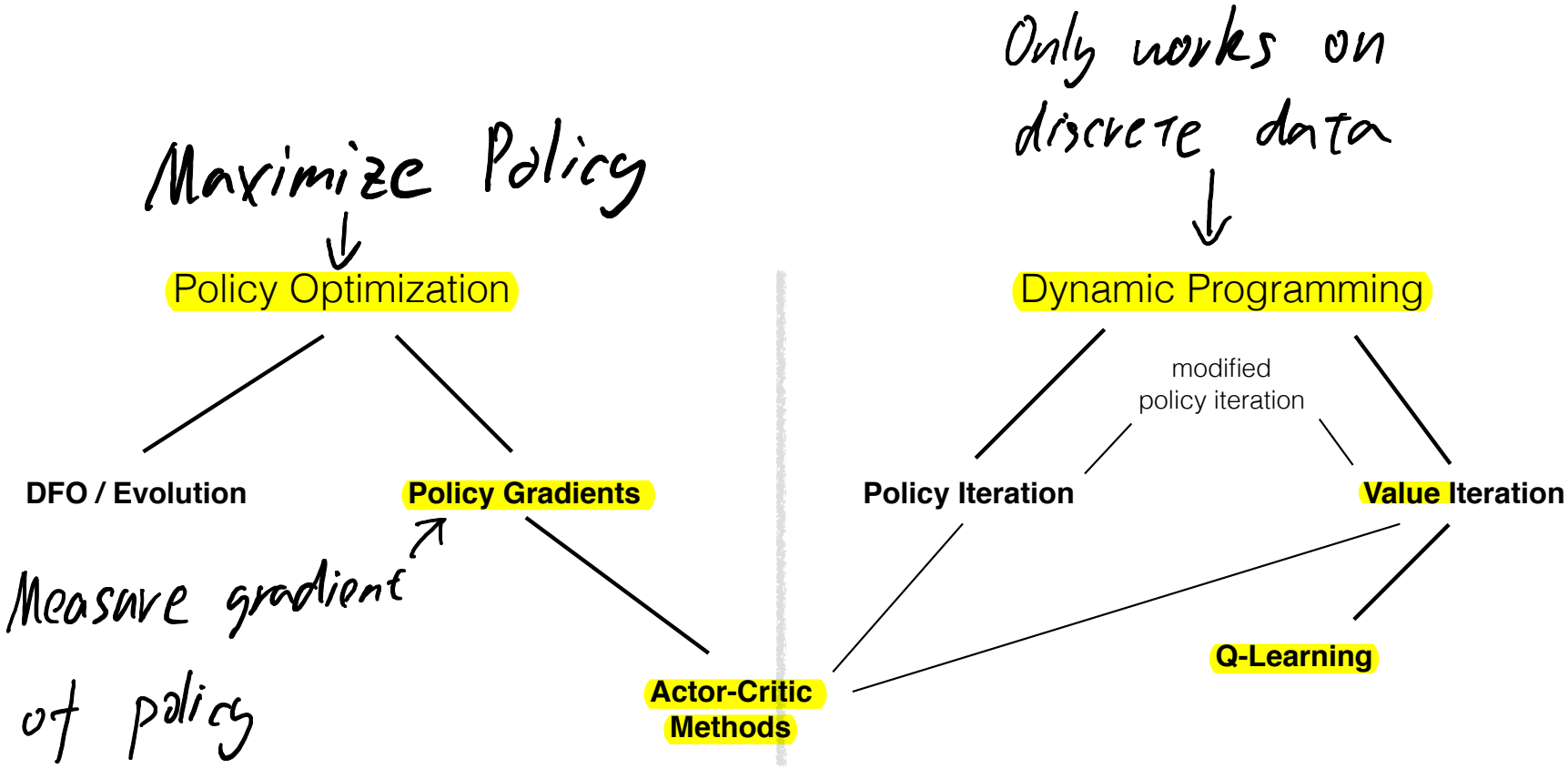
A different kind of optimization problem (min-max) but still considered to be RL.

- Go (complete information, deterministic) – *AlphaGo*[2]
- Backgammon (complete information, stochastic) – *TD-Gammon*[3]
- Stratego (incomplete information, deterministic)
- Poker (incomplete information, stochastic)

[2] David Silver, Aja Huang, et al. "Mastering the game of Go with deep neural networks and tree search". In: *Nature* 529.7587 (2016), pp. 484–489.

[3] Gerald Tesauro. "Temporal difference learning and TD-Gammon". In: *Communications of the ACM* 38.3 (1995), pp. 58–68.

Approaches to RL

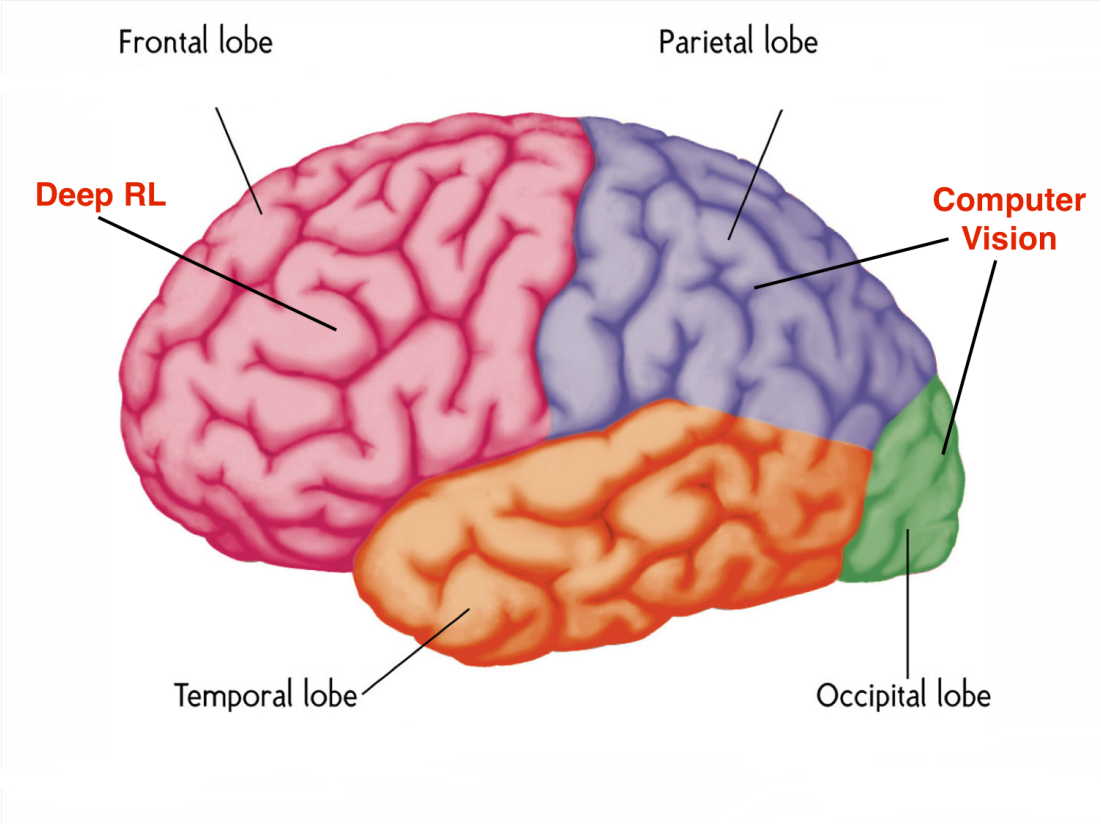


What is Deep RL?

- RL using nonlinear function approximators
- Usually, updating parameters with stochastic gradient descent

What's Deep RL?

Whatever the front half of the cerebral cortex does (motor and executive cortices)



Markov Decision Processes

Definition

- Markov Decision Process (MDP) defined by $(\mathcal{S}, \mathcal{A}, P)$, where
  - $\mathcal{S}$: **state space**
  - $\mathcal{A}$: **action space**
  - $P(r, s' \mid s, a)$: a transition probability distribution
- Extra objects defined depending on problem setting
  - μ: Initial state distribution
  - γ: discount factor

Episodic Setting

- In each episode, the initial state is sampled from μ, and the process proceeds until the *terminal state* is reached. For example:
  - Taxi robot reaches its destination (termination = good)
  - Waiter robot finishes a shift (fixed time)
  - Walking robot falls over (termination = bad)
- Goal: maximize expected reward per episode

Policies

- Deterministic policies: $a = \pi(s)$
- Stochastic policies: $a \sim \pi(a \mid s)$

- Parameterized policies: π_θ it is a function

Episodic Setting

sample first state

$$s_0 \sim \mu(s_0)$$
$$a_0 \sim \pi(a_0 \mid s_0)$$
$$s_1, r_0 \sim P(s_1, r_0 \mid s_0, a_0)$$
$$a_1 \sim \pi(a_1 \mid s_1)$$
$$s_2, r_1 \sim P(s_2, r_1 \mid s_1, a_1)$$
$$\dots$$
$$a_{T-1} \sim \pi(a_{T-1} \mid s_{T-1})$$
$$s_T, r_{T-1} \sim P(s_T \mid s_{T-1}, a_{T-1})$$

s_T: terminal state

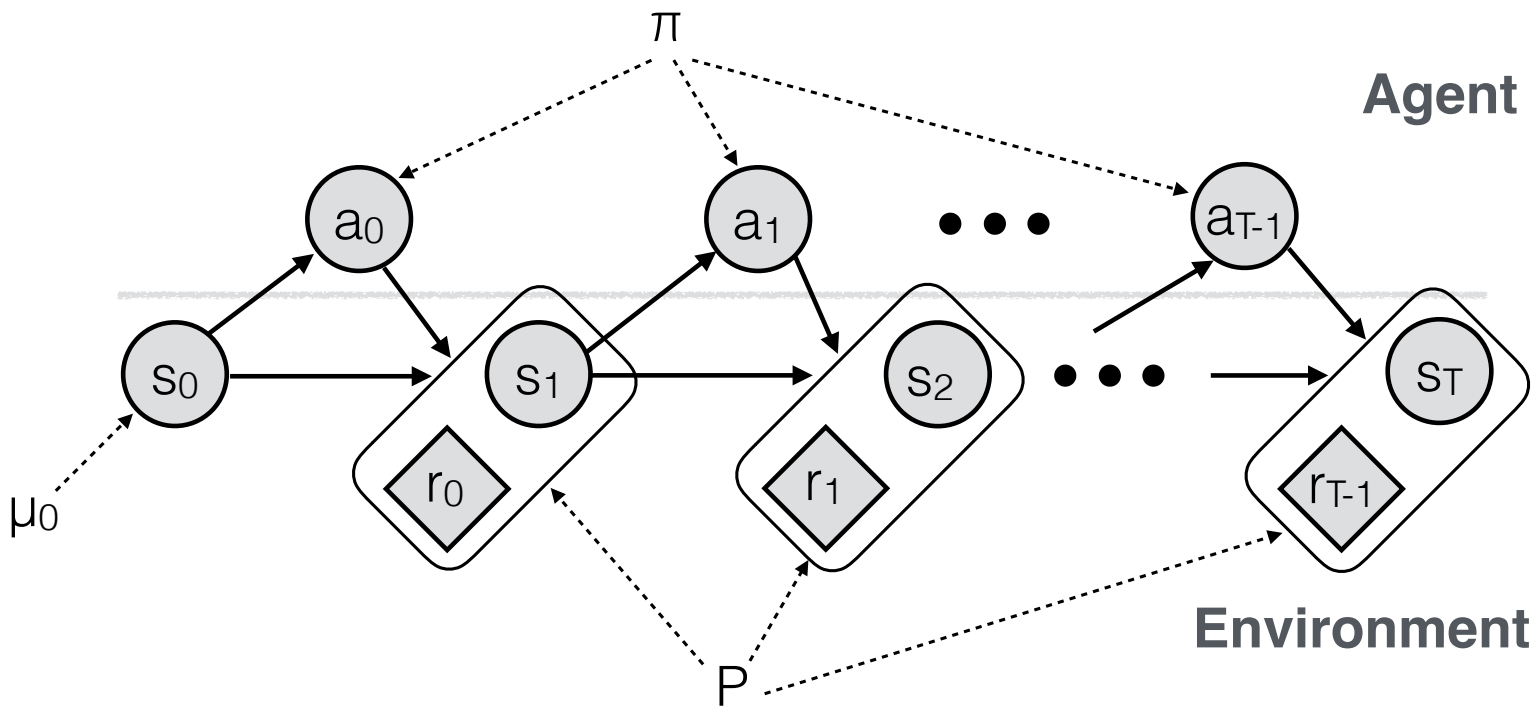
Objective:

given a police, the average reward (current + future)

maximize $\eta(\pi)$, where

$$\eta(\pi) = E[r_0 + r_1 + \cdots + r_{T-1} \mid \pi]$$

Episodic Setting



Objective:

$$\text{maximize}\, \eta(\pi), \text{ where}$$
$$\eta(\pi) = E[r_0 + r_1 + \cdots + r_{T-1} \mid \pi]$$

Parameterized Policies

- A family of policies indexed by parameter vector $\theta \in \mathbb{R}^d$
  - Deterministic: $a = \pi(s, \theta)$
  - Stochastic: $\pi(a \mid s, \theta)$
- Analogous to classification or regression with input s, output a. E.g. for neural network stochastic policies:
  - Discrete action space: network outputs vector of probabilities
  - Continuous action space: network outputs mean and diagonal covariance of Gaussian

Reinforcement Learning via Black-Box Optimization

Derivative Free Optimization Approach

- Objective:

  Maximize current + future rewards

  maximize $E[R \mid \pi(\cdot, \theta)]$

- View $\theta \rightarrow \blacksquare \rightarrow R$ as a black box
- Ignore all other information other than R collected during episode

Cross-Entropy Method

- Evolutionary algorithm
- Works **embarrassingly** well

Method	Mean Score	Reference
Nonreinforcement learning		
Hand-coded	631,167	Dellacherie (Fahey, 2003)
Genetic algorithm	586,103	(Böhm et al., 2004)
Reinforcement learning		
Relational reinforcement learning+kernel-based regression	≈50	Ramon and Driessens (2004)
Policy iteration	3183	Bertsekas and Tsitsiklis (1996)
Least squares policy iteration	<3000	Lagoudakis, Parr, and Littman (2002)
Linear programming + Bootstrap	4274	Farias and van Roy (2006)
Natural policy gradient	≈6800	Kakade (2001)
CE+RL	21,252	
CE+RL, constant noise	72,705	
CE+RL, decreasing noise	348,895	

Approximate Dynamic Programming Finally Performs Well in the Game of Tetris

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Mohammad Ghavamzadeh*
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Bruno Scherrer
INRIA Nancy - Grand Est, Team Maia, FRANCE
bruno.scherrer@inria.fr

István Szita and András Lörincz. "Learning Tetris using the noisy cross-entropy method". In: *Neural computation* 18.12 (2006), pp. 2936–2941

DPL works better on high-dimen space

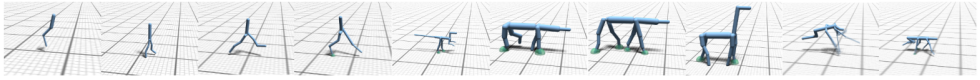
Victor Gabillon, Mohammad Ghavamzadeh, and Bruno Scherrer. "Approximate Dynamic Programming Finally Performs Well in the Game of Tetris". In: *Advances in Neural Information Processing Systems*. 2013

Cross-Entropy Method

- Evolutionary algorithm
- Works **embarrassingly** well
- A similar algorithm, Covariance Matrix Adaptation, has become standard in graphics:

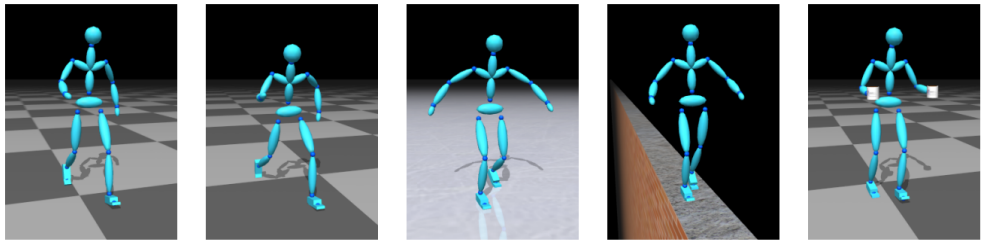
Optimal Gait and Form for Animal Locomotion

Kevin Wampler* Zoran Popović
University of Washington



Optimizing Walking Controllers for Uncertain Inputs and Environments

Jack M. Wang David J. Fleet Aaron Hertzmann
University of Toronto



Cross-Entropy Method

Initialize $\mu \in \mathbb{R}^d, \sigma \in \mathbb{R}^d$
for iteration $= 1, 2, \ldots$ **do**
    Collect n samples of $\theta_i \sim N(\mu, \text{diag}(\sigma))$
    Perform a noisy evaluation $R_i \sim \theta_i$
    Select the top $p\%$ of samples (e.g. $p = 20$), which we'll call the **elite set**
    Fit a Gaussian distribution, with diagonal covariance, to the elite set, obtaining a new μ, σ.
end for
Return the final μ.

Cross-Entropy Method

- Analysis: a very similar algorithm is an minorization-maximization (MM) algorithm, guaranteed to monotonically increase expected reward
- Recall that Monte-Carlo EM algorithm collects samples, reweights them, and them maximizes their logprob
- We can derive MM algorithm where each iteration you maximize $\sum_i \log p(\theta_i) R_i$

Policy Gradient Methods

Policy Gradient Methods: Overview

Problem:

Trajectory? Page 31

$$\text{maximize } E[R \mid \pi_\theta]$$

Intuitions: collect a bunch of trajectories, and ...

rely on luck
high var

1. Make the good trajectories more probable
2. Make the good actions more probable (actor-critic, GAE)

reduce variance

3. Push the actions towards good actions (DPG, SVG)

Score Function Gradient Estimator

- Consider an expectation $E_{x \sim p(x \mid \theta)}[f(x)]$. Want to compute gradient wrt θ

$$\begin{aligned}
\nabla_\theta E_x[f(x)] &= \nabla_\theta \int \mathrm{d}x \; p(x \mid \theta) f(x) \\
&= \int \mathrm{d}x \; \nabla_\theta p(x \mid \theta) f(x) \\
&= \int \mathrm{d}x \; p(x \mid \theta) \frac{\nabla_\theta p(x \mid \theta)}{p(x \mid \theta)} f(x) \\
&= \int \mathrm{d}x \; p(x \mid \theta) \nabla_\theta \log p(x \mid \theta) f(x) \\
&= E_x[f(x) \nabla_\theta \log p(x \mid \theta)].
\end{aligned}$$

- Last expression gives us an unbiased gradient estimator. Just sample $x_i \sim p(x \mid \theta)$, and compute $\hat{g}_i = f(x_i) \nabla_\theta \log p(x_i \mid \theta)$.
- Need to be able to compute and differentiate density $p(x \mid \theta)$ wrt θ

Derivation via Importance Sampling

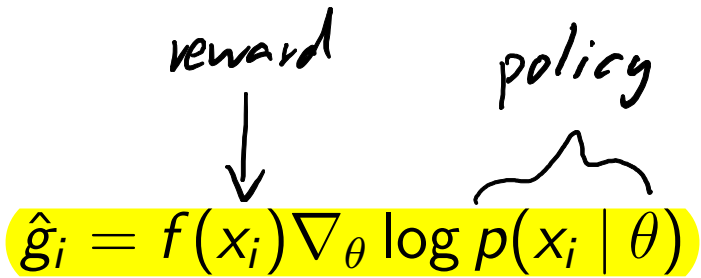
Alternate Derivation Using Importance Sampling

$$\mathbb{E}_{x\sim\theta}\left[f(x)\right] = \mathbb{E}_{x\sim\theta_{\text{old}}}\left[\frac{p(x\mid\theta)}{p(x\mid\theta_{\text{old}})}f(x)\right]$$

$$\nabla_\theta\mathbb{E}_{x\sim\theta}\left[f(x)\right] = \mathbb{E}_{x\sim\theta_{\text{old}}}\left[\frac{\nabla_\theta p(x\mid\theta)}{p(x\mid\theta_{\text{old}})}f(x)\right]$$

$$\begin{aligned}\nabla_\theta\mathbb{E}_{x\sim\theta}\left[f(x)\right]\Big|_{\theta=\theta_{\text{old}}} &= \mathbb{E}_{x\sim\theta_{\text{old}}}\left[\frac{\nabla_\theta p(x\mid\theta)\big|_{\theta=\theta_{\text{old}}}}{p(x\mid\theta_{\text{old}})}f(x)\right] \\ &= \mathbb{E}_{x\sim\theta_{\text{old}}}\left[\nabla_\theta \log p(x\mid\theta)\big|_{\theta=\theta_{\text{old}}}f(x)\right]\end{aligned}$$

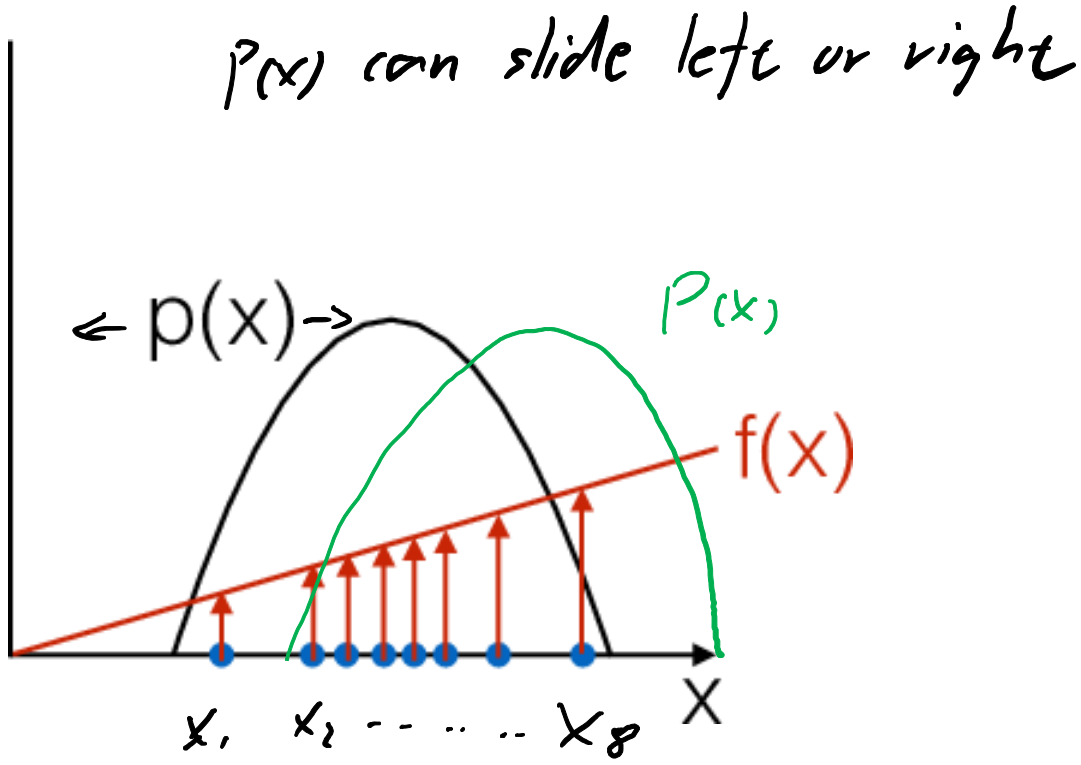
Score Function Gradient Estimator: Intuition



- Let's say that $f(x)$ measures how good the sample x is.
- Moving in the direction $\hat{g}_i$ pushes up the logprob of the sample, in proportion to how good it is
- *Valid even if $f(x)$ is discontinuous,* and *unknown, or sample space (containing x) is a discrete set*

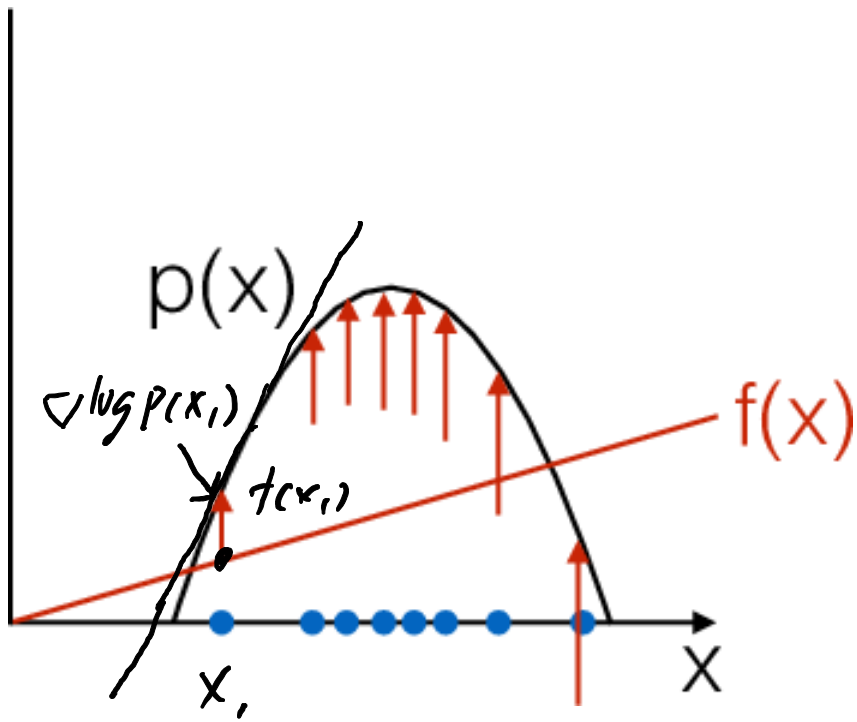
Score Function Gradient Estimator: Intuition

$$\hat{g}_i = f(x_i)\nabla_\theta \log p(x_i \mid \theta)$$



Score Function Gradient Estimator: Intuition

$$\hat{g}_i = f(x_i)\nabla_\theta \log p(x_i \mid \theta)$$



$x = \ell$

next page

Score Function Gradient Estimator for Policies

- Now random variable x is a whole trajectory $\tau = (s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T)$

$$\nabla_\theta E_\tau[R(\tau)] = E_\tau[\nabla_\theta \log p(\tau \mid \theta) R(\tau)]$$

- Just need to write out $p(\tau \mid \theta)$:

$$p(\tau \mid \theta) = \mu(s_0) \prod_{t=0}^{T-1} [\pi(a_t \mid s_t, \theta) P(s_{t+1}, r_t \mid s_t, a_t)]$$

$$\log p(\tau \mid \theta) = \cancel{\log \mu(s_0)} + \sum_{t=0}^{T-1} [\log \pi(a_t \mid s_t, \theta) + \cancel{\log P(s_{t+1}, r_t \mid s_t, a_t)}]$$

any term w/o θ drop out

$$\nabla_\theta \log p(\tau \mid \theta) = \nabla_\theta \sum_{t=0}^{T-1} \log \pi(a_t \mid s_t, \theta)$$

$$\nabla_\theta \mathbb{E}_\tau [R] = \mathbb{E}_\tau \left[R \nabla_\theta \sum_{t=0}^{T-1} \log \pi(a_t \mid s_t, \theta) \right]$$

- Interpretation: using good trajectories (high R) as supervised examples in classification / regression

Policy Gradient–Slightly Better Formula

- Previous slide:

$$\nabla_\theta \mathbb{E}_\tau [R] = \mathbb{E}_\tau \left[\left(\sum_{t=0}^{T-1} r_t \right) \left(\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t \mid s_t, \theta) \right) \right]$$

- But we can cut trajectory to t steps and derive gradient estimator for one reward term $r_{t'}$.

$$\nabla_\theta \mathbb{E} [r_{t'}] = \mathbb{E} \left[r_{t'} \sum_{t=0}^{t} \nabla_\theta \log \pi(a_t \mid s_t, \theta) \right]$$

- Sum this formula over t, obtaining

$$\nabla_\theta \mathbb{E} [R] = \mathbb{E} \left[\sum_{t=0}^{T-1} r_{t'} \sum_{t=0}^{t'} \nabla_\theta \log \pi(a_t \mid s_t, \theta) \right]$$

Estimate ✓

$$= \mathbb{E} \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t \mid s_t, \theta) \sum_{t'=t}^{T-1} r_{t'} \right]$$

Adding a Baseline

- Suppose $f(x) \geq 0, \quad \forall x$
- Then for every x_i, gradient estimator $\hat{g}_i$ tries to push up it's density
- We can derive a new unbiased estimator that avoids this problem, and only pushes up the density for better-than-average x_i.

$$\begin{aligned}\nabla_\theta \mathbb{E}_x [f(x)] &= \nabla_\theta \mathbb{E}_x [f(x) - b] \\ &= \mathbb{E}_x [\nabla_\theta \log p(x \mid \theta)(f(x) - b)]\end{aligned}$$

- A near-optimal choice of b is always $\mathbb{E}[f(x)]$ (which must be estimated)

Policy Gradient with Baseline

- Recall

$$\nabla_\theta \mathbb{E}_\tau [R] = \sum_{t'=0}^{T-1} r_{t'} \sum_{t=t}^{T-1} \nabla_\theta \log \pi(a_t \mid s_t, \theta)$$

- Using the $\mathbb{E}_{a_t} [\nabla_\theta \log \pi(a_t \mid s_t, \theta)] = 0$, we can show

$$\nabla_\theta \mathbb{E}_\tau [R] = \mathbb{E}_\tau \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t \mid s_t, \theta) \left(\sum_{t=t'}^{T-1} r_{t'} - b(s_t) \right) \right]$$

  for any "baseline" function $b : \mathcal{S} \to \mathbb{R}$
- Increase logprob of action a_t proportionally to how much returns $\sum_{t=t'}^{T-1} r_{t'}$ are better than expected
- Later: use *value functions* to further isolate effect of action, at the cost of bias
- For more general picture of score function gradient estimator, see *stochastic computation graphs*[4].

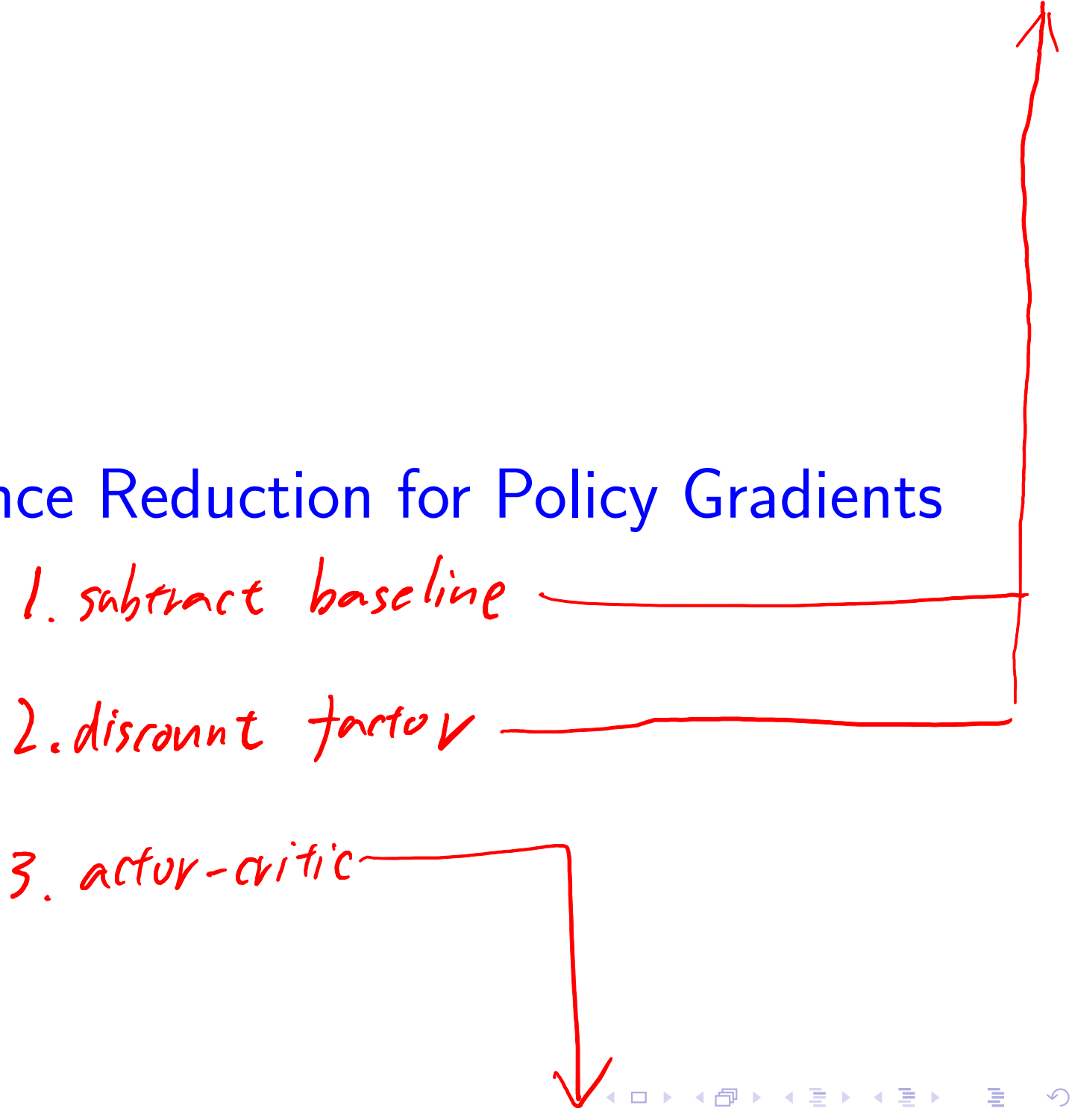
[4] John Schulman, Nicolas Heess, et al. "Gradient Estimation Using Stochastic Computation Graphs". In: *Advances in Neural Information Processing Systems*. 2015, pp. 3510–3522.

That's all for today

Course Materials: goo.gl/5wsgbJ

Variance Reduction for Policy Gradients

1. subtract baseline
2. discount factor
3. actor-critic



Review (I)

- Process for generating trajectory
  $\tau = (s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T)$

$$
\begin{aligned}
s_0 &\sim \mu(s_0) \\
a_0 &\sim \pi(a_0 \mid s_0) \\
s_1, r_0 &\sim P(s_1, r_0 \mid s_0, a_0) \\
a_1 &\sim \pi(a_1 \mid s_1) \\
s_2, r_1 &\sim P(s_2, r_1 \mid s_1, a_1) \\
&\ldots \\
a_{T-1} &\sim \pi(a_{T-1} \mid s_{T-1}) \\
s_T, r_{T-1} &\sim P(s_T \mid s_{T-1}, a_{T-1})
\end{aligned}
$$

- Given parameterized policy $\pi(a \mid s, \theta)$, the optimization problem is

$$\underset{\theta}{\text{maximize}}\, \mathbb{E}_\tau \left[R \mid \pi(\cdot \mid \cdot, \theta) \right]$$

where $R = r_0 + r_1 + \cdots + r_{T-1}$.

Review (II)

- In general, we can compute gradients of expectations with the *score function gradient estimator*

$$\nabla_\theta \mathbb{E}_{x \sim p(x \mid \theta)} [f(x)] = \mathbb{E}_x [\nabla_\theta \log p(x \mid \theta) f(x)]$$

- We derived a formula for the policy gradient

$$\nabla_\theta \mathbb{E}_\tau [R] = \mathbb{E}_\tau \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t \mid s_t, \theta) \left(\sum_{t=t'}^{T-1} r_{t'} - b(s_t) \right) \right]$$

Value Functions

- The *state-value function* V^π is defined as:

$$V^\pi(s) = E[r_0 + r_1 + r_2 + \ldots \mid s_0 = s]$$

$= E[R \mid s_0 = s]$

Measures *expected future return, starting with state s*

- The *state-action value function* Q^π is defined as

$$Q^\pi(s, a) = E[r_0 + r_1 + r_2 + \ldots \mid s_0 = s, a_0 = a]$$

- The *advantage function* A^π is

Reward with action 'a' taken

Average of R with all possible actions considered

$$A^\pi(s, a) = Q^\pi(s, a) - V^\pi(s)$$

Measures *how much better is action a than what the policy* π *would've done.*

Refining the Policy Gradient Formula

- Recall

$$\nabla_\theta \mathbb{E}_\tau [R] = \mathbb{E}_\tau \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t \mid s_t, \theta) \left(\sum_{t=t'}^{T-1} r_{t'} - b(s_t) \right) \right]$$

$$= \sum_{t=0}^{T-1} \mathbb{E}_\tau \left[\nabla_\theta \log \pi(a_t \mid s_t, \theta) \left(\sum_{t=t'}^{T-1} r_{t'} - b(s_t) \right) \right]$$

$$= \sum_{t=0}^{T-1} \mathbb{E}_{s_0 \ldots a_t} \left[\nabla_\theta \log \pi(a_t \mid s_t, \theta) \mathbb{E}_{r_t s_{t+1} \ldots s_T} \left[\left(\sum_{t=t'}^{T-1} r_{t'} - b(s_t) \right) \right] \right]$$

$$= \sum_{t=0}^{T-1} \mathbb{E}_{s_0 \ldots a_t} \left[\nabla_\theta \log \pi(a_t \mid s_t, \theta) \mathbb{E}_{r_t s_{t+1} \ldots s_T} [Q^\pi(s_t, a_t) - b(s_t)] \right]$$

- Where the last equality used the fact that

$$\mathbb{E}_{r_t s_{t+1} \ldots s_T} \left[\sum_{t=t'}^{T-1} r_{t'} \right] = Q^\pi(s_t, a_t)$$

Refining the Policy Gradient Formula

- From the previous slide, we've obtained

$$\nabla_\theta \mathbb{E}_\tau [R] = \mathbb{E}_\tau \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t \mid s_t, \theta)(Q^\pi(s_t, a_t) - b(s_t)) \right]$$

- Now let's define $b(s) = V^\pi(s)$, which turns out to be near-optimal[5]. We get

since we want to max R, we want A to be pos

$$\nabla_\theta \mathbb{E}_\tau [R] = \mathbb{E}_\tau \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t \mid s_t, \theta) A^\pi(s_t, a_t) \right]$$

- Intuition: increase the probability of good actions (positive advantage) decrease the probability of bad ones (negative advantage)

[5] Evan Greensmith, Peter L Bartlett, and Jonathan Baxter. "Variance reduction techniques for gradient estimates in reinforcement learning". In: *The Journal of Machine Learning Research* 5 (2004), pp. 1471–1530.

Variance Reduction

- Now, we have the following policy gradient formula:

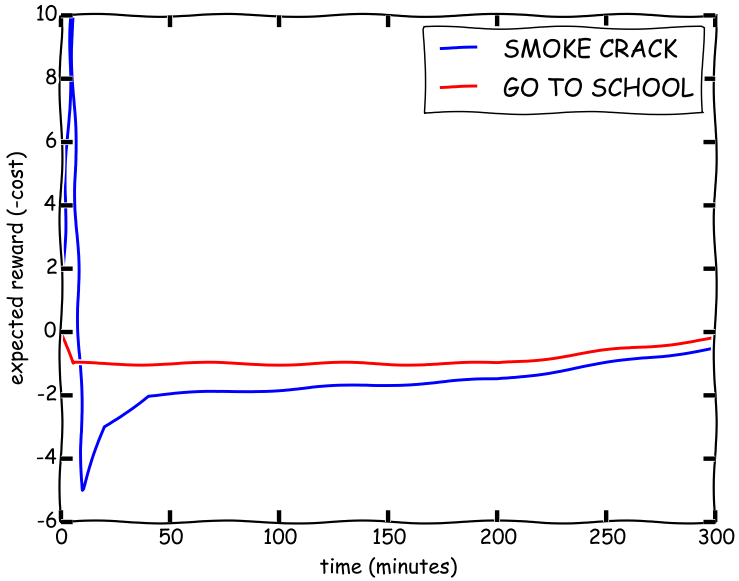
$$\nabla_\theta \mathbb{E}_\tau [R] = \mathbb{E}_\tau \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t \mid s_t, \theta) A^\pi(s_t, a_t) \right]$$

- A^π is not known, but we can plug in a random variable $\hat{A}_t$, an *advantage estimator*
- Previously, we showed that taking

$$\hat{A}_t = r_t + r_{t+1} + r_{t+2} + \cdots - b(s_t)$$

for any function $b(s_t)$, gives an unbiased policy gradient estimator. $b(s_t) \approx V^\pi(s_t)$ gives variance reduction.

The Delayed Reward Problem

- One reason RL is difficult is the long delay between action and reward
- 

The Delayed Reward Problem

- With policy gradient methods, we are confounding the effect of multiple actions:

$$\hat{A}_t = r_t + r_{t+1} + r_{t+2} + \cdots - b(s_t)$$

  mixes effect of $a_t, a_{t+1}, a_{t+2}, \ldots$
- SNR of $\hat{A}_t$ scales roughly as $1/T$
  - Only a_t contributes to *signal* $A^\pi(s_t, a_t)$, but $a_{t+1}, a_{t+2}, \ldots$ contribute to noise.

Var. Red. Idea 1: Using Discounts

- Discount factor γ, $0 < \gamma < 1$, downweights the effect of rewars that are far in the future—ignore long term dependencies
- We can form an advantage estimator using the *discounted return*:

$$\hat{A}_t^\gamma = \underbrace{r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \dots}_{\text{discounted return}} - b(s_t)$$

  reduces to our previous estimator when $\gamma = 1$.
- So advantage has expectation zero, we should fit baseline to be *discounted value function*

$$V^{\pi,\gamma}(s) = \mathbb{E}_\tau \left[r_0 + \gamma r_1 + \gamma^2 r_2 + \dots \mid s_0 = s \right]$$

- $\hat{A}_t^\gamma$ is a biased estimator of the advantage function

Var. Red. Idea 2: Value Functions in the Future

- Another approach for variance reduction is to use the value function to estimate future rewards

$r_t + r_{t+1} + r_{t+2} + \ldots$ use empirical rewards

$\Rightarrow$

$r_t + V(s_{t+1})$ cut off at one timestep

$r_t + r_{t+1} + V(s_{t+2})$ cut off at two timesteps

....

Adding the baseline again, we get the advantage estimators

$\hat{A}_t = r_t + V(s_{t+1}) - V(s_t)$ cut off at one timestep

$\hat{A}_t = r_t + r_{t+1} + V(s_{t+2}) - V(s_t)$ cut off at two timesteps

... Low var, high bias, since lots of R is ignored

Combining Ideas 1 and 2

- Can combine discounts and value functions in the future, e.g., $\hat{A}_t = r_t + \gamma V(s_{t+1}) - V(s_t)$, where V approximates discounted value function $V^{\pi,\gamma}$.
- The above formula is called an *actor-critic* method, where *actor* is the policy π, and *critic* is the value function V.[6]
- Going further, the *generalized advantage estimator*[7]

$$\hat{A}_t^{\gamma,\lambda} = \delta_t + (\gamma\lambda)\delta_{t+1} + (\gamma\lambda)^2\delta_{t+2} + \dots$$
$$\text{where} \quad \delta_t = r_t + \gamma V(s_{t+1}) - V(s_t)$$

- Interpolates between two previous estimators:

PG

$\lambda = 0$: $r_t + \gamma V(s_{t+1}) - V(s_t)$ (low v, high b)

$\lambda = 1$: $r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \dots - V(s_t)$ (low b, high v)

AC

[6] Vijay R Konda and John N Tsitsiklis. "Actor-Critic Algorithms." In: *Advances in Neural Information Processing Systems*. Vol. 13. Citeseer. 1999, pp. 1008–1014.

[7] John Schulman, Philipp Moritz, et al. "High-dimensional continuous control using generalized advantage estimation". In: *arXiv preprint arXiv:1506.02438* (2015).

Alternative Approach: Reparameterization

PG is deterministic, no estimator is needed

- Suppose problem has continuous action space, $a \in \mathbb{R}^d$
- Then $\frac{d}{da} Q^\pi(s, a)$ tells use how to improve our action
- We can use reparameterization trick, so a is a deterministic function $a = f(s, z)$, where z is noise. Then,

$$\nabla_\theta \mathbb{E}_\tau [R] = \nabla_\theta Q^\pi(s_0, a_0) + \nabla_\theta Q^\pi(s_1, a_1) + \dots$$

- This method is called the deterministic policy gradient[8]
- A generalized version, which also uses a dynamics model, is described as the stochastic value gradient[9]

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[8] David Silver, Guy Lever, et al. "Deterministic policy gradient algorithms". In: *ICML*. 2014; Timothy P Lillicrap et al. "Continuous control with deep reinforcement learning". In: *arXiv preprint arXiv:1509.02971* (2015).

[9] Nicolas Heess et al. "Learning continuous control policies by stochastic value gradients". In: *Advances in Neural Information Processing Systems*. 2015, pp. 2926–2934.

Trust Region and Natural Gradient Methods

Optimization Issues with Policy Gradients

PG requires largest number of episodes.

- Hard to choose reasonable stepsize that works for the whole optimization hard to do line search to find optimal stepsize
  - we have a gradient estimate, no objective for line search
  - statistics of data (observations and rewards) change during learning
- They make inefficient use of data: each experience is only used to compute one gradient. next episode doesn't use previous data
  - Given a batch of trajectories, what's the most we can do with it?

Policy Performance Function

- Let $\eta(\pi)$ denote the performance of policy π

$$\eta(\pi) = \mathbb{E}_\tau [R|\pi]$$

- The following neat identity holds:

$$\eta(\tilde{\pi}) = \eta(\pi) + \mathbb{E}_{\tau\sim\tilde{\pi}} [A^\pi(s_0, a_0) + A^\pi(s_1, a_1) + A^\pi(s_2, a_2) + \dots]$$

- Proof: consider nonstationary policy $\pi_0\pi_1\pi_2, \dots$

$$\begin{aligned}\eta(\tilde{\pi}\tilde{\pi}\tilde{\pi}\cdots) = \eta(\pi\pi\pi\cdots) \\ + \eta(\tilde{\pi}\pi\pi\cdots) - \eta(\pi\pi\pi\cdots) \\ + \eta(\tilde{\pi}\tilde{\pi}\pi\cdots) - \eta(\tilde{\pi}\pi\pi\cdots) \\ + \eta(\tilde{\pi}\tilde{\pi}\tilde{\pi}\cdots) - \eta(\tilde{\pi}\tilde{\pi}\pi\cdots) \\ + \dots\end{aligned}$$

- t^{th} difference term equals $A^\pi(s_t, a_t)$

Local Approximation

- We just derived an expression for the performance of a policy $\tilde{\pi}$ relative to π

$$\begin{aligned}\eta(\tilde{\pi}) &= \eta(\pi) + \mathbb{E}_{\tau\sim\tilde{\pi}}\left[A^{\pi}(s_0,a_0) + A^{\pi}(s_1,a_1) + \dots\right] \\ &= \eta(\pi) + \mathbb{E}_{s_{0:\infty}\sim\tilde{\pi}}\left[\mathbb{E}_{a_{0:\infty}\sim\tilde{\pi}}\left[A^{\pi}(s_0,a_0) + A^{\pi}(s_1,a_1) + \dots\right]\right]\end{aligned}$$

- Can't use this to optimize $\tilde{\pi}$ because state distribution has complicated dependence.
- Let's define L_{π} the *local approximation*, which ignores change in state distribution—can be estimated by sampling from π

$$\begin{aligned}L_{\pi}(\tilde{\pi}) &= \mathbb{E}_{s_{0:\infty}\sim\pi}\left[\mathbb{E}_{a_{0:\infty}\sim\tilde{\pi}}\left[A^{\pi}(s_0,a_0) + A^{\pi}(s_1,a_1) + \dots\right]\right] \\ &= \mathbb{E}_{s_{0:\infty}}\left[\sum_{t=0}^{T-1}\mathbb{E}_{a\sim\tilde{\pi}}\left[A^{\pi}(s_t,a_t)\right]\right] \\ &= \mathbb{E}_{s_{0:\infty}}\left[\sum_{t=0}^{T-1}\mathbb{E}_{a\sim\pi}\left[\frac{\tilde{\pi}(a_t\mid s_t)}{\pi(a_t\mid s_t)}A^{\pi}(s_t,a_t)\right]\right] \\ &= \mathbb{E}_{\tau\sim\pi}\left[\sum_{t=0}^{T-1}\frac{\tilde{\pi}(a_t\mid s_t)}{\pi(a_t\mid s_t)}A^{\pi}(s_t,a_t)\right]\end{aligned}$$

Local Approximation

- Now let's consider parameterized policy, $\pi(a \mid s, \theta)$. Sample with θ_{old}, now write local approximation in terms of θ.

$$L_\pi(\tilde{\pi}) = \mathbb{E}_{s_{0:\infty}} \left[\sum_{t=0}^{T-1} \mathbb{E}_{a \sim \pi} \left[\frac{\tilde{\pi}(a_t \mid s_t)}{\pi(a_t \mid s_t)} A^\pi(s_t, a_t) \right] \right]$$

$$\Rightarrow L_{\theta_{\text{old}}}(\theta) = \mathbb{E}_{s_{0:\infty}} \left[\sum_{t=0}^{T-1} \mathbb{E}_{a \sim \theta} \left[\frac{\pi(a_t \mid s_t, \theta)}{\pi(a_t \mid s_t, \theta_{\text{old}})} A^\theta(s_t, a_t) \right] \right]$$

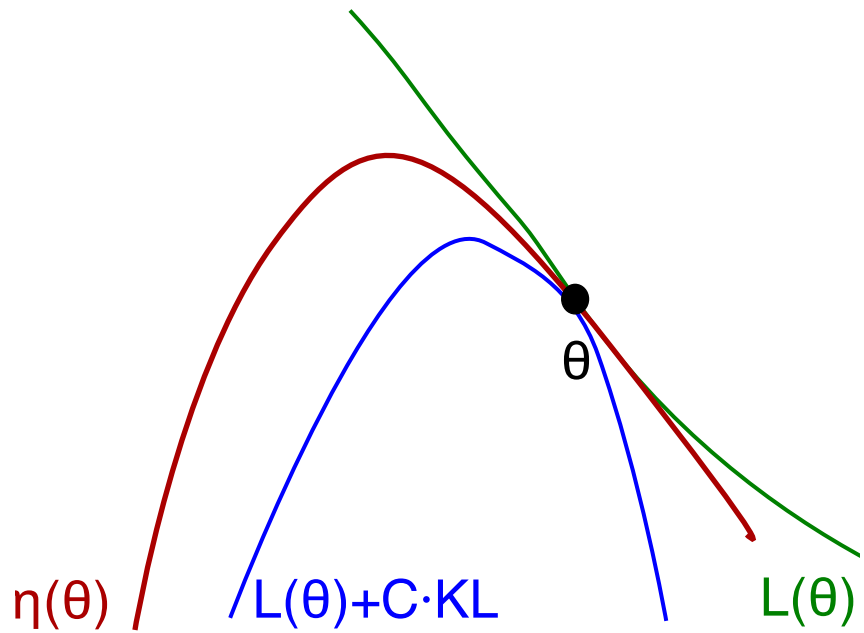
- $L_{\theta_{\text{old}}}(\theta)$ matches $\eta(\theta)$ to first order around θ_{old}.

$$\begin{aligned}
\nabla_\theta L_{\theta_{\text{old}}}(\theta)\big|_{\theta=\theta_0} &= \mathbb{E}_{s_{0:\infty}} \left[\sum_{t=0}^{T-1} \mathbb{E}_{a \sim \theta} \left[\frac{\nabla_\theta \pi(a_t \mid s_t, \theta)}{\pi(a_t \mid s_t, \theta_{\text{old}})} A^\theta(s_t, a_t) \right] \right] \\
&= \mathbb{E}_{s_{0:\infty}} \left[\sum_{t=0}^{T-1} \mathbb{E}_{a \sim \theta} \left[\nabla_\theta \log \pi(a_t \mid s_t, \theta) A^\theta(s_t, a_t) \right] \right] \\
&= \nabla_\theta \eta(\theta)\big|_{\theta=\theta_{\text{old}}}
\end{aligned}$$

MM Algorithm

- Theorem (ignoring some details)[10]

$$\eta(\theta) \geq \underbrace{L_{\theta_{\text{old}}}(\theta)}_{\text{local approx. to } \eta} - \underbrace{C \max_s D_{KL}\left[\pi(\cdot \mid \theta_{\text{old}}, s) \parallel \pi(\cdot \mid \theta, s)\right]}_{\text{penalty for changing policy}}$$



- If $\theta_{\text{old}} \to \theta_{\text{new}}$ improves lower bound, it's guaranteed to improve η

[10] John Schulman, Sergey Levine, et al. "Trust Region Policy Optimization". In: *arXiv preprint arXiv:1502.05477* (2015).

Review

- Want to optimize $\eta(\theta)$. Collected data with policy parameter θ_{old}, now want to do update
- Derived local approximation $L_{\theta_{\text{old}}}(\theta)$
- Optimizing KL penalized local approximation givesn guaranteed improvement to η
- More approximations gives practical algorithm, called TRPO

TRPO—Approximations

- Steps:
  - Instead of max over state space, take mean
  - Linear approximation to L, quadratic approximation to KL divergence
  - Use hard constraint on KL divergence instead of penalty
- Solve the following problem approximately

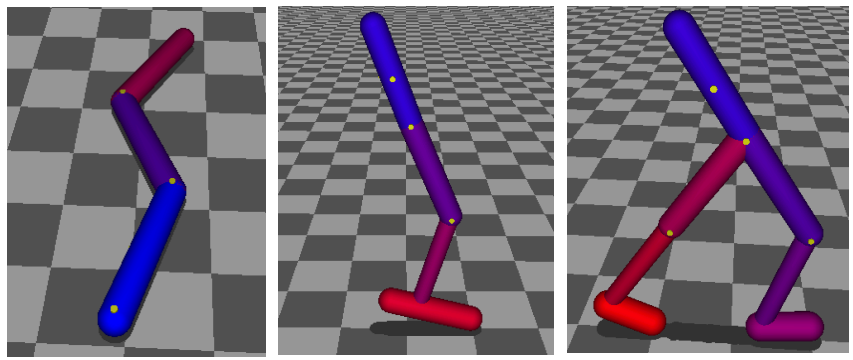
$$\begin{aligned} &\text{maximize } L_{\theta_{\text{old}}}(\theta) \\ &\text{subject to } \quad \overline{D}_{KL}[\theta_{\text{old}} \parallel \theta] \leq \delta \end{aligned}$$

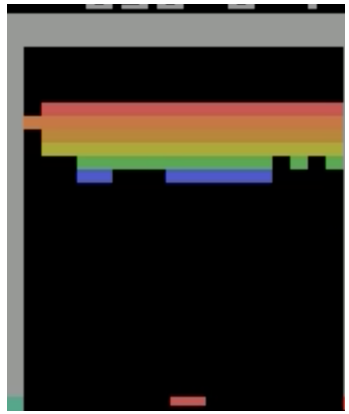
- Solve approximately through line search in the *natural gradient* direction $s = F^{-1}g$
- Resulting algorithm is a refined version of *natural policy gradient*[11]

[11] Sham Kakade. "A Natural Policy Gradient." In: *NIPS*. vol. 14. 2001, pp. 1531–1538.

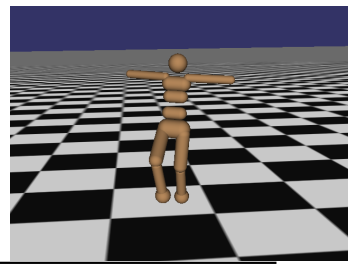
Empirical Results: TRPO + GAE

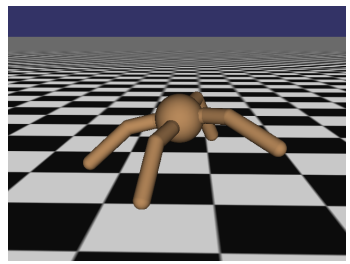
- TRPO, with neural network policies, was applied to learn controllers for 2D robotic swimming, hopping, and walking, and playing Atari games[12]





- Used TRPO along with generalized advantage estimation to optimize locomotion policies for 3D simulated robots[13]





[12] John Schulman, Sergey Levine, et al. "Trust Region Policy Optimization". In: *arXiv preprint arXiv:1502.05477* (2015).

[13] John Schulman, Philipp Moritz, et al. "High-dimensional continuous control using generalized advantage estimation". In: *arXiv preprint arXiv:1506.02438* (2015).

Putting In Perspective

Quick and incomplete overview of recent results with deep RL algorithms

- Policy gradient methods
  - TRPO + GAE
  - Standard policy gradient (no trust region) + deep nets + parallel implementation[14]
  - Repar trick[15]
- Q-learning[16] and modifications[17]
- Combining search + supervised learning[18]

Not suitable for continuous domain

[14] V. Mnih et al. "Playing Atari with Deep Reinforcement Learning". In: *arXiv preprint arXiv:1312.5602* (2013).

[15] Nicolas Heess et al. "Learning continuous control policies by stochastic value gradients". In: *Advances in Neural Information Processing Systems*. 2015, pp. 2926–2934; Timothy P Lillicrap et al. "Continuous control with deep reinforcement learning". In: *arXiv preprint arXiv:1509.02971* (2015).

[16] V. Mnih et al. "Playing Atari with Deep Reinforcement Learning". In: *arXiv preprint arXiv:1312.5602* (2013).

[17] Ziyu Wang, Nando de Freitas, and Marc Lanctot. "Dueling Network Architectures for Deep Reinforcement Learning". In: *arXiv preprint arXiv:1511.06581* (2015); Hado V Hasselt. "Double Q-learning". In: *Advances in Neural Information Processing Systems*. 2010, pp. 2613–2621.

[18] X. Guo et al. "Deep learning for real-time Atari game play using offline Monte-Carlo tree search planning". In: *Advances in Neural Information Processing Systems*. 2014, pp. 3338–3346; Sergey Levine et al. "End-to-end training of deep visuomotor policies". In: *arXiv preprint arXiv:1504.00702* (2015); Igor Mordatch et al. "Interactive Control of Diverse Complex Characters with Neural Networks". In: *Advances in Neural Information Processing Systems*. 2015, pp. 3114–3122.

Algorithm sometimes just choose the simplest way to gain imediate reward (local minimum)

Define reward is VERY IMPORTANT

Open Problems

Entropy regularization on policy

Time step is an important hyperparameter.

We can try semi-supervized / pre-trained method we play a few games first. (Be careful with local minimum)

What's the Right Core Model-Free Algorithm?

- Policy gradients (score function vs. reparameterization, natural vs. not natural) vs. Q-learning vs. derivative-free optimization vs others
- Desiderata
  - scalable
  - sample-efficient
  - robust
  - learns from *off-policy* data

Exploration

- Exploration: actively encourage agent to reach unfamiliar parts of state space, avoid getting stuck in local maximum of performance
- Can solve finite MDPs in polynomial time with exploration[19]
  - optimism about new states and actions
  - maintain distribution over possible models, and plan with them (Bayesian RL, Thompson sampling)
- How to do exploration in deep RL setting? Thompson sampling[20], novelty bonus[21]

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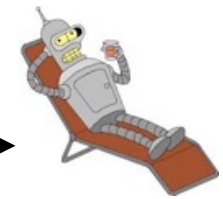
[19] Alexander L Strehl et al. "PAC model-free reinforcement learning". In: *Proceedings of the 23rd international conference on Machine learning*. ACM. 2006, pp. 881–888.

[20] Ian Osband et al. "Deep Exploration via Bootstrapped DQN". . In: *arXiv preprint arXiv:1602.04621* (2016).

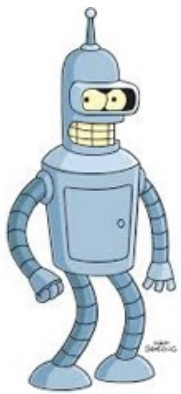
[21] Bradly C Stadie, Sergey Levine, and Pieter Abbeel. "Incentivizing Exploration In Reinforcement Learning With Deep Predictive Models". In: *arXiv preprint arXiv:1507.00814* (2015).

Hierarchy



task 1 … task 2 … task 3 … task 4 … 10 timesteps / day



walk to x … fetch object y … say z … .01 hz: 10^3 time steps per day

footstep planning: 1hz: 10^5 timesteps / day

torque control: 100hz: 10^7 timesteps /day

More Open Problems

- Using learned models
- Learning from demonstrations

The End

Questions?