Development of Optimal Cruise Control

EEE 587: Optimal Control

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Contents

References

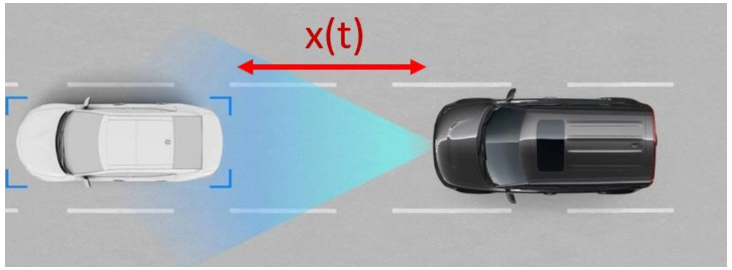
- [1] Zhuwei Wang, Yu Gao and Chao Fang ‘Optimal Control Design for Connected Cruise Control with Edge Computing, Caching, and Control’
  - The experimental values for design was analyzed and performed recursion relations

- [2] Donald E. kirk reference book ‘Optimal Control Theory’, Dover publications, New York, 1970.
  - Solving recursion equation

Abstract

- An Optimal control design is proposed for Connected cruise control(CCC) of autonomous vehicle.
- The optimal control strategy is iteratively solved using backward recursion in order to study headway and velocity.
- Work flow:

  1. Dynamics of the platoon
  2. Linear quadratic optimization problem
  3. Optimal control strategy
  4. Numerical simulation

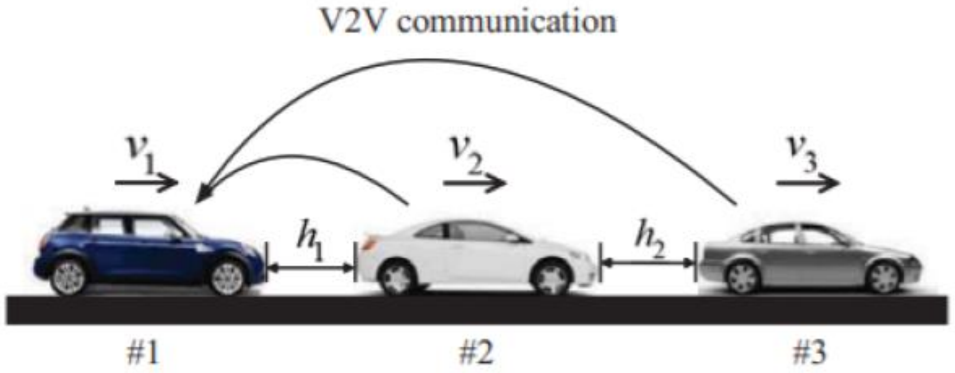


Motivation

- To further increase the safety and convenience features with increasing levels of automation.
- To assist driver through safe control, adverse weather and road conditions including traffic.
- The long-term trend is towards partial or even fully autonomous operation of single or group of vehicle.

Cruise Control

- Cruise control – controls the speed of motor vehicle.
- Fuel efficiency and driver comfort in steady traffic conditions.
- ACC - is an active safety system that automatically controls the acceleration and braking of a vehicle
- CCC – Implementing V2V for better control



$$\dot{x}(t) = dx_1/dt = v_1$$
$$h_1 = x_2 - x_1$$

Methodology – Dynamical Model

- The Dynamics state equation for vehicle platoon model

$\dot{x}$ (t) = Ax(t) + Bu(t − τ)

Where coefficient matrices are

$$A = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & \beta f^* & -\alpha - \beta \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

- Then, during the kth time period [kT, (k + 1)T], the discrete-time system can be derived as

$$x_{k+1} = A_k x_k + B_{k1} u_k + Bk_2 u_{k-1}$$

$$A_k = e^{AT}, \; x_k = x(kT), \; u_k = u(kT)$$

T – Settling time

T – Time delay

t = time period

f^* – range policy

α, β – constant parameter

$\alpha = \beta = 0.6$

ε – positive constant

(ε → 0)

x – displacement

$x(\dot{t})$– velocity

Dynamical Model Contd…

- The Dynamics state equation for vehicle platoon model

$\dot{x}$ (t) = Ax(t) + Bu(t − τ)

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α, β – constant parameter
$\alpha = \beta = 0.6$
ε – positive constant
(ε → 0)
x – displacement
$\dot{x}$ → v- velocity

Updated State Equation

To make the system controllable ,the Discrete time equation is changed to ,

$z_{k+1} = E_k z_k + P_k u_k$

With z_k as the new state variable

Where $z_k = [x_k^T \; u_{k-1}]^T$, $E_k = \begin{bmatrix} Ak & Bk2 \\ 0 & 0 \end{bmatrix}$, $P_k = \begin{bmatrix} Bk1 \\ 1 \end{bmatrix}$

$Ak = e^{AT}$, $x_k = x(kT)$, $u_k = u(kT)$,

$\mathrm{Bk1} = \int_0^{\mathrm{T}-\tau} e^{\mathrm{At}} \mathrm{dt}\, \mathrm{B}$, $\mathrm{Bk2} = \int_{\mathrm{T}-\tau}^{\mathrm{T}} e^{\mathrm{At}} \mathrm{dt}\, \mathrm{B}$

- $Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \varepsilon & 0 \\ 0 & 0 & 0 & \varepsilon \end{bmatrix}$; $R = 1$

Functional Equation

- **Performance Measure:**

The problem is to find optimal policy & hence cost function is given as the typical cost function in a quadratic form ,

ie) defined as a sum of the deviations of the velocity and headway & the magnitude of the control input.

$$J_N = x_N^{\mathrm{T}} Q x_N + \sum_{k=0}^{N-1} (x_k^{\mathrm{T}} Q x_k + u_k^{\mathrm{T}} R u_k),$$

Where,

N - Finite Time Origin

$\bar{Q}$ - new coefficient matrix

- $\bar{Q} = \begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix}$

Optimal Control Law:

The optimal control strategy for CCC vehicle is given as;

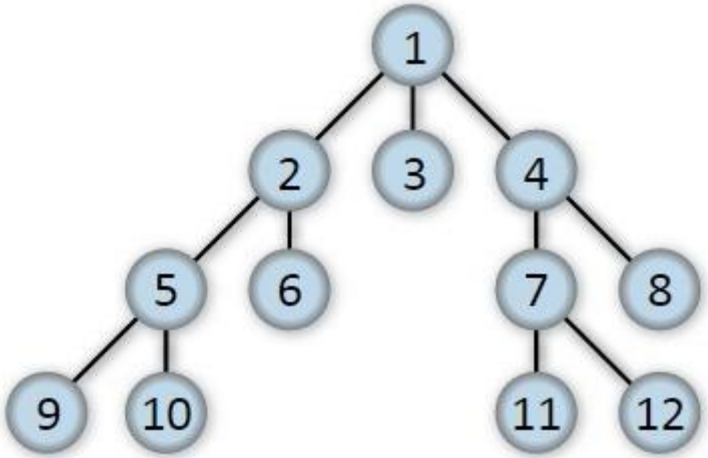
$$u_k = -L_k z_k$$

Where,

$$L_k = [P^T_k\ S_{k+1} P_k + R]^{-1}\ P^T_k\ S_{k+1} E_k$$

$$S_k = E^T_k\ S_{k+1} E_k + \bar{Q} - L^T_k\ P^T_k\ S_{k+1} E_k$$

$$s_N = \bar{Q}$$



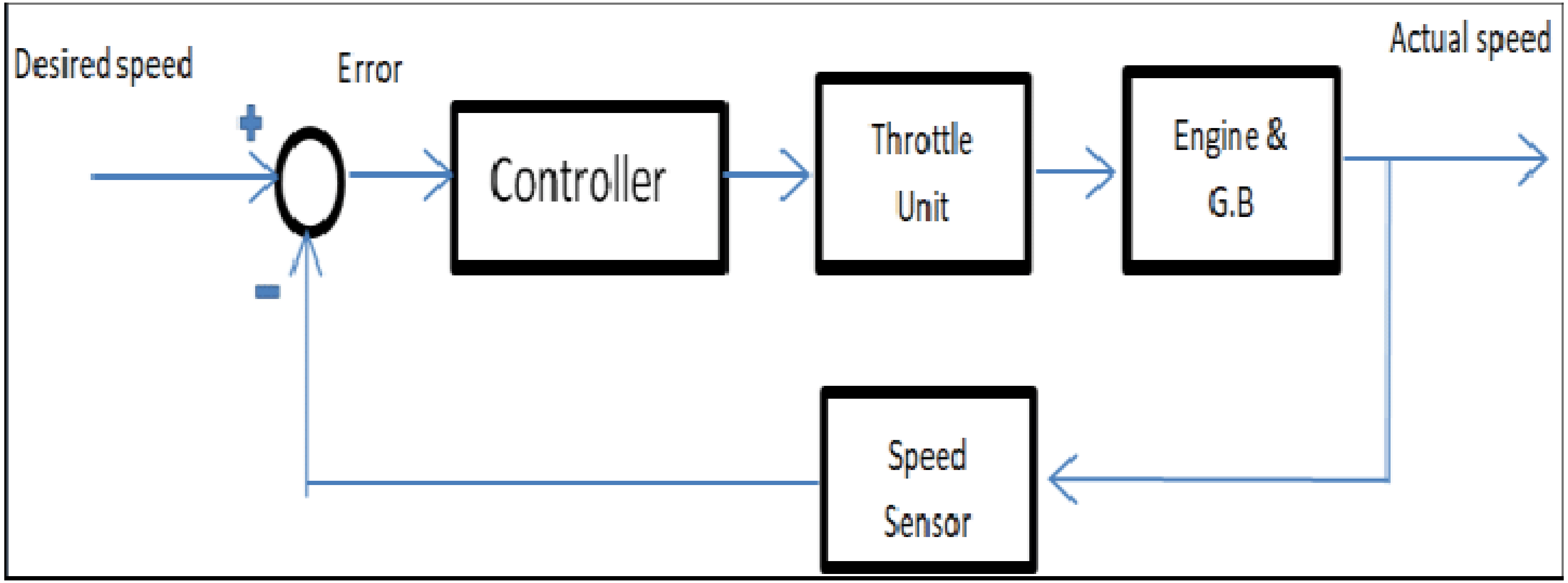
The residual cost function is given as;

$$V_j = z_N^T \overline{Q} z_N + \sum_{k=j}^{N-1} (z_k^T \overline{Q} z_k + u_k^T R u_k).$$

The headway of CCC vehicle is given as;

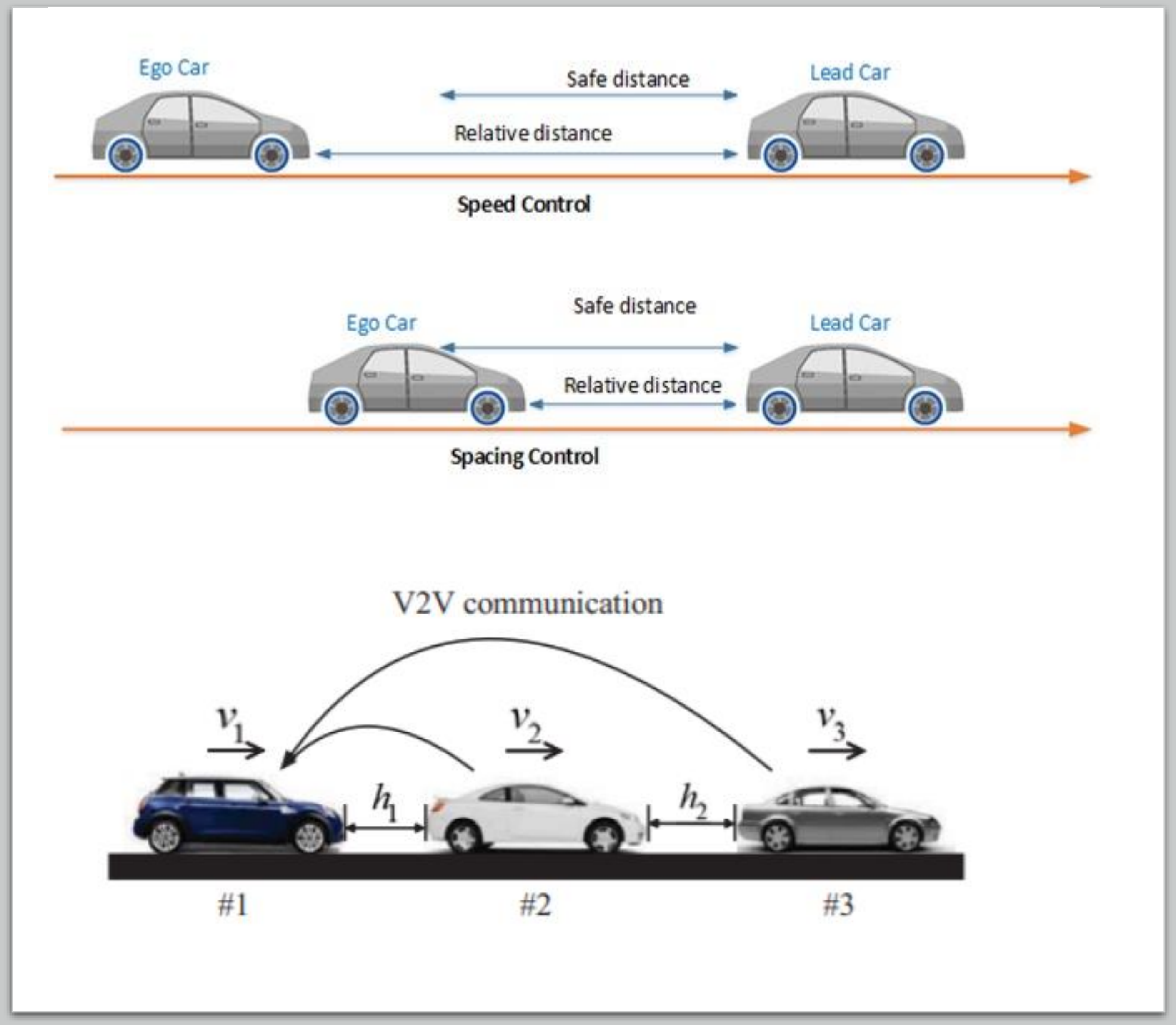
$$H = E^T{}_{N-1} * S + E_{N-1} + Q$$

Block diagram for Connected cruise control



Working Principle

- Works by detecting the distance and speed of the vehicles ahead by using either LIDAR and RADAR system.
- Brake and throttle controls are done to keep the vehicle in safe position.



Algorithm

//RELATION BETWEEN POSITION & VELOCITY(MATLAB)

OPTIMAL_CRUISE_CONTROL.m

1. Setup - Initial conditions [A,B,Ak,Bk1,Bk2] ← [ ]
2. Weighting and Dynamic Steady State Constants
3. [E,P,U,O] ← [ ]
4. Discretize E,P
5. Check for Controllability control = ctrb(E,P)
6. Q and S initial Values [Q,S] ← [ ].
7. Iterative Process
8. for K ← N-1:-1:1
9. do
10. Riccati Recurrence
11. u(K) ← -L(K,:)*X(:,K);
12. L(:,K+1) ← E*X(:,K) + P*L(K,:)*X(:,K);
13. End - Iterative Process
14. Plot Result

Headway Vs Time, Velocity Vs Time

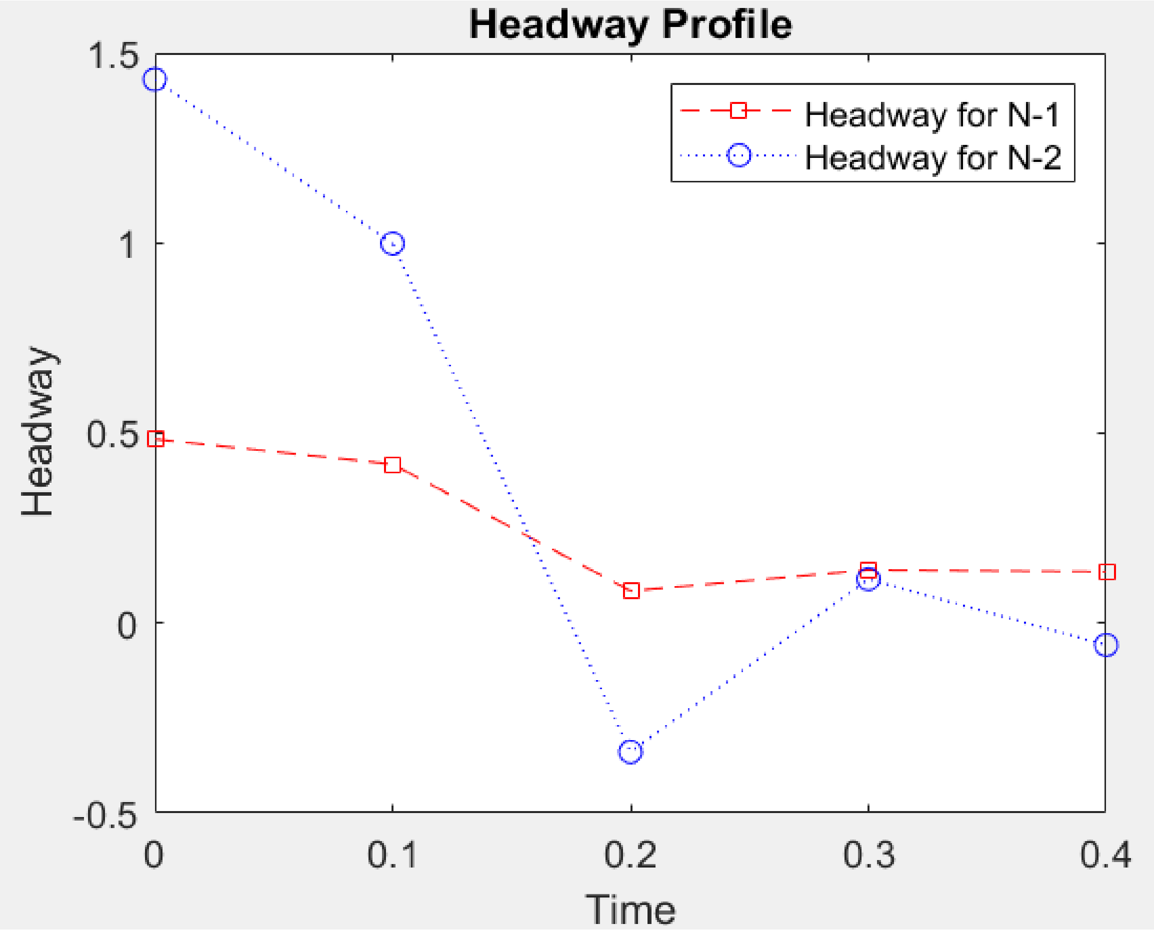


Fig. 1. Headway of the CCC vehicle with various sampling periods and time delays using the proposed algorithm.

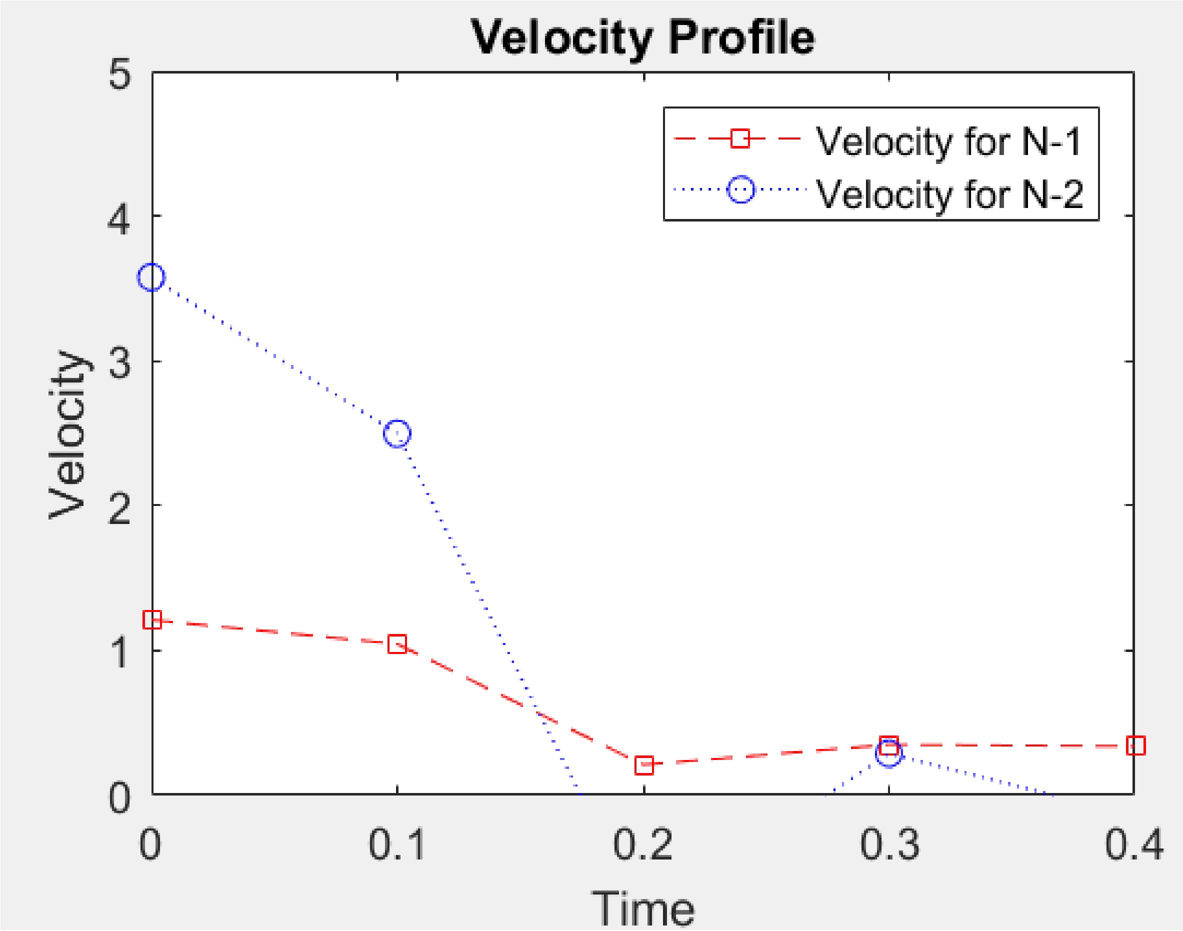


Fig. 1. Headway of the CCC vehicle with various sampling periods and time delays using the proposed algorithm.

Optimal control signal and Position Vs time

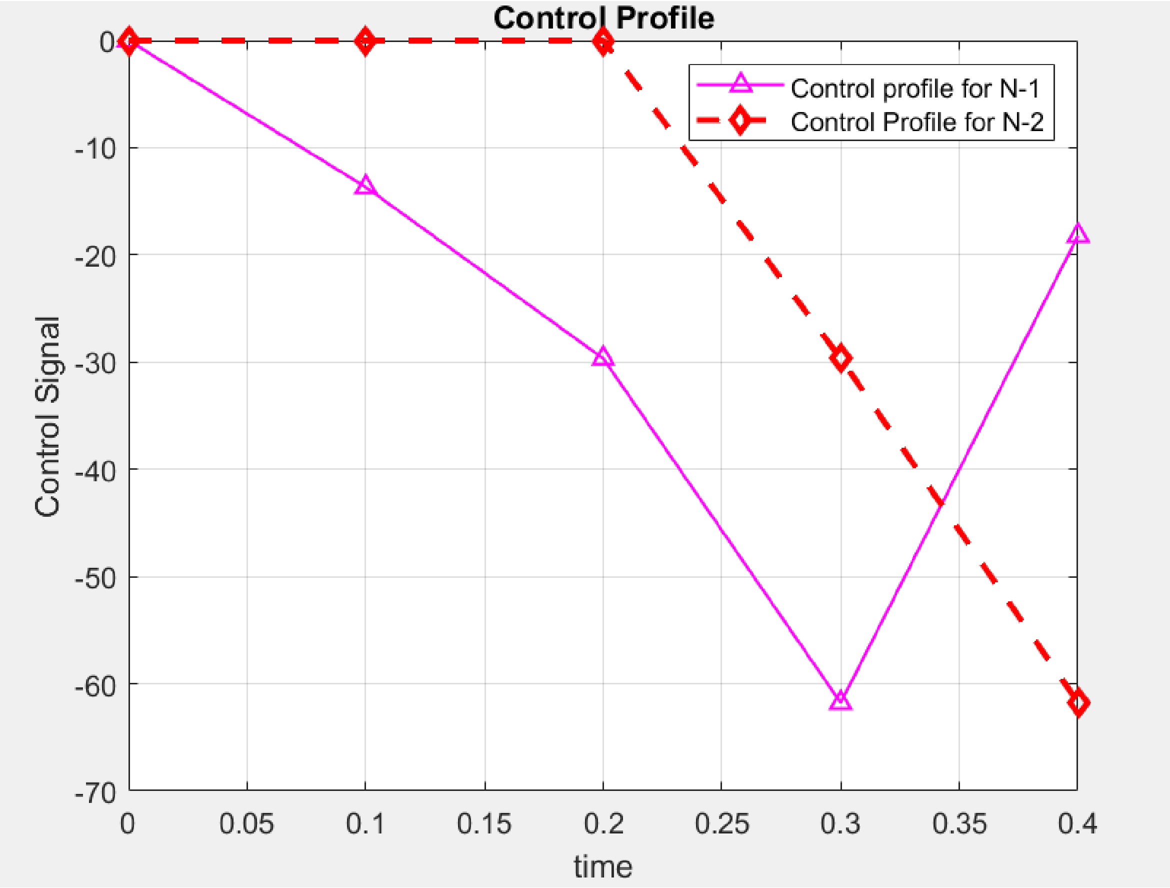


Fig. 3. Control Signal of the CCC vehicle with various sampling periods and time delays using the proposed algorithm

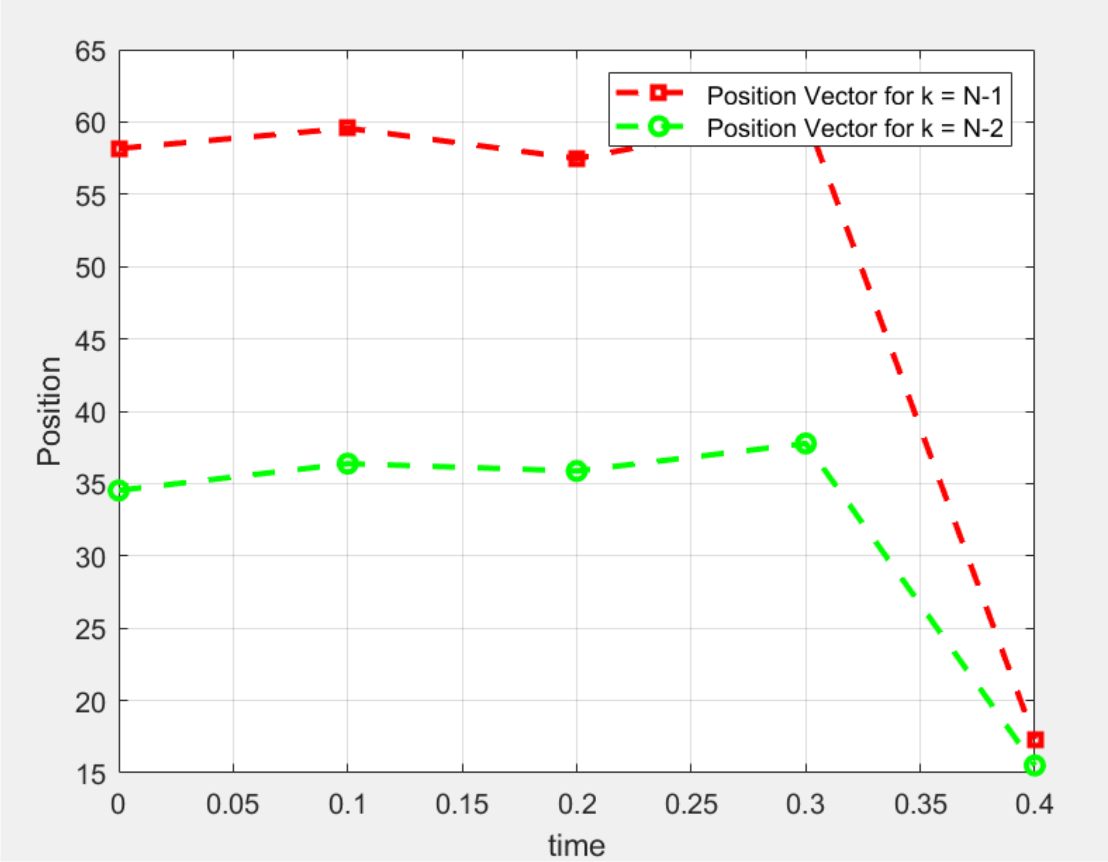


Fig. 4. Position of the CCC vehicle with various sampling periods and time delays using the proposed algorithm

Summary and conclusion

- The CCC problem considering V2V communication delays are analyzed.
- A linear quadratic optimal control problem is formulated in order to minimize the deviations of the CCC vehicle's headway and velocity.
- The optimal control strategy can be iteratively solved using the backwards recursion.
- Numerical simulations indicate that the proposed control algorithm can provide better performances compared to the existing algorithms when the iterations are increased.
- It is evident that profiles of Position, headway , velocity and Control Signal obey the optimal Control law from the plot.

THANK YOU

Queries are welcome