Simulation of Spin Stabilization

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I. Nomenclature

A	=	moment of inertial along x in body fixed frame
B	=	moment of inertial along y in body fixed frame
C	=	moment of inertial along z in body fixed frame
$\overrightarrow{H_G}$	=	Angular momentum vector about the center of gravity
I_G	=	Moment of Inertia matrix about the center of gravity
$\hat{k}$	=	z direction unit vector of the body fixed frame
M_{Gx}	=	Moment about the center of gravity about the x axis of the body fixed frame
M_{Gy}	=	Moment about the center of gravity about the y axis of the body fixed frame
M_{Gz}	=	Moment about the center of gravity about the z axis of the body fixed frame
$\overrightarrow{M_{gg}}$	=	Gravity gradient torque
q	=	Quaternion matrix
r_{Gx}	=	x component of the radius of the orbit in the body fixed frame
r_{Gy}	=	y component of the radius of the orbit in the body fixed frame
r_{Gz}	=	z component of the radius of the orbit in the body fixed frame
$\overrightarrow{\omega_B}$	=	Angular velocity of the body
$\overrightarrow{\omega_R}$	=	Angular velocity of the body
θ	=	Nutation angle
$\dot{\psi}$	=	Spin rate
$\dot{\varphi}$	=	Precession rate

II. Introduction

Here, the effects of a small disturbance on a spin stabilized spacecraft were simulated. This was done to study how a spacecraft would react to a disturbance while spin stabilized. This was done for three cases. In the first case, the space craft was spinning about its third axis. In the second case, the spacecraft is spinning about its second axis. In the third case, the spacecraft is also spinning about its second axis, but the effects of gravity gradient are included. In all cases, the spacecraft is spinning about its spin axis at 5 rad/s. The body fixed frame of each spacecraft is initially aligned with the LVLH frame. In cases 1 and 2, the LVLH frame is initially aligned with the ECI frame (the inclination of their orbits is 0). For case 3, the orbit has an inclination of 30º. The moment of inertia follows an A B C structure for each craft. In cases 1 and 2, A=100 kg*m^2, B=200 kg*m^2, and C=400 kg*m^2. For case 3, A=C=400 kg*m^2 and B=200 kg*m^2. As defined, A is along the x axis, B is along the y axis and C is along the z axis. These are for the body fixed frame. The orbits of each case are circular with an altitude of 400 km.

III. Analysis

A. Case 1

As mentioned above, the first case was initially spinning about the C axis at 5 rad/s. To simulate a small disturbance, a small torque was applied over a short time. The Torque was 1 Nm about the A axis and 2 Nm about the B axis. No torque was applied in the spinning direction as it would only make the craft spin faster. In the simulation, this torque was applied for 0.1s. Using this torque, the angular velocity can be found using equation 1.

$$\dot{\overrightarrow{\omega_B}} = I_G^{-1}[\overrightarrow{\omega_B}\, x\, (I_G\overrightarrow{\omega_B}) + \overrightarrow{M_G}] \tag{1}$$

Equation 1 can be simplified into 3 equations specifically for case 1.

$$\overrightarrow{\omega_B} = \begin{bmatrix} M_{Gx} + \frac{B-C}{A}\omega_y\omega_z \\ M_{Gy} + \frac{C-A}{B}\omega_x\omega_z \\ M_{Gz} + \frac{A-B}{C}\omega_x\omega_y \end{bmatrix} \tag{2}$$

Quaternions were used to describe the rotation of the body with respect to the ECI frame. Initially the space craft is aligned with the ECI frame. This means that the initial quaternion values in order are 1,0,0,0. Using angular velocity, the quaternions can be found with equation 3.

$$\dot{q_{BR}} = \frac{1}{2}\begin{bmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{bmatrix} * \overrightarrow{\omega_{B/R}} \tag{3}$$

$$\overrightarrow{\omega_{B/R}} = \overrightarrow{\omega_B} - \overrightarrow{\omega_R} \tag{4}$$

Equation 4 is the definition of $\overrightarrow{\omega_{B/R}}$. Observe that ω_B can be used in equation 3 because the ECI frame is a inertial frame and does not rotate. MATLAB was utilized to solve these two equations simultaneously. The function ode45 was used for this. The results were plotted vs time.

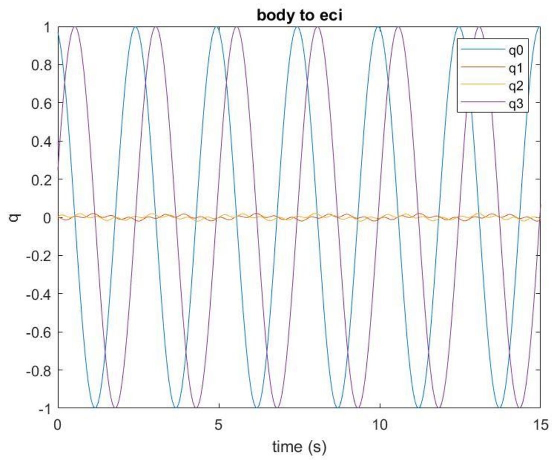


Fig. 1 Case 1 quaternions with respect to time

An ω vs time chart was also made to show the motion in more detail.

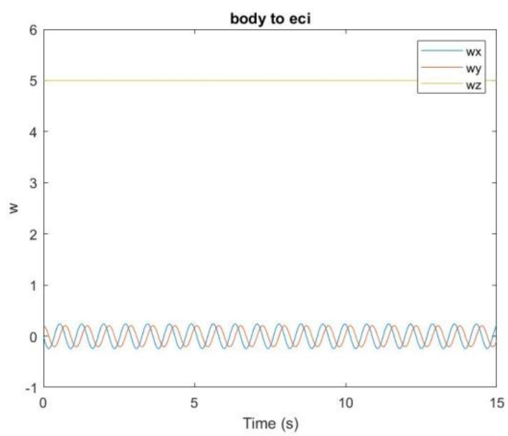


Fig. 2 Case 2 ω vs time

With figure 2 the results can be seen more clearly. The motion is a spin and precession motion, after a disturbance was applied. To find spin and precession rate, angular momentum must first be found.

$$\overrightarrow{H_G} = I_G\overrightarrow{\omega_B} \tag{5}$$

Nutation angle must be found next.

$$\theta = \cos^{-1}\left(\frac{\overrightarrow{H_L} \cdot \hat{k}}{\|\overrightarrow{H_L}\|}\right) \tag{6}$$

The "*" indicates a dot product. For case 1, θ was found to be 0.3690 rad. Now that θ has been found, the constant Ω from the angular velocity is needed for spin and precession rates. The angular velocity solutions are sinusoidal waves, so they are of the following form.

$$\omega = \Omega\cos(\lambda t) \tag{7}$$

$$\lambda = \frac{(A-C)^2}{A^2}\omega_{spin} \tag{8}$$

Equation 7 is the definition of λ. The ω_{spin} is the spin about the C axis. With these equations Ω can be found and then used to find spin rate and precession rate.

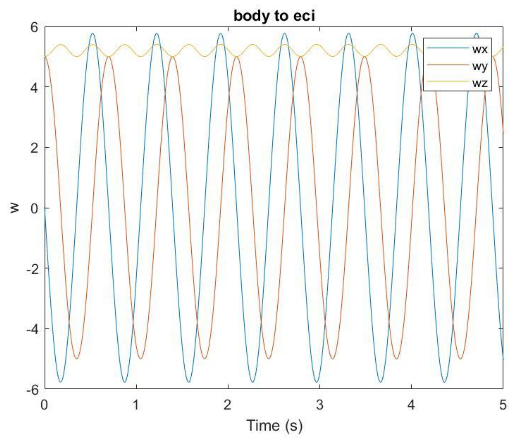
$$\dot{\psi} = \omega_{spin} - \frac{\Omega}{\tan 9} \tag{9}$$

$$\dot{\varphi} = \frac{C}{A-C} * \frac{\dot{\psi}}{\cos(9)} \tag{10}$$

Using these, spin rate was found to be -12.4868 rad/s and precession rate was found to be 16.6514 rad/s.

B. Case 2.

For case the same spacecraft is simulated. Equations 1-3 are still used but their conditions are different. Now it is initially spinning about the y axis of the body fixed frame at 5 rad/s. The torque applied for the disturbance is also different. It is now 1 Nm about the A axis and 2 Nm about the C axis. This results in a different motion.



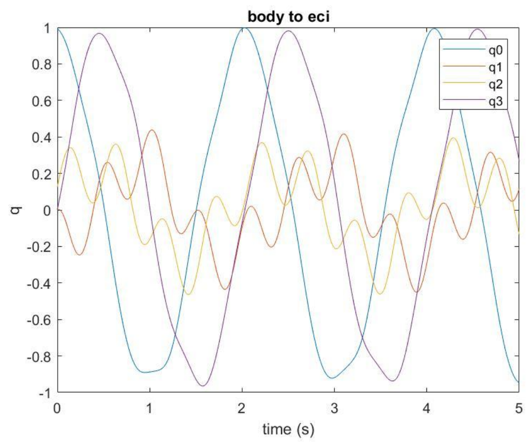


Fig. 3 Case 2 ω vs time Fig. 4 Case 2 q's with respect to time

This motion appears much more chaotic than case 1. Since there is no constant ω, this is not pure spin and precession. Because of this, no spin rate or precession rate can be found. When the results of this are visualized using a given program, the motion resembles exaggerated spin and precession. Because of this, no spin rate or precession rate can be found.

C. Case 3

For case 3, the effects of gravity gradient torque are studied. It can be found through the following equation.

$$\overrightarrow{M_{gg}} = \begin{matrix} \frac{3\mu}{r_L^5} r_{Gy} r_{Gz}(C-B) \\ 0 \\ \frac{3\mu}{r_L^5} r_{Gy} r_{Gx}(B-A) \end{matrix} \tag{11}$$

Here the second component of $\overrightarrow{M_{gg}}$ is 0 because it is multiplied by (A-C), which is equal to 0. To find $\overrightarrow{r_G}$, part of a pre-written code was used. The original code can be found in the appendix. To find $\overrightarrow{r_G}$ in the body fixed frame, quaternion multiplication is used. This process can be found in the handwritten calculations in the appendix.

IV. Conclusion

In case 1, the small disturbance resulted in a small spin and precession motion. This does not happen in case 2. Case 2 has a more intense motion that resembles spin and precession. The major difference between the two cases is the axis it's spinning about. In case one it is spinning about the C axis, which has the largest moment of inertia. It is spinning about the B axis in case 2. Based on this, spin stabilization is most effective when the spacecraft is spinning about its axis with the largest moment of inertia. For case 3, simulation results are inconclusive.

Appendix

A. OEtoRV code used for Radius of 400 km circular orbit

```
% -----------------------------------------------
% Program Number: RVtoOE and OEtoRV (ECI coords)
% Program Purpose: for use with non hyperbolic
%
% Created By: Lee Weidow
%
% Created On: Oct 25 2021
% Last Modified On:
%
% Credit to: the Smart Dude
% By submitting this program with my name,
% I affirm that the creation and modification/
% of this program is primarily my own work.
%
% Comments: hard coded for problem 1 on homework 4 of ae313
% to use for different vectors or oes, remove '%' from input sections
% -----------------------------------------------

clc
clear
close all
k=1;
I=[1 0 0];
J=[0 1 0];
K=[0 0 1];
mu=398600;
while k==1
    fprintf('RVtoOE (1) or OEtoRV (0)\n')
    %choice=input('Input corrosponding number: ');
    choice=0;
    if choice==1 %RVtoOE
        k=0;
        rvec=[-2436.45 -2436.45 6891.0379];
        vvec=[5.088611 -5.088611 0];
        %rvec(1)=input('I component of r: ');
        %rvec(2)=input('J component of r: ');
        %rvec(3)=input('K component of r: ');
        %vvec(1)=input('I component of v: ');
        %vvec(2)=input('J component of v: ');
        %vvec(3)=input('K component of v: ');
        r=norm(rvec);
        v=norm(vvec);
        hvec=cross(rvec,vvec);
        h=norm(hvec);
        energy=.5*v^2-mu/r;
        a=-mu/(2*energy);
        p=h^2/mu;
        hhat=hvec./h;
        i=acos(dot(hhat,K));
        rhat=rvec./r;
        evec=cross(vvec,hvec)/mu-rhat;
        e=norm(evec);
        ehat=evec./e;
        if dot(rvec,vvec)>=0
            f=acos(dot(ehat,rhat));
```

```
        elseif dot(rvec,vvec)<0
            f=2*pi-acos(dot(ehat,rhat));
        end
        nhat=cross(K,hvec)./norm(cross(K,hvec));
        if dot(ehat,K)>=0
            w=acos(dot(nhat,ehat));
        elseif dot(ehat,K)<0
            w=2*pi-acos(dot(nhat,ehat));
        end
        omega=atan2(nhat(2),nhat(1));
        fprintf('OEs:\na=%f\nf=%f\ne=%f\ni=%f\nw=%f\nomega=%f\n',a,f,e,i,w,omega)
    elseif choice ==0 %OEtoRV
        k=0;
        a=7.7122e+03;
        f=1.4901e-08;
        e=.001;
        i=1.1071;
        w=1.5708;
        omega=2.3562;
        %a=input('semi-major axis(a): ');
        %f=input('true anomoly(f): ');
        %e=input('eccentricity(e): ');
        %i=input('inclination(i): ');
        %w=input('argument of periapsis(w): ');
        %omega=input('RA of ascending nodes(capital omega): ');
        p=a*(1-e^2);
        if p==0
            p=a;
        end
        h=sqrt(p*mu);
        r=p/(1+e*cos(f));
        rpqw=[r*cos(f) r*sin(f) 0]';
        vpqw=(mu/h).*[-sin(f) e+cos(f) 0];
        vpqw=vpqw';
        v=norm(vpqw);
        rot=[cos(omega) -sin(omega) 0; sin(omega) cos(omega) 0; 0 0 1]*[1 0 0; 0 cos(i) -s
in(i); 0 sin(i) cos(i)]*[cos(w) -sin(w) 0; sin(w) cos(w) 0;0 0 1];
        %for reference:
        %rot1=[cos(omega) -sin(omega) 0; sin(omega) cos(omega) 0; 0 0 1];
        %rot2=[1 0 0; 0 cos(i) -sin(i); 0 sin(i) cos(i)];
        %rot3=[cos(w) -sin(w) 0; sin(w) cos(w) 0;0 0 1];
        vvec=rot*vpqw;
        rvec=rot*rpqw;
        fprintf('Velocity in ECI = %.4fI %.4fJ %.4fK\n',vvec(1),vvec(2),vvec(3))
        fprintf('Radius in ECI = %.2fI %.2fJ %.2fK\n',rvec(1),rvec(2),rvec(3))
    else
        fprintf('invalid input\n')
    end
end
```

B. Hand Calculations

Project Start

$\dot{q}_{BR} = \frac{1}{2} B(q_{BR}) \vec{\omega}_{BR}$

$\dot{\vec{\omega}}_B = I_G^{-1} \left[-\vec{\omega}_B \times (I_G \vec{\omega}_B) + \right.$

$\omega_B(0) = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$ rad/s

for disturbance (1)

apply a Torque for a very short time

3 ode 45's

→ use 1 for before torque (ex Time = (0:5)) → (const...

use 1 for during torque (Time = (5:5.1)) → perturb...

use for after torque (Time = (5.1:10)) → effect...

initial conditions will be taken from previous solutions for these

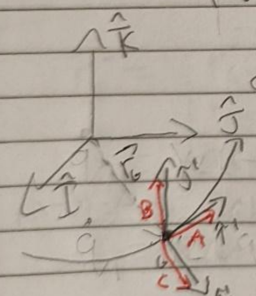
$\vec{\omega}_B = \begin{bmatrix} \omega_A \\ \omega_B \\ \omega_C \end{bmatrix}$ in body frame

$\vec{\omega}_R = \begin{bmatrix} 0 \\ \omega_R \\ 0 \end{bmatrix}$ in LVLH frame (and initially in body fra...

$\vec{\omega}_{B/R} = \vec{\omega}_B - \vec{\omega}_R$

Initially

$q_{BR}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$



A in R' frame   B in R frame   C in R frame

$$Q_{BR} = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_0^2 & 2(q_0 q_3 + q_1 q_2) & 2(q_1 q_3 - \ldots \\ 2(q_1 q_2 - q_0 q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2 q_3 + \ldots \\ 2(q_0 q_2 + q_1 q_3) & 2(q_2 q_3 - q_0 q_1) & q_0^2 - q_1^2 - q_2^2 \ldots \end{bmatrix}$$

(IE) $\omega_R|_{ABC} = Q_{RB}\, \omega_{R|R} = Q_{BR}^{T}\, \omega_{R|R}$

here, $I_G = \begin{vmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{vmatrix}$ $\rightarrow$ these values are chosen by me based on project

for $A < B < C$ config

$A = 100$ kg m²
$B = 200$ kg m²
$C = 400$ kg m²

Again

$\dot{\vec{\omega}}_B = I_G^{-1}[-\vec{\omega}_B \times I_G \vec{\omega}_B + \vec{M}_G]$ $\qquad I_G^{-1} = \begin{vmatrix} 1/A & 0 & 0 \\ 0 & 1/B & 0 \\ 0 & 0 & 1/C \end{vmatrix}$

$I_G \vec{\omega}_B = \begin{vmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{vmatrix} \begin{vmatrix} \omega_x \\ \omega_y \\ \omega_z \end{vmatrix} = \begin{vmatrix} A\omega_x \\ B\omega_y \\ C\omega_z \end{vmatrix}$

$\dot{\vec{\omega}}_B = I_G^{-1}[\begin{vmatrix} -\omega_x \\ -\omega_y \\ -\omega_z \end{vmatrix} \times \begin{vmatrix} A\omega_x \\ B\omega_y \\ C\omega_z \end{vmatrix} + \vec{M}_G]$

Cross $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\omega_x & -\omega_y & -\omega_z \\ A\omega_x & B\omega_y & C\omega_z \end{vmatrix} = (-C\omega_y\omega_z + B\omega_y\omega_z)\hat{i} - (-C\omega_x\omega_z + A\omega_x\omega_z)\hat{j} + (-B\omega_x\omega_y + A\omega_x\omega_y)\hat{k}$ $\quad -(A-C) = (C-A)$

$\dot{\vec{\omega}}_B = I_G^{-1}\left[\begin{vmatrix} (B-C)\omega_y\omega_z \\ (C-A)\omega_x\omega_z \\ (A-B)\omega_x\omega_y \end{vmatrix} + \begin{vmatrix} M_{Gx} \\ M_{Gy} \\ M_{Gz} \end{vmatrix}\right] = I_G^{-1} \begin{vmatrix} (B-C)\omega_y\omega_z + M_{Gx} \\ (C-A)\omega_x\omega_z + M_{Gy} \\ (A-B)\omega_x\omega_y + M_{Gz} \end{vmatrix}$

$\dot{\vec{\omega}}_B = \begin{vmatrix} 1/A & 0 & 0 \\ 0 & 1/B & 0 \\ 0 & 0 & 1/C \end{vmatrix} \begin{vmatrix} (B-C)\omega_y\omega_z + M_{Gx} \\ (C-A)\omega_x\omega_z + M_{Gy} \\ (A-B)\omega_x\omega_y + M_{Gz} \end{vmatrix} =$

Project cont.

$$\dot{\vec{\omega}}_B = \left| \begin{matrix} \frac{1}{A}((B-C)\omega_y\omega_z + M_{Gx}) \\ \frac{1}{B}((C-A)\omega_x\omega_z + M_{Gy} \\ \frac{1}{C}((A-B)\omega_x\omega_y + M_{Gz}) \end{matrix} \right| \rightarrow$$

W/ A<B<C config from before (A=100 B=200 C=400)

$$\vec{\omega}_B = \left| \begin{matrix} \frac{1}{100}(200-400)\omega_y\omega_z + M_{Gx} \\ \frac{1}{200}(400-100)\omega_x\omega_z + M_{Gy} \\ \frac{1}{400}(100-200)\omega_x\omega_y + M_{Gz} \end{matrix} \right|$$

$$= \left| \begin{matrix} -2\omega_y\omega_z + M_{Gx} \\ 1.5\omega_x\omega_z + M_{Gy} \\ -.25\omega_x\omega_y + M_{Gz} \end{matrix} \right|$$

$$\dot{\omega}_x = M_{Gx} - 2\omega_y\omega_z$$

$$\dot{\omega}_y = M_{Gy} + 1.5\omega_x\omega_z$$

$$\dot{\omega}_z = M_{Gz} - .25\omega_x\omega_y$$

a small disturbance is going to be represente as a torque for a short period of time.

@ here the sat. is spinning about the C (3rd) axis

$$\therefore \vec{\omega}_B = \left| \begin{matrix} 0 \\ 0 \\ \omega_0 \end{matrix} \right|$$

and $$\vec{M}_G = \left| \begin{matrix} M_{Gx} \\ M_{Gy} \\ 0 \end{matrix} \right|$$

Project cont

$Q_{RB} = Q_{BR}^T =$

$$\begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_0^2 & 2(q_1 q_2 - q_0 q_3) & 2(q_0 q_2 + q_1 q_3) \\ 2(q_0 q_3 + q_1 q_2) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2 q_3 - q_0 q_1) \\ 2(q_1 q_3 - q_0 q_2) & 2(q_2 q_3 + q_0 q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

$$\vec{\omega}_R\Big|_{ABC} = Q_{RB}\,\vec{\omega}_R\Big|_R = Q_{RB}\begin{vmatrix} 0 \\ \omega_R \\ 0 \end{vmatrix}$$

b/c ω_R is in the LVLH frame, it must be converted into ABC Frame using Q_{RB}. This can be done b/c # of eq is still the same as # of unknowns

$$\vec{\omega}_R = \begin{vmatrix} 0 + 2\omega_R(q_1 q_2 - q_0 q_3) + 0 \\ 0 + \omega_R(q_0^2 - q_1^2 + q_2^2 - q_3^2) + 0 \\ 0 + 2\omega_R(q_2 q_3 + q_0 q_1) + 0 \end{vmatrix}$$

$$\vec{\omega}_R\Big|_{ABC} = \begin{vmatrix} 2\omega_R(q_1 q_2 - q_0 q_3) \\ \omega_R(q_0^2 - q_1^2 + q_2^2 - q_3^2) \\ 2\omega_R(q_2 q_3 + q_0 q_1) \end{vmatrix}$$

$$B(q_{BR}) = \begin{vmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{vmatrix}$$

Project cont.

$$\dot{q}_{B/R} = \frac{1}{2}\begin{bmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{bmatrix}\begin{bmatrix} \omega_{B/R x} \\ \omega_{B/R y} \\ \omega_{B/R z} \end{bmatrix}$$

$$\dot{q}_{B/R} = \frac{1}{2}\begin{bmatrix} -q_1\omega_{B/R x} - q_2\omega_{B/R y} - q_3\omega_{B/R z} \\ q_0\omega_{B/R x} - q_3\omega_{B/R y} + q_2\omega_{B/R z} \\ q_3\omega_{B/R x} + q_0\omega_{B/R} - q_1\omega_{B/R z} \\ -q_2\omega_{B/R x} + q_1\omega_{B/R y} + q_0\omega_{B/R z} \end{bmatrix}$$

all eq used for Ode 45 (try both defining $\vec{\omega}_{B/R}$ in Matlab or directly replacing it w/ $\vec{\omega}_B - \vec{\omega}_R$)

$$\dot{q}_0 = (-q_1\omega_{B/R x} - q_2\omega_{B/R y} - q_3\omega_{B/R z})\tfrac{1}{2}$$

$$\dot{q}_1 = (q_0\omega_{B/R x} - q_3\omega_{B/R y} + q_2\omega_{B/R z})\tfrac{1}{2}$$

$$\dot{q}_2 = (q_3\omega_{B/R x} + q_0\omega_{B/R y} - q_1\omega_{B/R z})\tfrac{1}{2}$$

$$\dot{q}_3 = (-q_2\omega_{B/R x} + q_1\omega_{B/R y} + q_0\omega_{B/R z})\tfrac{1}{2}$$

$\omega_{B/R x} = \omega_{B_A} - \omega_{R_A}$; $\omega_{B/R y} = \omega_{B_B} - \omega_{R_B}$; $\omega_{B/R z} = \omega_{B_C} - \omega_{R_C}$

$\dot{\omega}_{B_A} = M_{B_A} - 2\omega_{B_B}\omega_{B_C}$; $\dot{\omega}_{B_B} = M_{B_B} + 1.5\omega_{B_A}\omega_{B_C}$

$\dot{\omega}_{B_C} = M_{B_C} - .25\omega_{B_A}\omega_{B_B}$

Project cont

$$\omega_{RA} = 2\omega_R(a_1 a_2 - a_0 a_3)$$

$$\omega_{RB} = \omega_R(a_0^2 - a_1^2 + a_2^2 - a_3^2)$$

$$\omega_{RC} = 2\omega_R(a_2 a_3 + a_0 a_1)$$

Unknowns: $\omega_{RA}, \omega_{RB}, \omega_{RC}, \omega_{BA}, \omega_{BB}, \omega_{BC}, a_0, a_1, a_2, a_3, \omega_{B/R x}, \omega_{B/R y}, \omega_{B/R z}$

13 unknowns 13 equations ✓

$Y_1 = \omega_{BA}$ $Y_4 = a_0$ $Y_8 = \omega_{RA}$ $Y_{11} = \omega_{B/R x}$

$Y_2 = \omega_{BB}$ $Y_5 = a_1$ $Y_9 = \omega_{RB}$ $Y_{12} = \omega_{B/R y}$

$Y_3 = \omega_{BC}$ $Y_6 = a_2$ $Y_{10} = \omega_{RC}$ $Y_{13} = \omega_{B/R z}$

$Y_7 = a_3$

$\dot{Y}_1 = M_{GA} - 2Y_2Y_3$

$\dot{Y}_2 = M_{GB} + 1.5Y_1Y_3$

$\dot{Y}_3 = M_{GC} - .25Y_1Y_2$

$\dot{Y}_4 = (-Y_5Y_{11} - Y_6Y_{12} - Y_7Y_{13})\frac{1}{2}$

$\dot{Y}_5 = (Y_4Y_{11} - Y_7Y_{12} + Y_6Y_{13})\frac{1}{2}$

$\dot{Y}_6 = (Y_7Y_{11} + Y_4Y_{12} - Y_5Y_{13})\frac{1}{2}$

$\dot{Y}_7 = (-Y_6Y_{11} + Y_5Y_{12} + Y_4Y_{13})\frac{1}{2}$

$Y_8 = 2\omega_R(Y_5Y_6 - Y_4Y_7)$

$Y_9 = \omega_R(Y_4^2 - Y_5^2 + Y_6^2 - Y_7^2)$

$Y_{10} = 2\omega_R(Y_6Y_7 + Y_4Y_5)$

$Y_{11} = Y_1 - Y_8$

$Y_{12} = Y_2 - Y_9$

$Y_{13} = Y_3 - Y_{10}$

$M_G =$	.1	N.m
	.2	
	0	

$\omega_R = .0012$

Reorientation project cont.

$$T = \frac{2\pi r}{V}$$

$$V = \sqrt{\frac{G m_E}{r}} = \sqrt{\frac{6.67\times10^{-11}\ N\cdot m^2/kg^2\ 5.97\times10^{24}\ kg}{6.74\times10^{6} + .04\times10^{6}}}$$

$$V = 7663.6\ m/s$$

$$T = \frac{2\pi r}{V} = \frac{2\pi(6.78\times10^{6}\ m)}{7664\ m/s} = 5558.7\ s$$

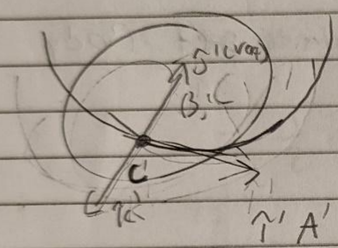
$$\omega = \frac{2\pi\ rad}{5558.7\ s} = .00113\ rad/s = \omega_R$$

Project M_{gg}

initially aligned w/ LVLH

$A, C > B$

$$\underline{I}_G = \begin{vmatrix} 400 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 400 \end{vmatrix}$$



Princ frame → LVLH

$C' \parallel \hat{k}'$ $A' \parallel \hat{i}'$ $B' \parallel \hat{j}'$

From before, 4400 km circular has a period of 5558.7s ~~ea.~~ $\therefore \omega_B = .00113 \frac{rad}{s} B'$

$\omega_R = .00113 \frac{rad}{s} \hat{j}' = .00113 \frac{rad}{s} B'$

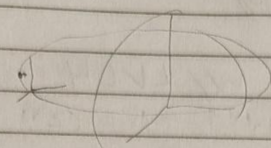
$$\vec{M}_{gg} = \begin{vmatrix} \frac{3\mu}{r_G^5} r_{Gy} r_{Gz} (C-B) \\ \frac{3\mu}{r_G^5} r_{Gx} r_{Gz} (A-C) \\ \frac{3\mu}{r_G^5} r_{Gx} r_{Gy} (B-A) \end{vmatrix}$$

$\vec{\omega}_{G/R} = \vec{0}$

q

Proj ect

initially,



$\|\vec{r}_G\| = 6378+400 = 6778km$

Find $\vec{r}_G$ in ECI, Then use $\dot{q}$ eq to find $\vec{r}_G$ in LVLH Then use it again to get Body frame

$$\vec{r}_G\big|_{Body} = q_{Br}^{-1} \otimes r_G\big|_{ref} \otimes q_{Br}$$

$$r_G\big|_{ref} = \begin{vmatrix} 0 \\ \vec{r}_{G\,LVLH} \end{vmatrix} \rightarrow \text{same format for all}$$

$$\vec{r}_{G\,LVLH} = q_{LE}^{-1} \otimes r_G\big|_{ECI} \otimes \underbrace{q_{LE}^{*}}_{\rightarrow \text{LVLH to ECI}}$$

$\dot{q}$: same as earlier

$$\dot{\omega}_x = \frac{B-C}{A}\omega_y\omega_z + M_{Gx} = M_{Gx} - .5\omega_y\omega_z$$
$$\dot{\omega}_y = \frac{C-A}{B}\overbrace{\omega_x\omega_z}^{0 \text{ b/c } C=A} + M_{Gy} = M_{Gy}$$
$$\dot{\omega}_z = \frac{A-B}{C}\omega_x\omega_y + M_{Gz} = M_{Gz} + .5\omega_x\omega_y$$

$M_G = M_{gg} + \underbrace{\text{distrubance}}$ → will be 0 after $t = .1s$

Project cont

$$r|_{LVLH} = q_{LE}^{-1} \otimes r_G|_{ECI} \otimes q_{LE}$$

$$=$$

$$r_G|_{ECI} = \begin{vmatrix} 0 \\ \vec{r_G} \end{vmatrix}$$

$$r|_{LVLH} = \begin{vmatrix} q_0 \\ -q_1 \\ -q_2 \\ -q_3 \end{vmatrix} \otimes \begin{vmatrix} 0 \\ r_{\hat{i}} \\ r_{\hat{j}} \\ r_{\hat{k}} \end{vmatrix} \otimes \begin{vmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -r_{\hat{i}} & -r_{\hat{j}} & r_{\hat{k}} \\ r_{\hat{i}} & 0 & r_{\hat{k}} & -r_{\hat{j}} \\ r_{\hat{j}} & -r_{\hat{k}} & 0 & r_{\hat{i}} \\ r_{\hat{k}} & r_{\hat{j}} & -r_{\hat{i}} & 0 \end{vmatrix} \begin{vmatrix} q_0 \\ -q_1 \\ -q_2 \\ -q_3 \end{vmatrix} \otimes \begin{vmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{vmatrix}$$

$$= \begin{vmatrix} q_1 r_{\hat{i}} + q_2 r_{\hat{j}} - q_3 r_{\hat{k}} \\ q_0 r_{\hat{i}} - q_2 r_{\hat{k}} + q_3 r_{\hat{j}} \\ q_0 r_{\hat{j}} + q_1 r_{\hat{k}} - q_3 r_{\hat{i}} \\ q_0 r_{\hat{k}} - q_1 r_{\hat{j}} + q_2 r_{\hat{i}} \end{vmatrix} \otimes \begin{vmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{vmatrix}$$

$$= \begin{vmatrix} q_0 & -q_1 & -q_2 & q_3 \\ q_1 & q_0 & q_3 & -q_2 \\ q_2 & -q_3 & q_0 & q_1 \\ q_3 & q_2 & -q_1 & q_0 \end{vmatrix} \begin{vmatrix} q_1 r_{\hat{i}} + q_2 r_{\hat{j}} - q_3 r_{\hat{k}} \\ q_0 r_{\hat{i}} - q_2 r_{\hat{k}} + q_3 r_{\hat{j}} \\ q_0 r_{\hat{j}} + q_1 r_{\hat{k}} - q_3 r_{\hat{i}} \\ q_0 r_{\hat{k}} - q_1 r_{\hat{j}} + q_2 r_{\hat{i}} \end{vmatrix}$$

similar for $r|_{BODY}$

$$r\big|_{body} = q_{BR}^{-1}\; r\big|_{LVLH}\; q_{BR}$$

$$= \begin{bmatrix} q_{v_1} r_{\hat{i}'} + q_{v_2} r_{\hat{j}'} - q_{v_3} r_{\hat{k}'} \\ q_{v_0} r_{\hat{i}'} - q_{v_2} r_{\hat{k}'} + q_{v_3} r_{\hat{j}'} \\ q_{v_0} r_{\hat{j}'} + q_{v_1} r_{\hat{k}'} - q_{v_3} r_{\hat{i}'} \\ q_{v_0} r_{\hat{k}'} - q_{v_1} r_{\hat{j}'} + q_{v_2} r_{\hat{i}'} \end{bmatrix} \otimes \begin{bmatrix} q_{v_0} \\ q_{v_1} \\ q_{v_2} \\ q_{v_3} \end{bmatrix}$$

$$\vec{r}_{body} = \begin{bmatrix} q_{v_0} & -q_{v_1} & -q_{v_2} & -q_{v_3} \\ q_{v_1} & q_{v_0} & q_{v_3} & -q_{v_2} \\ q_{v_2} & -q_{v_3} & q_{v_0} & q_{v_1} \\ q_{v_3} & q_{v_2} & -q_{v_1} & q_{v_0} \end{bmatrix} \cdot$$

not part of $\vec{r}_{body}$

$$r\big|_{body} = \begin{bmatrix} r_{\hat{i}'}(q_{v_1}^2 + q_{v_0}^2 - q_{v_3}^2 - q_{v_2}^2) + r_{\hat{j}'}(2q_{v_1}q_{v_2} + 2q_{v_0}q_{v_3}) + r_{\hat{k}'}(-2q_{v_0}q_{v_2} + q_{v_0}q_{v_2} - q_{v_1}q_{v_3}) \\ r_{\hat{i}'}(2q_{v_2}q_{v_1} - 2q_{v_0}q_{v_3}) + r_{\hat{j}'}(q_{v_2}^2 + q_{v_3}^2 + q_{v_0}^2 - q_{v_1}^2) + r_{\hat{k}'}(-2q_{v_0}q_{v_1} + 2q_{v_0}q_{v_1}) \\ r_{\hat{i}'}(2q_{v_0}q_{v_2}) + r_{\hat{j}'}(2q_{v_2}q_{v_3} - 2q_{v_0}q_{v_1}) + r_{\hat{k}'}(-q_{v_3}^2 - q_{v_2}^2 + q_{v_1}^2 + q_{v_0}^2) \end{bmatrix}$$

$$= \begin{bmatrix} r_{bx} \\ r_{by} \\ r_{bz} \end{bmatrix}$$

$$\begin{vmatrix} M_{ggx} \\ M_{ggy} \\ M_{ggz} \end{vmatrix} = \begin{vmatrix} \frac{3\mu}{6778^5} r_{Gy} r_{Gz} (200) \\ 0 \\ \frac{3\mu}{6778^5} r_{Gx} r_{Gy} (-200) \end{vmatrix}$$

For disturbance

$$\vec{M_G} = \vec{M_G}_{disturbance} + \vec{M_{gg}}$$

For "Torque free"

$$\vec{M_G} = \vec{M_{gg}}$$

Project

Finding nutation angle and Precession Rate

$$\theta = \cos^{-1}\left(\frac{\vec{H_G} \cdot k}{\|\vec{H_G}\|}\right) ; \vec{H_G} = I\vec{\omega_B}$$

From previous work, it is known that

$\omega_x = \Omega \cos(\lambda t)$

$$\therefore \Omega = \frac{\omega_x}{\cos(\lambda t)} \rightarrow \lambda = \frac{(A-C)^2}{A^2}\omega_z$$

Spin rate

$$\lambda = \frac{(100-400)^2}{100^2}\left(5\ \tfrac{rad}{s}\right) = 9$$

$$\Omega = \frac{\omega_x}{\cos(9t)} \rightarrow \text{from Matlab}$$

Spin Rate

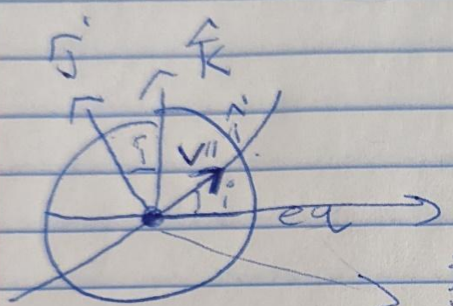
$$\dot{\psi} = \omega_z - \frac{\Omega}{\tan\theta} = 5 - \frac{\Omega}{\tan\theta}$$

$$\dot{\phi} = \frac{C}{A-C}\frac{\dot{\psi}}{\cos\theta}$$

$$\frac{v^2}{2} = -\frac{\mu}{2r} + \frac{\mu}{r}$$

$$v^2 = -\frac{\mu}{r} + \frac{2\mu}{r}$$

$$v = \sqrt{\frac{\mu}{r}}$$



X is out of page

$\hat{I} \parallel \hat{i}'$ initially

DCM =

initial DCM $Q_{i'I} = \begin{array}{c|ccc|} & \hat{i}' & \hat{j}' & \hat{k}' \\ \hline & 0 & 0 & 1 \\ & -\sin i & \cos i & 0 \\ & \cos i & \sin i & 0 \\ \end{array}$

Reorientation project cont.

$$T = \frac{2\pi r}{V}$$

$$V = \sqrt{\frac{G m_E}{r}} = \sqrt{\frac{6.67\times10^{-11}\ N\cdot m^2/kg^2\ 5.97\times10^{24}\ kg}{6.38\times10^{6} + .4\times10^{6}}}$$

$$V = 7667.6\ m/s$$

$$T = \frac{2\pi r}{V} = \frac{2\pi(6.78\times10^{6} m)}{7664\ m/s} = 5558.7 s$$

$$\omega = \frac{2\pi\ rad}{5558.7 s} = .00113\ rad/s = \omega_R$$

```
clc
clear
close all
%case 1
time=[0:0.01:0.1];
time2=[0:.05:15];
[T,X]=ode45(@diffeq,time,[0;0;5;1;0;0;0]);
[T2,X2]=ode45(@diffeq2,time2,[X(end,1);X(end,2);5;X(end,4);X(end,5);X(end,6);X(end,7)]);
[T3,X3]=ode45(@diffeq3,time,[0;0;5;1;0;0;0]);
[T4,X4]=ode45(@diffeq4,time2,[X3(end,1);X3(end,2);5;X3(end,4);X3(end,5);X3(end,6);X3(end,7
)]);
yourMatrix=[T4, X4(:,4), X4(:,5), X4(:,6), X4(:,7),zeros(301,1)+6600, zeros(301,1), zeros(
301,1) ];

writematrix(yourMatrix, 'output2.csv', 'Delimiter', 'comma')

%precession and spin rate
Hg=[100*X4(:,1)'; 200*X4(:,2)'; 400*X4(:,3)';];
theta=[1:301];
for k=1:301
    thetavec(k)=acos(dot(Hg(:,k),[0,0,1])/norm(Hg(:,k)));
end
%since ode45 is numerical, it is not exact so there is some variation in
%theta
theta=mean(thetavec);
%same as above for omega
omegavec=X4(:,1)./cos(9.*T4);
omega=mean(omegavec);
%spin rate
psidot=5-omega/tan(theta);
%precession rate
phidot=400/(100-400)*psidot/cos(theta);

figure()
plot(T2,X2(:,1))
title('Solution w');
xlabel('Time (s)');
ylabel('w');
hold on
plot(T2,X2(:,2))
hold on
plot(T2,X2(:,3));
legend('wx','wy','wz')
title('body to lvlh')
hold off

figure()
plot(T2,X2(:,4))
hold on
plot(T2,X2(:,5))
hold on
plot(T2,X2(:,6))
hold on
plot(T2,X2(:,7))
title('q WRT time')
```

```
xlabel('time (s)')
ylabel('q')
legend('q0','q1','q2','q3')
title('body to lvlh')

figure()
plot(T4,X4(:,1))
title('Solution w');
xlabel('Time (s)');
ylabel('w');
hold on
plot(T4,X4(:,2))
hold on
plot(T4,X4(:,3));
legend('wx','wy','wz')
title('body to eci')
hold off

figure()
plot(T4,X4(:,4))
hold on
plot(T4,X4(:,5))
hold on
plot(T4,X4(:,6))
hold on
plot(T4,X4(:,7))
title('q WRT time')
xlabel('time (s)')
ylabel('q')
legend('q0','q1','q2','q3')
title('body to eci')

%case 2
%same I as 1 but different M=[1;0;2]
time=[0:0.01:0.05];
time2=[0:.01:5];
[T,X5]=ode45(@diffeq5,time,[0;5;0;1;0;0;0]);
[T6,X6]=ode45(@diffeq6,time2,[X5(end,1);X5(end,2);5;X5(end,4);X5(end,5);X5(end,6);X5(end,7
)]);
[T7,X7]=ode45(@diffeq7,time,[0;5;0;1;0;0;0]);
[T8,X8]=ode45(@diffeq8,time2,[X7(end,1);X7(end,2);5;X7(end,4);X7(end,5);X7(end,6);X7(end,7
)]);

figure()
plot(T6,X6(:,1))
title('Solution w');
xlabel('Time (s)');
ylabel('w');
hold on
plot(T6,X6(:,2))
hold on
plot(T6,X6(:,3));
legend('wx','wy','wz')
title('body to lvlh')
hold off

figure()
```

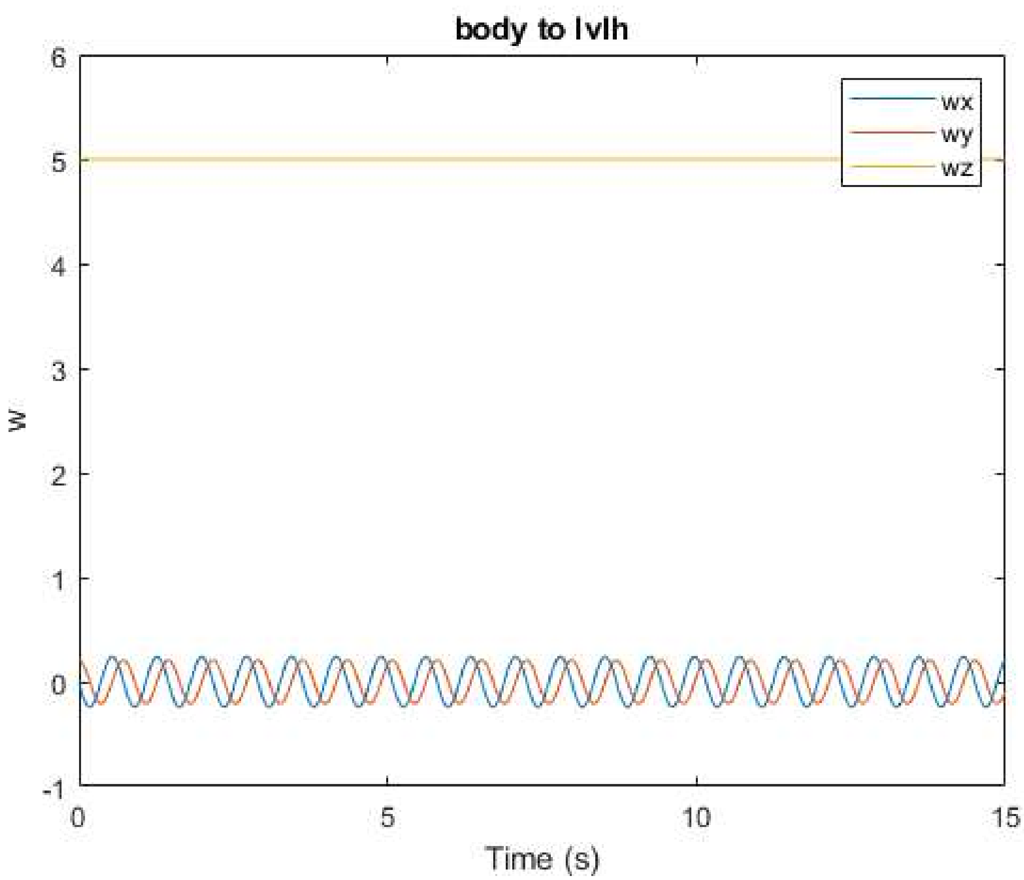
```
plot(T6,X6(:,4))
hold on
plot(T6,X6(:,5))
hold on
plot(T6,X6(:,6))
hold on
plot(T6,X6(:,7))
title('q WRT time')
xlabel('time (s)')
ylabel('q')
legend('q0','q1','q2','q3')
title('body to lvlh')

figure()
plot(T8,X8(:,1))
title('Solution w');
xlabel('Time (s)');
ylabel('w');
hold on
plot(T8,X8(:,2))
hold on
plot(T8,X8(:,3));
legend('wx','wy','wz')
title('body to eci')
hold off

figure()
plot(T8,X8(:,4))
hold on
plot(T8,X8(:,5))
hold on
plot(T8,X8(:,6))
hold on
plot(T8,X8(:,7))
title('q WRT time')
xlabel('time (s)')
ylabel('q')
legend('q0','q1','q2','q3')
title('body to eci')

%for visulaizer
% yourMatrix=[T7, X7(:,4), X7(:,5), X7(:,6), X7(:,7), zeros(6,1)+6600, zeros(6,1), zeros(6
,1);T8, X8(:,4), X8(:,5), X8(:,6), X8(:,7),zeros(501,1)+6600, zeros(501,1), zeros(501,1) ]
;
% writematrix(yourMatrix, 'output.csv', 'Delimiter', 'comma')

function dydt5=diffeq5(t,y)
dy1=1-2*y(2)*y(3);
dy2=1.5*y(1)*y(3);
dy3=2-.25*y(1)*y(2);
dy4=.5*(-y(5)*(y(1)-2*.00113*(y(5)*y(6)-y(4)-y(7)))-y(6)*(y(2)-.00113*(y(4)^2-y(5)^2+y(6)^
2-y(7)^2))-y(7)*(y(3)-2*.00113*(y(6)*y(7)+y(4)*y(5))));
dy5=.5*(y(4)*(y(1)-2*.00113*(y(5)*y(6)-y(4)-y(7)))-y(7)*(y(2)-.00113*(y(4)^2-y(5)^2+y(6)^2
-y(7)^2))+y(6)*(y(3)-2*.00113*(y(6)*y(7)+y(4)*y(5))));
dy6=.5*(y(7)*(y(1)-2*.00113*(y(5)*y(6)-y(4)-y(7)))+y(4)*(y(2)-.00113*(y(4)^2-y(5)^2+y(6)^2
-y(7)^2))-y(5)*(y(3)-2*.00113*(y(6)*y(7)+y(4)*y(5))));
dy7=.5*(-y(6)*(y(1)-2*.00113*(y(5)*y(6)-y(4)-y(7)))+y(5)*(y(2)-.00113*(y(4)^2-y(5)^2+y(6)^
```

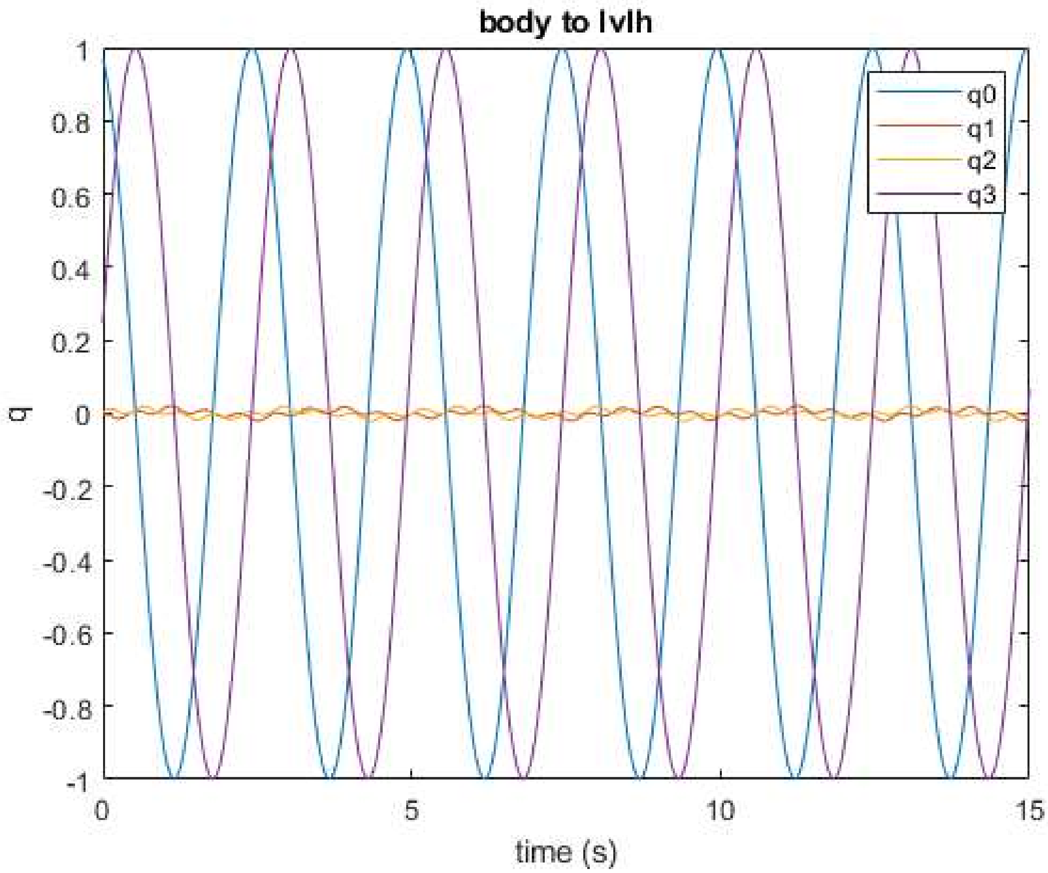
```
2-y(7)^2))+y(4)*(y(3)-2*.00113*(y(6)*y(7)+y(4)*y(5))));
% y(8)=2*.00113*(y(5)*y(6)-y(4)-y(7));
% y(9)=.00113*(y(4)^2-y(5)^2+y(6)^2-y(7)^2);
% y(10)=2*.00113*(y(6)*y(7)+y(4)*y(5));
% y(11)=y(1)-y(8);
% y(12)=y(2)-y(9);
% y(13)=y(3)-y(10);
dydt5=[dy1;dy2;dy3;dy4;dy5;dy6;dy7];
end

function dydt6=diffeq6(t,y)
dy1=-2*y(2)*y(3);
dy2=1.5*y(1)*y(3);
dy3=-.25*y(1)*y(2);
dy4=.5*(-y(5)*(y(1)-2*.00113*(y(5)*y(6)-y(4)-y(7)))-y(6)*(y(2)-.00113*(y(4)^2-y(5)^2+y(6)^
2-y(7)^2))-y(7)*(y(3)-2*.00113*(y(6)*y(7)+y(4)*y(5))));
dy5=.5*(y(4)*(y(1)-2*.00113*(y(5)*y(6)-y(4)-y(7)))-y(7)*(y(2)-.00113*(y(4)^2-y(5)^2+y(6)^2
-y(7)^2))+y(6)*(y(3)-2*.00113*(y(6)*y(7)+y(4)*y(5))));
dy6=.5*(y(7)*(y(1)-2*.00113*(y(5)*y(6)-y(4)-y(7)))+y(4)*(y(2)-.00113*(y(4)^2-y(5)^2+y(6)^2
-y(7)^2))-y(5)*(y(3)-2*.00113*(y(6)*y(7)+y(4)*y(5))));
dy7=.5*(-y(6)*(y(1)-2*.00113*(y(5)*y(6)-y(4)-y(7)))+y(5)*(y(2)-.00113*(y(4)^2-y(5)^2+y(6)^
2-y(7)^2))+y(4)*(y(3)-2*.00113*(y(6)*y(7)+y(4)*y(5))));
dydt6=[dy1;dy2;dy3;dy4;dy5;dy6;dy7];
end

function dydt7=diffeq7(t,y)
%angular velocity
dy1=1-2*y(2)*y(3);
dy2=1.5*y(1)*y(3);
dy3=2-.25*y(1)*y(2);
%qs. this format is same for all diffeq functions
dy4=.5*(-y(5)*y(1)-y(6)*y(2)-y(7)*y(3));
dy5=.5*(y(4)*y(1)-y(7)*y(2)+y(6)*y(3));
dy6=.5*(y(7)*y(1)+y(4)*y(2)-y(5)*y(3));
dy7=.5*(-y(6)*y(1)+y(5)*y(2)+y(4)*y(3));
dydt7=[dy1;dy2;dy3;dy4;dy5;dy6;dy7];
end
%torque free
function dydt8=diffeq8(t,y)
dy1=-2*y(2)*y(3);
dy2=1.5*y(1)*y(3);
dy3=-.25*y(1)*y(2);
dy4=.5*(-y(5)*y(1)-y(6)*y(2)-y(7)*y(3));
dy5=.5*(y(4)*y(1)-y(7)*y(2)+y(6)*y(3));
dy6=.5*(y(7)*y(1)+y(4)*y(2)-y(5)*y(3));
dy7=.5*(-y(6)*y(1)+y(5)*y(2)+y(4)*y(3));
dydt8=[dy1;dy2;dy3;dy4;dy5;dy6;dy7];
end

%distrubance
function dydt=diffeq(t,y)
dy1=1-2*y(2)*y(3);
dy2=2+1.5*y(1)*y(3);
dy3=-.25*y(1)*y(2);
dy4=.5*(-y(5)*(y(1)-2*.00113*(y(5)*y(6)-y(4)-y(7)))-y(6)*(y(2)-.00113*(y(4)^2-y(5)^2+y(6)^
2-y(7)^2))-y(7)*(y(3)-2*.00113*(y(6)*y(7)+y(4)*y(5))));
dy5=.5*(y(4)*(y(1)-2*.00113*(y(5)*y(6)-y(4)-y(7)))-y(7)*(y(2)-.00113*(y(4)^2-y(5)^2+y(6)^2
```

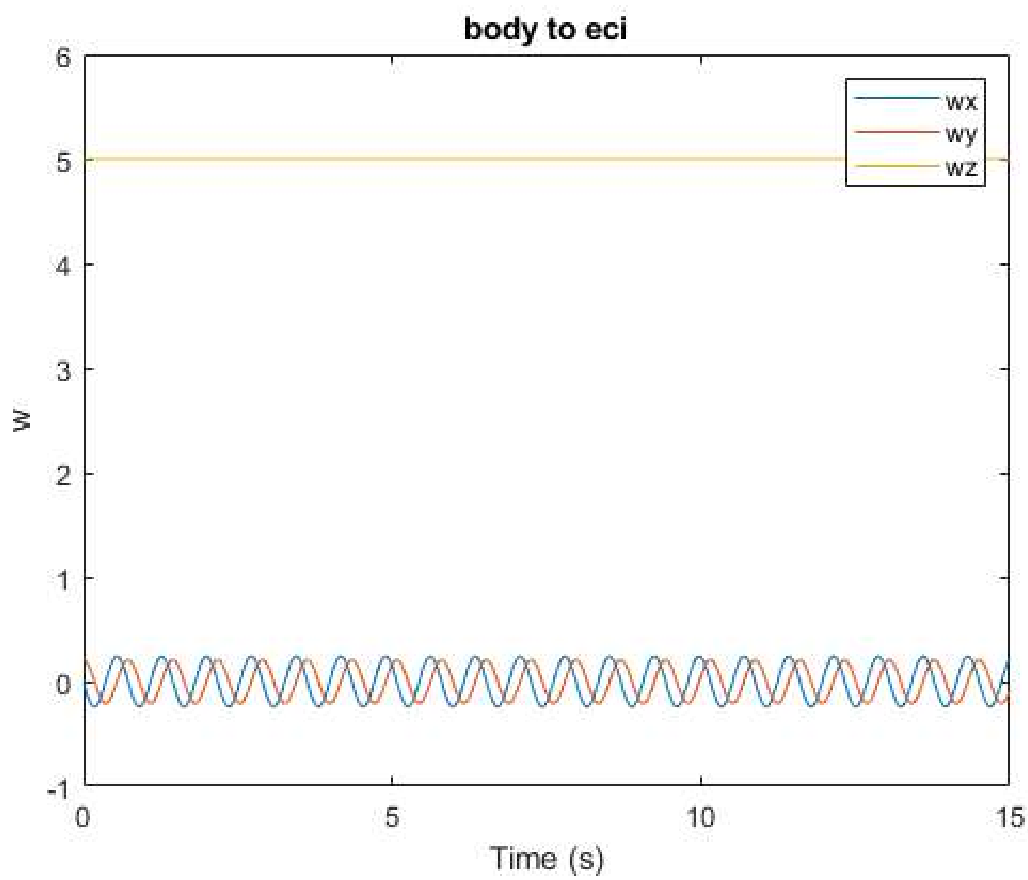
```
-y(7)^2))+y(6)*(y(3)-2*.00113*(y(6)*y(7)+y(4)*y(5))));
dy6=.5*(y(7)*(y(1)-2*.00113*(y(5)*y(6)-y(4)-y(7)))+y(4)*(y(2)-.00113*(y(4)^2-y(5)^2+y(6)^2
-y(7)^2))-y(5)*(y(3)-2*.00113*(y(6)*y(7)+y(4)*y(5))));
dy7=.5*(-y(6)*(y(1)-2*.00113*(y(5)*y(6)-y(4)-y(7)))+y(5)*(y(2)-.00113*(y(4)^2-y(5)^2+y(6)^
2-y(7)^2))+y(4)*(y(3)-2*.00113*(y(6)*y(7)+y(4)*y(5))));
% y(8)=2*.00113*(y(5)*y(6)-y(4)-y(7));
% y(9)=.00113*(y(4)^2-y(5)^2+y(6)^2-y(7)^2);
% y(10)=2*.00113*(y(6)*y(7)+y(4)*y(5));
% y(11)=y(1)-y(8);
% y(12)=y(2)-y(9);
% y(13)=y(3)-y(10);
dydt=[dy1;dy2;dy3;dy4;dy5;dy6;dy7];
end
%torque free
function dydt2=diffeq2(t,y)
dy1=-2*y(2)*y(3);
dy2=1.5*y(1)*y(3);
dy3=-.25*y(1)*y(2);
dy4=.5*(-y(5)*(y(1)-2*.00113*(y(5)*y(6)-y(4)-y(7)))-y(6)*(y(2)-.00113*(y(4)^2-y(5)^2+y(6)^
2-y(7)^2))-y(7)*(y(3)-2*.00113*(y(6)*y(7)+y(4)*y(5))));
dy5=.5*(y(4)*(y(1)-2*.00113*(y(5)*y(6)-y(4)-y(7)))-y(7)*(y(2)-.00113*(y(4)^2-y(5)^2+y(6)^2
-y(7)^2))+y(6)*(y(3)-2*.00113*(y(6)*y(7)+y(4)*y(5))));
dy6=.5*(y(7)*(y(1)-2*.00113*(y(5)*y(6)-y(4)-y(7)))+y(4)*(y(2)-.00113*(y(4)^2-y(5)^2+y(6)^2
-y(7)^2))-y(5)*(y(3)-2*.00113*(y(6)*y(7)+y(4)*y(5))));
dy7=.5*(-y(6)*(y(1)-2*.00113*(y(5)*y(6)-y(4)-y(7)))+y(5)*(y(2)-.00113*(y(4)^2-y(5)^2+y(6)^
2-y(7)^2))+y(4)*(y(3)-2*.00113*(y(6)*y(7)+y(4)*y(5))));
dydt2=[dy1;dy2;dy3;dy4;dy5;dy6;dy7];
end

%body to eci. since wr=0 wb/r=wb
function dydt3=diffeq3(t,y)
dy1=1-2*y(2)*y(3);
dy2=2+1.5*y(1)*y(3);
dy3=-.25*y(1)*y(2);
dy4=.5*(-y(5)*y(1)-y(6)*y(2)-y(7)*y(3));
dy5=.5*(y(4)*y(1)-y(7)*y(2)+y(6)*y(3));
dy6=.5*(y(7)*y(1)+y(4)*y(2)-y(5)*y(3));
dy7=.5*(-y(6)*y(1)+y(5)*y(2)+y(4)*y(3));
dydt3=[dy1;dy2;dy3;dy4;dy5;dy6;dy7];
end
%torque free
function dydt4=diffeq4(t,y)
dy1=-2*y(2)*y(3);
dy2=+1.5*y(1)*y(3);
dy3=-.25*y(1)*y(2);
dy4=.5*(-y(5)*y(1)-y(6)*y(2)-y(7)*y(3));
dy5=.5*(y(4)*y(1)-y(7)*y(2)+y(6)*y(3));
dy6=.5*(y(7)*y(1)+y(4)*y(2)-y(5)*y(3));
dy7=.5*(-y(6)*y(1)+y(5)*y(2)+y(4)*y(3));
dydt4=[dy1;dy2;dy3;dy4;dy5;dy6;dy7];
end

%this section was used for gravity gradient calculations but does not work
% %lvlh to eci
% %initial q
% time=[0:50]; %s
% Qle=[0 0 1; cosd(30) -sind(30) 0; sind(30) cosd(30) 0];
```

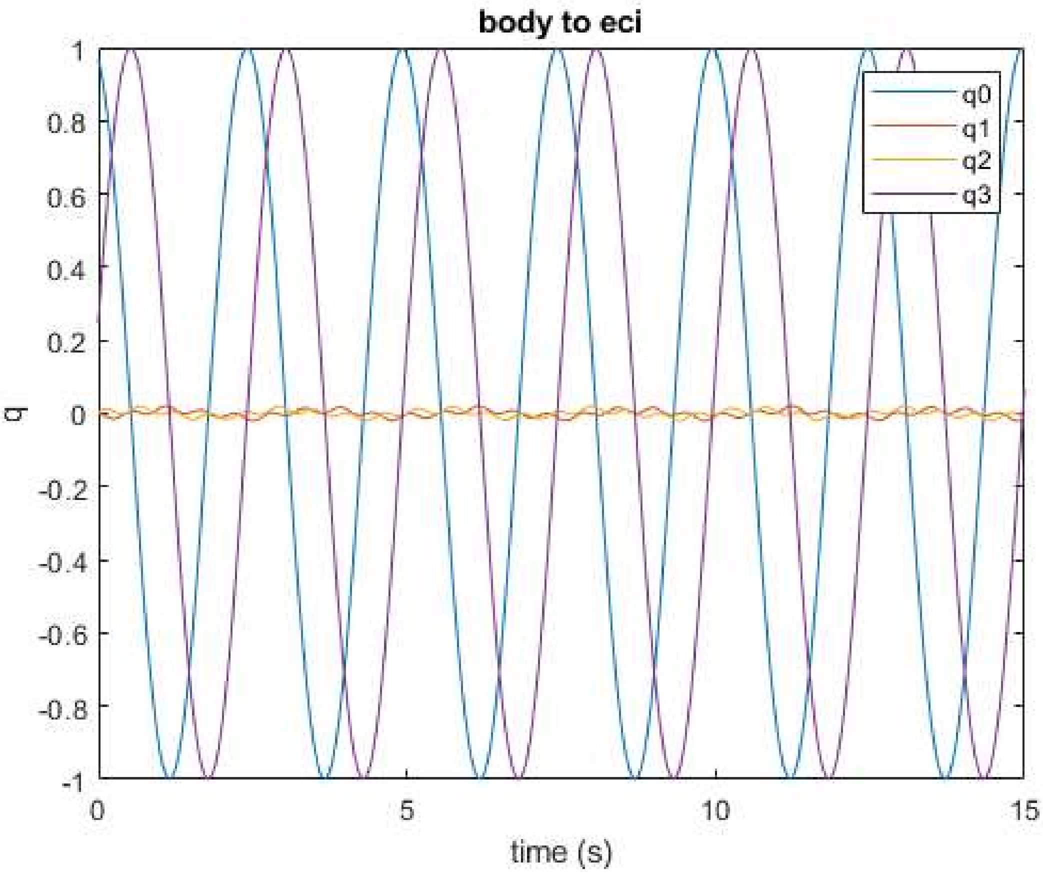
```
% [V,D]=eig(Qle);
% %D(:,1)=1 0 0 therfore V(:,1) is the axis of rotation
% rotangle=acos(.189); %rad
% uhat=V(:,1);
% qinitial=[cos(rotangle/2);sin(rotangle).*uhat];
% %lvlh to eci
% [T,X]=ode45(@diffeq,time,[0;.00113;0;qinitial(1);qinitial(2);qinitial(3);qinitial(4)]);
% %body to lvlh
% %ae313 calculations taken from a previously written code
% mu=398600;
% a=6778;
% f=[0:1:51];
% e=0;
% i=30;
% w=0;
% omega=0;
% p=a*(1-e^2);
% if p==0
%     p=a;
% end
% h=sqrt(p*mu);
% r=p./(1+e.*cos(f));
% rpqw=[r.*cos(f); r.*sin(f); zeros(1,length(f))];
% vpqw=(mu/h).*[-sin(f); e+cos(f); zeros(1,length(f))];
% v=[];
% for k=1:length(f)
% v(k)=norm(vpqw(:,k));
% end
%
% rot=[cos(omega) -sin(omega) 0; sin(omega) cos(omega) 0; 0 0 1]*[1 0 0; 0 cos(i) -sin(i);
 0 sin(i) cos(i)]*[cos(w) -sin(w) 0; sin(w) cos(w) 0;0 0 1];
% %for reference:
% %rot1=[cos(omega) -sin(omega) 0; sin(omega) cos(omega) 0; 0 0 1];
% %rot2=[1 0 0; 0 cos(i) -sin(i); 0 sin(i) cos(i)];
% %rot3=[cos(w) -sin(w) 0; sin(w) cos(w) 0;0 0 1];
%
% for o=1:length(f)
% vvec(:,o)=rot*vpqw(:,o);
% rvec(:,o)=rot*rpqw(:,o);
% end
%
% for j=1:51
% rlvlh(1,j)=rvec(1,j).*(X(j,4)^2-X(j,7)^2-X(j,6)^2+X(j,5)^2)+rvec(2,j).*(2*X(j,5)*X(j,6)-
2*X(j,4)*X(j,7))+rvec(3,j).*2*X(j,4)*X(j,6);
% rlvlh(2,j)=rvec(1,j).*(2*X(j,6)*X(j,5)-2*X(j,4)*X(j,7))+rvec(2,j).*(X(j,6)^2-X(j,7)^2+X(
j,4)^2-X(j,5)^2)+rvec(3,j).*(2*X(j,4)*X(j,5)-2*X(j,6)*X(j,7));
% rlvlh(3,j)=rvec(1,j).*2*X(j,4)*X(j,6)+rvec(2,j).*(2*X(j,6)*X(j,7)-2*X(j,4)*X(j,5))+rvec(
3,j).*(-X(j,7)^2-X(j,6)^2+X(j,5)^2+X(j,4)^2);
% end
% rlvlh=[zeros(1,51);rlvlh]; %from def of q multiplication
%
% [T2,X2]=ode45(@diffeq2,time,[0;0;0;1;0;0;0]);
%
% %lvlh to eci (will be using opposite)
% function dydt=diffeq(t,y)
% dy1=0;
% dy2=0;
```

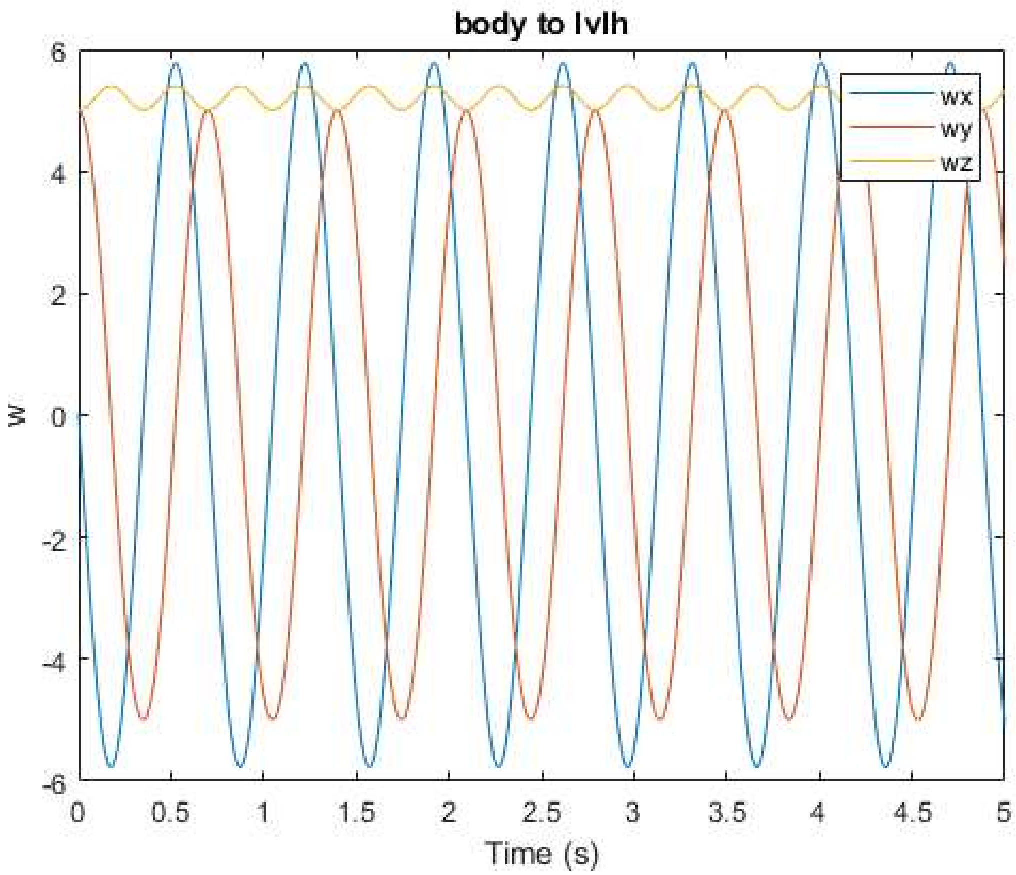
```
% dy3=0;
% dy4=.5*(-y(5)*y(1)-y(6)*y(2)-y(7)*y(3));
% dy5=.5*(y(4)*y(1)-y(7)*y(2)+y(6)*y(3));
% dy6=.5*(y(7)*y(1)+y(4)*y(2)-y(5)*y(3));
% dy7=.5*(-y(6)*y(1)+y(5)*y(2)+y(4)*y(3));
% dydt=[dy1;dy2;dy3;dy4;dy5;dy6;dy7];
% end
% %for body to lvlh disturbance
% function dydt2=diffeq2(t,y)
% %angular velocity
% dy1=1+3*mu/(6558*10^3)^5.*(rlvlh(2,1).*(2*y(6)*y(5)-2*y(4)*y(7))+rlvlh(3,1).*(y(6)^2-y(7
)^2+y(4)^2-y(5)^2)+rlvlh(4,1).*(2*y(4)*y(5)-2*y(6)*y(7))).*(rlvlh(2,1).*2*y(4)*y(6)+rlvlh(
3,1).*(2*y(6)*y(7)-2*y(4)*y(5))+rlvlh(4,1).*(-y(7)^2-y(6)^2+y(5)^2+y(4)^2))-2*y(2)*y(3);
% dy2=1+1.5*y(1)*y(3);
% dy3=2+3*mu/(6558*10^3)^5.*(rlvlh(2,1).*(2*y(6)*y(5)-2*y(4)*y(7))+rlvlh(3,1).*(y(6)^2-y(7
)^2+y(4)^2-y(5)^2)+rlvlh(4,1).*(2*y(4)*y(5)-2*y(6)*y(7))).*(rlvlh(2,1).*(y(4)^2-y(7)^2-y(6
)^2+y(5)^2)+rlvlh(3,1).*(2*y(5)*y(6)-2*y(4)*y(7))+rlvlh(4,1).*2*y(4)*y(6))-.25*y(1)*y(2);
% %qs. this format is same for all diffeq functions
% dy4=.5*(-y(5)*y(1)-y(6)*y(2)-y(7)*y(3));
% dy5=.5*(y(4)*y(1)-y(7)*y(2)+y(6)*y(3));
% dy6=.5*(y(7)*y(1)+y(4)*y(2)-y(5)*y(3));
% dy7=.5*(-y(6)*y(1)+y(5)*y(2)+y(4)*y(3));
% dydt2=[dy1;dy2;dy3;dy4;dy5;dy6;dy7];
% end
```

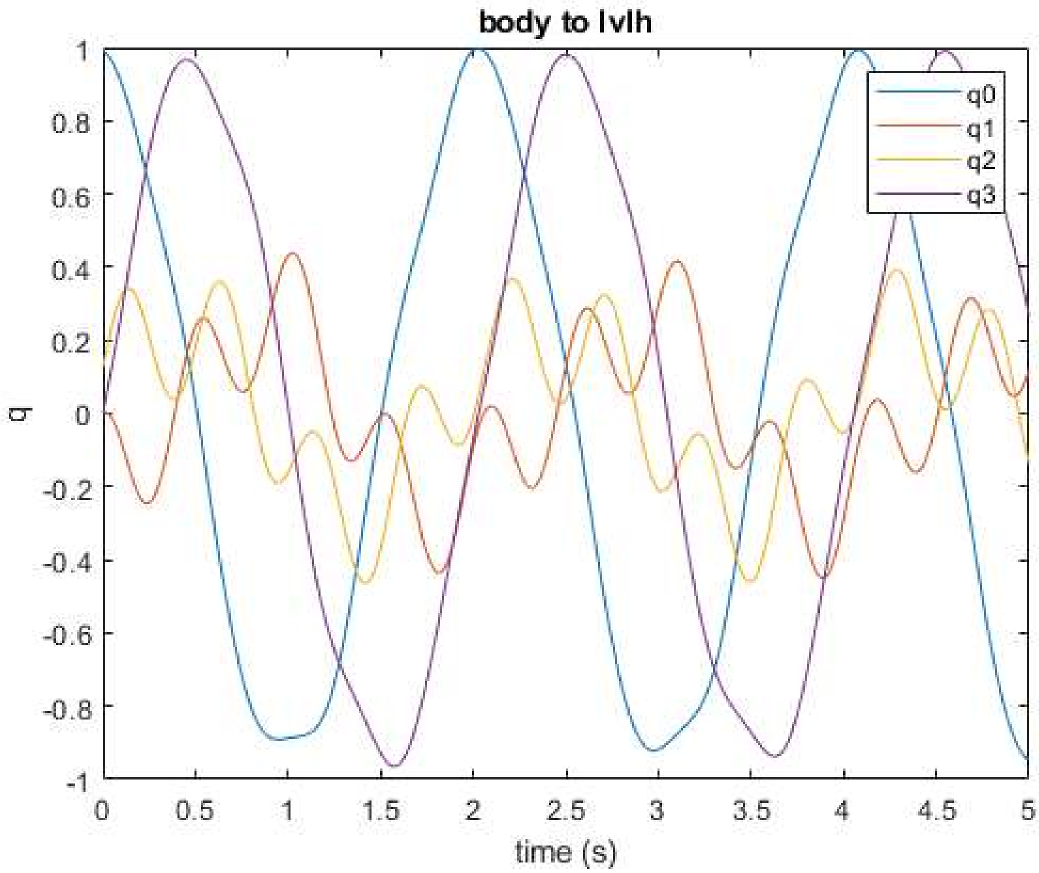


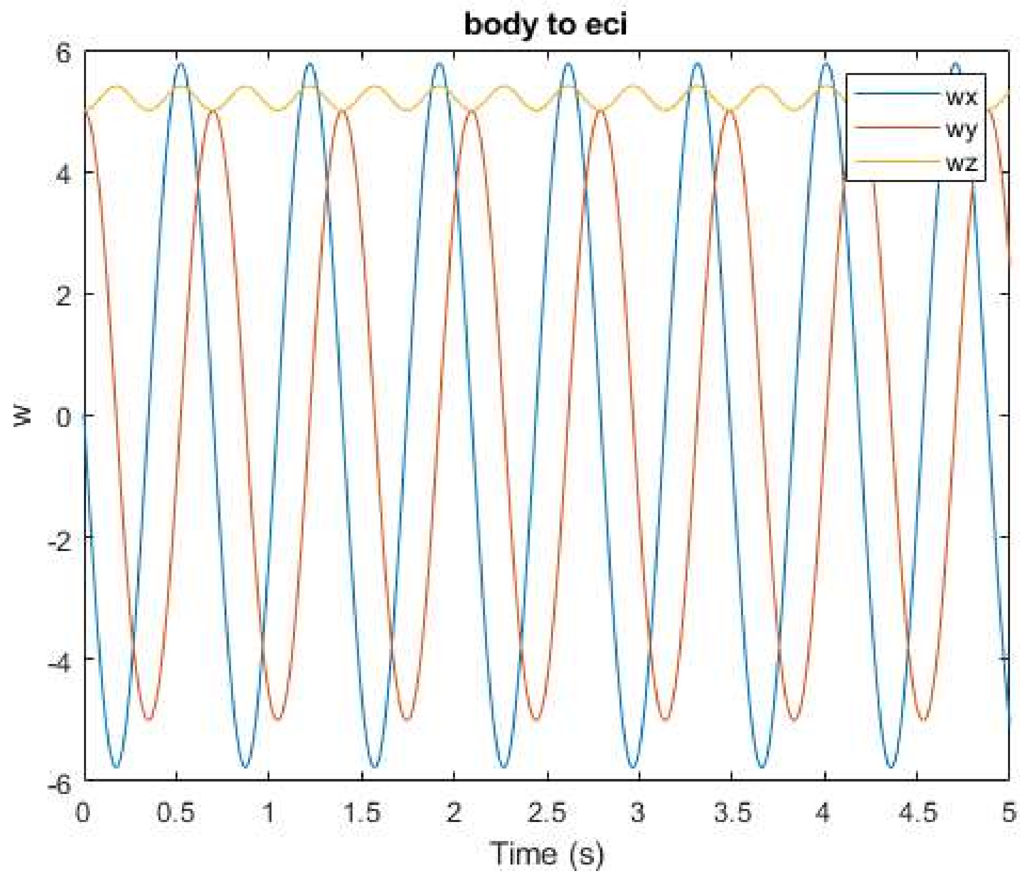

body to lvlh
q0
q1
q2
q3
q
time (s)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15

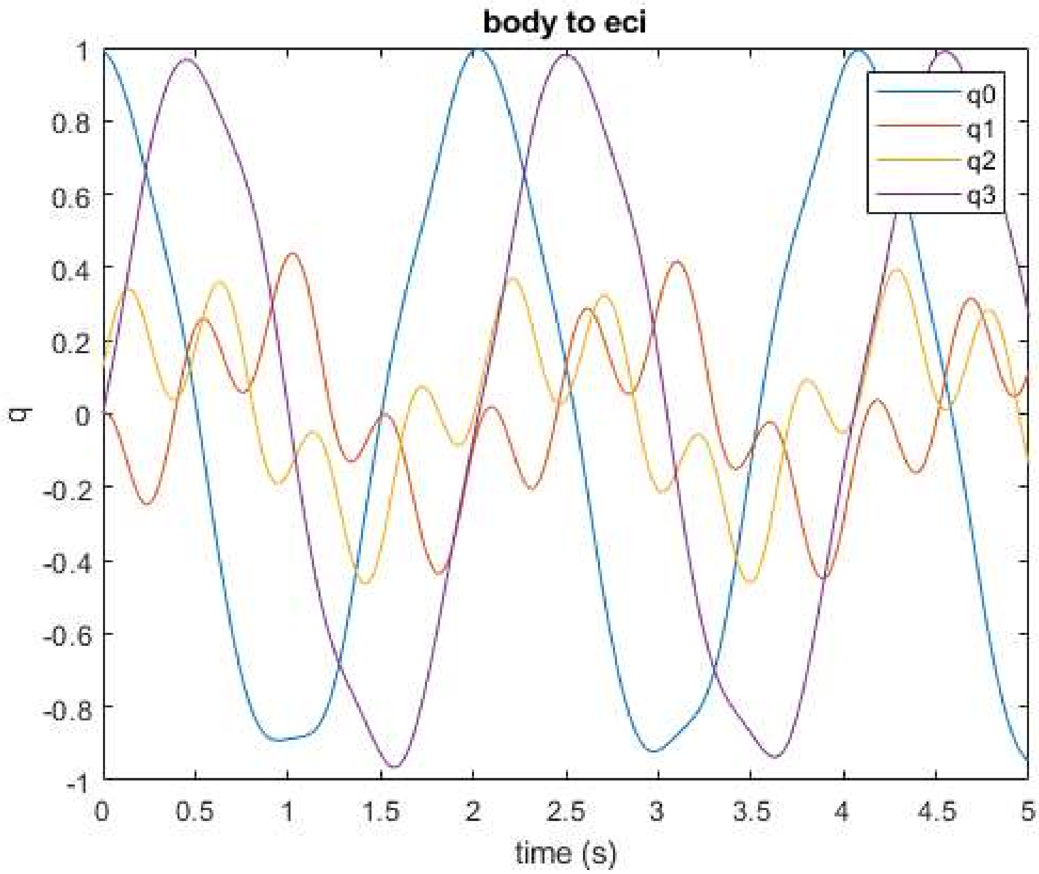

body to eci
wx
wy
wz
w
Time (s)
6
5
4
3
2
1
0
-1
0
5
10
15


body to eci
q0
q1
q2
q3
q
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
time (s)


body to lvlh
wx
wy
wz
w
6
4
2
0
-2
-4
-6
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Time (s)


body to lvlh
q
time (s)
q0
q1
q2
q3
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


body to eci
w
Time (s)
wx
wy
wz
6
4
2
0
-2
-4
-6
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5



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References

[1] Curtis, Howard D., *Orbital Mechanics for Engineering Students low*, 4th ed Elsevier, 2009.