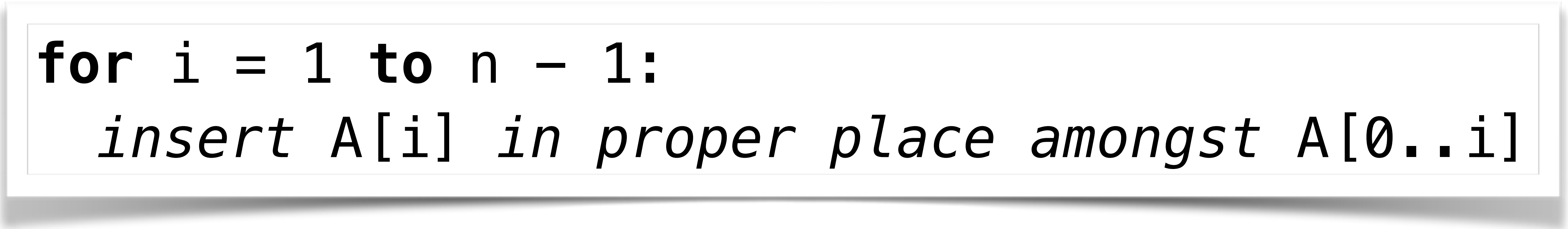
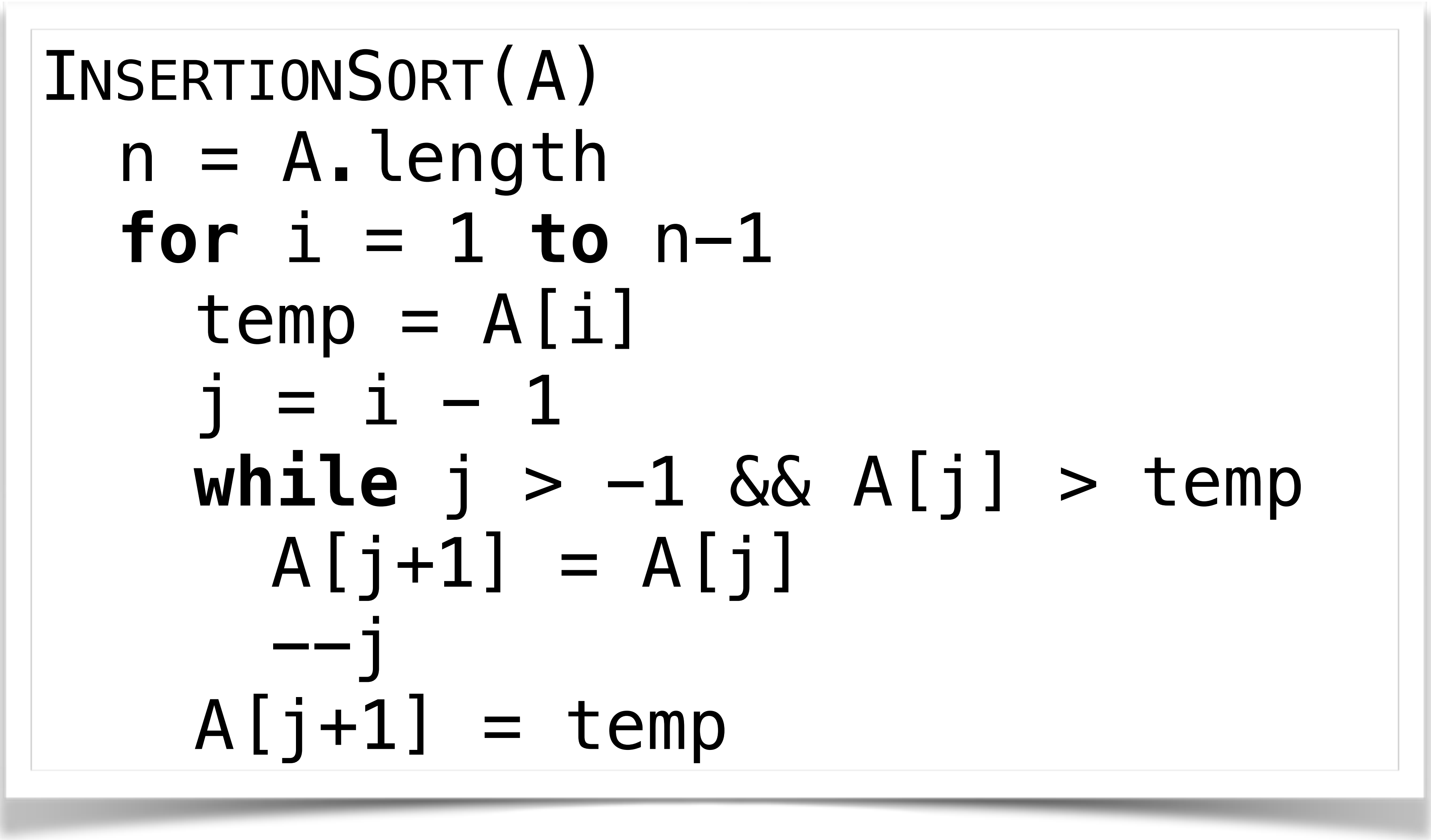
Insertion Sort

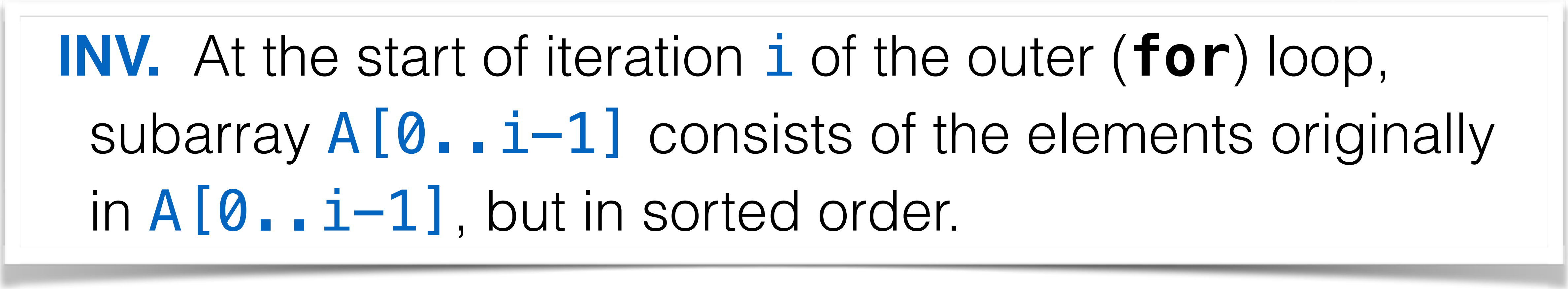


```
for i = 1 to n − 1:
  insert A[i] in proper place amongst A[0..i]
```



```
InsertionSort(A)
  n = A.length
  for i = 1 to n−1
    temp = A[i]
    j = i − 1
    while j > −1 && A[j] > temp
      A[j+1] = A[j]
      −−j
    A[j+1] = temp
```

Loop Invariant



INV. At the start of iteration `i` of the outer (**`for`**) loop, subarray `A[0..i-1]` consists of the elements originally in `A[0..i-1]`, but in sorted order.

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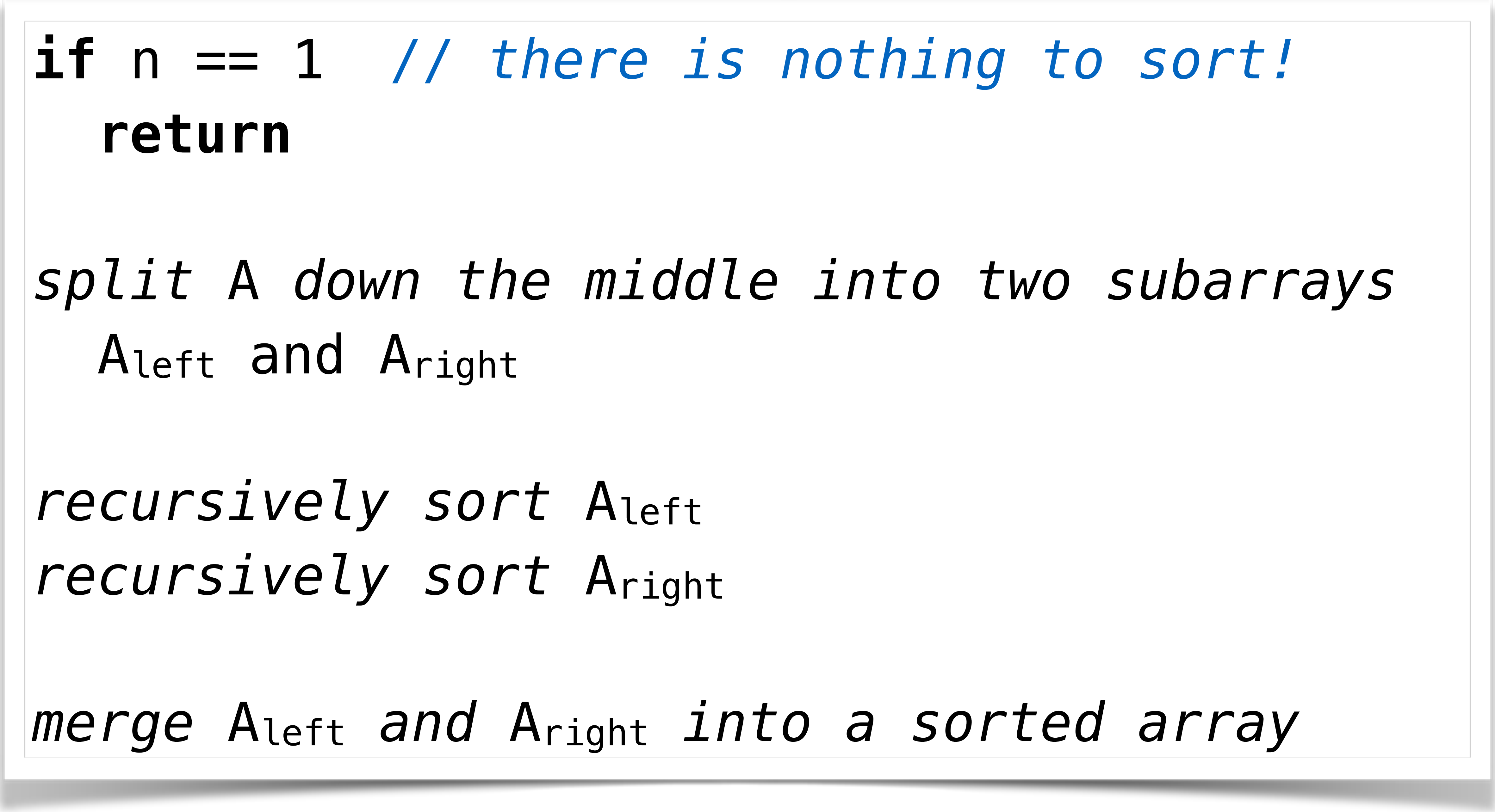
- (Initialization) INV is true at outset.
- (Maintenance) Each iteration of **`for`** loop maintains INV through shifting and insertion.
- (Termination) INV && (`i = n`)
  ⇒ `A[0..n−1]` — *the whole array* — is sorted.

Merge Sort



```
if n == 1  // there is nothing to sort!
   return

split A down the middle into two subarrays
   $A_{left}$ and $A_{right}$

recursively sort $A_{left}$
recursively sort $A_{right}$

merge $A_{left}$ and $A_{right}$ into a sorted array
```

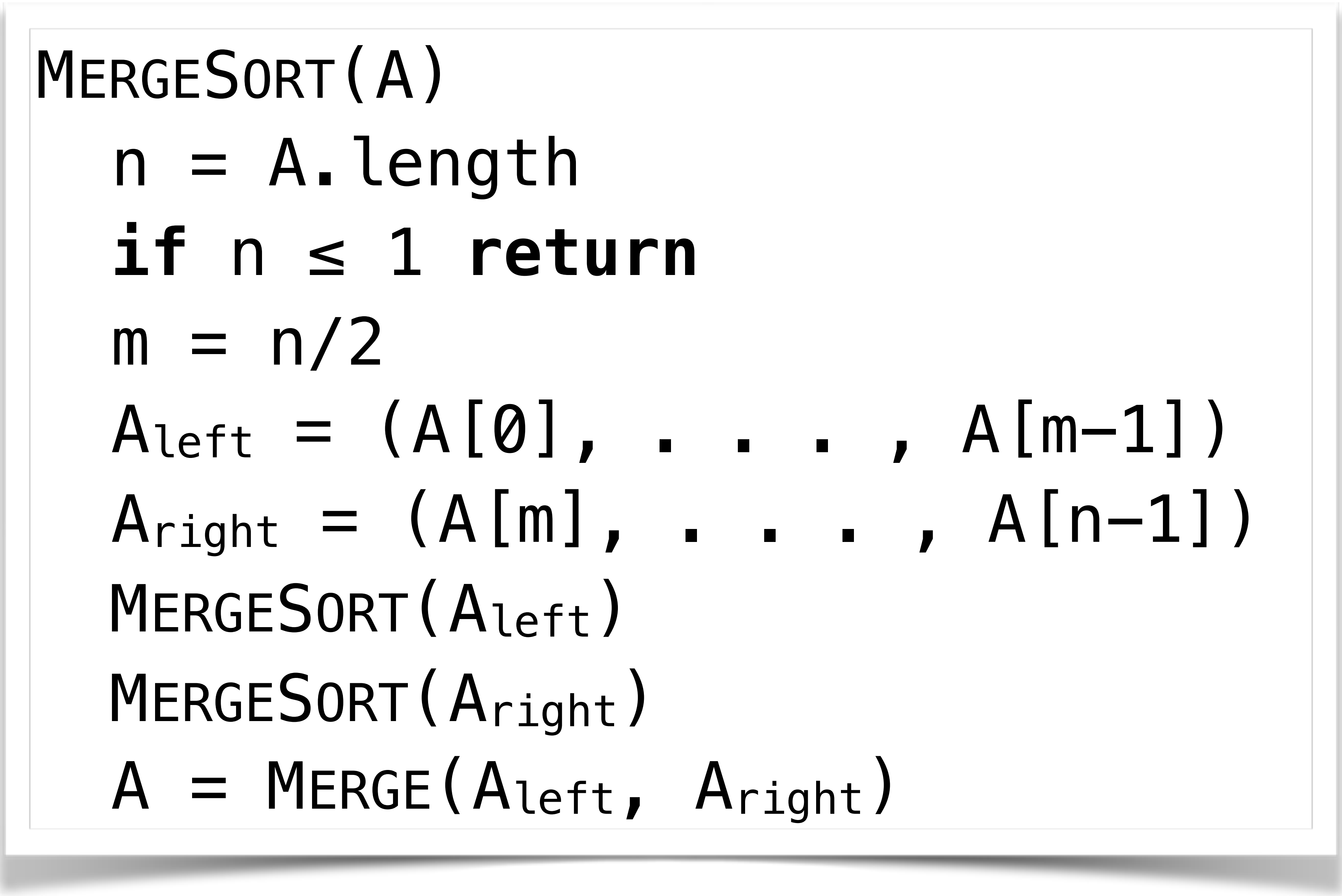
Input 8 7 3 5 2 1

Input	8	7	3	5	2	1
1. Split array down the middle	8	7	3	5	2	1

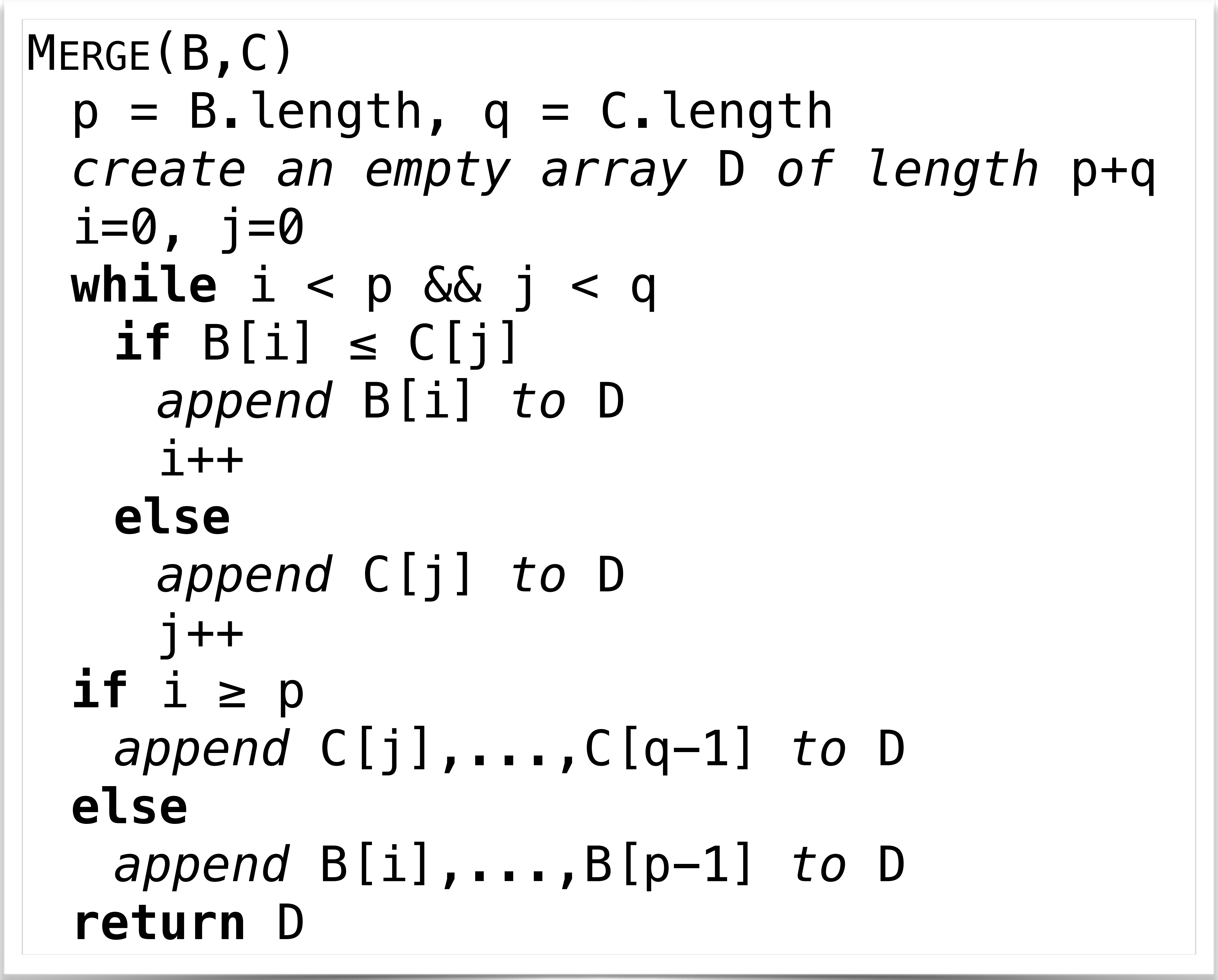
Input	8	7	3	5	2	1
1. Split array down the middle	8	7	3	5	2	1
2. Recursively sort left half	3	7	8	5	2	1

Input	8	7	3	5	2	1
1. Split array down the middle	8	7	3	5	2	1
2. Recursively sort left half	3	7	8	5	2	1
3. Recursively sort right half	3	7	8	1	2	5

Input	8	7	3	5	2	1
1. Split array down the middle	8	7	3	5	2	1
2. Recursively sort left half	3	7	8	5	2	1
3. Recursively sort right half	3	7	8	1	2	5
4. Merge two halves to make a sorted whole	1	2	3	5	7	8

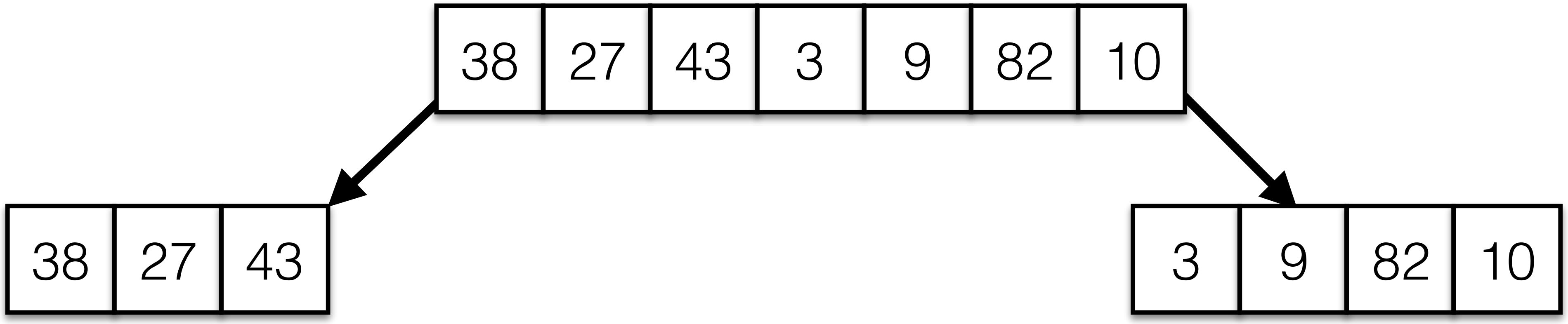


```
MERGESORT(A)
    n = A.length
    **if** $n \le 1$ **return**
    m = n/2
    $A_{left}$ = (A[0], . . . , A[m−1])
    $A_{right}$ = (A[m], . . . , A[n−1])
    MERGESORT($A_{left}$)
    MERGESORT($A_{right}$)
    A = MERGE($A_{left}$, $A_{right}$)
```



```
MERGE(B,C)
  p = B.length, q = C.length
  create an empty array D of length p+q
  i=0, j=0
  while i < p && j < q
    if B[i] ≤ C[j]
      append B[i] to D
      i++
    else
      append C[j] to D
      j++
  if i ≥ p
    append C[j],...,C[q−1] to D
  else
    append B[i],...,B[p−1] to D
  return D
```

38	27	43	3	9	82	10



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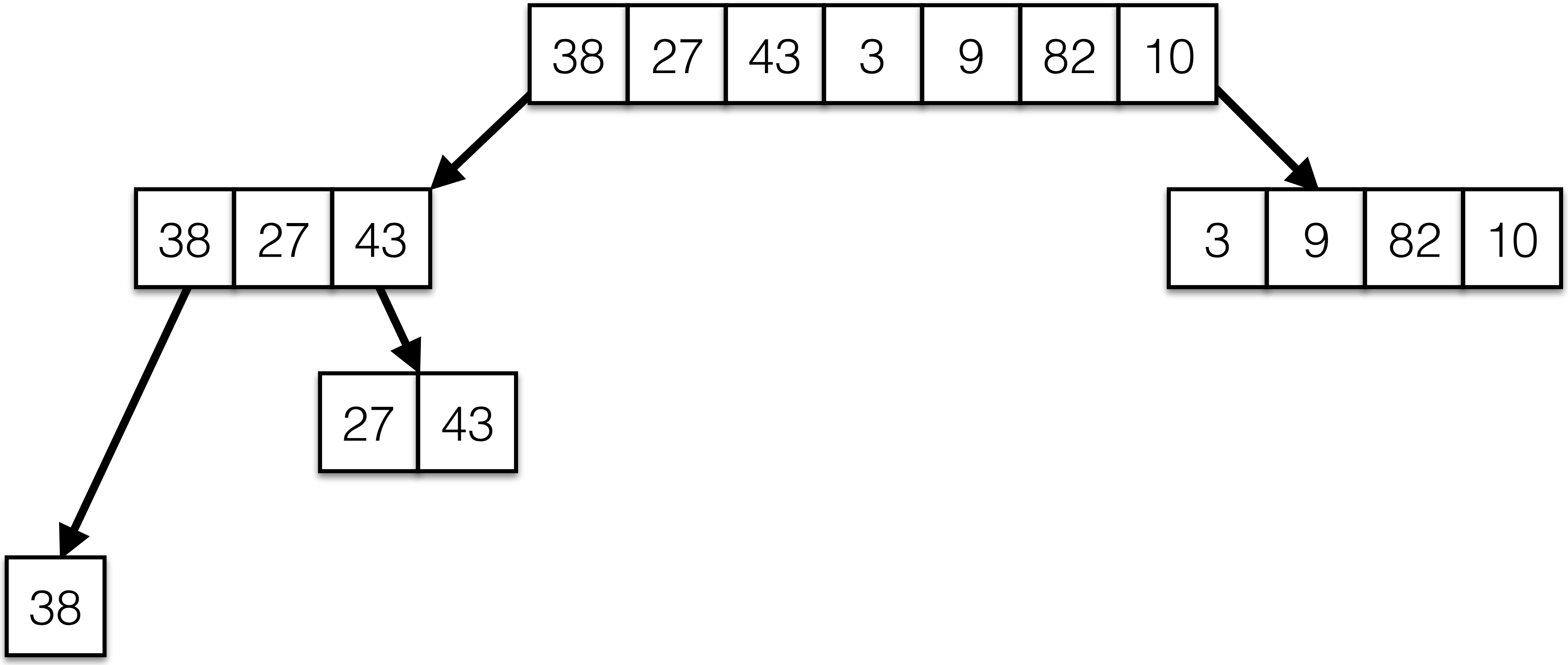

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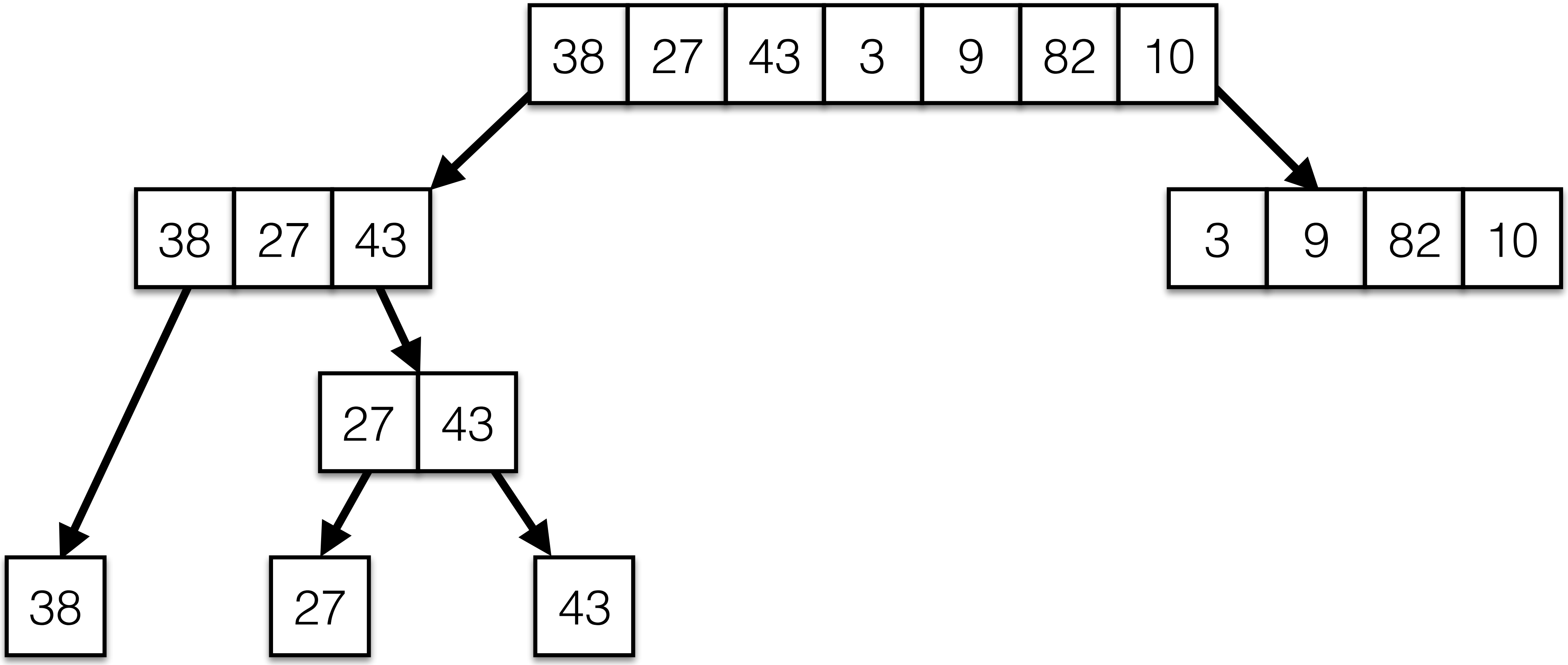

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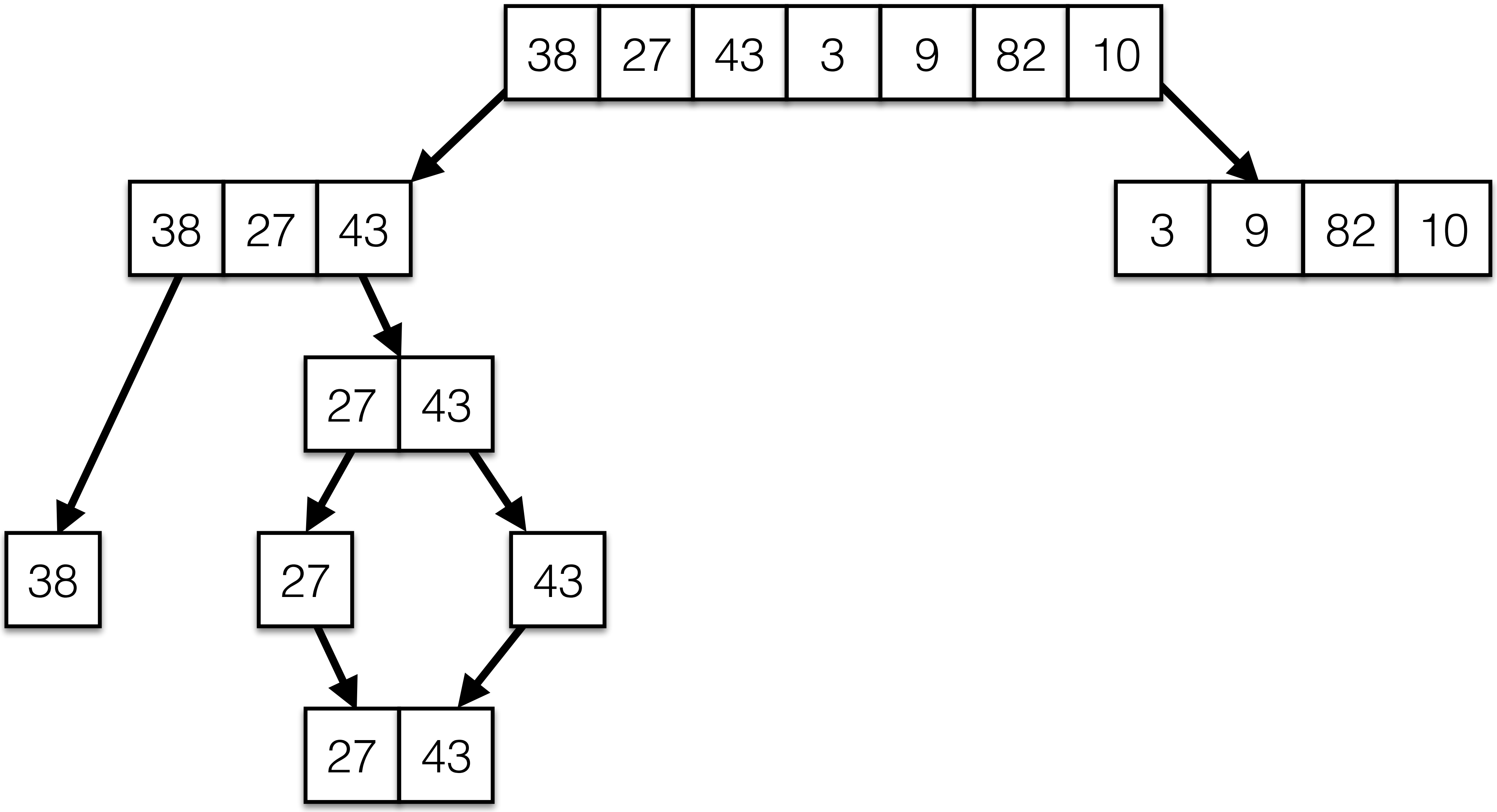

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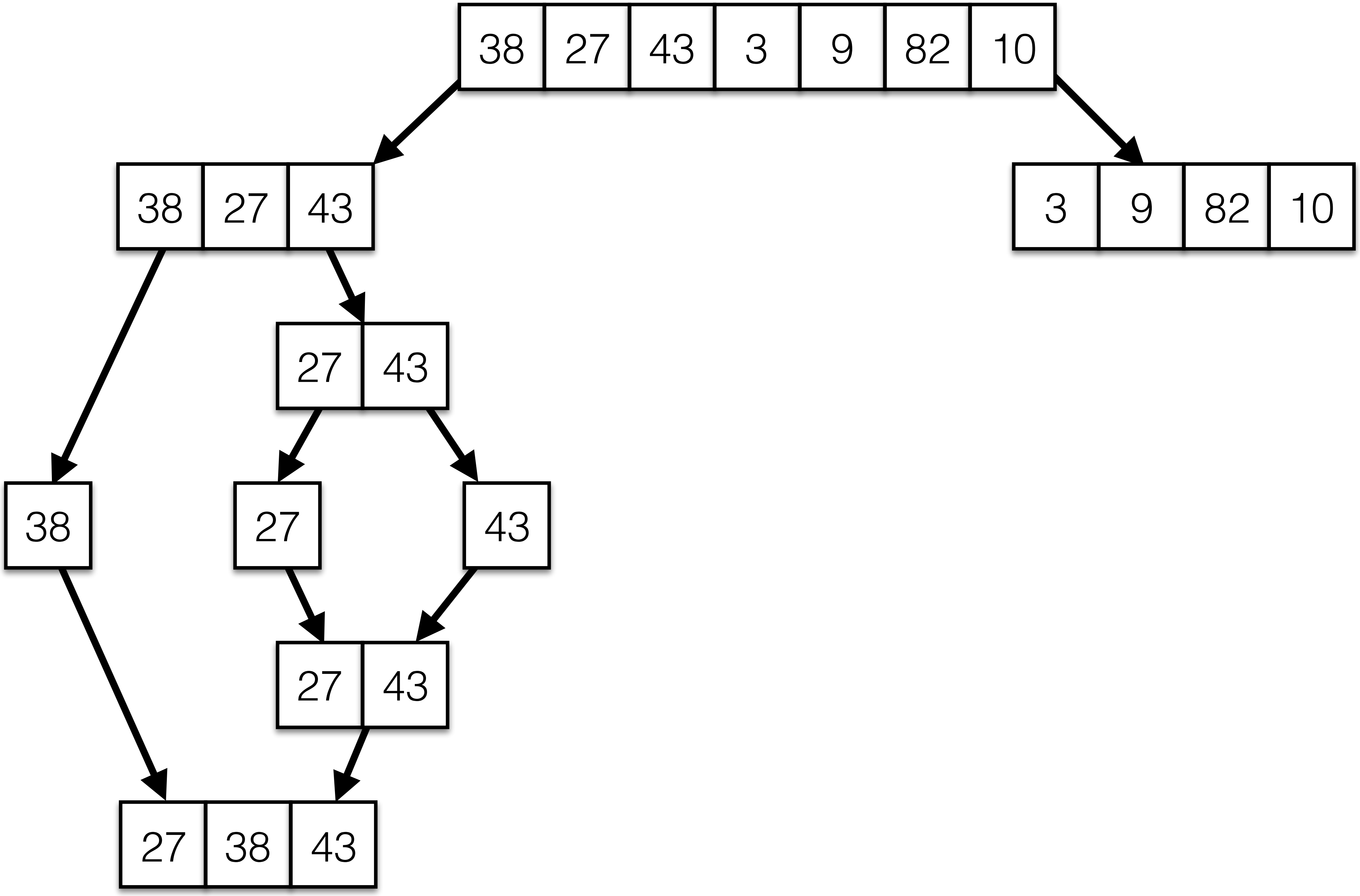

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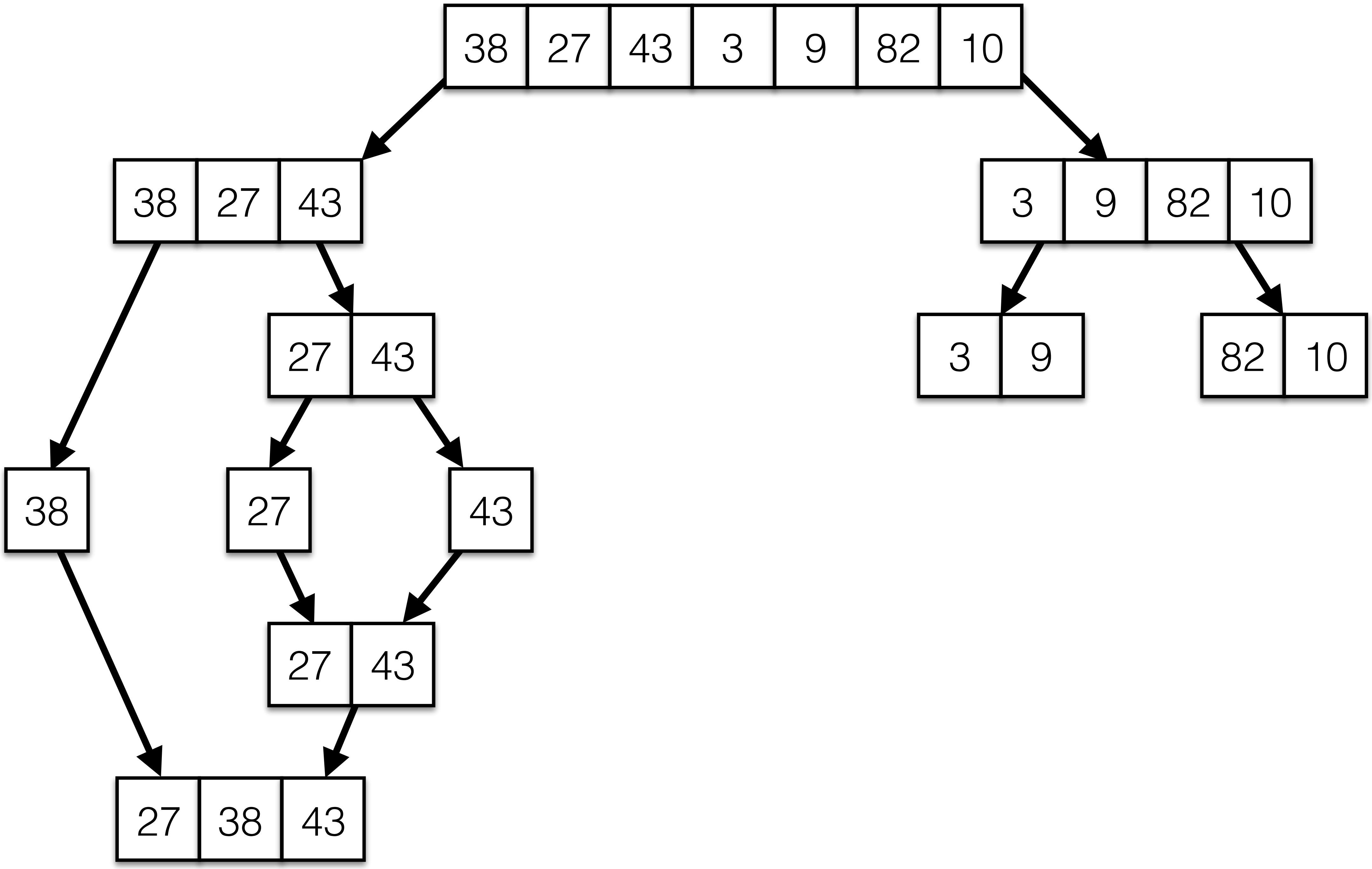
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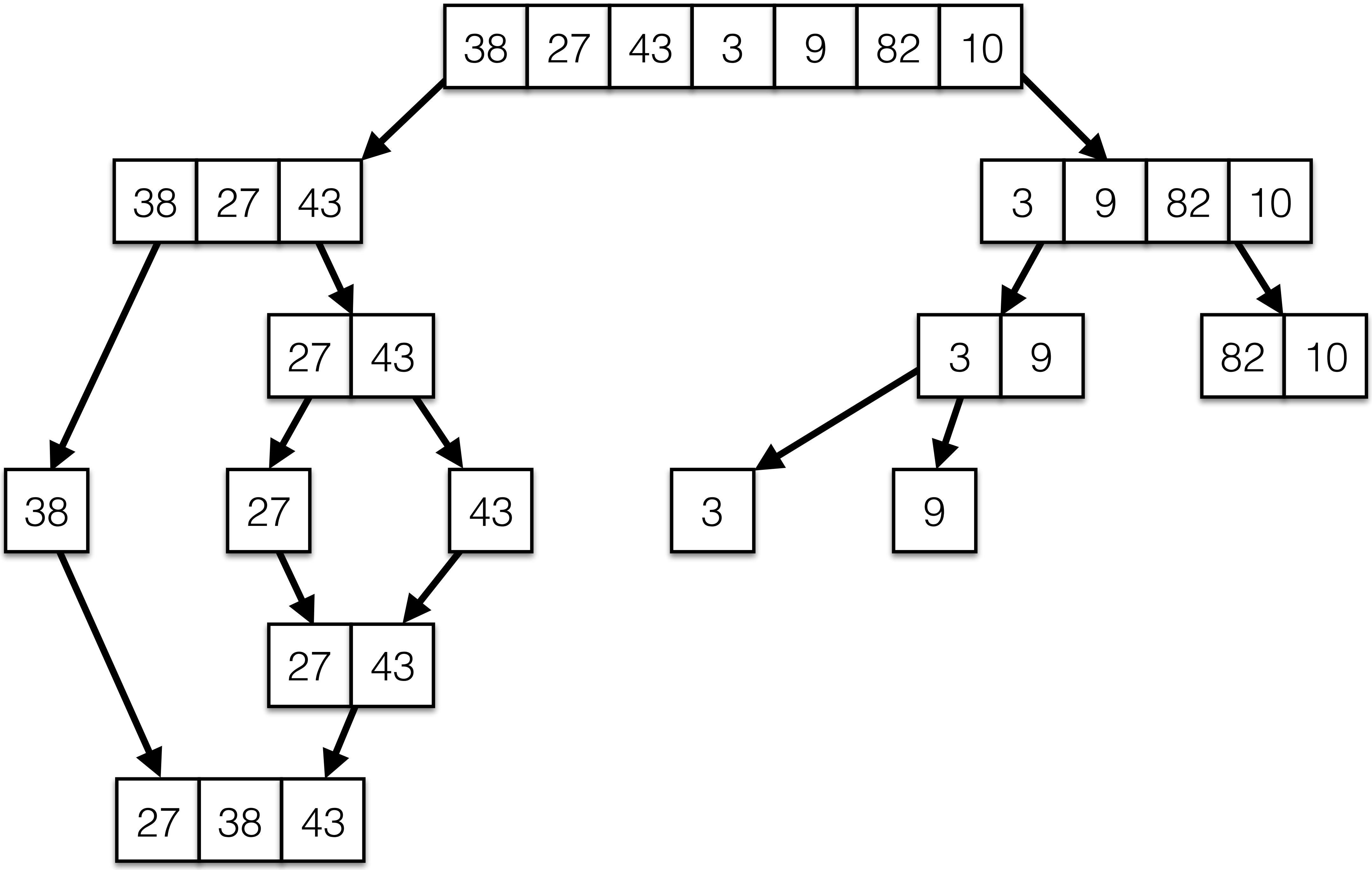

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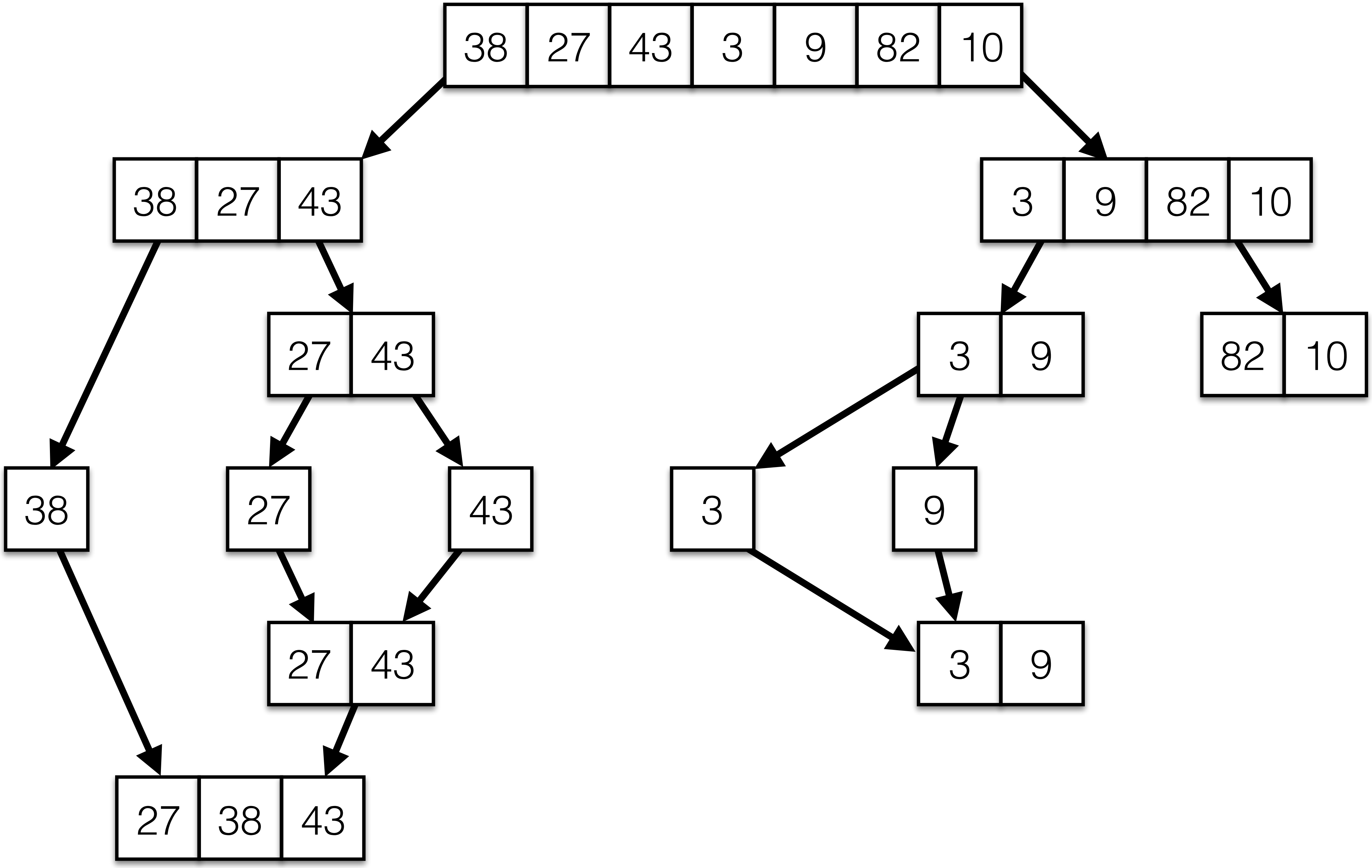
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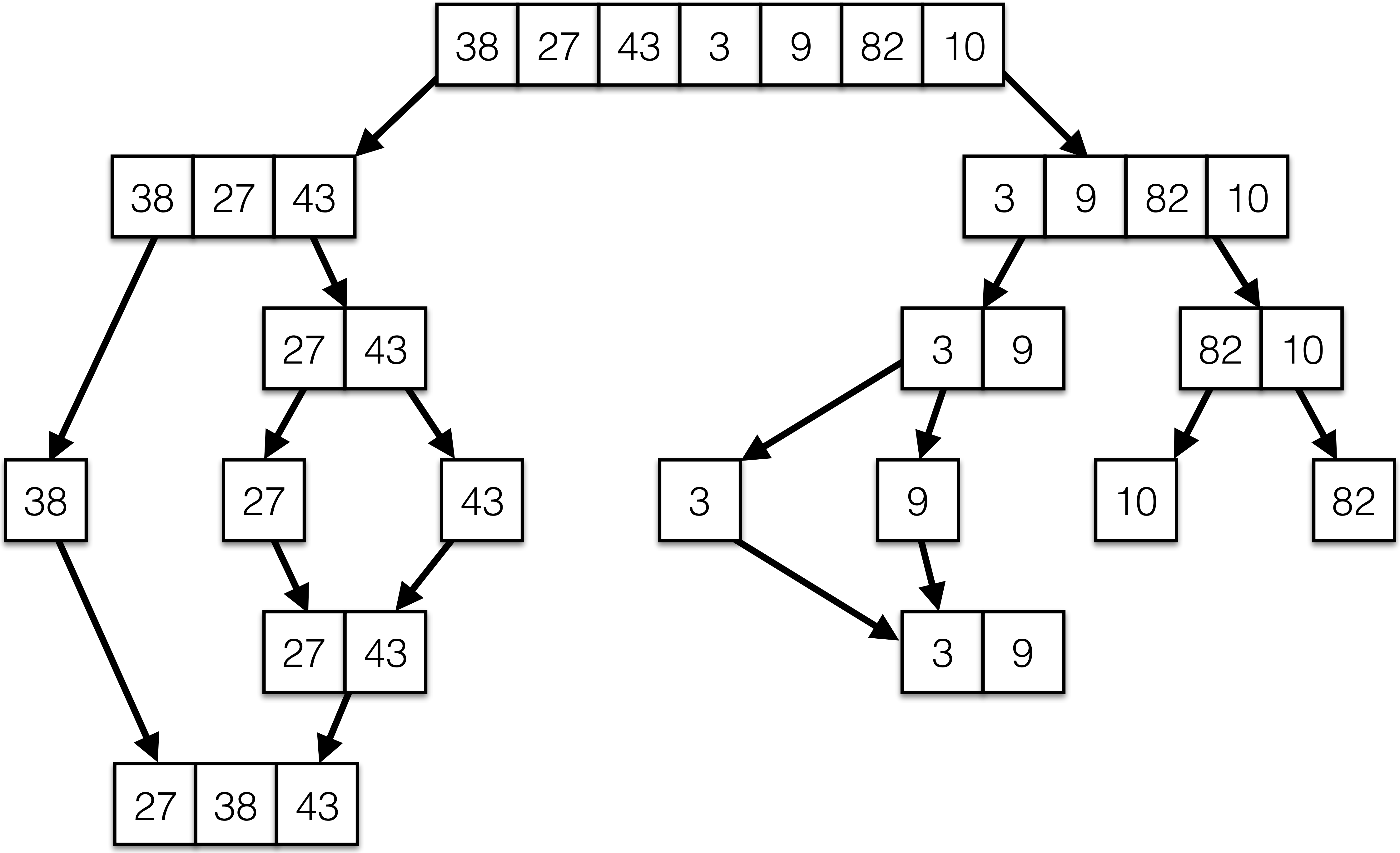

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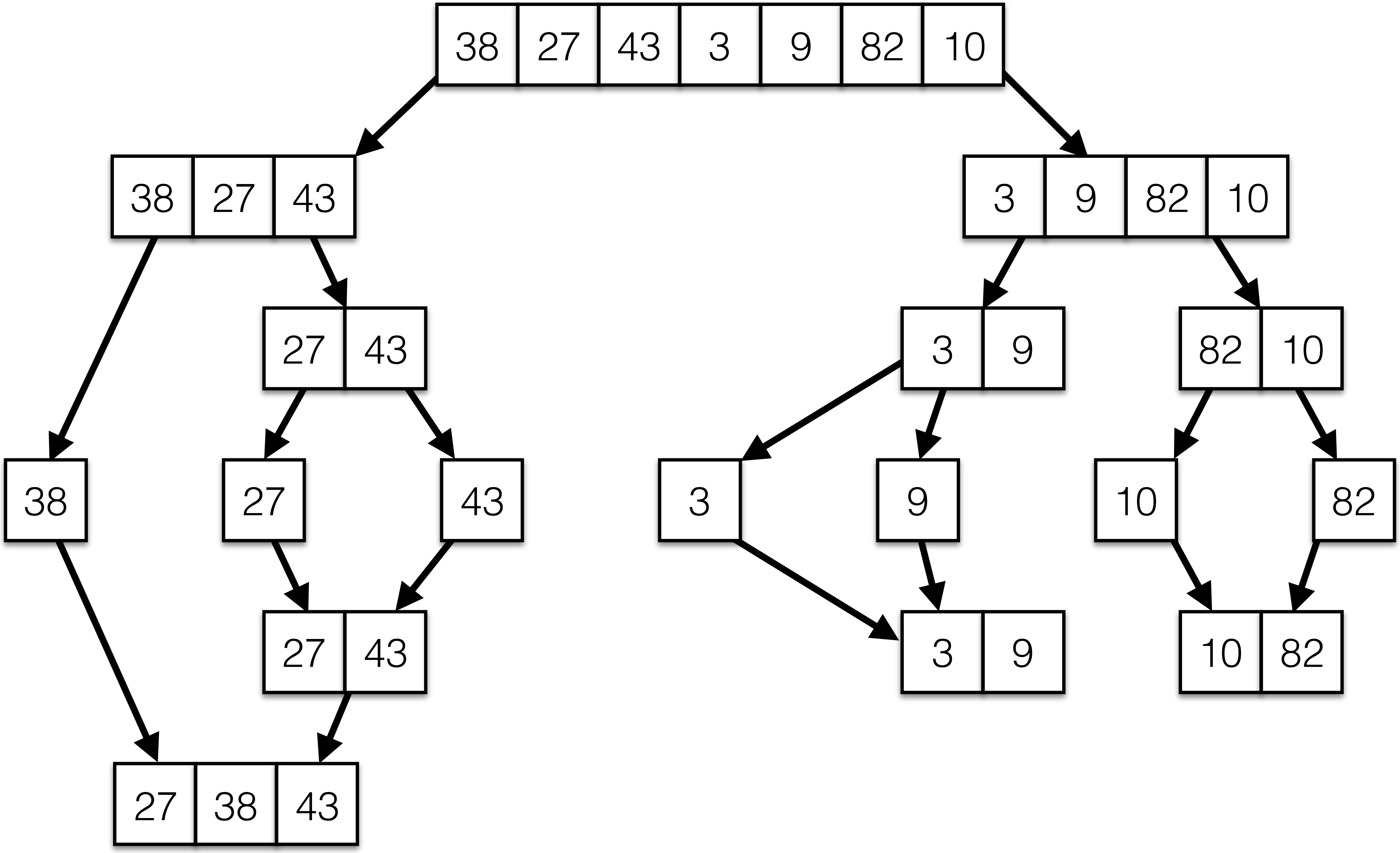
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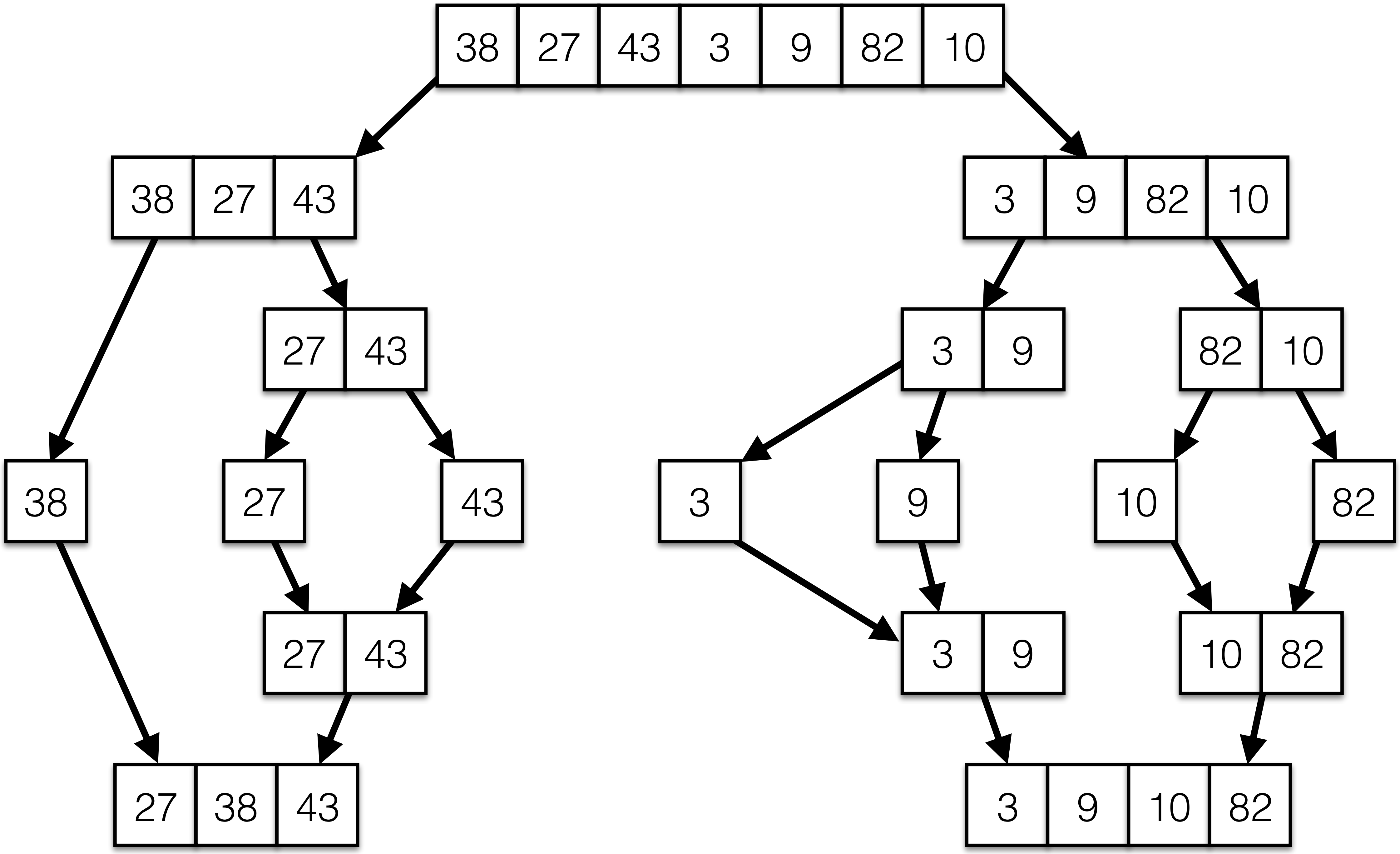
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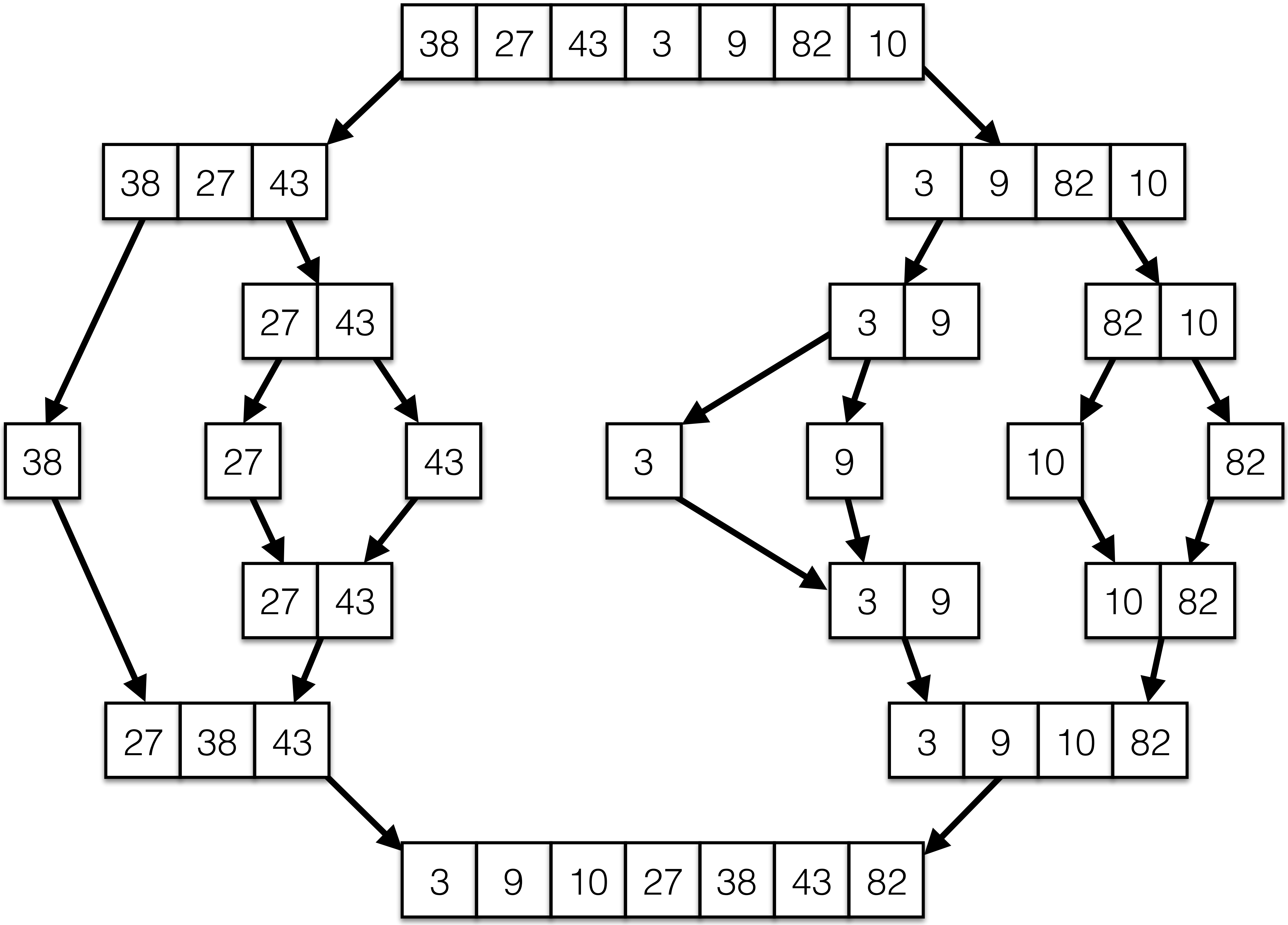

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