18.2 Question:- In one state, 52% of the voters are Republicans and 48% are Democrats. In a second state, 47% of the voters are Republicans and 53% are Democrats. Suppose a simple random sample of 100 voters are surveyed from each state. What is the probability that the survey will show a greater percentage of Republican voters in the second state than in the first state?

Solution:-

Let us denote state1 as S1 and state2 as S2

Mean of Republican voters in first state $= \mu_{S1} = 0.52$

Mean of Republican voters in second state $= \mu_{S2} = 0.47$

No. of samples of first state $= n_{S1} = 100$

No. of samples of second state $= n_{S2} = 100$

Mean of difference between samples of two states $= \mu_{S1-S2}$
$$= 0.52 - 0.47 = 0.05$$

Standard deviation of difference between samples of state1 and state2

$$= \sqrt{\frac{\mu_{S1}(1-\mu_{S1})}{n_{S1}} + \frac{\mu_{S2}(1-\mu_{S2})}{n_{S2}}}$$

$$= \sqrt{\frac{0.52(1-0.52)}{100} + \frac{0.47(1-0.47)}{100}}$$

$$= \sqrt{0.002496 + 0.002491}$$

$$= \sqrt{0.004987} = 0.07061$$

We need to find the probability that mean of state1 samples of (S1) is less than the mean of state2 (S2) samples of state 2.

This is equivalent to finding that mean of samples of state1 $(\overline{x}_{S1})$ is less than mean of samples of state 2 $(\overline{x}_{S2})$

Let us find the Z value for difference of samples between two states

$$Z_{\bar{x}_{S1} - \bar{x}_{S2}} = \frac{x - \mu_{S1-S2}}{\text{stand deviation of difference of sample mean}}$$

$$= \frac{0 - 0.05}{0.07061}$$

$$= \frac{-0.05}{0.07061}$$

$$= -0.7081$$

From Z table, we find

Probability $(Z < -0.7081)$

$$= 0.2389$$

So, probability that survey will show greater percentage of Republican voters in the second state than in first state is 0.2389, which in in terms of percentage is 23.89%.