Learner Name (s): Binyameen Mohamedy_______________ Grade(s): 11_______
School Name: UJ Academy________________ Region: Gauteng South__________
Project Title: Recursive Prime Generating Functions________________________

Purpose

Let a(n) = a(n-1) + b(n), b(n) = GCD(a(n-1),n)

a(h)=k, initial condition.

GCD(a,b) is the greatest common divisor of a and b.

The aim of the project was to prove a mathematical conjecture. A function, b(n), was discovered in 2003, conjectured to only produce 1 or prime numbers as values. This problem ended up being surprisingly difficult to prove, remaining unsolved until this day. This project is focused on 2 Conjectures, including the one proposed in 2003. Conjecture 2 states, the function, b(n), produces infinitely many distinct prime numbers.

Method

Initial investigations were conducted by analysing numerical and graphical data produced through a graphing calculator and code. To prove Conjecture 1, the problem was reformulated into instead proving a certain sum is positive/ greater then 0. Doing so requires proving certain inequalities for the function which were done through estimating sums of b(n). For the second conjecture, a heuristic proof is given in which it is proved that for infinitely many p, the function, b(p)=p. This shows the function is prime for infinitely many prime numbers. That was done by showing it happens when the function a(n) reaches its minimum value, this prime is generated and has the next term a(n+1) achieve its maximum possible value.

Results

Conjecture 1 is solved and proved true, thus for example if a(1)=7, than b(n) is prime or 1 for every n>1. A heuristic proof is developed that shows Conjecture 2 is true. Minor identities are discovered on the limits of the function, as well as a criteria determining what the value $\lim_{n\to\infty} \frac{a(n)}{n}$ is. This criterion determines the types of primes produced.

Conclusion

2, previously unsolved, mathematical problems were solved, furthering a new theory and the study of prime numbers whilst also furthering South African mathematics. Pure Mathematics has applications everywhere, such as prime numbers in encryption systems, non-Euclidean geometry in Einsteins theory of relativity, complex analysis in the physics of the atomic bomb, Navier-Stokes equations in Computational Fluid Dynamics/ CFD simulations etc. progress in mathematics is to the interest of every field.