Proceedings of Computational Heat and Fluid Flow
November 10, 2017, IIT, Madras

DIFFUSION ON TWO-DIMENSIONAL UNSTRUCTURED ORTHOGONAL AND NON-ORTHOGONAL MESHES IN A SQUARE DOMAIN

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ABSTRACT

Unstructured meshes are quite common for representing complex geometries in realistic fluid flow problems ,but formulation of these meshes for Finite Volume method is not as simple as structured meshes.In this project a FORTRAN code is developed for the solution of diffusion equation in square domain for both orthogonal and non-orthogonal unstructured meshes. A numerical example is solved to validate the code.

NOMENCLATURE

Γ Diffusivity
Φ Diffusion variable
S_Φ Source term
$\mathbf{A}_f$ Vector area of face
Φ_ξ Differentiation of Φ with respect to ξ
k Thermal conductivity
T Temperature

INTRODUCTION

When we talk about more realistic domain for fluid flows, as the geometric irregularity increases, it is impossible to approximate the domain with the use of structured meshes. There is necessity of the mesh which can handle geometric complexity more naturally, which is fulfilled by unstructured grid [1]. Unstructured mesh can be considered as the limiting case of multi block grid where each cell works as one block. Unstructured mesh does not have any implicit relationship between coordinate axis. It can be considered as one of the most important attributes of this mesh because we can easily mesh any complex geometries. As we can include more than one type of elements in mesh, the cells do not have fixed number of neighbours. Their element connectivity can not be easily expressed as two dimensional array like structured mesh. This mesh is highly inefficient in terms of storage capacity as it requires explicit storage of cell neighbourhood relationship.

GOVERNING EQUATIONS

The discretisation in unstructured meshes can be developed from the basic control volume technique [2, 3] . Steady state diffusion equation can be written as,

$$\nabla \cdot (\Gamma \nabla \Phi) + S_\Phi = 0 \tag{1}$$

Taking integration over control volume,

$$\int_{cv} \nabla \cdot (\Gamma \nabla \Phi)\, dv + \int_{cv} S_\Phi\, dv = 0 \tag{2}$$

Using Gauss Divergence theorem,

$$\int_{cs} (\Gamma \nabla \Phi) \cdot d\mathbf{A} + \int_{cv} S_\Phi\, dv = 0 \tag{3}$$

Taking linear profile assumption for source term.

$$\sum_f (\Gamma \nabla \Phi)_f \cdot \mathbf{A}_f + (S_u + S_p \Phi_p)\Delta v = 0 \tag{4}$$

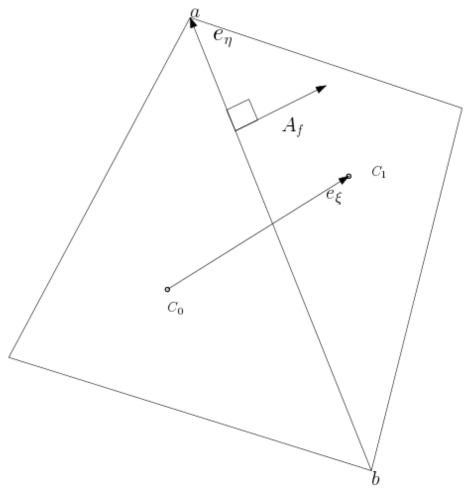

FIGURE 1. Cell centred control volume arrangement

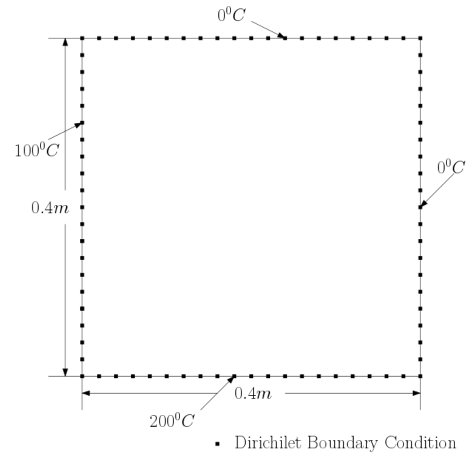

FIGURE 2. Problem figure

Non-Orthogonal mesh

$$\nabla\Phi = \frac{\partial\Phi}{\partial x} i + \frac{\partial\Phi}{\partial y} j \tag{5}$$

using chain rule we can write,

$$\Phi_\xi = \Phi_x x_\xi + \Phi_y y_\xi, \quad \Phi_\eta = \Phi_x x_\xi + \Phi_y y_\eta \tag{6}$$

Area of the face ab,

$$\mathbf{A}_f = A_x i + A_y j \tag{7}$$

putting equation 5,6 and 7 in equation 4 we get

$$\sum_f (\Gamma\nabla\Phi)_f \cdot \mathbf{A}_f = \Gamma_f \left(\frac{A_x y_\eta - A_y x_\eta}{J}\right)\Phi_\xi + \Gamma_f \left(\frac{A_y x_\xi - A_x y_\xi}{J}\right)\Phi_\eta \tag{8}$$

where $J = x_\xi y_\eta - x_\eta y_\xi$
Applying geometric simplification from figure we can reduce the above equation,

$$\sum_f (\Gamma\nabla\Phi)_f \cdot \mathbf{A}_f = \Gamma_f \left(\frac{A_f \cdot A_f}{A_f \cdot e_\xi}\right)\Phi_\xi - \Gamma_f \left(\frac{A_f \cdot A_f}{A_f \cdot e_\xi}\right)(e_\xi \cdot e_\eta)\Phi_\eta \tag{9}$$

Here,the first term in equation 9 is called as Direct Gradient term and the second term is called as Cross Diffusion term, which is usually being treated as source term by calculating its value using the last iteration values.

By taking approximation $\Phi_\xi = \frac{\Phi_1 - \Phi_0}{\Delta\xi}$ and $\Phi_\eta = \frac{\Phi_a - \Phi_b}{\Delta\eta}$, we can write our discretised equation

$$a_P \Phi_P = \sum_{nb} a_{nb}\Phi_{nb} + b \tag{10}$$

where,

$$a_{nb} = \left(\frac{\Gamma_f}{\Delta\xi}\frac{A_f \cdot A_f}{A_f \cdot e_\xi}\right)_{nb}, \qquad a_P = \sum_{nb} a_{nb} - S_p\Delta v, \qquad b = S_u\Delta v$$

Orthogonal mesh

For orthogonal meshes, e_ξ and e_η are perpendicular, which vanishes the second term and makes quantity inside bracket in the first term equal to $\mathbf{A}_f$.

RESULTS AND DISCUSSION

Numerical example of pure diffusion is solved. Square domain of 0.4 m side length is taken as problem domain. Thermal conductivity is $1000kW/m$. Left side and bottom side have constant temperature of 100^0C and 200^0C respectively, while other two sides are at 0^0C.Figure 2 shows the domain under consideration. Pure diffusion and steady state conditions are assumed. Three different type of meshes are taken to solve the problem (a)Structured Mesh(Quadrilateral elements) (b) Orthogonal unstructured mesh (Trianlge elements) (c) Non-orthogonal unstructured mesh (Mixture of triangle and quadrilateral elements).

Considering our assumptions,the governing equation re-

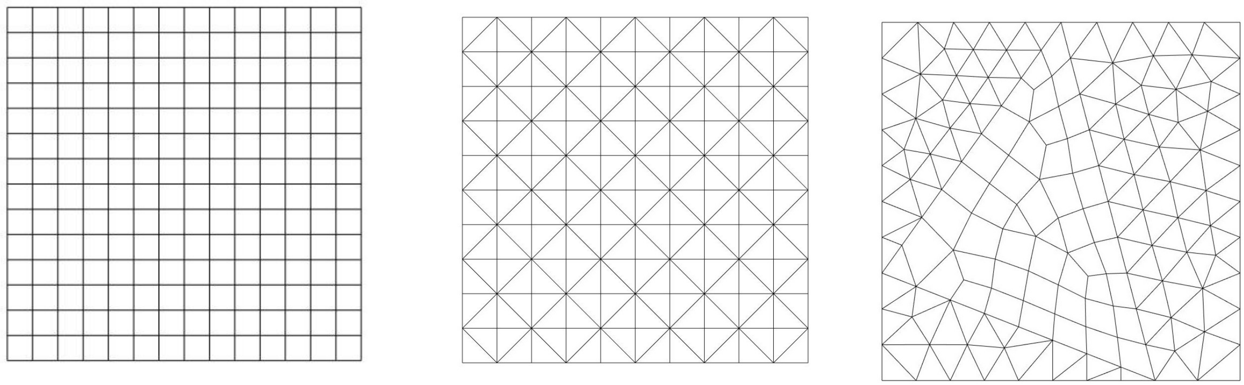

FIGURE 3. Mesh (a)Structured Mesh(Quadrilateral elements) (b) Orthogonal unstructured mesh (Trainlge elements) (c) Non-orthogonal unstructured mesh (Mixture of triangle and quadrilateral elements)

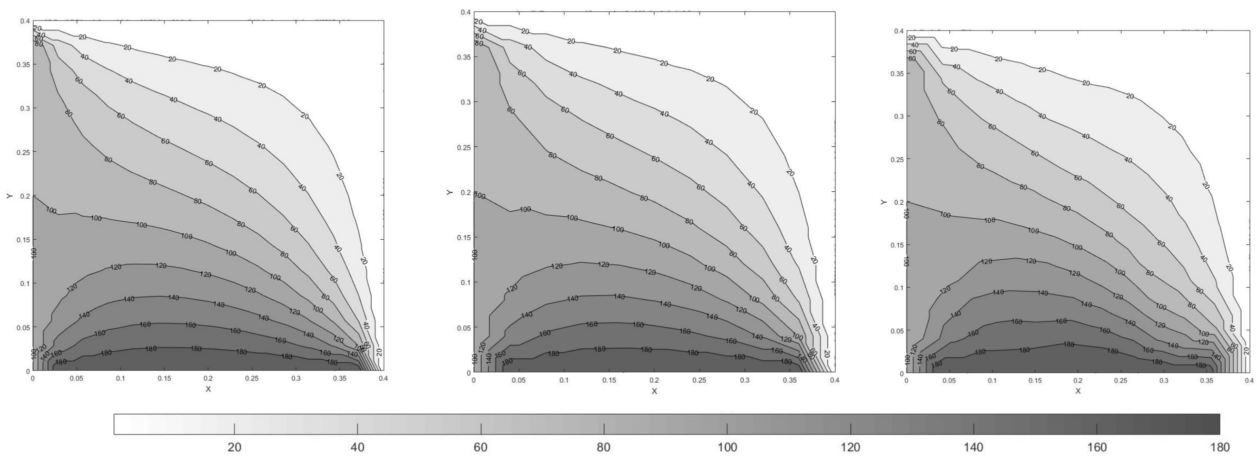

FIGURE 4. Temperature distribution for (a)Structured Mesh (Quadrilateral elements) (b) Orthogonal unstructured mesh (Triangle elements) (c) Non-orthogonal unstructured mesh (Mixture of triangle and quadrilateral elements)

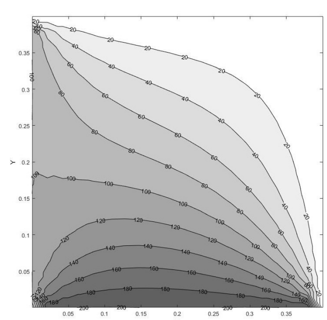

FIGURE 5. Approximate analytical solution

duces,

$$k(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) = 0 \tag{11}$$

which is basically Laplace equation. Analytical solution for Laplace over square domain is available. The analytical solution for our problem will be,

$$T_{analy} = \sum_{n=1}^{\infty} A_n sinh(2.5n\pi(x-0.4))sin(2.5n\pi y)$$

$$+\sum_{n=1}^{\infty} B_n sinh(2.5n\pi(y-0.4))sin(2.5n\pi x) \tag{12}$$

where,

$$A_n = \frac{200(1-cos(n\pi)}{n\pi sinh(-n\pi)}, \quad B_n = \frac{400(1-cos(n\pi)}{n\pi sinh(-n\pi)}$$

Since the solution is summation of infinite exponential sine series, there is the need to approximate the exact solution. We considered first 300 iterations for both the terms(for large value of n, coefficients A_n and B_n approach zero values). Approximate solution is plotted in figure 5. From figure 4 we can say that there is good agreement found between different meshes for temperature distribution. These results also look physically correct by comparing with approximate analytical solution as shown figure 5. Some deviation can be observed at the corners which is due to the application of two dirichlet boundary conditions at single point and interpolation used for plotting contour plots.

CONCLUSION

A FORTRAN code is developed for finite volume method to solve steady state diffusion equation over unstructured mesh. One numerical example over square domain is solved with three different meshes and compared with approximate analytical solution, which validates the code written.

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[3] H.K: Versteeg and W. Malalasekera. *Introduction to Computational Fluid Dynamics*, volume 44. 2005.