Shortest path to reach one prime to other by changing single digit at a time

GROUP - 30

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Problem Statement:

Shortest path to reach one prime to other by changing single digit at a time. Given two four-digit prime numbers, suppose 1033 and 8179, we need to find the shortest path from 1033 to 8179 by altering only single digit at a time such that every number that we get after changing a digit is prime. For example, a solution is 1033, 1733, 3733, 3739, 3779, 8779, 8179

Introduction:

In this question, we have to find the shortest distance between two prime numbers by changing a single digit at a time.To solve this Problem we need the understanding of graph traversal (BFS) and Sieve of Eratosthenes.

- **BFS:** Breadth-first search is an algorithm for traversing or searching tree or graph data structures
- **Sieve of Eratosthenes:** It is an algorithm for finding all the prime numbers in a segment (form 1 to n) using O(nloglogn) operations.

Algorithm Analysis :

1. **Breadth-first search (BFS):** It starts at the tree root (or some arbitrary node of a graph, sometimes referred to as a 'search key'), and explores all of the neighbor nodes at the present depth prior to moving on to the nodes at the next depth level. It uses the opposite strategy of depth-first search, which instead explores the node branch as far as possible before being forced to backtrack and expand other nodes.
2. **Sieve of Eratosthenes:** The algorithm is very simple: at the beginning we write down all numbers between 2 and n. We mark all proper multiples of 2 (since 2 is the smallest prime number) as composite. A proper multiple of a number x, is a number greater than x and divisible by x. Then we find the next number that hasn't been marked as composite, in this case it is 3. Which means 3 is prime, and we mark all proper multiples of 3 as composite. The next unmarked number is 5, which is the next prime number, and we mark all proper multiples of it. And we continue this procedure until we processed all numbers in the row.

Algorithm Design

Step by step approach Of Algorithms:

- First we, find and store all the prime numbers of 4 digit using Sieve of Eratosthenes
- Make a graph using those prime numbers. Vertex will be the prime numbers and two primes u,v will have an edge if they can be converted into one another by changing a single digit. Ex. 1373 and 1303 will have an edge connecting both the nodes in our graph.
- If we have to find the shortest distance between a and b, then we will root our graph at node a and initialize its distance as 0.
- Then apply bfs on our graph to find the shortest distance between given two prime numbers. The relation used will be distance[child] = distance[parent] + 1, where child and parent have a direct edge and parent has a height smaller than child.

Time Complexity:

The Time complexity of our program is : **O(N(log(log(N))) + P 2)**

Because of the three functions used-

- sieve(): : to generate prime numbers up to N. it take *O(N(log(log(N))))* time.
- makeGraph() : to add edges in our graph. It takes *O(P 2)* time.
- bfs : to find shortest distance between given two 4 digit primes. It takes *O(N + P 2)* time

Space Complexity:

The space complexity of the program is **O(N + P 2)** .

Because of the three functions-

- sieve(): : to store prime numbers up to N. it take *O(N)* space.
- makeGraph() : to add edges in our graph. At max we will have P ∗ (P − 1)/2 edges. So storing N nodes and P 2 edges Which is *O(N + P 2)* space.
- bfs : in bfs, our queue can have a maximum of P values at a time. So, It takes *O(P)* time.

Applications:

Graphs are widely used to represent the flow of computation . In World Wide Web, web pages are considered to be the vertices. There is an edge from a page u to other page v if there is a link of page v on page u. This is an example of Directed graph. It was the basic idea behind Google Page Ranking Algorithm.Few important real life applications of graph data structures are:

- Facebook
- Google Maps
- Graph theory is also used to study molecules in chemistry and physics.

Conclusion:

- We have solved our problem using BFS and Sieve of Eratosthenes, which will take over all time of O(N(log(log(N))) + P 2) .

  Where,

  N: largest 4 digit number.

  P: number of prime numbers of 4 digits.

References:

1. https://en.wikipedia.org/wiki/Breadth-first search

2.https://www.geeksforgeeks.org/shortest-path-reach-one-prime-changing-single-digit-time/

3. Cormen, Leiserson, Rivest, and Stein (2009). Introduction to Algorithms, 3rd edition.