GATE MADE SIMPLE

9911704170

9899961331

@chewithankursir

t.me/ankur073

@ankurbansalsir

ANKUR BANSAL

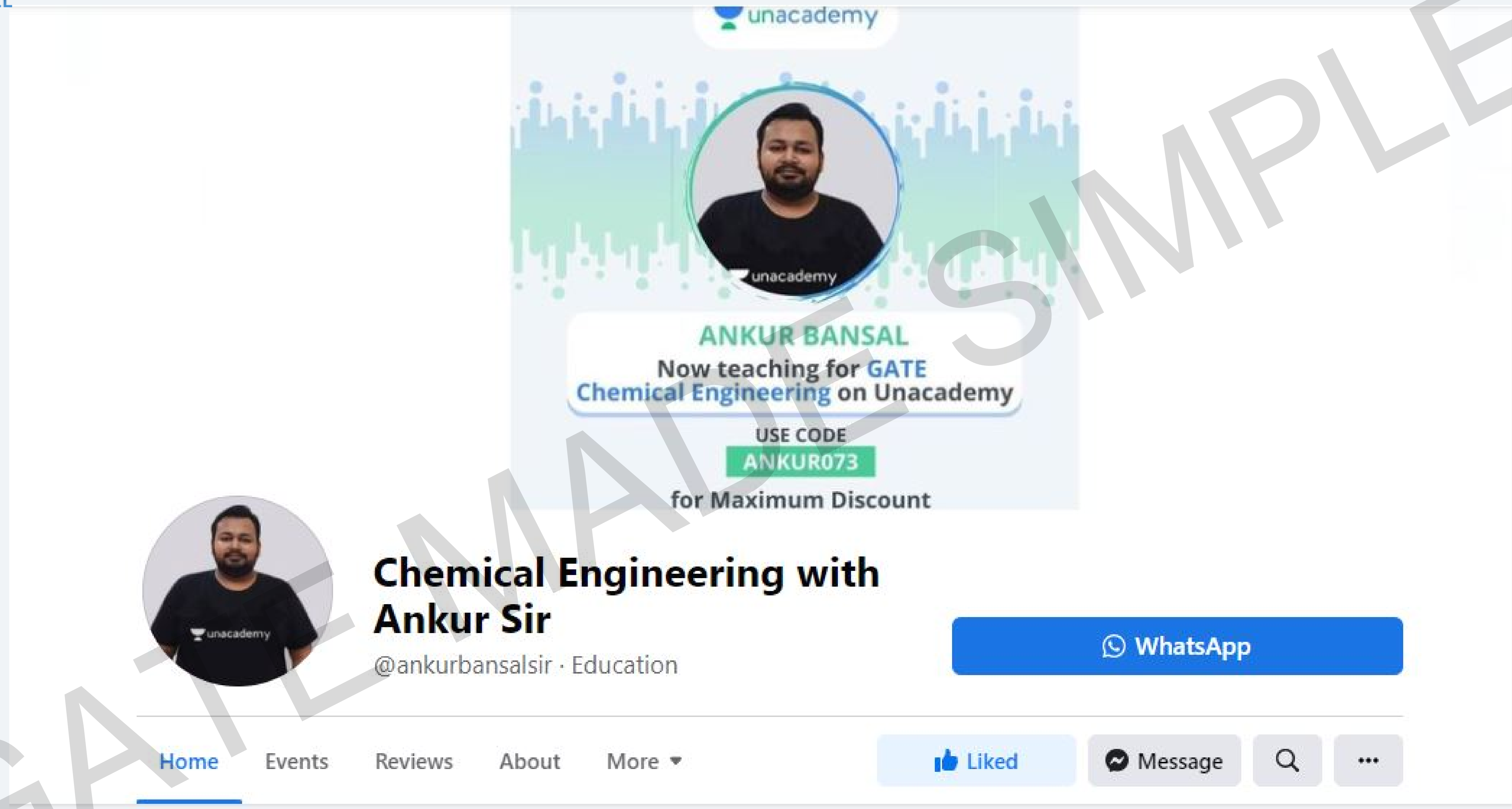
M.Tech IIT DELHI

10 Years teaching experience in GATE Chemical Engg.
Most experienced faculty in this field

GATE CHEMICAL ENGINEERING

Differential Equation PYQs

Ankur Bansal



[Click Here for Like and follow my Facebook page for more updates]

Q4. The differential equation

$$\frac{d^2y}{dx^2} + \sin x \, \frac{dy}{dx} + ye^x = \sinh x \ is$$

(A) first order and linear

(B) first order and non linear

(C) second order and linear

(D) second order and non linear

Ans. (C)

Sol.

$$\frac{d^2y}{dx^2} + \sin x \frac{dy}{dx} + ye^x = \sinh x$$

Is of second order linear differential equation (each derivative being of one degree).

Q10. The differential equation for the variation of the amount of salt x in a tank with time t is given by $\frac{dx}{dt} + \frac{x}{20} = 10$. x is in kg and t is in minute. Assuming that there is no salt in the tank initially, the time (in minute) at which the amount of salt increases to 100 kg is

(A) 10 ln 2 (B) 20 ln 2

(C) 50 ln 2 (D) 100 ln 2

Ans. (B)
Sol.

Given Differential equation

$$\frac{dx}{dt} + \frac{x}{20} = 10$$

X is in kg t is in minute it is a linear differential equation of first order and first degree

Its integrating factor (IF)

$$\int \frac{1}{20} dt = e^{t/20}$$

$$xe^{t/20} = \int e^{t/dt} 10dt + c$$

$$xe^{t/20} = 10.\frac{e^{t/20}}{1/20} + c = 200e^{t/20} + \text{c}$$

$$\rightarrow X = 200 + x = 200 + ce^{-t/20}$$

C being constant of integration. When t = 0, x = 0 kg (given initial condition) with it gives

$$0 = 200 + c^{-0/20} = 200 + c$$

’ c = - 200

Problem is to find t, when x = 100 kg using this value of x in which we have

$$100 = 200 - 200e^{-\frac{t}{20}}$$

$$\rightarrow -100 = -200e^{-\frac{t}{20}}$$

$$or\ e^{-t/20} = \frac{1}{2} \rightarrow \log e^{-t/20} = \log \frac{1}{2}$$

$$\rightarrow -\frac{t}{20} = -\log 2$$

$$t = 20 \log 2$$

Q12. The differential equations $\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} = 0$ can be reduced to (where, α is a constant)

(A) $\left(\frac{dy}{dx}\right)^3 = \alpha - \frac{3y^2}{2}$

$(B)\left(\frac{dy}{dx}\right)^2 \alpha - 2y$

(C) $\frac{dy}{dx} = \frac{\alpha}{y^2}$

$(D)\frac{dy}{dx} = \frac{\alpha}{y}$

Ans. (D)
Sol.
We take,

$$\frac{dy}{dx} = \frac{\alpha}{y}$$

$$\rightarrow y\frac{dy}{dx} = \alpha$$

On differentiating Eq. we get

$$\frac{dy}{dx}\frac{dy}{dx} + y\frac{d^2y}{dx^2} = 0$$

$$\rightarrow \left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} = 0$$

→ on integrating can be reduced to on integrating

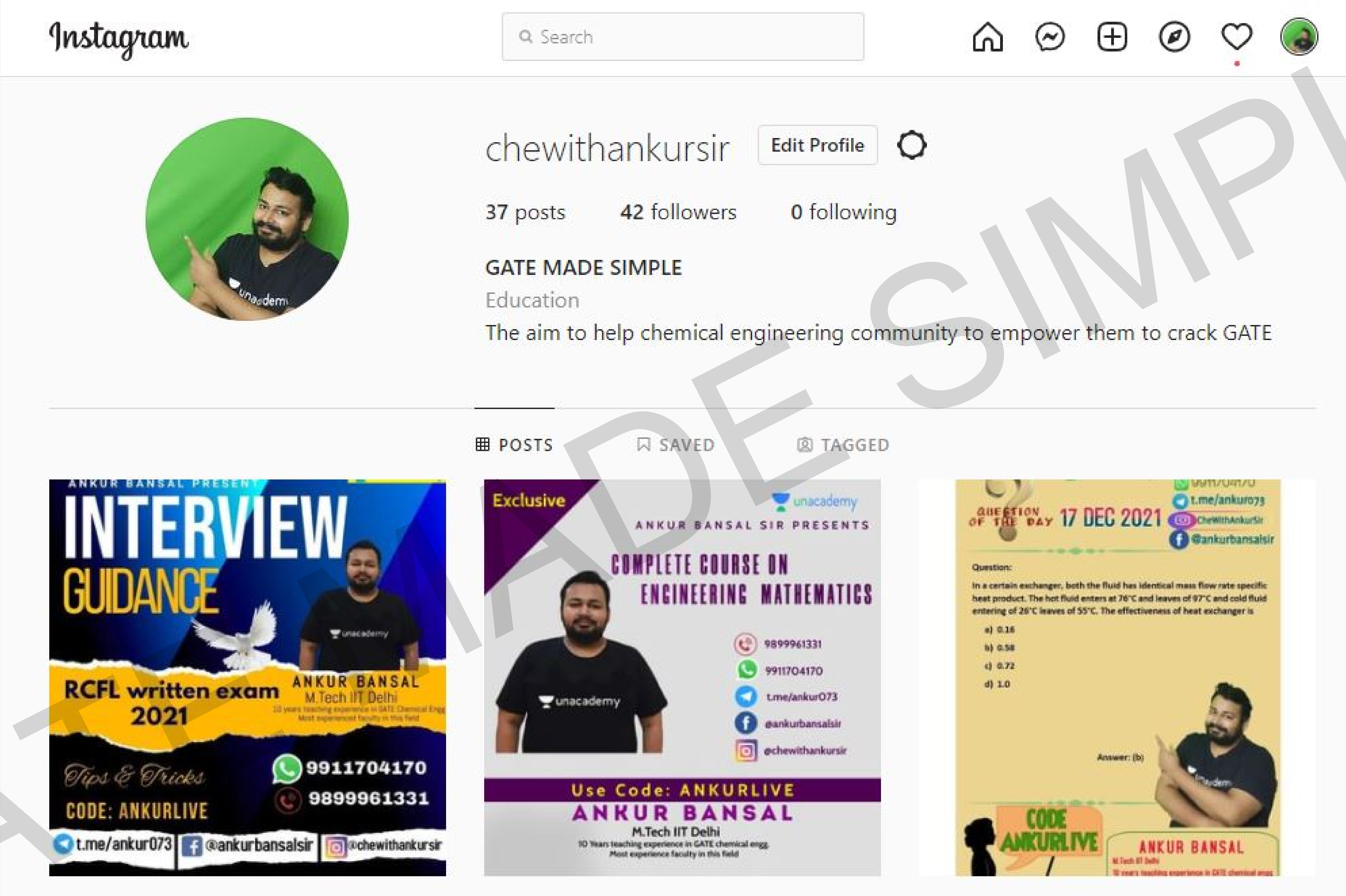
Q14. Match the following, where x is the spatial coordinate and t is time

Group I	Group II
P. Wave equation	1. $\frac{\partial c}{\partial t} = \alpha \frac{\partial c}{\partial x}$
Q. Heat equation	2. $\frac{\partial c}{\partial t} = \alpha^2 \frac{\partial^2 c}{\partial x^2}$
	3. $\frac{\partial^2 c}{\partial t^2} = \alpha^2 \frac{\partial c}{\partial x}$
	4. $\frac{\partial^2 c}{\partial t^2} = \alpha^2 \frac{\partial^2 c}{\partial x^2}$

(A) P – 4, Q – 2 (B) P – 2, Q – 4

(C) P – 3, Q – 1 (D) P – 1, Q – 3

Ans. (A)
Sol.
Option (A) is correct.



Click Here for Like and follow my Instagram profile for more updates

Q22. What condition is to be satisfied so that the solution of the differential equation $\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0$ is of the form $y = (C_1 + C_2x)e^{mx}$ where C_1 & C_2 are constants of integration?

(A) $a^2 = b$

(B) $b^2 = a$

(C) $a^2 = 4b$

(D) $b^2 = 4a$

Ans. (C)

Sol.

Given $\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0 \; \ldots\ldots\ldots (i)$

Its solution is $y = (C_1 + xC_2)e^{mx}$

C_1 and C_2 being constants of integration. Problem is to find the condition for this solution the solution $y = (C_1 + xC_2)e^{mx}$ shows the roots of the auxiliary eq. (i) must be equal. Auxiliary Eq. (i) is

$$m^2 + am + b = 0$$

$$m = \frac{-a \pm \sqrt{a^2 - 4b}}{2a}$$

Q24. If f(x) is the solution of the equation $\frac{dy}{dx} + 2xy + 2x = 0$ and g(x) is the solution of the equation $\frac{dy}{dx} + 2xy - 2x = 0$ and the constant of integration in $f(x)$ is equal to that in $g(x)$ then which of the following is true?

(A) $g(x) = f(x) + 2$
(B) $g(x) = f(x) + 1$
(C) $g(x) = f(x) - 1$
(D) $g(x) = f(x) - 2$

Ans. (A)

Sol.

$$\frac{dy}{dx} + 2xy = -2x \quad .....(i)$$

$$\text{If} = \int 2x dx = e^{x^2}$$

$$\text{Solution is } ye^{x^2} = \int e^{x^2} \cdot (-2x) dx + c$$

$$= -\mathrm{e}^{x^2} + c_1$$

$$f(x) = y - 1 + c_1 e^{-x^2}$$

[Given. F(x) is solution of Eq. (i)]

$$\frac{dy}{dx} + 2xy = 2x$$

$$\text{if } = \int 2xdx = e^{x^2}$$

Solution is $ye^{x^2} = \int e^{x^2} . 2xdx + c_2$

$$= e^{x^2} + c_2$$

$$g(x) = y = 1 + c_2 e^{-x^2}$$

[given, g(x) is solution of Eq

Also, $c_1 = c_2$

Click Here to watch All Sessions of Guidance from Ankur Sir

$G(x) = c_2 e^{-x^2} = 1 + c_1 e^{-x^2}$

$= 1 + f(x) + 1$

$= 2 + f(x)$

Or $g(x) = f(x) + 2$

Q33. With $y = e^{ax}$,

if the sum $s = \frac{dy}{dx} + \frac{d^2y}{dx^2} + \cdots + \frac{d^ny}{dx^n}$ approaches $2y$ as $n \to \infty$, then the value of a is

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) 2

Ans. (C)

Sol.

$$Y = e^{ax}$$

$$S = \frac{dy}{dx} + \frac{d^2y}{dx^2} + \cdots + \frac{d^ny}{dx^n}$$

$$= ae^{ax} + a^2e^{ax} + \cdots + a^ne^{ax}$$

$$= ae^{ax}(1 + a + a^2 + \cdots + a^{n-1})$$

$$= ae^{ax}\frac{1 - a^n}{(1 - a)}$$

Given, $\lim_{x\to\infty} s = 2y = 2e^{ax}$

$$\Rightarrow \lim_{x\to\infty} ae^{ax}\frac{(1-a^{an})}{(1-a)} = 2e^{ax}$$

$$\Rightarrow \lim_{x\to\infty} \frac{(1-a^{n})}{1-a} = \frac{2}{a}$$

this is possible when $0 < a < 1$.

and then

$$\frac{1}{1-a} = \frac{2}{a}\left(as \lim_{x\to\infty} a^{n} = 0\right)$$

$$\Rightarrow a = 2(1 - a) = 2 - 2a$$

$$3a = 2 \Rightarrow a = \frac{2}{3}$$

Q36. The solution to the following equation $x^2\frac{d^3y}{dx^3} + 2x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$ is given by

(A) $y = C_1x + C_2x^{-2} + C_3$

(B) $y = C_1x^2 + C_2x^{-2} + C_3$

(C) $y = C_1x^2 + C_2x^{-1} + C_3$

(D) $y = C_1x + C_2x^{-1} + C_3$

Ans. (C)

Sol.

$$x^2\frac{d^3y}{dx^3}+2x\frac{d^2y}{dx^2}-2\frac{dy}{dx}=0 \quad \dots\dots(\text{i})$$

On multiplying Eq. (i) by x, we get

$$x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}=0 \quad \dots\dots(\text{ii})$$

It is homogeneous linear differential equation of 3rd order and first degree

Putting $x=e^t$, then $x\frac{d}{dx}=\frac{d}{dt}=D$

$$x^2\frac{d^2}{dx^2} = D(D-1) \Rightarrow x^3\frac{d^3}{dx^3} = D(D-1)(D-2)$$

Using these in Eq. (ii) we get

$$[D(D-1)(D-2) + 2D(D-1) - 2D]y = 0$$

$$\Rightarrow D[D^2 - 3D + 2 + 2D - 2 - 2 - 2]Y = 0$$

$$\Rightarrow D[D^2 - D - 2]y = 0$$

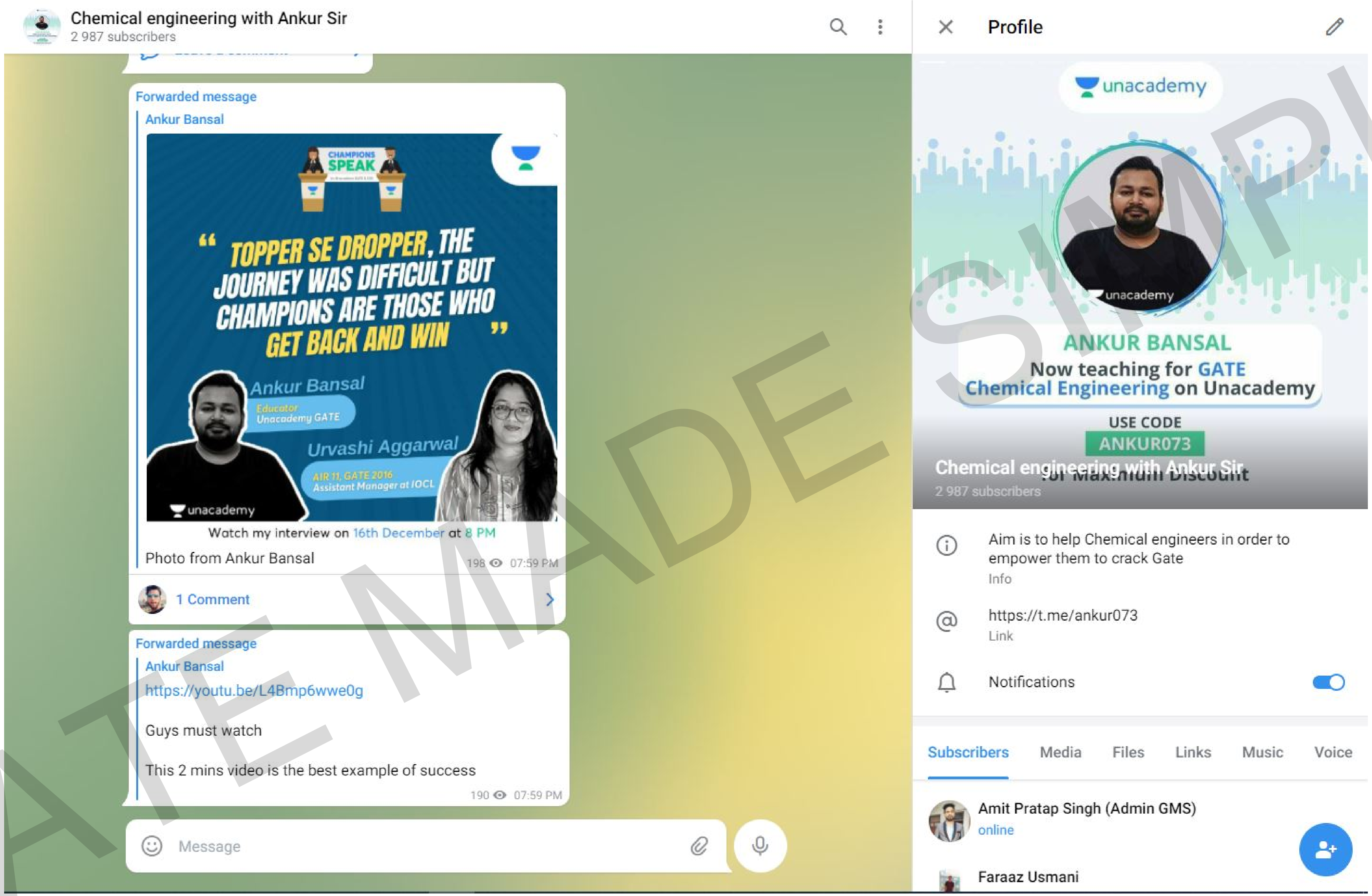
Its auxiliary equation is

$$\mathrm{m}(\mathrm{m}^2 - \mathrm{m} - 2) = 0 \Rightarrow \mathrm{m} = 0, -1, 2$$

$\mathrm{CF} = \mathrm{a}_1\mathrm{e}^{0t} + \mathrm{a}_2\mathrm{e}^{-\mathrm{t}} + \mathrm{a}_3\mathrm{e}^{2\mathrm{t}}$ So, $\mathrm{y} = \mathrm{a}_1 + \mathrm{a}_2\mathrm{e}^{-\mathrm{t}} + \mathrm{a}_3\mathrm{e}^{2\mathrm{t}} = \mathrm{a}_1 + \mathrm{a}_2\mathrm{x}^{-1} + \mathrm{a}_3\mathrm{x}^2$

$$\mathrm{PI} = 0$$

$= C_1x^2 + C_2x^{-1} + C_3$ {where , $a_3 = C_1, a_2 = C_2, a_1 = C_3$ are constant }



[Click Here for join my Telegram group for more updates]

Q43. The initial condition for which the following equation

$$(x^2 + 2x)\frac{dy}{dx} = 2(x+1)y; y(x_0) = y_0$$

has infinitely many solutions, is

(A) y (x = 0) = 5

(B) y (x = 0) = 1

(C) y (x = 2) = 1

(D) y (x = -2) = 0

Ans. (D)

Sol.

$$(x^2 + 2x)\frac{dy}{dx} = 2(x+1)y; y(x_0) = y_0$$

$$\Rightarrow x(x+2)\frac{dy}{dx} = (2xy + 2y)$$

$$\Rightarrow \frac{dy}{y} = \frac{2(x+1)dx}{x(x+2)}$$

$$\Rightarrow \frac{dy}{2y} = \frac{1}{2}\left[\frac{1}{2} + \frac{1}{x+2}\right]dx$$

Integrating on both sides,

$$\Rightarrow \frac{1}{2}\log y = \frac{1}{2}[\log x + \log(x+2)] + \frac{1}{2}\log C \Rightarrow \log y = \log x(x+2)C \Rightarrow$$
$$= y(x^2 + 2x)C$$

Given at $x = x_0 \Rightarrow y = y_0$

$$\Rightarrow y_0 = (x_0^1 + 2x_0)C \Rightarrow C = \frac{y_0}{x_0^2 + 2x_0}$$

From Eq (i), $y \frac{(x^2+2x)}{(x_0^2+2x_0)} y_0$

At $x = -2, y = 0$

Q47. The solution of the following differential equation $x\frac{dy}{dx} + y(x^2 - 1) = 2x^3$ is

(A) 0

(B) $2 + ce^{-\frac{x^2}{2}}$

(C) $c_1 x + c_2 x^2$

(D) $2x + Cxe^{-\frac{x^2}{2}}$

Ans. (D)

Sol.

$$x\frac{dy}{dx} + (x^2 - 1)y = 2x^3$$

$$\Rightarrow \frac{dy}{dx} + \frac{x^2 - 1}{x}y = 2x^2 \quad \ldots\ldots(i)$$

Integrating factor,

$$\text{If} = \int_e \frac{x^2 - 1}{x}dx = \int_e \left(x - \frac{1}{x}\right)dx$$

$$= e^{\frac{x^2}{2}} - \log x = e^{\frac{x^2}{2}} e^{\log\frac{1}{x}} = \frac{e^{x^2/2}}{x}$$

olution of. (i)

$$y\frac{e^{x^2/2}}{x} = \int \frac{e^{x^2/2}}{x} \cdot 2x^2 dx + c$$

$$= 2\int e^{x^2/2} dx + c$$

Putting $\frac{x^2}{2} = t \Rightarrow \frac{2x}{2} dx = dt$

$$xdx = dt$$

So, $y\frac{e^{x^2/2}}{x} = 2\int e^t dt + c = 2e^t + c$

$$= 2e^{x^2/2} + C$$

$$y = 2X + CXe^{-x^2/2}$$

Q52. Which one of the following is not an integrating factor for the differential equation $xdy - ydx = 0$?

(A) $\frac{1}{x^2}$

(B) $\frac{1}{y^2}$

(C) $\frac{1}{xy}$

(D) $\frac{1}{(x+y)}$

Ans. (D)

Sol.

Given, differential equation

$$xdy - ydx = 0 \quad \ldots\ldots (i)$$

$$\frac{\text{Eq.(i)}}{x^2} \Rightarrow \frac{xdy - ydx}{x^2} = 0 \Rightarrow d\left(\frac{y}{x}\right) = 0$$

$$\Rightarrow \frac{y}{x} = \text{constants } C_1$$

$$\Rightarrow \frac{1}{x^2} \text{ is integrating factor}$$

$$\frac{\text{Eq. (i)}}{y^2} \Rightarrow -\frac{(ydx - xdy)}{y^2} = 0$$

$$\Rightarrow -d\left(\frac{x}{y}\right) = 0 \Rightarrow \frac{-x}{y} = C_2$$

$\Rightarrow \frac{1}{y^2}$ is integrating factor

$$\frac{\mathrm{Eq}\cdot(\mathrm{i})}{\mathrm{xy}} \Rightarrow \frac{\mathrm{dy}}{\mathrm{y}} - \frac{\mathrm{dx}}{\mathrm{x}} = 0$$

$$\Rightarrow \log \mathrm{y} - \log \mathrm{x} = \mathrm{C}_1$$

$\Rightarrow \frac{1}{\mathrm{xy}}$ is integrating factor.

$$\frac{\text{Eq}\cdot(\text{i})}{x+y} \Rightarrow \frac{xdy - ydx}{x+y} = 0$$

$$\Rightarrow \frac{x}{x+y}dy - \frac{y}{x+y}dx = 0$$

$$M = \frac{-y}{x+y}$$

$$\Rightarrow \frac{\partial M}{\partial y} = \frac{-(1\cdot(x+y) - y\cdot 1)}{(x+y)^2} = \frac{-x}{(x+y)^2}$$

$$N = \frac{x}{x+y}$$

$$\Rightarrow \frac{\partial N}{\partial x} = \frac{1\cdot(x+y) - x\cdot 1}{(x+y)^2} = \frac{y}{(x+y)^2}$$

$$As\ \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\Rightarrow \frac{xdy}{x+y} - \frac{y}{x+y}dx = 0 \text{ is not integrable}$$

$$\Rightarrow \frac{1}{x+y} \text{ is not int integrating factor}$$

Q53. Which one of the following is not a solution of the differential equation $\frac{d^2y}{dx^2} + y = 1$?

(A) $y = 1$

(B) $y = 1 + \cos x$

(C) $y = 1 + \sin x$

(D) $y = 2 + \sin + \cos x$

Ans. (D)

Sol.

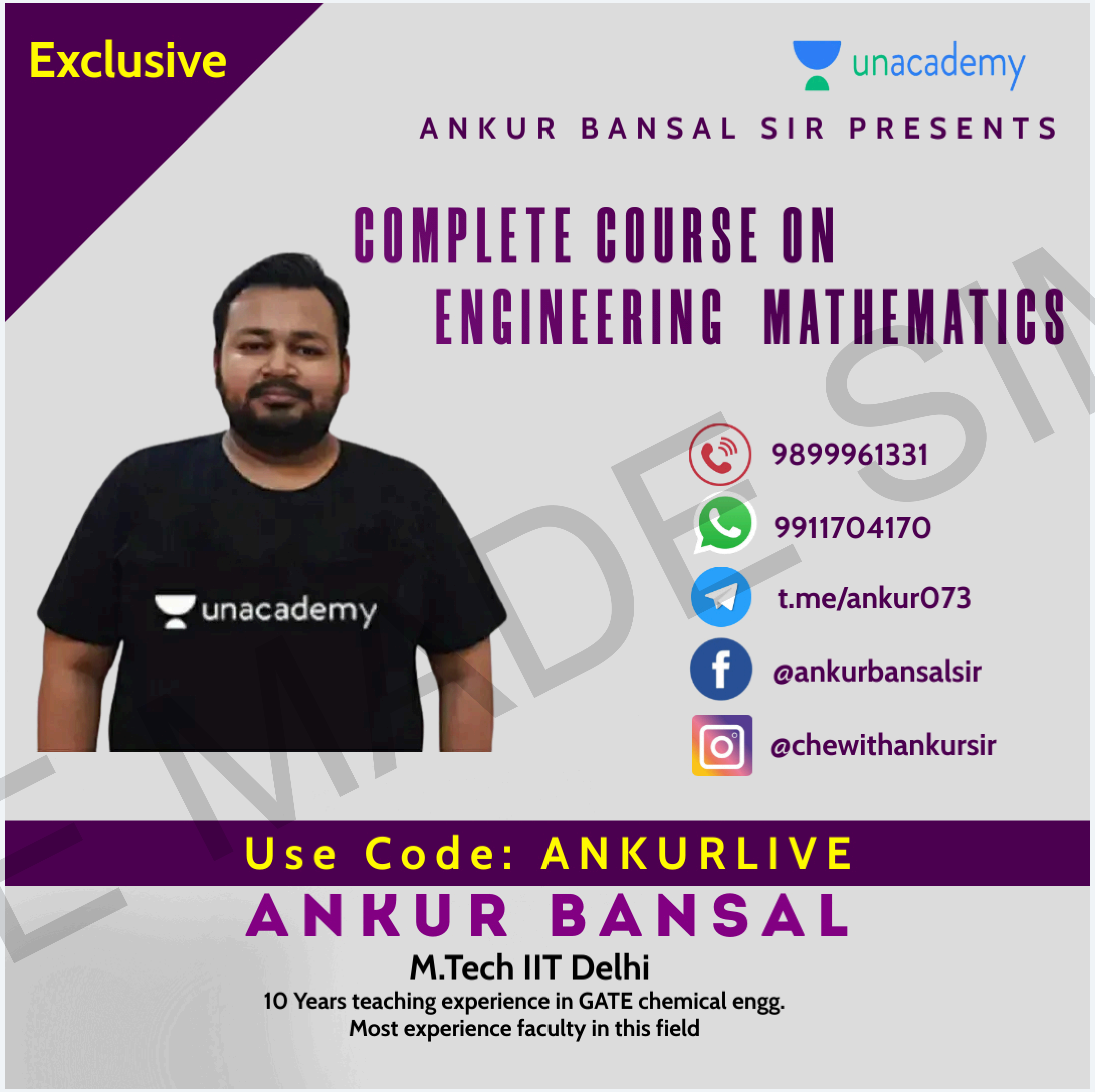
$$\frac{d^2y}{dx^2} + y = 1 \ldots\ldots (i)$$

We consider, $y = 2 + \sin x + \cos x$

$$\frac{dy}{dx} = \cos x - \sin x$$

$\frac{d^2y}{dx^2} = -\sin x - \cos x$ LHS of Eq. (i) $\Rightarrow -\sin x - \cos x + 2\sin x + \cos x =$ $2 \neq 1$ Thus, $y = 2 + \sin x + \cos x$ does not satisfy Eq. (i)

$\Rightarrow$ it is not the solution of Eq (i).



Click Here to watch Complete course on Engineering Mathematics

Q57. Which one of the following transformations reduces $\frac{dy}{dx} + Ay^3 + By = 0$ to a linear differential equation? (A & B are positive constants)

(A) $u = y^{-3}$

(B) $u = y^{-2}$

(C) $u = y^{-1}$

(D) $u = y^2$

Ans. (B)

Sol.

$$\frac{dy}{dx} + Ay^3 + By = 0$$

A and B being constant

Putting $u = 1/y^2 \Rightarrow \frac{du}{dx} = -2y^{-3}\frac{dy}{dx}$

Now, Eq (i) v

Using alone values in it, we get $-\frac{1}{2}\frac{du}{dx} + A + Bu = 0$

or $\frac{du}{dx} - 2Bu = 2A$

It is required linear differential equation.

Q68. The general solution of the differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$, where C_1 and C_2 are constants of integration, is

(A) $C_1e^{-3x} + C_2e^{-2x}$

(B) $C_1e^{3x} + C_2e^{-2x}$

(C) $C_1e^{3x} + C_2e^{2x}$

(D) $C_1e^{-3x} + C_2e^{2x}$

Ans. (B)

Sol.

Given, $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y$

Its auxiliary equation is $\mathrm{m}^2 - \mathrm{m} - 6 = 0$

$$\Rightarrow (m-3)(m+2) = 0 \Rightarrow m = 3 \cdot -2$$

$$CF = C_1 e^{3x} + C_2 e^{-2x}$$

$$PI = 0$$

$$So, y = CF + PI = C_1 e^{3x} + C_2 e^{-2x}$$

Q81. The solution of the differential equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 0$

With the initial conditions $y(0) = 0, \left.\frac{dy}{dt}\right|_{t=0} = -1$, is

(A) $-\mathrm{t}\sin \mathrm{t}$

(B) $-\mathrm{e}^{-t}(1 - \cos t)$

(C) $-(\mathrm{t} + \sin \mathrm{t}/2)$

(D) $-\mathrm{e}^{-\mathrm{t}}\sin \mathrm{t}$

Ans. (B)

Sol.

$$\text{Given, } \frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 0 \text{ .... (i)}$$

$$\text{With } y(0) = 0, \left.\frac{dy}{dt}\right|_{t=0} = -1$$

Auxiliary eq. of (i)

$$m^2 + 2\ m + 2 = o$$

$$\Rightarrow m = \frac{-2 + \sqrt{4-8}}{2} = -1 \pm i$$

$$CF = e^{-t}(C_1 \cos t + C_2 \sin t)$$

$$\text{so, } y(t) = e^{-t}(C_1 \cos t + C_2 \sin t)$$

Click Here to watch PYQs with discussion of CT

$$\text{At } t = 0, y(0) = e^{-0}(C_1 \cos 0 + C_2 \sin 0)$$

$$0 = C_1$$

$$y'(t) = e^{-t} - (t)(C_1 \cos t + C_2 \sin t)$$

$$+e^{-t}(-C_1 \sin t + C_2 \cos t)$$

$$\text{At } t = 0, y'(0) = -1 \cdot (0) + e^{-0} C_2$$

$$-1 = C_2$$

$$\text{So } y(t) = -e^{-t} \sin t$$

Q90. Which one of following choices is a solution of the differential equation given below? $\frac{dy}{dx} = \frac{y^2}{x} + \frac{y}{x} - \frac{2}{x}$. Note c is real constant.

(A) $y = \frac{c-x^2}{c+2x^2}$

(B) $y = \frac{c+2x^2}{c-x^2}$

(C) $y = \frac{c-x^3}{c+2x^3}$

(D) $y = \frac{c+2x^3}{c-x^3}$

Ans. (D)

Sol.

$$\text{Given, } \frac{dy}{dx} = \frac{y^2}{x} + \frac{y}{x} - \frac{2}{x}$$

$$= \frac{y^2 + y - 2}{x}$$

$$\Rightarrow \frac{dy}{y^2 + y - 2} = \frac{dx}{x}$$

$$\Rightarrow \frac{dy}{y^2 + 2y - y - 2} = \frac{dx}{x}$$

$$\Rightarrow \frac{dy}{(y+2)(y-1)} = \frac{dx}{x}$$

$$\Rightarrow \frac{1}{3}\left[\frac{1}{y-1} - \frac{1}{y+2}\right] dy = \frac{dx}{x}$$

On integrating

$$\frac{1}{3}[\log(y-1) - \log(y+2)] = \log x + \log c$$

$$\Rightarrow \log\frac{(y-1)}{(y+2)} = \log(c_1 x)^3$$

$$\Rightarrow \frac{y-1}{y+2} = (c_1 x)^3 = c_2 x^3 \;\; \left[\text{putting } c_1^3 = c_2\right]$$

$$\Rightarrow \; y - 1 = (y+2)c_2 x^3$$

$$y - yc_2x^3 = 1 + 2c_2x^3$$

$$y(1 - c_2x^3) = 1 + 2c_2x^3$$

$$y = \frac{1 + 2c_2x^3}{1 - c_2x^3} = \frac{c_2\left(\frac{1}{c_2} + 2x^3\right)}{c_2\left(\frac{1}{c_2} - x^3\right)}$$

$$y = \frac{c + 2x^3}{c - x^3} = \left(\text{putting } \frac{1}{c_2} = c\right)$$

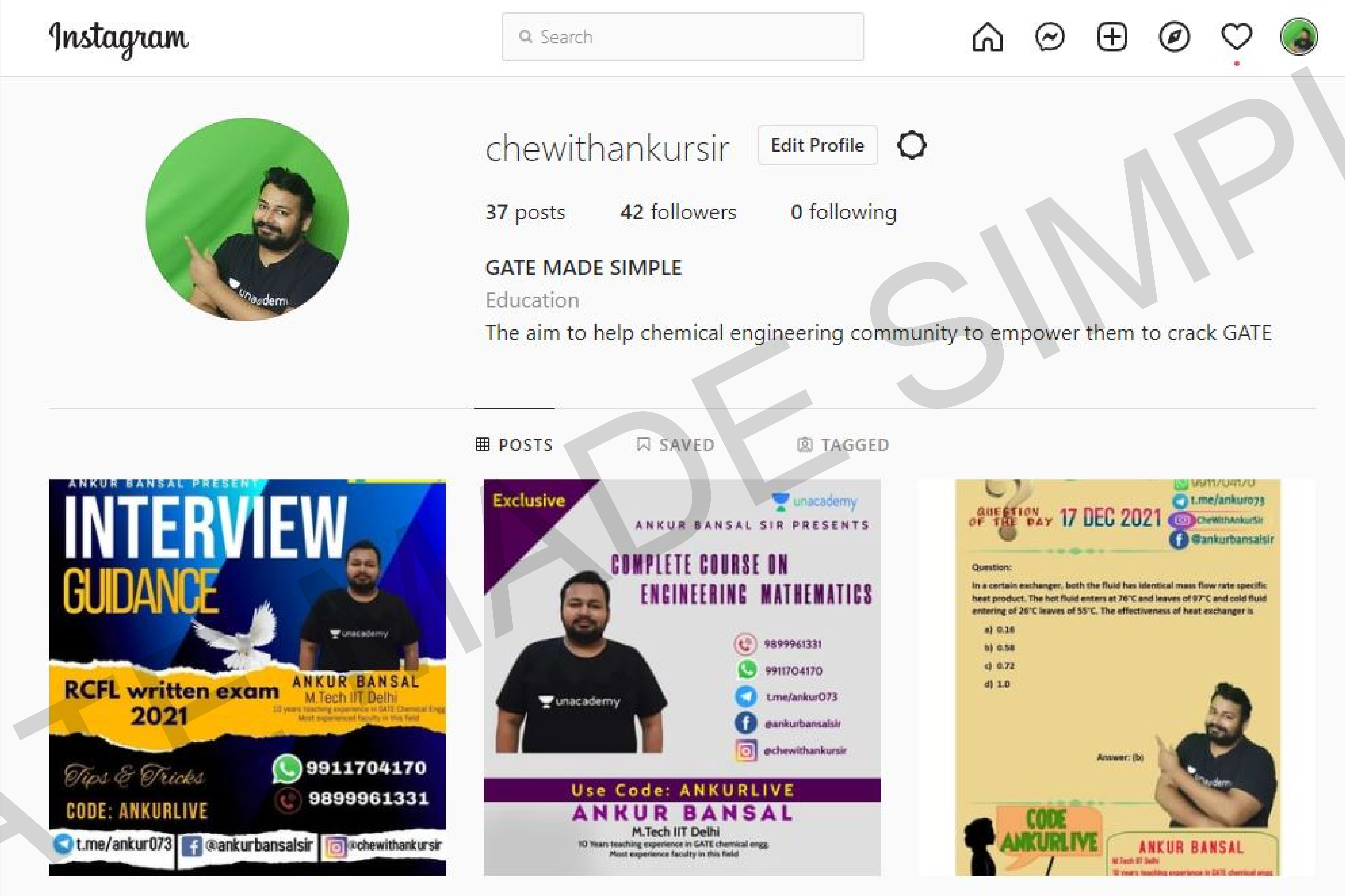
Q94. If a and b are arbitrary constants, then the solution to the ordinary differential equation $\frac{d^2y}{dx^2} - 4y = 0$ is

(A) $y = ax + b$

(B) $y = ae^{-x}$

(C) $y = a\sin 2x + b\cos 2x$

(D) $y = a\cosh 2x + b\sinh 2x$



Click Here for Like and follow my Instagram profile for more updates

Ans. (D)

Sol.

We have

$$(D^2 - 4) = 0 \Rightarrow D^2 - 4 = 0$$

$$\Rightarrow D^2 = 4 \Rightarrow D = \pm 2$$

$$y = C_1 e^{2x} + C_2 e^{-2x}$$

$$= ae^{2x} + be^{-2x}$$

$$\text{Cosh}\, 2x = \frac{e^{2x} + e^{-2x}}{2}$$

$$\sinh 2x = \frac{e^{2x} - e^{-2x}}{2}$$

Q104. Evaluate $\int \frac{dx}{e^x - 1}$ (Note: C is a constant of integration)

(A) $\frac{e^x}{e^x+1} + C$

(B) $\ln \frac{(e^x-1)}{e^x} + C$

(C) $\ln\left(\frac{e^x}{e^x-1}\right) + C$

(D) $\ln(1 - e^{-x}) + C$

Ans. (B)

Sol.

$$\int \frac{dx}{e^x - 1}$$

$$e^x - 1 = t$$

$$e^{x\,dx} = dt$$

$$dx = \frac{dt}{t+1}$$

$$\ln\frac{t}{t+1} + c = \ln\left|\frac{e^x - 1}{e^x}\right| + c$$

[Click Here to watch All PYQs with discussion of Mechanical Operation]

Q106. The solution of the differential equation $\frac{dy}{dx} - y^2 = 0$, given $y = 1$ at $x = 0$ is

(A) $\frac{1}{1+x}$

(B) $\frac{1}{1-x}$

(C) $\frac{1}{(1-x)^2}$

(D) $\frac{x^3}{3} + 1$

Ans. (B)

Sol.

$$\frac{dy}{dx} = y^2$$

$$\int \frac{dy}{y^2} = \int dx$$

$$\frac{-1}{y} = x + C \quad C = -1$$

$$y(0) = 1$$

$$\frac{-1}{y} = x - 1$$

$$y =$$

$$\frac{-1}{x-1} = \frac{1}{1-x}$$

Q107. The solution of the differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} + 0.25y = 0$, given y = 0 at x = 0 and $\frac{dy}{dx} = 1$ at $x = 0$ is

(A) $xe^{0.5x} - xe^{-0.5x}$

(B) $0.5xe^{x} - 0.5xe^{-x}$

(C) $xe^{0.5x}$

(D) $-xe^{0.5x}$

[Click Here to watch All Session's of Champion's Speak]

Ans. (C)

Sol.

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} + 0.25y = 0$$

$$(D^2 - D + .25)y = 0$$

$$D^2 - D + .25 = 0$$

$$D = 1 \pm \frac{\sqrt{1 - 4x \cdot 25}}{2}$$

$$= \frac{1}{2}, \frac{1}{2}$$

$$y = (C_1 x + C_2)e^{1/2x}$$

$$y(0) = 0$$

$$y^1 = C_1\left[x\, e^{1/2} \cdot \frac{1}{2} + e^{1/2} \cdot 1\right] C_2 = 0$$

$$y^1(0) = 1$$

$$y = xe^{1/2x} \quad C_1 = 1$$

Q114. The integrating factor for the differential equation $\frac{dy}{dx} - \frac{y}{1+x} = (1+x)$ is

(A) $\frac{1}{1+x}$

(B) $(1+x)$

(C) $x(1+x)$

(D) $\frac{x}{1+x}$

Ans. (A)

Sol.

$$\frac{dy}{dx} - \frac{y}{1+x} = (1+x)$$

$$\text{I.F.} = e^{\int \frac{-1}{1+x}dx} = e^{\ln(1+x)^{-1}} = \frac{1}{1+x}$$

Q115. The differential equation $\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} + x^3y = e^x$ is a

(a) nonlinear differential equation of first degree

(b) linear differential equation of first degree

(c) linear differential equation of second degree

(d) nonlinear differential equation of second degree

Ans. (B)

Sol.

$$\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} + x^3y = e^x$$

GATE MADE SIMPLE

[Click Here to Learn from the Mock Interviews]

Q120. Consider a linear ordinary differential equation $\frac{dy}{dx} + p(x)y = r(x)$. Functions $p(x)$ and r (x) are defined and have a continuous first derivative. The integrating factor of this equation is non zero. Multiplying this equation by its integrating factor converts this into a:

(A) homogeneous differential equation

(B) nonlinear differential equation

(C) second order differential equation

(D) exact differential equation

Ans. (D)

Sol.

Option (D) is correct.

Q129. What is the solution for the second order differential equation $\frac{d^2y}{dx^2} + y = 0$, with the initial conditions? $y|_{x-0} = 5$ and $\left.\frac{dy}{dx}\right|_{x=0} = 10$?

(A) $y = 5 + 10\sin x$

(B) $y = 5\cos x - 5\sin x$

(C) $y = 5\cos x + 10x$

(D) $y = 5\cos x + 10\sin x$

Ans. (D)

Sol.

$$D^2 + 1 = o$$

$$D = \pm i$$

$$Y = C_1 \cos x + C_2 \sin x$$

By using the boundary condition

$$y = 5\cos x + 10\sin x$$

Q138. For the initial value problem

$$\frac{dx}{dt} = \sin(t), x(0) = 0$$

The value of x at $t = \pi/3$, is ___________

Ans. 0.5

Sol.

$$\frac{dx}{dt} = \sin t \Rightarrow \int dx = \int \sin t dt \Rightarrow x = -\cos t + C$$

Given,

$$x(0) \Longrightarrow \text{ at } t = 0 \Rightarrow x = 0 \Rightarrow 0 = -1 + C == 1$$

$$\therefore x = -\cos t + 1$$

$$\text{at t} = \frac{\pi}{3}, \text{x} - \cos\frac{\pi}{3} + 1 = \frac{-1}{2} + 1 = \frac{1}{2} = 0.5$$

Q143. Consider the following two equations:

$$\frac{dx}{dt} + x + y = 0$$

$$\frac{dy}{dt} - x = 0$$

The above set of equations is represented by

(A) $\frac{d^2y}{dt^2} - \frac{dy}{dt} - y = 0$

(B) $\frac{d^2x}{dt^2} - \frac{dx}{dt} - y = 0$

(C) $\frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 0$

(D) $\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 0$

Ans. (C)

Sol.

Given $\frac{dx}{dt} + x + y = 0 \ldots \ldots (1)$

$\frac{dx}{dt} - x = 0$ …… (2)

From (2); $x = \frac{dy}{dt} \ldots \ldots \ldots (3)$

Differentiate both sides w. x. t 't', then $\frac{dx}{dt} = \frac{d^2y}{dt^2}$

$$\frac{dx}{dt} = \frac{d^2y}{dt^2} \ldots \ldots \ldots \ldots \ldots (4)$$

∴ From (1); $\frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 0 \left[\because x = \frac{dy}{dt} \,\&\, \frac{dx}{dt} = \frac{d^2y}{dt^2}\right]$

Q145. If $y = e^{-x^2}$ then the value of

$$\lim_{x \to \infty} \frac{1}{x} \frac{dy}{dx}$$

is ________

Ans. 0

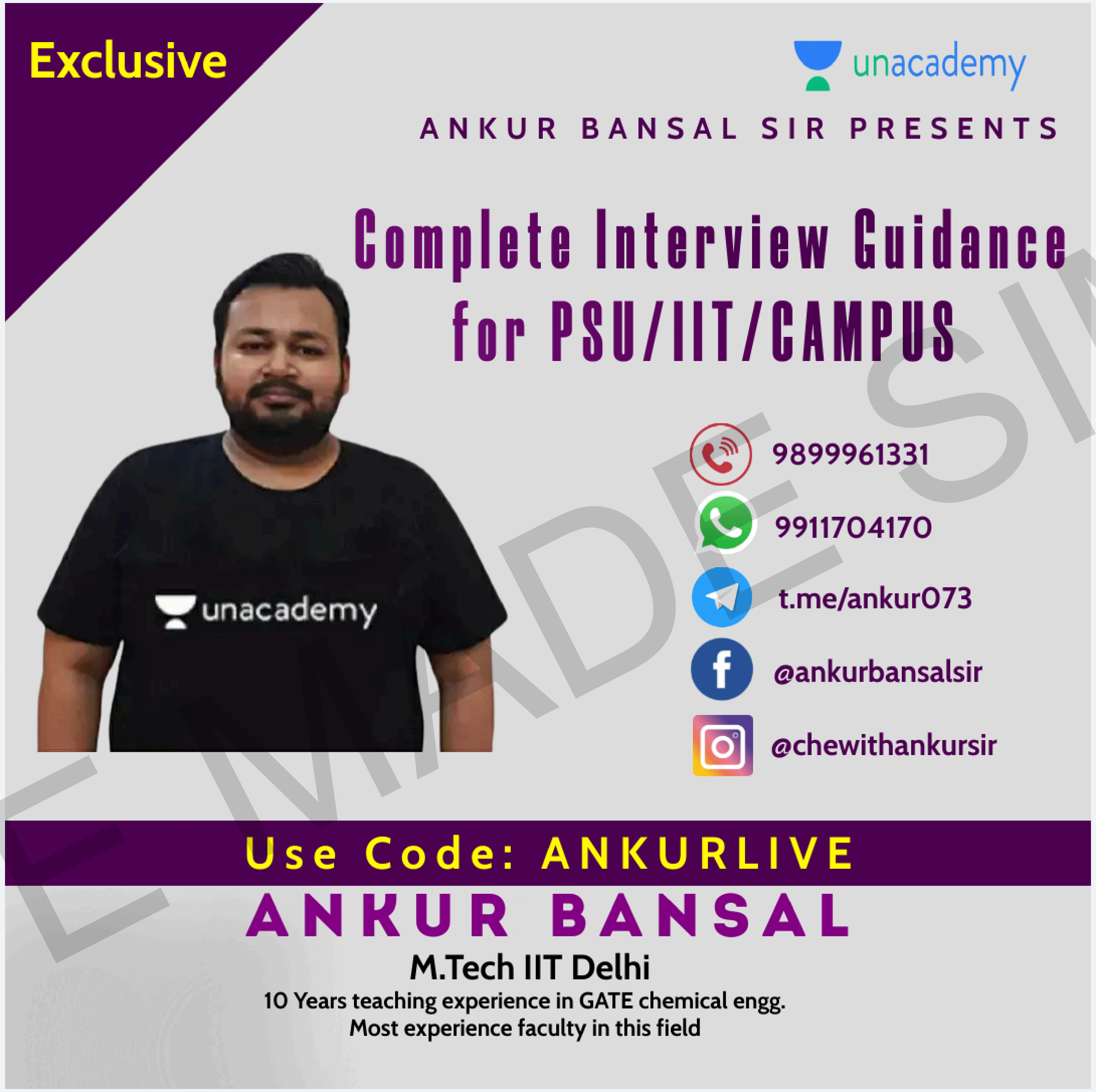
Sol.

$$y = e^{-x^2} \Rightarrow \frac{dy}{dx} = e^{-x^2}(-2x)$$

$$\therefore \lim_{x\to\infty} \frac{1dy}{xdx} = \lim_{x\to\infty} \frac{1}{x}\left[e^{-x^2}(-2x)\right]\left[\because \frac{dy}{dx} = e^{-x^2}(-2x)\right]$$

$$= \lim_{x\to\infty} -2e^{-x^2}$$

$$= -2 \times 0\left[\because \lim_{x\to\infty} e^{-x^2} = 0\right] = 0$$



Click Here to watch Complete Interview Guidance

Q153. Given $\frac{dy}{dx} = y - 20$, and $y|_{x=0} = 40$, the value of y at $x = 2$ is ______________ (round off to nearest integer).

(GATE – 2020)

Ans. (167.78)

Sol.

$$\frac{dy}{dx} = y - 20$$

$$\int_{40}^{y} \frac{dy}{y-20} = \int_{0}^{2} dx$$

$$\ln\left(\frac{y-202}{40-20}\right) = 2$$

$$y - 20 = 20e^2 = 167.78$$

Q158. The solution of the ordinary differential equation $\frac{dy}{dx} + 3y = 1$, subject to the initial condition $y = 1$ at $x = 0$, is

(GATE – 2019)

(a) $\frac{1}{3}\left(1 + 2e^{-x/3}\right)$ (b) $\frac{1}{3}\left(5 - 2e^{-x/3}\right)$

(c) $\frac{1}{3}\left(5 - 2e^{-3x}\right)$ (d) $\frac{1}{3}\left(1 + 2e^{-3x}\right)$

Ans. (d)

Sol.

$$\frac{dy}{dx} + 3y = 1$$

$$\int_{1}^{y} \frac{dy}{1-3y} = \int_{0}^{x} dx$$

$$-\frac{1}{3}\ln\left[\frac{1-3y}{1-3}\right] = x$$

$$3y - 1 = 2e^{-3x}$$

$$y = [1 + 2e^{-3x}].1/3$$

Q166.The value of y as $t \rightarrow \infty$ for the following differential equation for an initial value of $y(1) = 0$ is: $(4t^2 + 1)\frac{dy}{dt} + 8yt - t = 0$

(GATE – 2003)

a) 1
b) 1/2
c) 1/4
d) 1/8

Ans. (d)

Sol.

$$(4t^2+1)\frac{dy}{dt}+8yt-t=0$$

$$\frac{dy}{dt}+\frac{8t}{(4t^2+1)}y=\frac{t}{(4t^2+1)}$$

$$IF=e^{\int\frac{8t}{(4t^2+1)}dt}=e^{\ln(4t^2+1)}=(4t^2+1)$$

$$y(4t^2+1)=\int\frac{t}{(4t^2+1)}.(4t^2+1)\mathrm{dt}+\mathrm{c}$$

$$y(4t^2+1) = \frac{t^2}{2} + c$$

y(1) = 0

$$0 = \frac{1}{2} + c \rightarrow c = -\frac{1}{2}$$

$$y = \frac{1}{(4t^2+1)}(t^2-1)\frac{1}{2}$$

$$y = \frac{1}{2}\frac{\left(1-\frac{1}{t^2}\right)}{\left(4+\frac{1}{t^2}\right)}$$

$$\lim_{t\to\infty} y = \frac{1}{2}.\frac{1}{4} = \frac{1}{8}$$

Q168. The differential equation, $\frac{d^2x}{dt^2} + 10\frac{dx}{dt} + 25x = 0$ will have a solution of the form

(GATE – 2003)

a) $(C_1 + C_2 t)e^{-5t}$
b) $C_1 e^{-2t}$
c) $C_1 e^{-5t} + C_2 e^{5t}$
d) $C^{5t} + C_2 e^{2t}$
where C_1 and C_2 are constants.

Ans. (b)

Sol.

$$(D^2 + 10D + 25)x = 0$$

$$Characteristic\ equation = m^2 + 10m + 25 = 0$$

$$m = -5, -5$$

x = C.F

$$\alpha = c_1 e^{5t} + c_2 t e^{-5t}$$

$$\alpha = (c_1 + c_2 t)e^{-5t}$$

Q175. (a) Reduce the following different equation to linear form

$$\frac{dz}{dt} + Zt = Z^{-1}t$$

(b) Find a general solution to the linearized equation

(c) Determine the integration constants if $z(0) = 0$

(GATE – 2002)

Ans. (a) → $\frac{da}{dt} + 2ta = 2t$, (b) → $z = \left[1 + ce^{-t^2}\right]^{1/2}$, (c)→ -1

Sol.

$$\frac{dz}{dt} + zt = z^{-1}t$$

$$z\frac{dz}{dt} + z^2t = t$$

$$z^2 = a \quad \rightarrow 2z\frac{dz}{dt} = \frac{da}{dt}$$

$$\frac{1}{2}\frac{da}{dt} + at = t$$

$$\frac{da}{dt} + 2ta = 2t \qquad (Linear\ differential\ Equation)$$

$$IF = e^{\int 2t\,dt} = e^{t^2}$$

$$ae^{t^2} = \int 2t\,e^{t^2}\,dt + c$$

$$ae^{t^2} = e^{t^2} + c$$

$$a = 1 + ce^{-t^2}$$

$$z^2 = 1 + ce^{-t^2}$$

$$z = \left[1 + ce^{-t^2}\right]^{1/2}$$

[Click Here to join Educator Community of Ankur Sir]

At t = 0 z = 0

$$0 = (1 + c)^{1/2} \rightarrow c = -1$$

Q186. The integrating factor for the differential equation $(\cos^2 x)\frac{dy}{dx} + y = \tan x$ is

(GATE – 2000)

A) $e^{\tan x}$

B) $\cos 2x$

C) $e^{-\tan x}$

D) $\sin 2x$

Ans. (a)

Sol.

$$cos^2x\frac{dy}{dx} + y = \tan x$$

Standard 1st order D.E.

$$\frac{dy}{dx} + \text{y} \sec^2 x = \frac{\tan x}{\cos^2 x}$$

$$IF = e^{\int \sec^2 x \, dx}$$

$$= e^{\tan x}$$

Q190. The general solution of $\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = 0$

(GATE – 2000)

(A) $(C_1x + C_2)e^x + (C_3 + C_4x)e^{-x}$

(B) $C_1\cos x + C_2\sin x + C_3e^x + C_4e^{-x}$

(C) $C_1e^{ix} + C_2e^{-ix}$

(D) $(C_1 + C_2x)\cos x + (C_3 + C_4x)\sin x$

Where $C_1, C_2, C_3,$ and C_4 are constants.

Ans. (a)

Sol.

$(D^4 + 2D^2 + 1)y = 0$

Characteristic equation

$m^4 + 2m^2 + 1 = 0$

$(m^2 + 1)^2 = 0$

$m \pm i, \pm i$

Two equal imaginary roots (pair)

General solution –

$y = (c_1 + c_2 x).e^{\alpha x}(c_3 \cos \beta x + c_4 \sin \beta x)$

$$\alpha = 0, \qquad \beta = 1$$

$$y = (c_1 + c_2 x)(c_3 \cos x + c_4 \sin x)$$

$$y = c_3(c_1 + c_2 x)\cos x + c_4(c_1 + c_2 x)\sin x$$

$$y = (c_1' + c_2' x)\cos x + (c_3' + c_4' x) sinx$$

Q200. Solve $\frac{dy}{dx} - 6xy = -6x$ by the following methods:

(a)variation of parameters

(b)separation of variables

(GATE – 1999)

Ans. (a)→ ($y = 1 + ce^{3x^2}$), (b)→ ($y = 1 + c'e^{3x^2}$)

Sol.

a) Variation of parameter

$$\frac{dy}{dx} - 6xy = -6x$$

$$IF = e^{\int p dx} = e^{-6\int x dx} = e^{-3x^2}$$

$$ye^{-3x^2} = \int e^{-3x^2}(-6x)dx + c$$

$$y = 1 + ce^{3x^2}$$

(b) Variable separable method –

$$\frac{dy}{dx} = 6x(y-1)$$

$$\int \frac{dy}{(y-1)} = \int 6x\,dx$$

$$\ln(y-1) = 3x^2 + c$$

$$y - 1 = e^{3x^2} * e^c$$

$$y = 1 + e^c e^{3x^2}$$

$$y = 1 + c' e^{3x^2}$$

Q206. The differential equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0$ will have a solution of the form

(GATE – 1998)

A) $C_1e^{-t} + C_2e^{2t}$
B) $C_1e^{-t} + C_2e^{-2t}$
C) $C_1e^{-3t} + C_2e^{-2t}$
D) C_1e^{-5t}

Ans. (b)

Sol.

$$(D^2 + 3D + 2)Y = 0$$

Characteristic equation –

$$m^2 + 3m + 2 = 0$$

$$m^2 + 2m + m + 2 = 0$$

$$m(m + 2) + (m + 2) = 0$$

$$m = -1, -2$$

Y=C.F.

$$Y = c_1e^{-x} + c_2e^{-2x}$$

Q216. Solve $\frac{d^2y}{dx^2} - m^2y = 0$ subject to $y = 1$ at $x = 0$ and $dy/dx = 0$ at $x = 1$.

(GATE – 1996)

Ans. $y = \frac{e^{mx}}{(1+e^{2m})} + \frac{e^{2m}}{(1+e^{2m})} e^{-mx}$

Sol.

$$(D^2 - m^2)y = 0$$

$$D = +m, -m$$

$$y = c_1 e^{mx} + c_2 e^{-mx}$$

@ x = 0, y = L

$1 = c_1 + c_2$ .... (i)

$$\frac{dy}{dx} = c_1 m e^{mx} + c_2(-m)e^{-mx}$$

@ x = 1 $\frac{dy}{dx} = 0$

$$0 = mc_1e^{m} - mc_2e^{-m} \rightarrow c_1e^{2m} = c_2 \ \ldots\ldots(ii)$$

From (i) & (ii)

$$c_1(1 + e^{2m}) = 1$$

$$c_1 = \frac{1}{(1 + e^{2m})}$$

$$c_2 = \frac{e^{2m}}{(1 + e^{2m})}$$

$$y = \frac{e^{mx}}{(1 + e^{2m})} + \frac{e^{2m}}{(1 + e^{2m})}e^{-mx}$$

Q218. Solve using $\frac{dy}{dx} + 0.6y = 6e^{-0.5x}$ the integrating factor method given y = 1 at x = 0.

(GATE – 1996)

Ans. $y = 60e^{-0.5x} - 59e^{-0.6x}$

Sol.

$$\frac{dy}{dx} + 0.6y = 6e^{-0.5x}$$

$$IF = e^{\int 0.6dx} = e^{0.6x}$$

$$y.e^{0..6x} = \int 6e^{-0.5x}e^{0.6x}dx + c$$

$$y.e^{0..6x} = 6\int e^{0.1x}dx + c$$

$$y.e^{0..6x} = \frac{6e^{0.1x}}{0.1} + c$$

$$y(0) = 1$$

$$1 = 60 + c \rightarrow c = -59$$

$$y = 60e^{0.1x}e^{-0.6x} - 59e^{-0.6x}$$

$$y = 60e^{-0.5x} - 59e^{-0.6x}$$

Q225. **Match the items in the left column with the appropriate items in the right column.**

(I) $y = x^2$

(II) $dy/dx = 2x$

(A) linear O.D.E.

(B) nonlinear O.D.E.

(C) linear algebraic equation

(D) nonlinear algebraic equation

(GATE – 1995)

Ans. $(I) \rightarrow (D), (II) \rightarrow (A)$

Sol.

$(i)\ y = x^2 \quad non\ linear\ algebric\ equation$

$(ii) \frac{dy}{dx} = 2x \quad linear\ O.D.E.$

Q226. **Match the items in the left column with the appropriate items in the right column.**

(I) $dy/dx + 5y = 0, y(0) = y_0$

(II) $dy/dx + 5 = 0, y(0) = y_0$

(A) $y = y_0 + 5x$

(B) $y = y_0 - 5x$

(C) $y = y_0 e^{-5x}$

(D) $y = y_0 e^{5x}$

(GATE – 1995)

Ans. $(I) \rightarrow y = y_0 e^{-5x}$ $(II) \rightarrow y = y_0 - 5x$

Sol.

(I) $\frac{dy}{dx} + 5y = 0$

$$-\int \frac{dy}{y} = \int 5dx + c$$

$$-\ln y = 5x + c$$

$$x = 0 \qquad y = y_0$$

$$-\ln y_0 = c$$

$$\ln \frac{y_0}{y} = 5x \rightarrow y = y_0 e^{-5x}$$

(II) $\frac{dy}{dx} + 5 = 0$

$$\int dy = -\int 5dx + c$$

$$y = -5x + c \qquad y(0) = y_0$$

$$y_0 = c$$

$$y = y_0 - 5x$$

Q230. Integrating factor for the differential equation, $\frac{dy}{dx} + P(x)y = Q(x)$

(GATE – 1994)

(a). $\exp[\int \mathrm{pdx}]$
(b). $\exp[-\int \mathrm{pdx}]$
(c). $\int \mathrm{Pdx}$
(d). $\mathrm{dP/dx}$

Ans. (a)

Sol.

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$I.F. = e^{\int P(x)dx}$$

Q231. The solution for the differential equation $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 0$

(GATE – 1994)

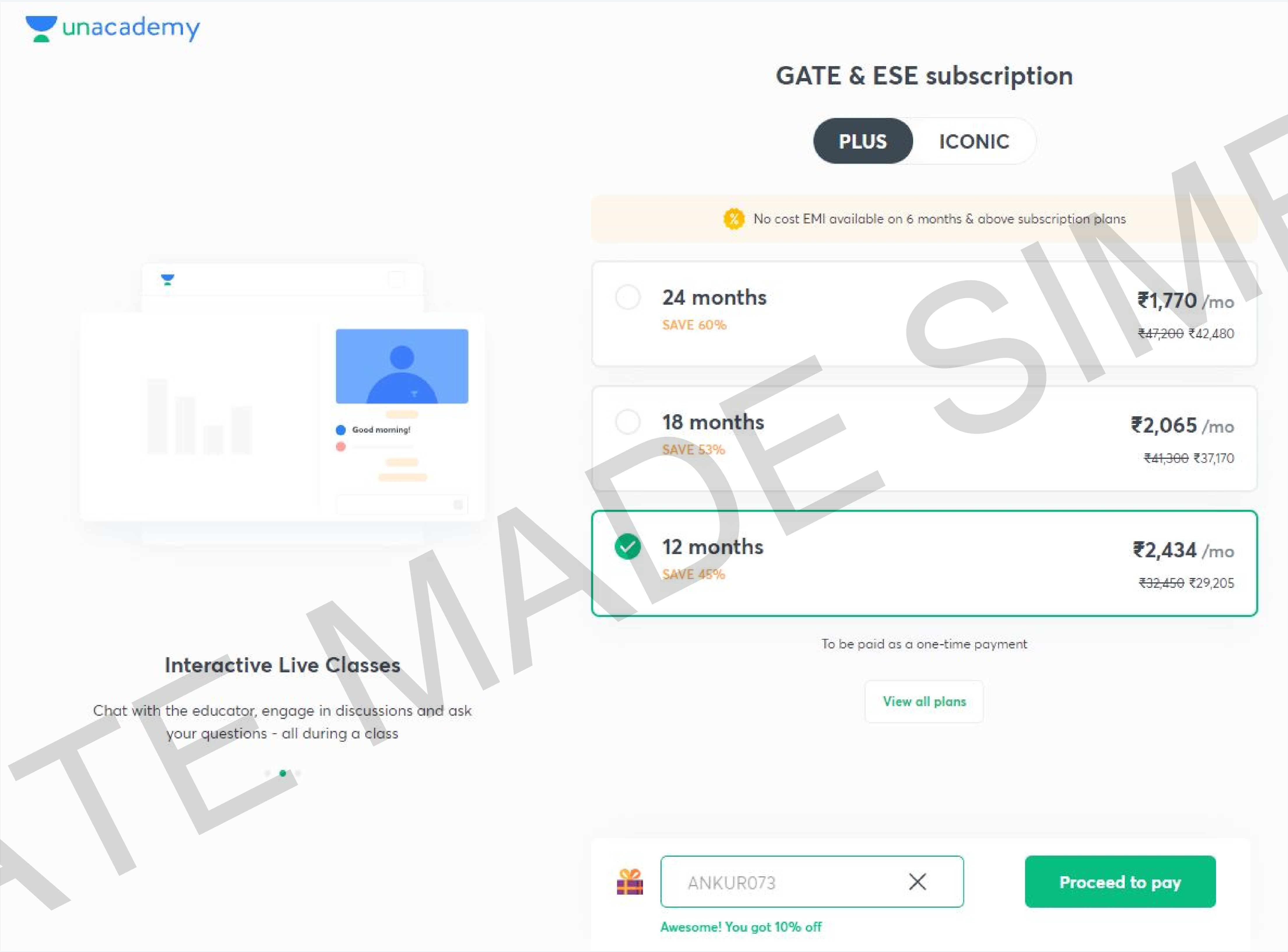
(a). $C_1 e^{-2t} + C_2 e^{3t}$

(b). $C_1 \sin 2t + C_2 \cos 2t$

(c). $C_1 e^{2t} + C_2 e^{-3t}$

(d). $C_1 e^{-2t} + C_2 e^{-3t}$

GATE MADE SIMPLE



Click Here to buy PLUS Plan

Ans. (d)

Sol.

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$$

$$(D^2 + 5D + 6)y = 0$$

$$M^2 + 5M + 6 = 0$$

$$M = -3, -2$$

Solution –

y= CF

$$y = c_1e^{-3x} + c_2e^{-2x}$$