

Question: 2.a)

The frequency response function is, $G(e^{j\omega}) = \frac{0.072 e^{j\omega} + 0.062}{e^{2j\omega} - 1.6 e^{j\omega} + 0.63}$

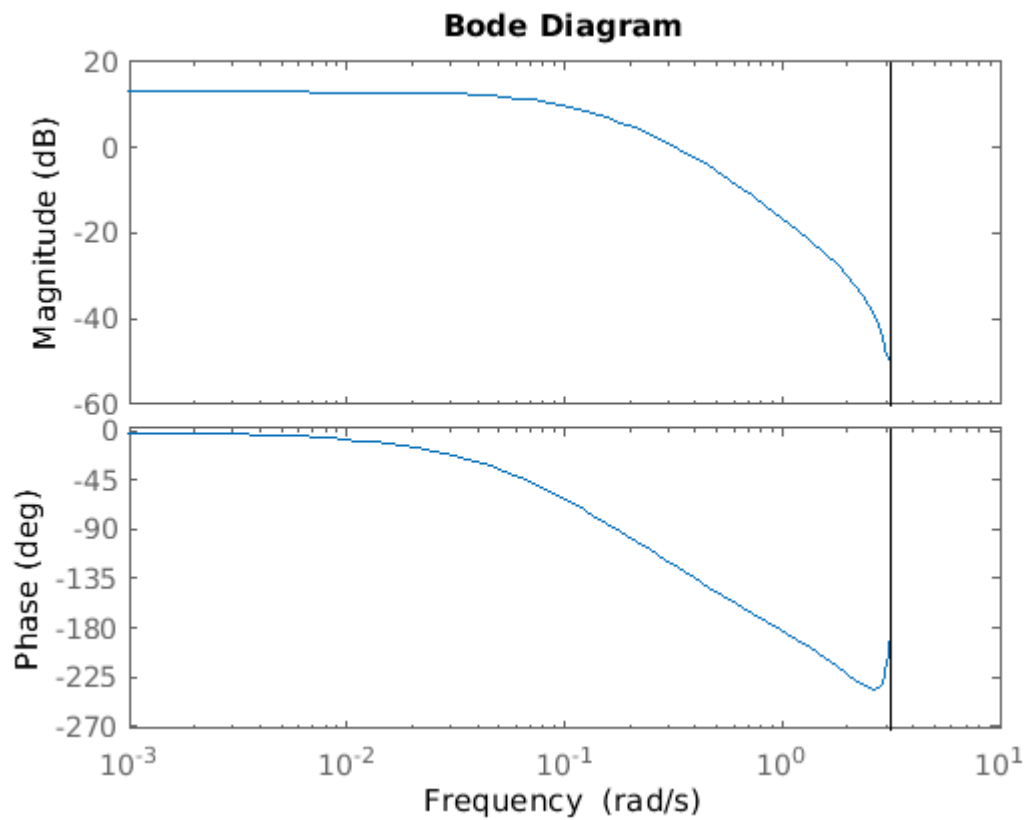
```
Gz = tf([0.072, 0.062], [1, -1.6, 0.63], 1)
```

```
Gz =
```

$$\frac{0.072 z + 0.062}{z^2 - 1.6 z + 0.63}$$

```
Sample time: 1 seconds  
Discrete-time transfer function.
```

```
bode(Gz)
```



Question: 2.b)

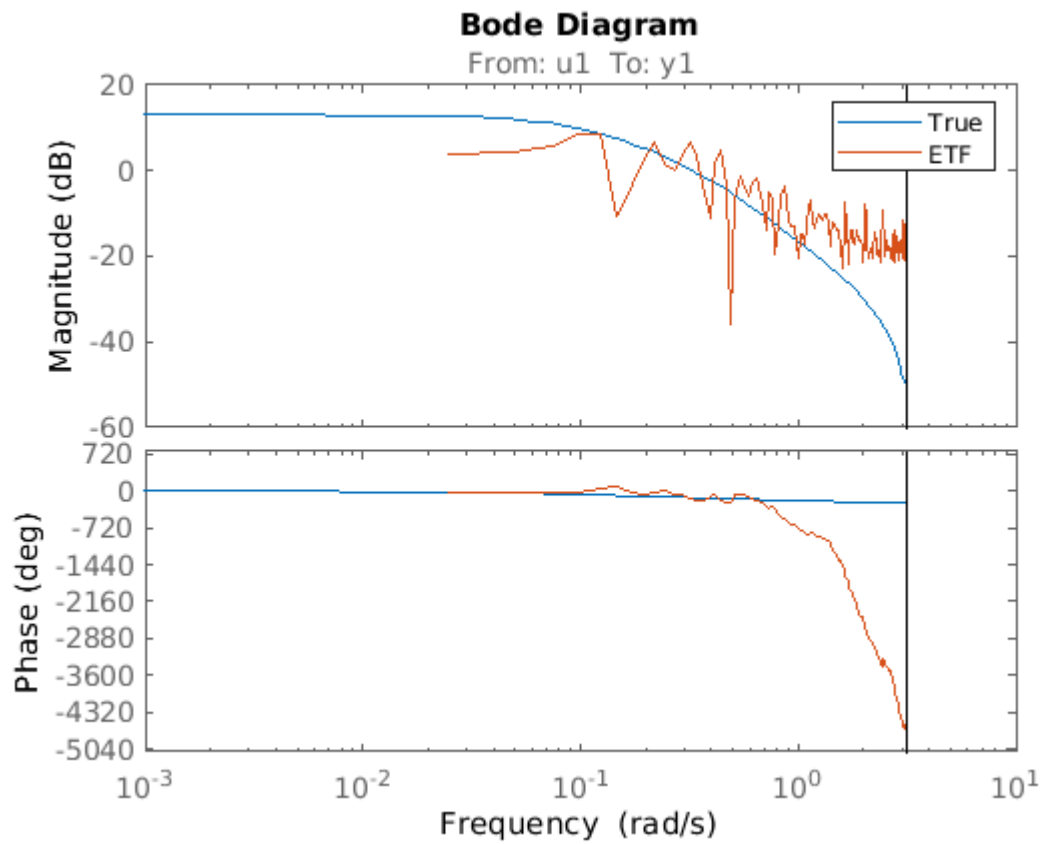
```
model_polynomial = idpoly(1, [0, 0.072, 0.062], 1, 1, [1, -1.6, 0.63]);  
  
uk_63 = idinput(63, 'prbs', [0, 1],[-1, 1]);  
uk_2047 = idinput(2047, 'prbs', [0, 1],[-1, 1]);  
uk_2047_coloured = idinput(2047, 'prbs', [0, 0.4],[-1, 1]);
```

Warning: The PRBS signal delivered is the 2047 first values of a full sequence of length 4094.

```
uk_2047_sine = idinput(2047, 'sine', [0, 0.4],[-1, 1]);  
  
ykstar_63 = sim(model_polynomial, uk_63);  
ykstar_2047 = sim(model_polynomial, uk_2047);  
ykstar_2047_coloured = sim(model_polynomial, uk_2047_coloured);  
ykstar_2047_sine = sim(model_polynomial, uk_2047_sine);  
  
dataset_63 = iddata(ykstar_63, uk_63, 1);  
dataset_2047 = iddata(ykstar_2047, uk_2047, 1);  
dataset_2047_coloured = iddata(ykstar_2047_coloured, uk_2047, 1);  
dataset_2047_sine = iddata(ykstar_2047_sine, uk_2047_sine, 1);  
  
dataset_63 = detrend(dataset_63,0);  
dataset_2047 = detrend(dataset_2047,0);  
dataset_2047_coloured = detrend(dataset_2047_coloured,0);  
dataset_2047_sine = detrend(dataset_2047_sine,0);  
  
G_hat_63 = etfe(dataset_63);  
G_hat_2047 = etfe(dataset_2047);  
G_hat_2047_coloured = etfe(dataset_2047_coloured);  
G_hat_2047_sine = etfe(dataset_2047_sine);
```

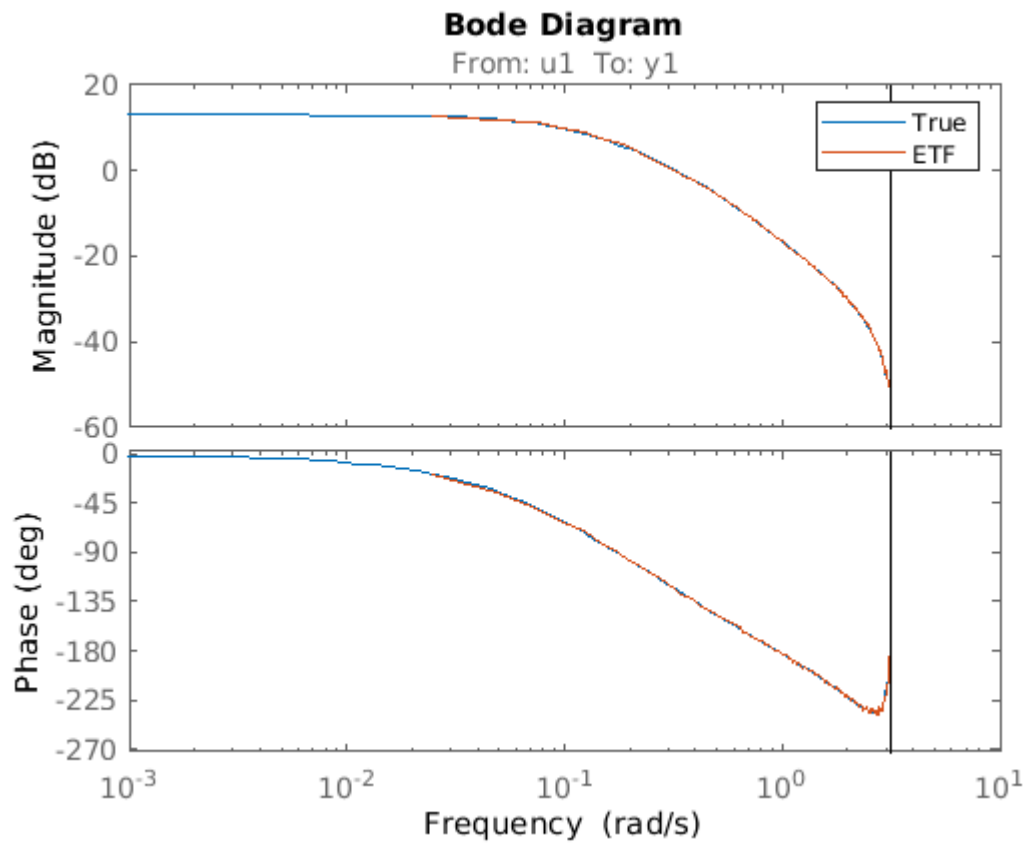
(i) Full-length PRBS of length $N = 63$

```
figure; bode(model_polynomial, G_hat_63);  
axes_handles = findall(gcf, 'type', 'axes');  
legend(axes_handles(3), 'True', 'ETF');
```



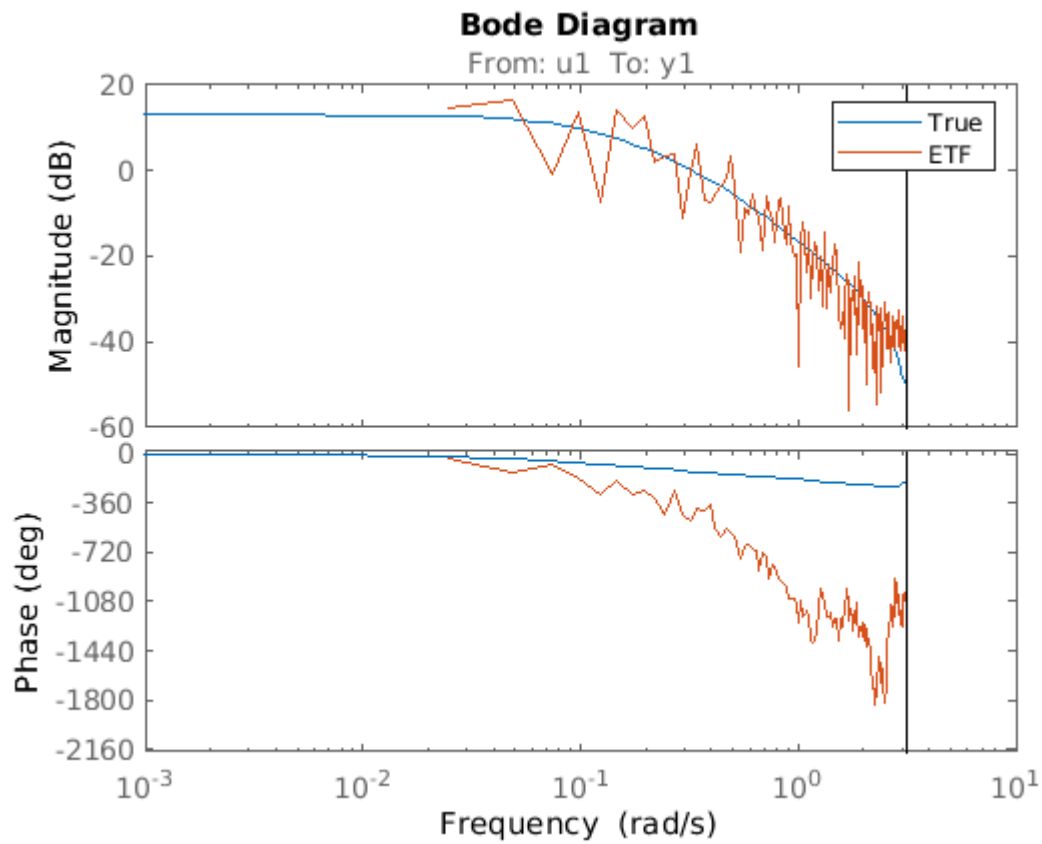
(ii) Full-length PRBS of length $N = 2047$

```
figure; bode(model_polynomial, G_hat_2047);
axes_handles = findall(gcf, 'type', 'axes');
legend(axes_handles(3), 'True', 'ETF');
```



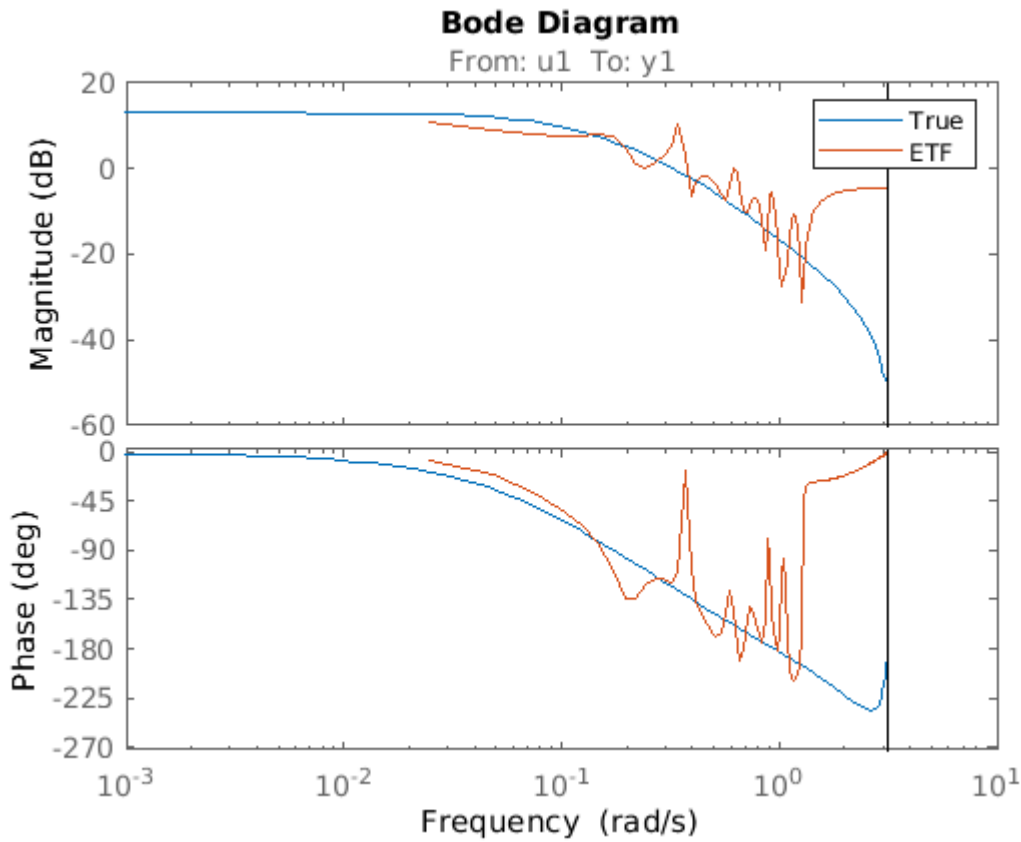
(iii) Coloured PRBS (using band = [0,0.4])

```
figure; bode(model_polynomial, G_hat_2047_coloured);
axes_handles = findall(gcf, 'type', 'axes');
legend(axes_handles(3), 'True', 'ETF');
```



(iv) Mixed sine in the frequency range $\omega \in [0, 0.4]$

```
figure; bode(model_polynomial, G_hat_2047_sine);
axes_handles = findall(gcf, 'type', 'axes');
legend(axes_handles(3), 'True', 'ETF');
```



Question: 2.c)

We know that,

$$|R_N(\omega)| \leq 2C_u \frac{C_G}{\sqrt{N}}$$

- Therefore for a large value of $N = 2047$ samples, R_N is very small. As a result the estimated ETF using a full-length PRBS, $\hat{G}(e^{j\omega_n})$ is nearly equal to the true FRR, $G(e^{j\omega_n})$.
- When a full-length PRBS with $N = 63$ samples is used, we can observe that the estimated ETF is way of from the true FRF.
- For $N = 2047$ samples, coloured PRBS and sine-wave with a band of $[0 \ 0.4]$ did not a good job in estimating the true FRF when compared to a full length PRBS.