Contents

Design Optimization

Homework No 5 Monish Dev Sudhakhar

```
%Implement an SQP algorithm with line search to solve this problem, starting from
%Incorporate the QP subproblem.
%Use BFGS approximation for the Hessian of the Lagrangian.
%Use the merit function and Armijo Line Search to find the step size.

%  objective function
obj = @(x) x(1)^2+(x(2)-3)^2; % replace with your objective function
% gradient of the objective function

df = @(x) [2*x(1) , 2*(x(2)-3)];

gradient = @(x) [x(2)^2-2*x(1) ; (x(2)-1)^2+5*x(1)-15];
dg = @(x) [-2 2*x(2) ; 5 2*(x(2)-1)];

% Note that explicit gradient and Hessian information is only optional.
% However, providing these information to the search algorithm will save
% computational cost from finite difference calculations for them.

% % algorithm specification is done here
opt.alg = 'myqp';

% Line Saerch algorithm can be accessed here
opt.linesearch = true;

%  Tolerance
opt.eps = 1e-3;

%  Initial Value
x0 = [1;1];

% Feasibility check
if max(gradient(x0)>0)
    errordlg('Infeasible intial point! You need to start from a feasible one!');
    return
end
```

Run optimization

Run your implementation of SQP algorithm. See mysqp.m

```
solution = mysqp(obj, df, gradient, dg, x0, opt);
x_solution = solution.x(:,end)
g_solution = gradient(solution.x(:,end))
f_solution = obj(solution.x(:,end))
```

```
x_solution =

    1.0604
    1.4563


g_solution =

    0.0001
   -9.4897


f_solution =

    3.5074
```

Report

```
report(solution,obj,gradient);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%



%%%%%%%%%%%%%% Sequential Quadratic Programming Implementation with BFGS %%%%%%%%%%%%%%%
%%%%%%%%%%%%%% By Max Yi Ren and Emrah Bayrak %%%%%%%%%%%%%%%%%%%%%%%%%%%%

function solution = mysqp(f, df, g, dg, x0, opt)
    % Set initial conditions

    x = x0; % Set current solution to the initial guess

    % Initialize a structure to record search process
    solution = struct('x',[]);
    solution.x = [solution.x, x]; % save current solution to solution.x

    % Initialization of the Hessian matrix
    W = eye(numel(x));             % Start with an identity Hessian matrix
    % Initialization of the Lagrange multipliers
    mu_old = zeros(size(g(x)));    % Start with zero Lagrange multiplier estimates
    % Initialization of the weights in merit function
    w = zeros(size(g(x)));         % Start with zero weights

    % Set the termination criterion
    gnorm = norm(df(x) + mu_old'*dg(x)); % norm of Largangian gradient

    while gnorm>opt.eps % if not terminated

        % Implement QP problem and solve
        if strcmp(opt.alg, 'myqp')
            % Solve the QP subproblem to find s and mu (using your own method)
            [s, mu_new] = solveqp(x, W, df, g, dg);
        else
            % Solve the QP subproblem to find s and mu (using MATLAB's solver)
            qpalg = optimset('Algorithm', 'active-set', 'Display', 'off');
            [s,~,~,~,lambda] = quadprog(W,[df(x)]',dg(x),-g(x),[], [], [], [], [],  qpalg);
            mu_new = lambda.ineqlin;
        end
```

```
        % opt.linesearch switches line search on or off.
        % You can first set the variable "a" to different constant values and see how it
        % affects the convergence.
        if opt.linesearch
            [a, w] = lineSearch(f, df, g, dg, x, s, mu_old, w);
        else
            a = 0.1;
        end

        % Update the current solution using the step
        dx = a*s;                % Step for x
        x = x + dx;              % Update x using the step

        % Update Hessian using BFGS. Use equations (7.36), (7.73) and (7.74)
        % Compute y_k
        y_k = [df(x) + mu_new'*dg(x) - df(x-dx) - mu_new'*dg(x-dx)]';
        % Compute theta
        if dx'*y_k >= 0.2*dx'*W*dx
            theta = 1;
        else
            theta = (0.8*dx'*W*dx)/(dx'*W*dx-dx'*y_k);
        end
        % Compute  dg_k
        dg_k = theta*y_k + (1-theta)*W*dx;
        % Compute new Hessian
        W = W + (dg_k*dg_k')/(dg_k'*dx) - ((W*dx)*(W*dx)')/(dx'*W*dx);

        % Update termination criterion:
        gnorm = norm(df(x) + mu_new'*dg(x)); % norm of Largangian gradient
        mu_old = mu_new;

        % save current solution to solution.x
        solution.x = [solution.x, x];
    end
end


% Armijo line search
function [a, w] = lineSearch(f, df, g, dg, x, s, mu_old, w_old)
% Initializiation of Scale factor and Size
    t = 0.1;
    b = 0.8;
    a = 1;

    D = s;


    w = max(abs(mu_old), 0.5*(w_old+abs(mu_old)));

    count = 0;
    while count<100
        % Calculate phi(alpha)
        phi_a = f(x + a*D) + w'*abs(min(0, -g(x+a*D)));

        % Caluclate psi(alpha) using phi(alpha)
        phi0 = f(x) + w'*abs(min(0, -g(x)));
        dphi0 = df(x)*D + w'*((dg(x)*D).*(g(x)>0));
        psi_a = phi0 +  t*a*dphi0;
        %
        if phi_a<psi_a;
```

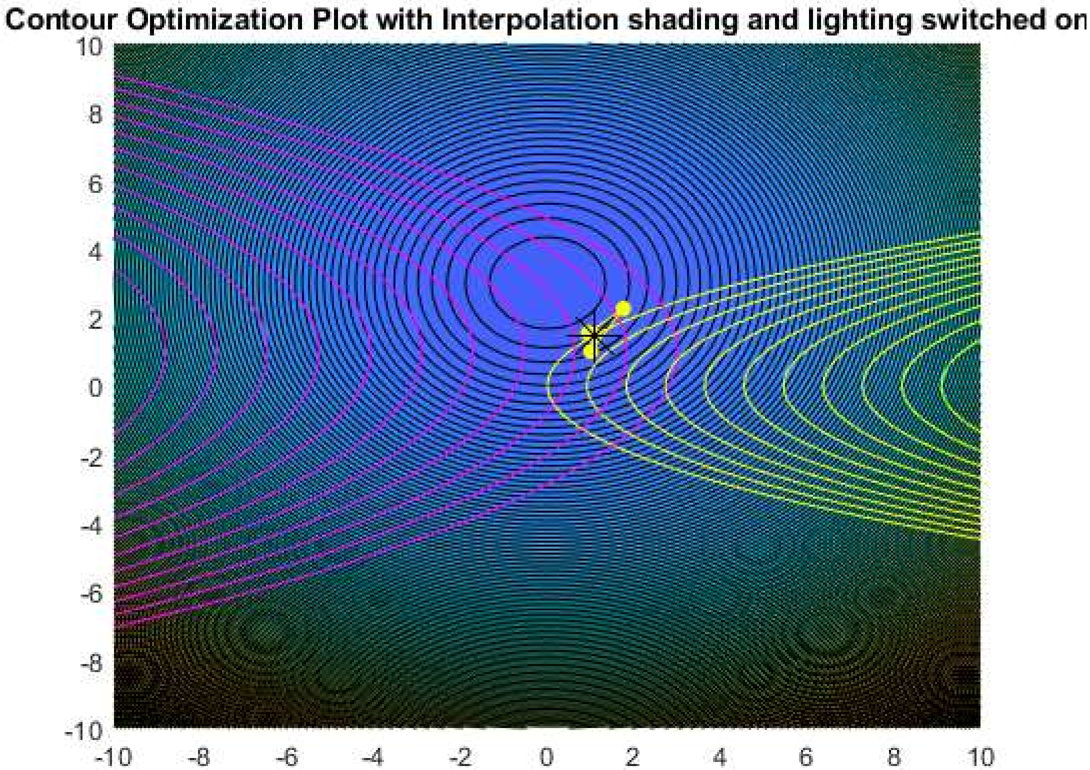
```
            break;
        else

            a = a*b;
            count = count + 1;
        end
    end
end



% The following code solves the QP subproblem using active set strategy


function [s, mu0] = solveqp(x, W, df, g, dg)
    % Implement an Active-Set strategy to solve the QP problem given by
    % min     (1/2)*s'*W*s + c'*s
    % s.t.    A*s-b <= 0



    % Strategy should be as follows:
    % 1-) Start with empty working-set
    % 2-) Solve the problem using the working-set
    % 3-) Check the constraints and Lagrange multipliers
    % 4-) If all constraints are staisfied and Lagrange multipliers are positive, terminate!
    % 5-) If some Lagrange multipliers are negative or zero, find the most negative one
    %     and remove it from the active set
    % 6-) If some constraints are violated, add the most violated one to the working set
    % 7-) Go to step 2


    % Compute c in the QP problem formulation
    c = [df(x)]';

    % Compute A in the QP problem formulation
    A0 = dg(x);

    % Compute b in the QP problem formulation
    b0 = -g(x);

    % Initialize variables for active-set strategy
    stop = 0;           % Start with stop = 0
    % Start with empty working-set
    A = [];         % A for empty working-set
    b = [];         % b for empty working-set
    % Indices of the constraints in the working-set
    active = [];    % Indices for empty-working set

    while ~stop  % Continue until stop = 1
        % Initialize all mu as zero and update the mu in the working set
        mu0 = zeros(size(g(x)));

        % Extact A corresponding to the working-set
        A = A0(active,:);
        % Extract b corresponding to the working-set
        b = b0(active);

        % Solve the QP problem given A and b
        [s, mu] = solve_activeset(x, W, c, A, b);
        % Round mu to prevent numerical errors (Keep this)
```

```
        mu = round(mu*1e12)/1e12;

        % Update mu values for the working-set using the solved mu values
        mu0(active) = mu;

        % Calculate the constraint values using the solved s values
        gcheck = A0*s-b0;

        % Round constraint values to prevent numerical errors (Keep this)
        gcheck = round(gcheck*1e12)/1e12;

        % Variable to check if all mu values make sense.
        mucheck = 0;        % Initially set to 0

        % Indices of the constraints to be added to the working set
        Iadd = [];              % Initialize as empty vector
        % Indices of the constraints to be added to the working set
        Iremove = [];           % Initialize as empty vector

        % Check mu values and set mucheck to 1 when they make sense
        if (numel(mu) == 0)
            % When there no mu values in the set
            mucheck = 1;         % OK
        elseif min(mu) > 0
            % When all mu values in the set positive
            mucheck = 1;         % OK
        else
            % When some of the mu are negative
            % Find the most negaitve mu and remove it from acitve set
            [~,Iremove] = min(mu);  % Use Iremove to remove the constraint
        end

        % Check if constraints are satisfied
        if max(gcheck) <= 0
            % If all constraints are satisfied
            if mucheck == 1
                % If all mu values are OK, terminate by setting stop = 1
                stop = 1;
            end
        else
            % If some constraints are violated
            % Find the most violated one and add it to the working set
            [~,Iadd] = max(gcheck); % Use Iadd to add the constraint
        end
        % Remove the index Iremove from the working-set
        active = setdiff(active, active(Iremove));
        % Add the index Iadd to the working-set
        active = [active, Iadd];

        % Make sure there are no duplications in the working-set (Keep this)
        active = unique(active);
    end
end

function [s, mu] = solve_activeset(x, W, c, A, b)
    % Given an active set, solve QP

    % Create the linear set of equations given in equation (7.79)
    M = [W, A'; A, zeros(size(A,1))];
    U = [-c; b];
    sol = M\U;          % Solve for s and mu
```

```
    s = sol(1:numel(x));                % Extract s from the solution
    mu = sol(numel(x)+1:numel(sol));    % Extract mu from the solution

end

function report(solution,f,g)
    figure; % Open an empty figure window
    hold on; % Hold on to the current figure

    % Draw a 2D contour plot for the objective functio
    drawContour(f,g);

    % Plot the search path
    x = solution.x;
    iter = size(x,2);
    plot(x(1,1),x(2,1),'.y','markerSize',20);

    for i = 2:iter

        line([x(1,i-1),x(1,i)],[x(2,i-1),x(2,i)],'Color','y');
        grid on
        plot(x(1,i),x(2,i),'.y','markerSize',20);

    end

    plot(x(1,i),x(2,i),'*k','markerSize',20);


    % Plot the convergence
    F = zeros(iter,1);
    for i = 1:iter
        F(i) = feval(f,x(:,i));
    end
    figure;
    axis([0 8 0 8])
    plot(1:iter, log(F-F(end)+eps),'g','lineWidth',1);
    grid on
    title('Convergence Plot')
end
function drawContour(f, g)
    x = -10:0.05:10;
    y = -10:0.05:10;
    Zf = zeros(length(y),length(x));
    Zg1 = Zf; Zg2 = Zf;
    for i = 1:length(x)
        for j = 1:length(y)
            Zf(j,i) = feval(f,[x(i);y(j)]);
            gall = feval(g,[x(i);y(j)]);
            Zg1(j,i) = gall(1);
            Zg2(j,i) = gall(2);
        end
    end

    % Contour Plot
    contourf(x, y, Zf,150);
    contour(x,y,Zg1,[0;0],'Color', [1, 1, 0])
    contour(x,y,Zg2,[0;0],'Color', [1, 0, 1])
    Zg1(Zg1>0) = NaN;
    Zg2(Zg2>0) = NaN;
    contour(x,y,Zg1, 10,'Color', [1, 1, 0])
```

```
    contour(x,y,Zg2, 10,'Color', [1, 0, 1])
    shading faceted;
    light('Position',[-1 0 0],'Style','local')
    title('Contour Optimization Plot with Interpolation shading and lighting switched on')
end
```

```
Warning: Imaginary parts of complex X and/or Y arguments ignored.
```



Contour Optimization Plot with Interpolation shading and lighting switched on
10
8
6
4
2
0
-2
-4
-6
-8
-10
-10
-8
-6
-4
-2
0
2
4
6
8
10

Convergence Plot
5
0
-5
-10
-15
-20
-25
-30
-35
-40
1
1.5
2
2.5
3
3.5
4
4.5
5