Quantum Communication: Role of Entanglement

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***Abstract*—This document discusses the potential benefit of the property of entanglement in quantum communication through case study and includes a comparative study of quantum and classical channels underlining the capacities, encoding-decoding and error mechanism,throwing light upon the advantage of quantum communication(Entanglement) over classical communication.**

***Index Terms*—Entanglement, Channel Capacity, Zero Error, Shannon Formula**

I. Introduction

Entanglement is the phenomenon wherein a pair of particles, such as photons interact physically i.e a change in one reflects in another. The pairs of particles interact in such a way that quantum state of each particle cannot be described independently of the state of other even when they are separated by large distance.
Classical Channel capacity is the amount of classical information that can be transferred from sender to receiver reliably, example would be light travelling over fiber optics lines or electricity travelling over phone lines. In contrast to classical channel which can transfer only classical information quantum channel can transfer both classical and quantum information, thus quantum channel has two types of capacity- Quantum capacity and Classical capacity.

II. task Assigned

- Case study on " The Power of Entangled Quantum Channles" by Seth Lloyd (MIT) [**1**]
- Capacities of Quantum Channel - Classical Capacity and Quantum Capacity.[**2**]
- Comparison between Classical channel and Quantum Channel.

III. Description of work carried out

A. Case Study

OBJECTIVE

M coupled, entangled modes can send M bits in the same time it takes a single mode to send a single bit, and at the same time it takes M uncoupled, unentangled modes to send root M bits.

Quantum mechanics restricts the rates at which the information can be transmitted down noiseless channel at finite power. Previous results have worked on enhancing communication capacity of uncoupled quantum channels by exploiting pre-existing quantum entanglement. This case study investigates how the capacity channels can be enhanced by coupling together the information propagated degrees of freedom to induce the entangle state in the process of transmission.

According to kholevo's theorem noiseless broadband bosanic channel can transmit $C_1 = \alpha\sqrt{\frac{P}{\hbar}}$ bits per second where P is Power and alpha is constant. If Power is spread among M unentangled broadband bosonic channel each with power P/M, rate of communication is $C = \sqrt{M * C1}$ this is the best rate known with using M unentangled channels. If As and Bs qubits are coupled by an intervening chaining of qubit $A_1B_1A_2B_2..A_nB_n$. Quantum Information can be sent along the chain by swapping A_i with B_i then swapping B_i with A_{i+1} and repeating until qubit is moved from A to B. Repeated swaps move qubits from A to B and B to A simultaneously so net transformation of energy down the channel is zero.

B. Capacities of Quantum Channel

- **Classical Capacity of Quantum Channel :**

  Quantum Communication steps :
  In the first phase, the sender, Alice, has to encode her information to compensate for the noise of the channel (i.e., for error correction) according to properties of the physical channel this step is called **channel coding**.
  After the sender has encoded the information into the appropriate form, it has to be put on the quantum channel, which transforms it according to its channel map this second phase is called the **channel evolution**.
  The quantum channel conveys the quantum state to the receiver, Bob; however, this state is still a superposed and probably mixed (according to the noise of the channel) quantum state.
  To extract the information that is encoded in the state, the receiver has to make a measurement this **measurement process** (with the error correction procedure) is the third phase of the communication over a quantum channel.

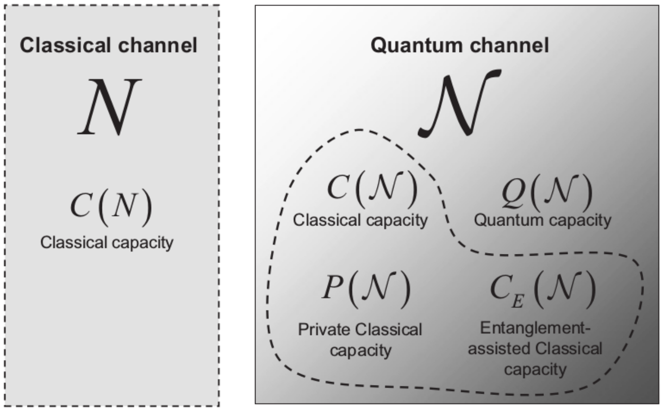

Fig. 1. Capacities of Classical and Quantum Channels

Different types of Classical Capacities of Quantum Channels

Considering quantum channels in order to describe the amount of transmittable information, we have to make a distinction between different capacity interpretations, namely the (unentangled) classical capacity C(N), private capacity P(N), and entanglement assisted classical capacity CE(N).
The **unentangled (HSW) classical capacity** is defined for product state inputs; however, it is possible to extend it for entangled input states.

The **private capacity P(N)** has deep relevance in secret quantum communications and quantum cryptography. It describes the rate at which Alice is able to send classical information through the channel in a secure manner. Security here means that an eavesdropper will not be able to access the encoded information without revealing her/himself.

The **entanglement-assisted classical capacity CE(N)** measures the classical information that can be transmitted through the channel, if Alice and Bob have already shared entanglement before the transmission.

Encoding-Decoding techniques

Encoder	Decoder	Description
Classical	Classical	Unentangled pure states with single measurement.
Classical	Quantum	Unentangled pure states with joint measurement.
Quantum	Classical	Entangled states with single measurement.
Quantum	Quantum	Entangled states with joint measurement.

Fig. 2. Encoder and Decoder Settings

There are two types of encoders that can be used- classical and quantum. Classical encoders gives product(unentangled) state as output and quantum encoder gives entangled state as output.
Possible encoder and decoder setting for the transmission of classical information through a quantum channel is described in the table.
HSW theorem quantifies explicitly the amount of classical information that can be transmitted through noisy quantum channel using product state as input where output is measured using joint measurement setting. Single measurement destroys possible benefits arising from entangled inputs.

Zero error capacity of Quantum Channel

It describes the amount of information that can be transmitted through zero probability of error. In case if classical information it can be reached if and only if input states are pure states and in case of quantum states it requires perfect distinguish ability and to achieve it they have to be pairwise orthogonal. The zero probability of error means that for the input code (X_i) the decoder has to identify the classical output code word $X_i^{'}$ with classical input code word X_i perfectly for each possible i, otherwise the quantum channel has no zero-error capacity; that is, for the zero-error quantum communication system.
In general, the following relation holds between these classical zero-error quantities:
C_0 (N) $<$ C_0 E(N) $<=$ C (N)

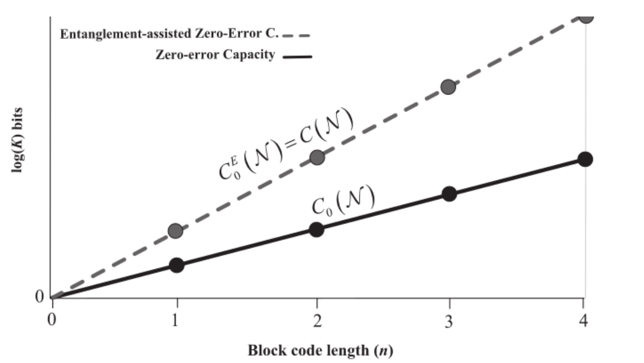

Fig. 3. Zero error Capacities without entanglement and with entanglement assisted Capacities

We note that the complete theoretical background on the possible impacts of entanglement on the zero-error capacities is not completely clarified, and research activities are currently in progress; on the other hand, one thing is certain: without entanglement, the zero-error capacities (classical or quantum) of quantum channels cannot be super activated.

- **Quantum Capacity of Quantum Channel**

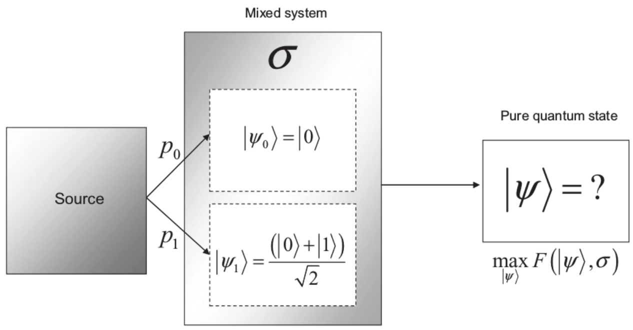

Fig. 4. Calculating fidelity using mixed state

If one would like to maximize the fidelity

$$\max_{|\psi\rangle} F(|\psi\rangle, \sigma),$$

then the following state $|\psi\rangle$ has to be considered:

$$|\psi\rangle = \cos\left(\frac{45^\circ}{2}\right)|0\rangle + \sin\left(\frac{45^\circ}{2}\right)|1\rangle.$$

In this case, the square of inner products for the states of the mixed $|\psi\rangle$ is

$$|\langle\psi|\psi_0\rangle|^2 = |\langle\psi|\psi_1\rangle|^2 = \cos^2\left(\frac{45^\circ}{2}\right) = \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right),$$

and the fidelity is

$$\begin{aligned} F(|\psi\rangle, \sigma) &= \frac{1}{2}|\langle\psi|\psi_0\rangle|^2 + \frac{1}{2}|\langle\psi|\psi_1\rangle|^2 \\ &= \frac{1}{2}\cdot\frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right) + \frac{1}{2}\cdot\frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right) = 0.8535. \end{aligned}$$

The classical capacity of a quantum channel is described by the maximum of quantum mutual information and the Holevo information. The quantum capacity of the quantum channels is described by the maximum of quantum coherent information. Basically, the quantum capacity of a quantum channel is lower than the classical capacity.
The sender transmits the encoded quantum state through the quantum channel, which will be modified typically according to the noise of the channel. Finally, the receiver decodes the quantum state. The reliable transmission of quantum information can be also measured by the fidelity (degree of exactness) of entanglement or **fidelity** of quantum information.

Quantum Coherent Information

In case of the classical capacity C(N), the correlation between the input and the output is measured by the Holevo information and the quantum mutual information function. In case of the quantum capacity Q(N), we have a completely different correlation measure with completely different behaviors: it is called the quantum coherent information. First, we compute the Holevo information between Alice and Bob. This quantity describes the classical information transmittable from Alice to Bob and gives an upper bound on the maximal classical information. Now, we have just the classical capacity of the channel. In the next step, we calculate the classical information that is passed to the environment. The difference between the two gives the coherent information.

IV. Comparison between Classical and Quantum Channel

A. Classical Channel

The Shannon-Hartley theorem states that the channel capacity is given by

$$C = Blog_2(1 + S/N) \tag{1}$$

where C is the capacity in bits per second, B is the bandwidth of the channel in Hertz, and S/N is the signal-to-noise ratio.

- As the bandwidth of the channel increases, it is possible to make faster changes in the information signal, thereby increasing the information rate.
- As S/N increases, one can increase the information rate while still preventing errors due to noise.
- For no noise, S/N and an infinite information rate is possible irrespective of bandwidth.

CLASSICAL CHANNEL	QUANTUM CHANNEL
1. There exists only one capacity i.e. classical capacity.	1.Quantum Capacity (unetangeld, private, entangled) and Classical Capacity.
2. Capacity depends on signal to noise ratio.	2. Capacity depends on power.
3. Communication is based on principles of Shannon's theory.	3. Communication is based on principles of quantum mechanics.
4. Can transmit only classical bits.	4. Can transmit classic bits as well as qbits.
5. Can send only single bit in the same time that it takes for M coupled modes to send M bits.	5. M coupled mose can send M bits at the same time it takes single mode to send single bit.

Fig. 5. Comparison between Classical and Quantum Channel

V. CONCLUSION

From the above discussions we can conclude that Entanglement is the property by which one can engineer interactions and thus can enhance the channel's capacity. Classical Channel is capable of transmitting only classical bits whereas Quantum Channel has four varieties of capacities that can be utilized for specific transmissions. We can also conclude that Zero Error capacities cannot be realized without Entanglement. The best encoder decoder setting is to use entangled states and joint measurement.

VI. Individual Contribution

- Aman choudhary-201551019
  Studied Shannon formula, Quantum Capacity of quantum channel and Entanglement.

- Monisha Wamankar-201551073
  Studied Classical Capacity of Quantum Channel and Entanglement.

VII. Future Scope

Future scope of this research work includes working on another ways to enhance capacities of quantum channel and overcome the limitations that are restricting the implementation of quantum communication in practical.

References

[1] The power of Entanglement Quantum channels, Seth Lloyd, 6 Dec, 2001.
[2] Advanced Quantum Communications, IEEE Press Editorial Board 2012.