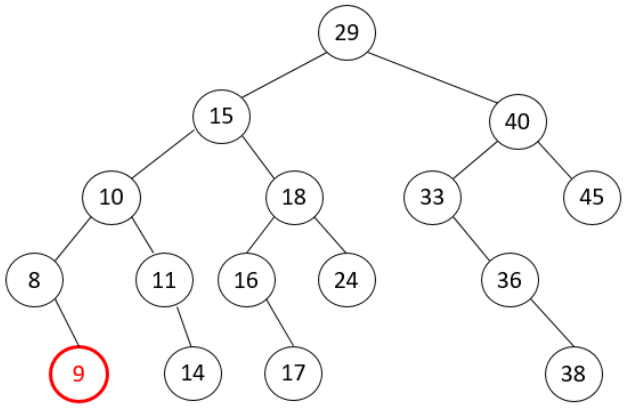
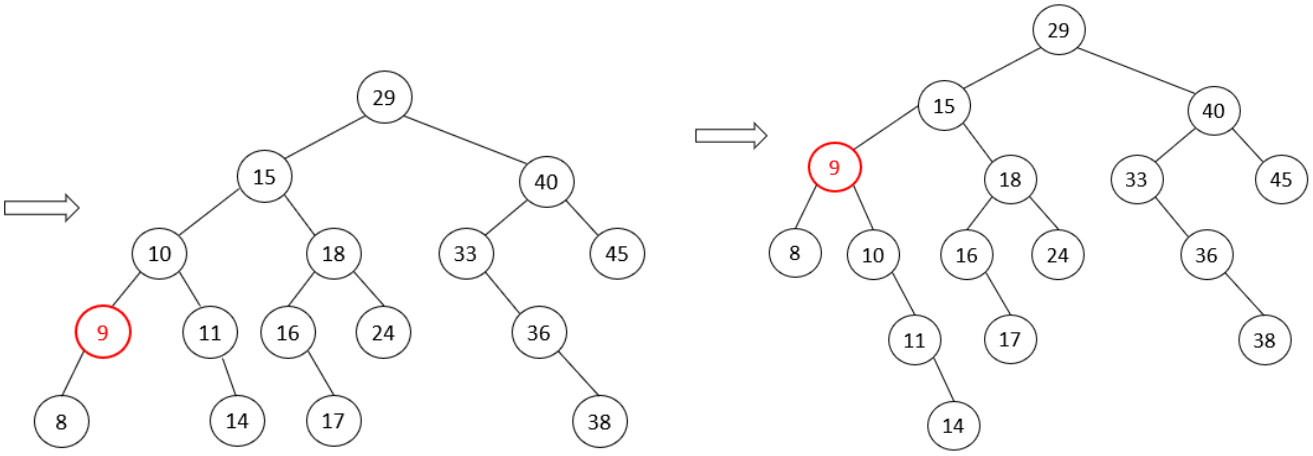
Com S 228
Fall 2016
Final Exam Sample Solution

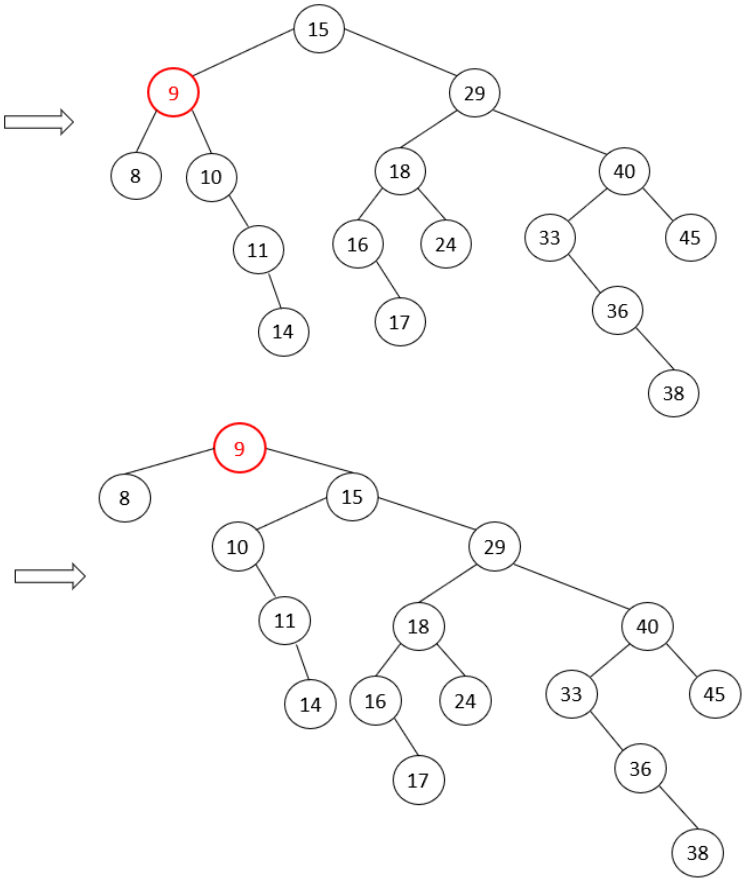
1a) 4
b) 0
c) 3
d) 40
e) 24
f) 8, 14, 11, 10, 17, 16, 24, 18, 15, 38, 36, 33, 45, 40, 29
g) BST insert:



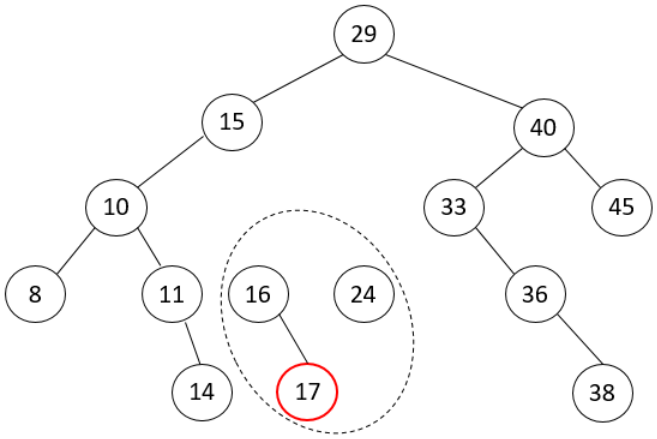

Zig-zag:



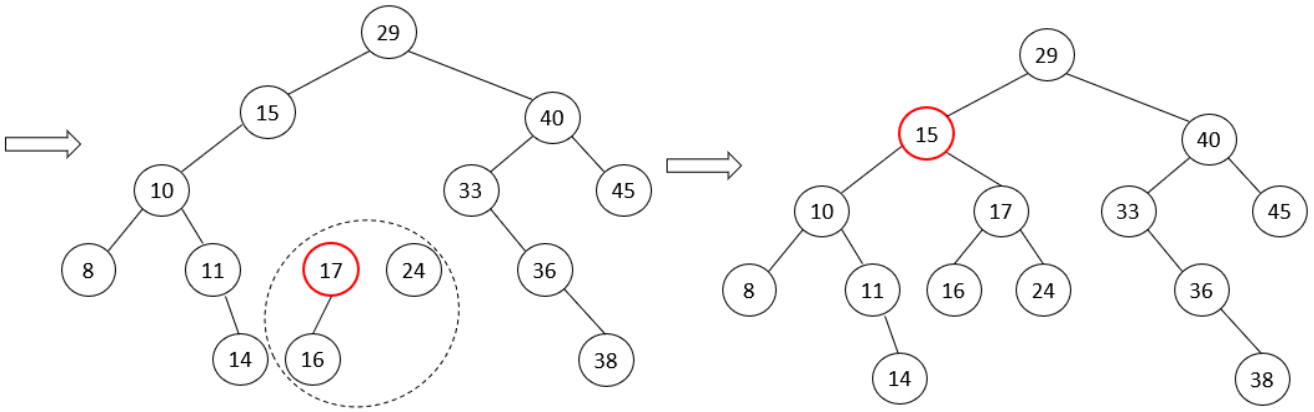

Zig-zig:



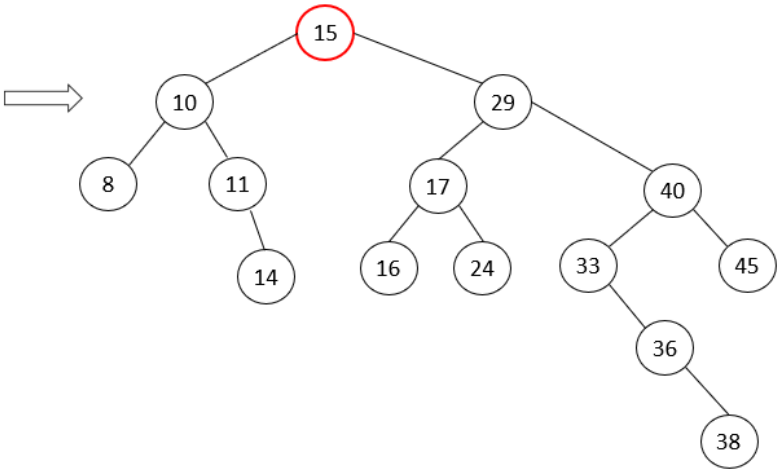

h) Remove 18:



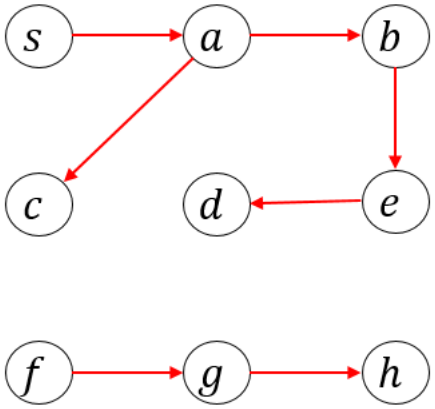

Join the subtrees of 18:



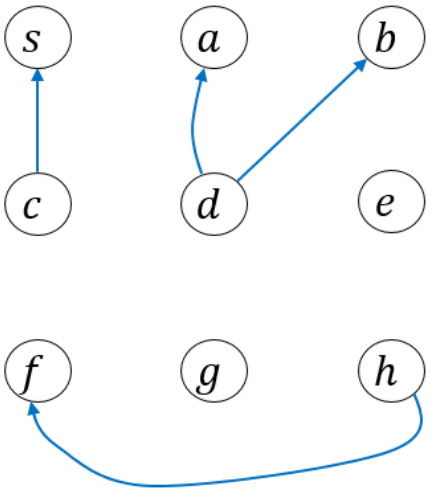

Splay at the (former) parent 15 (zig):



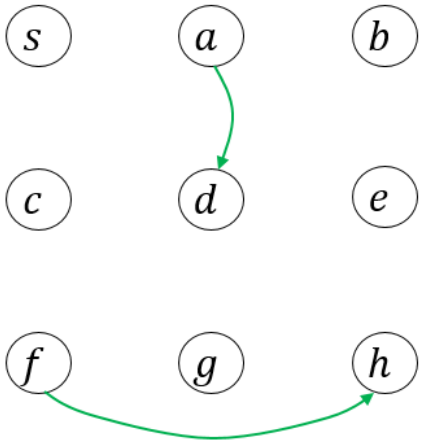

2a) 5
b) 2
c) Yes
d) DFS forest



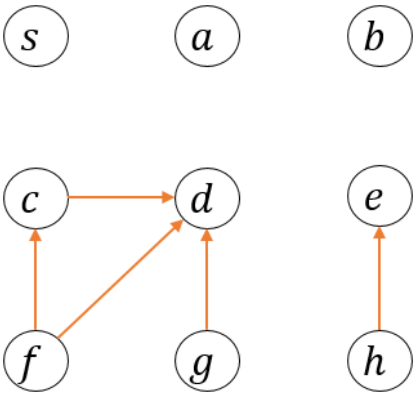
e) Back edges



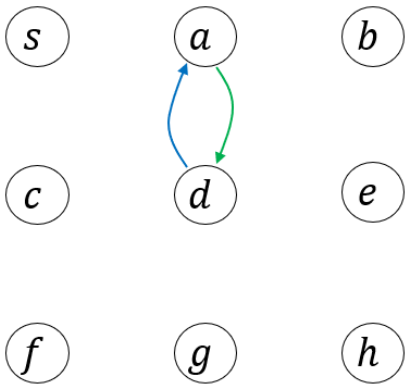

f) Forward edges




g) Cross edges




h) Seven simple cycles:




$a, d$
$f, h$
$s, a, c$
$a, c, d$
$a, b, e, d$
$b, e, d$
$f, g, h$

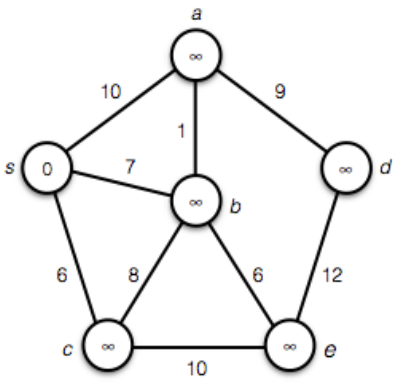
3. There exists a unique sorting result.

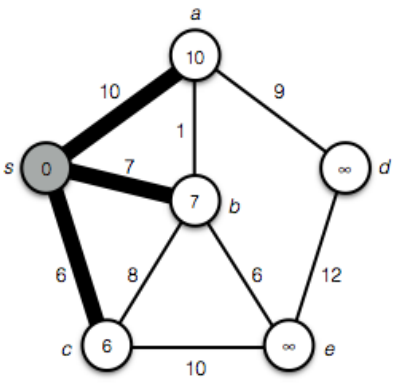
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $c$ | $f$ | $d$ | $g$ | $h$ | $e$ | $a$ | $b$ |

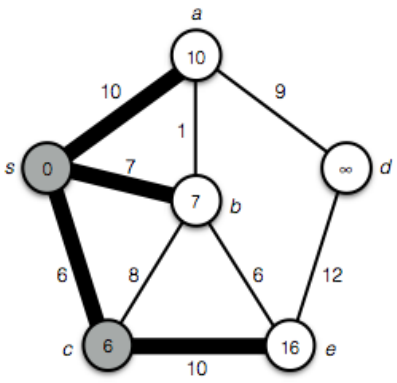
4.

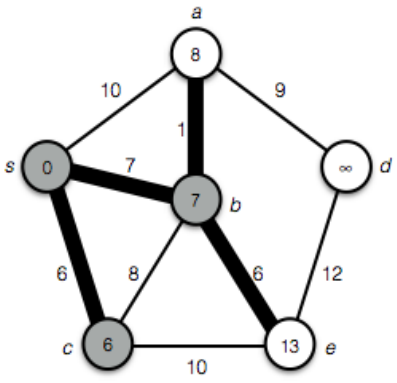
a) $O(V + E)$
b) $O(V + E)$
c) $O(V^2)$
d) $O(V + E)$
e) $O(V + E)$

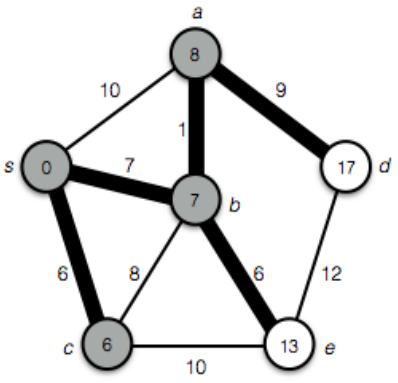
5.

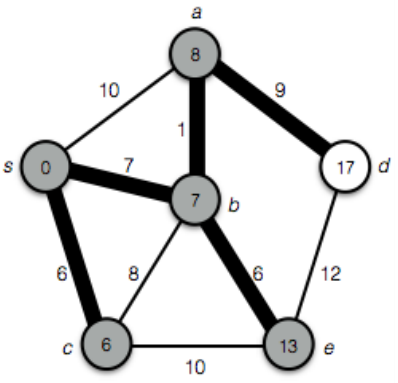

a
∞
10
9
1
s
0
7
∞
b
∞
d
6
8
6
12
c
∞
∞
e
10

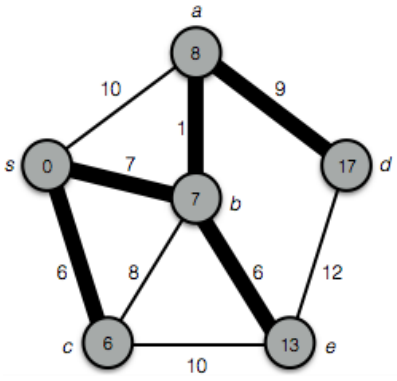

a
10
10
9
1
s
0
7
7
b
∞
d
6
8
6
12
c
6
∞
e
10


a
10
10
9
1
s
0
7
7
b
∞
d
6
8
6
12
c
6
16
e
10


a
8
10
9
1
s
0
7
7
b
∞
d
6
8
6
12
c
6
13
e
10


a
8
10
9
1
s
0
7
7
b
17
d
6
8
6
12
c
6
13
e
10


a
8
10
9
1
s
0
7
7
b
17
d
6
8
6
12
c
6
13
e
10


a
8
10
9
1
s
0
7
7
b
17
d
6
8
6
12
c
6
13
e
10

6.

| Row | | | | Array | | | |
|---|---|---|---|---|---|---|---|
| **0** | 3 | 4 | 6 | 5 | 2 | 0 | 1 |
| **1** | 3 | 5 | 6 | 4 | 2 | 0 | 1 |
| **2** | **6** | **5** | **3** | **4** | **2** | **0** | **1** |
| **3** | 1 | 5 | 3 | 4 | 2 | 0 | 6 |
| **4** | 5 | 1 | 3 | 4 | 2 | 0 | 6 |
| **5** | 5 | 4 | 3 | 1 | 2 | 0 | 6 |
| **6** | 0 | 4 | 3 | 1 | 2 | 5 | 6 |
| **7** | 4 | 0 | 3 | 1 | 2 | 5 | 6 |
| **8** | 4 | 2 | 3 | 1 | 0 | 5 | 6 |
| **9** | 0 | 2 | 3 | 1 | 4 | 5 | 6 |
| **10** | 3 | 2 | 0 | 1 | 4 | 5 | 6 |
| **11** | 1 | 2 | 0 | 3 | 4 | 5 | 6 |
| **12** | 2 | 1 | 0 | 3 | 4 | 5 | 6 |
| **13** | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| **14** | 1 | 0 | 2 | 3 | 4 | 5 | 6 |
| **15** | 0 | 1 | 2 | 3 | 4 | 5 | 6 |

7a)

```
/**
 * Perform a left rotation on the edge between the parent node p
 * and its right child node r.
 *
 * @param p      parent node
 * @param r      child node
 * @throws NullPointerException  if either p or r is null
 * @throws IllegalArgumentException  if neither p nor r is null
 *                                   but r is not a right child of p
 */
public void leftRotate(Node<E> p, Node<E> r)
          throws NullPointerException, IllegalArgumentException
{
     // handle exceptions.
     //
     // insert code below (4 pts)
     if (p == null || r == null)
          throw new NullPointerException();
     if (p.right != r)
          throw new IllegalArgumentException();

     // make the left subtree of r the new right subtree of p.
     //
     // insert code below (3 pts)
     p.right = r.left;
     if (r.left != null)
          r.left.parent = p;

     // establish the relationship between r and the parent of p
     // (if p has one).
     //
     // insert code below (6 pts)
     Node<E> g = p.parent;    // grandparent of r
     r.parent = g;
     if (g == null)           // p is the root.
          root = r;
     else
     {
          if (g.left == p)
               g.left = r;
          else
               g.right = r;
     }

     // reverse the parent-child relationship between p and r.
```

```
        //
        // insert code below (2 pts)
        r.left = p;
        p.parent = r;
    }
```

b)

```
    /**
     * Change the binary tree into its mirror image.
     */
    public void mirrorImage()
    {
        // insert code below (1 pt)

        mirrorImageRec(root);
    }

    /**
     * Replace the subtree rooted at n with its mirror image. Link
     * updates only. No creation of a new node.
     *
     * @param n  root of the subtree to be replaced with its mirror
     *           image.
     */
    private void mirrorImageRec(Node<E> n)
    {
        // handle the case n == null
        //
        // insert code below (1 pt)
        if (n == null) return;

        // swap the left and right subtrees and then generate their
        // mirror images.
        //
        // insert code below (5 pts)
        Node<E> tmp = n.left;
        n.left = n.right;
        n.right = tmp;

        mirrorImageRec(n.left);
        mirrorImageRec(n.right);
    }
```