$$T(n) = \begin{cases} 0, & n = 1 \\ (n+1) + \dfrac{1}{n}\displaystyle\sum_{k=0}^{n-1} T(k-1) + T(n-k), & n > 1 \end{cases}$$

$$T(n) = (n+1) + \frac{1}{n}\sum_{k=0}^{n-1} T(k-1) + T(n-k)$$

$$T(n) = (n+1) + \frac{2}{n}\sum_{k=0}^{n-1} T(n-k) \qquad \cdots (1) \qquad \because T(k-1) = T(n-k)$$

$$nT(n) = n(n+1) + 2\sum_{k=0}^{n-1} T(n-k) \qquad \Rightarrow \textit{multiplying by "n" into both side}$$

$$nT(n) = n(n+1) + 2[T(0) + T(1) + T(2) + \cdots + T(n-2) + T(n-1)] \qquad \cdots (2)$$

$$(n-1)T(n-1) = n(n-1) + 2[T(0) + T(1) + T(2) + \cdots + T(n-2)] \qquad \cdots (3) \;\Rightarrow \textit{replacing "n" by "} n-1 \textit{"}$$

$(2) - (3) \Rightarrow$

$$nT(n) - (n-1)T(n-1) = n(n+1) - n(n-1) + 2[T(0) + T(1) + T(2) + \cdots + T(n-2) + T(n-1) - T(0) - T(1) - T(2) - \cdots - T(n-2)]$$

$$nT(n) - (n-1)T(n-1) = 2n + 2T(n-1)$$

$$nT(n) = 2T(n-1) + (n-1)T(n-1) + 2n$$

$$nT(n) = (n+1)T(n-1) + 2n$$

$$\frac{nT(n)}{n(n+1)} = \frac{(n+1)T(n-1)}{n(n+1)} + \frac{2n}{n(n+1)} \qquad \Rightarrow \textit{divided by } n(n+1) \textit{ into both side}$$

$$\frac{T(n)}{(n+1)} = \frac{T(n-1)}{n} + \frac{2}{(n+1)} \qquad \cdots (4)$$

Using Substitution method, (into equation number 4)

$$\frac{T(n)}{(n+1)} = \frac{T(n-1)}{n} + \frac{2}{(n+1)} \qquad \cdots (4)$$

$$\Rightarrow \left[\frac{T(n-2)}{(n-1)} + \frac{2}{n}\right] + \frac{2}{(n+1)} \qquad \Rightarrow \textit{replacing "n" by } (n-1) \textit{ into eq.} (4)$$

$$\Rightarrow \left[\frac{T(n-3)}{(n-2)} + \frac{2}{(n-1)}\right] + \frac{2}{n} + \frac{2}{(n+1)} \qquad \Rightarrow replacing\ "n"\ by\ (n-2) into\ eq. (4)$$

$\vdots$

$$\Rightarrow \frac{T(1)}{2} + 2\left[\frac{1}{3} + \cdots + \frac{1}{(n-1)} + \frac{1}{n} + \frac{1}{(n+1)}\right] \qquad \Rightarrow replacing\ "n"\ by\ 2\ into\ eq. (4)$$

$$\Rightarrow \frac{T(1)}{2} + 2\sum_{k=3}^{n+1}\frac{1}{k}$$

$$\frac{T(n)}{(n+1)} \backsimeq 2\sum_{k=3}^{n+1}\frac{1}{k} \qquad \because T(1) = 0 \qquad \cdots (5)$$

From Calculus, we know,

$$\sum_{k=3}^{n+1}\frac{1}{k} \le \int_{2}^{n}\frac{1}{x}dx$$

$$\sum_{k=3}^{n+1}\frac{1}{k} \le \log_e n - \log_e 2$$

So, From eq. (5)

$$\frac{T(n)}{(n+1)} \backsimeq 2(\log_e n - \log_e 2)$$

$$T(n) \backsimeq 2(n+1)(\log_e n - \log_e 2)$$

$$\backsimeq O(n \log_2 n)$$