

## 1.1. Implement UNet

### Binary Cross Entropy Loss:

```
22 def CELoss(y, y_hat, cw):
23
24     y, y_hat, cw = y.numpy(), y_hat.numpy(), cw.numpy()
25     n, c, d1, d2 = y_hat.shape
26     loss = np.empty_like(y)
27     L=[]
28
29     for i in range(n):
30         for k in range(d1):
31             for l in range(d2):
32                 clas = y[i,k,l]
33                 num = np.exp(y_hat[i,clas,k,l])
34                 denom = np.exp(y_hat[i,:,k,l]).sum()
35                 ce = -np.log(num/denom)*cw[clas]
36                 L.append(ce)
37
38     loss = np.asarray(L).reshape(y.shape)
39
40     return loss
41
```

### Dice Loss:

```
1 class DiceLoss(nn.Module):
2
3     def __init__(self):
4
5         super(DiceLoss, self).__init__()
6
7     def forward(self, input, target):
8
9         smooth = 1.0
10
11         iflat = input.view(-1)
12         tflat = target.view(-1)
13
14         iflat_square = iflat * iflat
15         tflat_square = tflat * tflat
16
17         intersection = (iflat * tflat).sum()
18
19         return 1 - ( (2.0 * intersection + smooth) / (iflat_square.sum() + tflat_square.sum() + smooth) )
20
21
```

### UNet:

```
314 class DoubleConv(nn.Module):
315
316     def __init__(self, in_ch, out_ch):
317
318         super(DoubleConv, self).__init__()
319
320         self.conv = nn.Sequential(
321             nn.Conv2d(in_ch, out_ch, 3, padding=1),
322             nn.BatchNorm2d(out_ch),
323             nn.ReLU(inplace = True),
324
325             nn.Conv2d(out_ch, out_ch, 3, padding=1),
326             nn.BatchNorm2d(out_ch),
327             nn.ReLU(inplace = True)
328         )
329
330     def forward(self, x):
331         return self.conv(x)
332
```

```

334 class UNet(nn.Module):
335
336     def __init__(self, in_ch=3, out_ch=1):
337
338         super(UNet, self).__init__()
339
340         self.conv1 = DoubleConv(in_ch, 64)
341         self.pool1 = nn.MaxPool2d(2)
342
343         self.conv2 = DoubleConv(64, 128)
344         self.pool2 = nn.MaxPool2d(2)
345
346         self.conv3 = DoubleConv(128, 256)
347         self.pool3 = nn.MaxPool2d(2)
348
349         self.conv4 = DoubleConv(256, 512)
350         self.pool4 = nn.MaxPool2d(2)
351
352         self.conv5 = DoubleConv(512, 1024)
353
354         self.up6 = nn.ConvTranspose2d(1024, 512, 2, stride=2)
355         self.conv6 = DoubleConv(1024, 512)
356
357         self.up7 = nn.ConvTranspose2d(512, 256, 2, stride=2)
358         self.conv7 = DoubleConv(512, 256)
359
360         self.up8 = nn.ConvTranspose2d(256, 128, 2, stride=2)
361         self.conv8 = DoubleConv(256, 128)
362
363         self.up9 = nn.ConvTranspose2d(128, 64, 2, stride=2)
364         self.conv9 = DoubleConv(128, 64)
365
366         self.conv10 = nn.Conv2d(64, out_ch, 1)
367
368
369     def forward(self, x):
370
371         c1 = self.conv1(x)
372         p1 = self.pool1(c1)
373
374         c2 = self.conv2(p1)
375         p2 = self.pool2(c2)
376
377         c3 = self.conv3(p2)
378         p3 = self.pool3(c3)
379
380         c4 = self.conv4(p3)
381         p4 = self.pool4(c4)
382
383         c5 = self.conv5(p4)
384
385         up_6 = self.up6(c5)
386         merge6 = torch.cat([up_6, c4], dim=1)
387         c6 = self.conv6(merge6)
388
389         up_7 = self.up7(c6)
390         merge7 = torch.cat([up_7, c3], dim=1)
391         c7 = self.conv7(merge7)
392
393         up_8 = self.up8(c7)
394         merge8 = torch.cat([up_8, c2], dim=1)
395         c8 = self.conv8(merge8)
396
397         up_9 = self.up9(c8)
398         merge9 = torch.cat([up_9, c1], dim=1)
399         c9 = self.conv9(merge9)
400
401         c10 = self.conv10(c9)
402
403         out = nn.Sigmoid()(c10)
404
405         return out
406
407

```

Read Data:

```

28 transform = transforms.Compose([transforms.ToPILImage(),
29                                 transforms.Resize((128, 128)),
30                                 transforms.ToTensor()])
31
98 def load_transform_dataset(input_path, mask_path, transform):
99     images = []
100     masks = []
101     size = 0
102
103     for imagepath in input_path:
104         size = size + 1
105         color_img = cv2.imread(str(imagepath))
106         color_img = transform(color_img)
107         images.append(color_img)
108
109     for imagepath in mask_path:
110         img = cv2.imread(str(imagepath))
111         mask_img = cv2.cvtColor(img, cv2.COLOR_BGR2GRAY)
112         mask_img = transform(mask_img)
113         masks.append(mask_img)
114
115     return images, masks, size
116
117 class MyDataset:
118
119     def __init__(self, inputs, masks, size):
120         self.inputs = inputs
121         self.masks = masks
122         self.len = size
123
124     def __len__(self):
125         return self.len
126
127     def __getitem__(self, idx):
128         return self.inputs[idx], self.masks[idx].float()
129
[ ] 1 train_inputs, train_thres, train_size = load_transform_dataset(train_input_path, train_mask_path, transform)
2 test_inputs, test_thres, test_size = load_transform_dataset(test_input_path, test_mask_path, transform)
3
4 train_data = MyDataset(train_inputs, train_thres, train_size)
5 test_data = MyDataset(test_inputs, test_thres, test_size)
6
7 trainloader = torch.utils.data.DataLoader(train_data, batch_size=5, shuffle=True)
8 testloader = torch.utils.data.DataLoader(test_data, batch_size=1, shuffle=True)

```

Train:

```
13 def train(net, trainloader, epochs=10, lr=0.003, loss_fun='dice'):
14
15     if torch.cuda.is_available():
16         net = net.cuda()
17
18     criterion = loss_function(loss_fun)
19     optimizer = torch.optim.Adam(net.parameters())
20     result = []
21
22     for e in range(epochs):
23         net.train()
24         running_loss = 0
25
26         for images, labels in tqdm(trainloader):
27             if torch.cuda.is_available():
28                 images = images.cuda()
29                 labels = labels.cuda()
30
31                 log_ps = net(images)
32                 loss = criterion(log_ps, labels)
33
34                 optimizer.zero_grad()
35                 loss.backward()
36                 optimizer.step()
37
38                 running_loss += loss.item()
39                 result.append([log_ps, images, labels])
40
41         else:
42             print(f"Training loss: {running_loss/len(trainloader)}")
43
44     return result
45
```

Test:

```
12 def test(net, testloader, loss_fun='dice'):
13
14     net.eval()
15     accuracy = 0
16     result = []
17     criterion = loss_function(loss_fun)
18
19     for images, labels in testloader:
20         if torch.cuda.is_available():
21             images = images.cuda()
22             labels = labels.cuda()
23
24             log_ps = net(images)
25             loss = criterion(log_ps, labels)
26
27             accuracy += 1 - loss
28             result.append([log_ps, images, labels])
29
30     print("Test Accuracy: {:.3f}".format(accuracy / len(testloader)))
31
32     return result
```

Binary Cross Entropy Loss Performance:

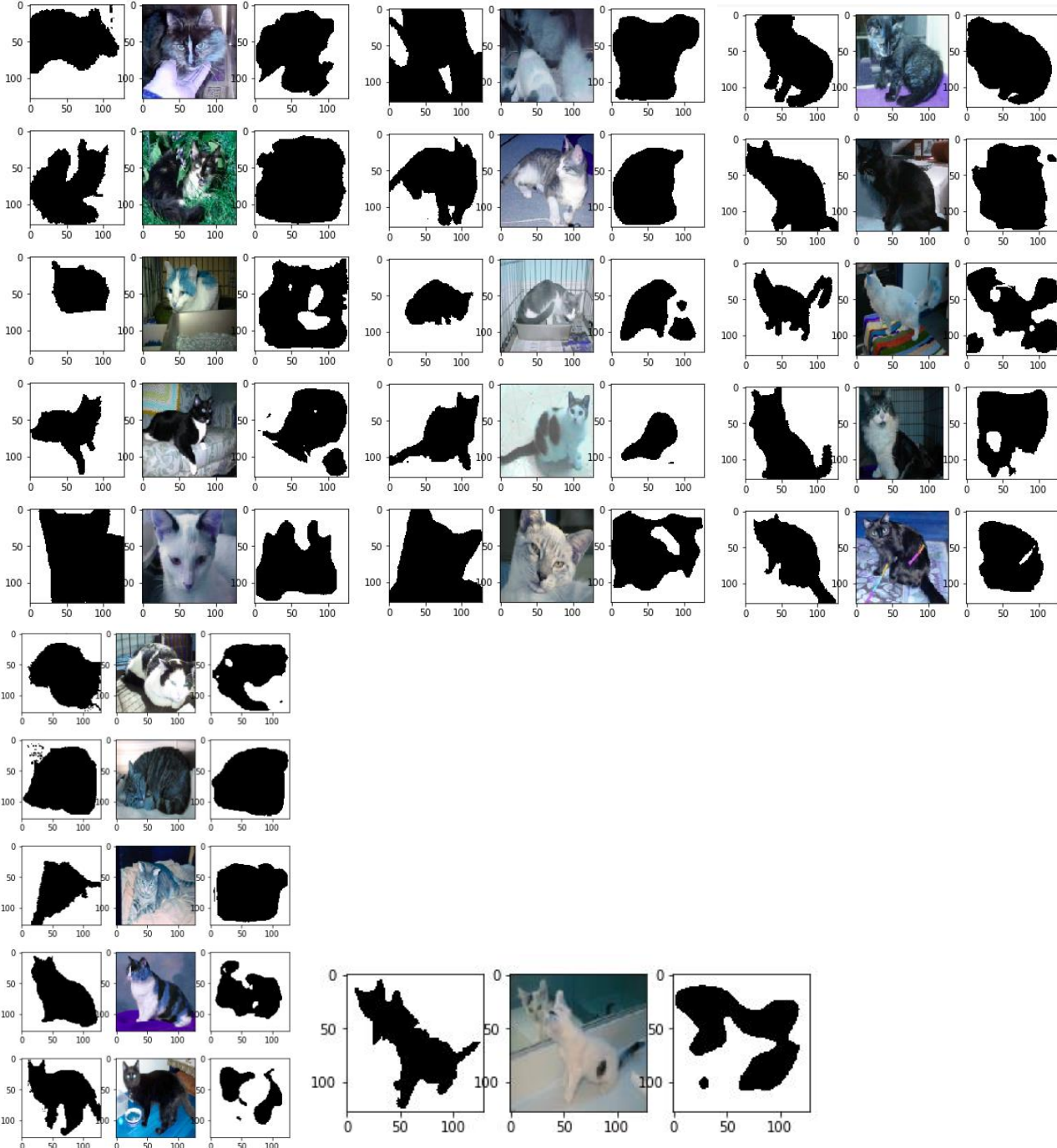
```
100% ██████████ 15/15 [00:03<00:00, 4.47it/s]
0% ██████████ 0/15 [00:00<?, ?it/s]Training loss: 0.7127752105394999
100% ██████████ 15/15 [00:03<00:00, 4.46it/s]
0% ██████████ 0/15 [00:00<?, ?it/s]Training loss: 0.6726645867029826
100% ██████████ 15/15 [00:03<00:00, 4.46it/s]
0% ██████████ 0/15 [00:00<?, ?it/s]Training loss: 0.6455337603886923
100% ██████████ 15/15 [00:03<00:00, 4.49it/s]
0% ██████████ 0/15 [00:00<?, ?it/s]Training loss: 0.6150294144948324
100% ██████████ 15/15 [00:03<00:00, 4.46it/s]
0% ██████████ 0/15 [00:00<?, ?it/s]Training loss: 0.6187748591105143
100% ██████████ 15/15 [00:03<00:00, 4.47it/s]
0% ██████████ 0/15 [00:00<?, ?it/s]Training loss: 0.5956886331240336
100% ██████████ 15/15 [00:03<00:00, 4.45it/s]
0% ██████████ 0/15 [00:00<?, ?it/s]Training loss: 0.5800564229488373
100% ██████████ 15/15 [00:03<00:00, 4.44it/s]
0% ██████████ 0/15 [00:00<?, ?it/s]Training loss: 0.5566850284735362
100% ██████████ 15/15 [00:03<00:00, 4.45it/s]
0% ██████████ 0/15 [00:00<?, ?it/s]Training loss: 0.5507455865542094
100% ██████████ 15/15 [00:03<00:00, 4.44it/s]
Training loss: 0.545043925444285
Test Accuracy: 0.656
```





## Dice Loss Performance:

```
100% ██████████ 15/15 [00:03<00:00, 4.47it/s]
0% ██████████ 0/15 [00:00<?, ?it/s]Training loss: 0.3616743922233582
100% ██████████ 15/15 [00:03<00:00, 4.45it/s]
0% ██████████ 0/15 [00:00<?, ?it/s]Training loss: 0.32314998706181847
100% ██████████ 15/15 [00:03<00:00, 4.44it/s]
0% ██████████ 0/15 [00:00<?, ?it/s]Training loss: 0.26556370258331297
100% ██████████ 15/15 [00:03<00:00, 4.46it/s]
0% ██████████ 0/15 [00:00<?, ?it/s]Training loss: 0.2352707068125407
100% ██████████ 15/15 [00:03<00:00, 4.43it/s]
0% ██████████ 0/15 [00:00<?, ?it/s]Training loss: 0.2460876742998759
100% ██████████ 15/15 [00:03<00:00, 4.44it/s]
0% ██████████ 0/15 [00:00<?, ?it/s]Training loss: 0.22416775226593016
100% ██████████ 15/15 [00:03<00:00, 4.45it/s]
0% ██████████ 0/15 [00:00<?, ?it/s]Training loss: 0.22270710865656534
100% ██████████ 15/15 [00:03<00:00, 4.42it/s]
0% ██████████ 0/15 [00:00<?, ?it/s]Training loss: 0.20345031817754108
100% ██████████ 15/15 [00:03<00:00, 4.43it/s]
Training loss: 0.20759664376576742
Test Accuracy: 0.742
```



Why dice loss is better than bce loss:

Because the actual goal of dice coefficient is to maximize those metrics, and cross-entropy is just a proxy which is easier to maximize using backpropagation. In addition, Dice coefficient performs better at class imbalanced problems by design.

## 1.2. Data Augmentation

I think there are two different ways to do the data augmentation. First, we can augment the images when loading the them. Second, we can augment the data when building the dataset (i.e. do the augmentation in “getitem”). I tried both and the second way is better in performance improvement. I used four different augmentation techniques: flip, changing the brightness, rotation, affine by randomly applying one of these to the image and mask synchronously when calling “getitem” from the dataset class. The reason of second way is better is because it can randomly assign one of the augmentations to one of the images of loaded data for every epoch (i.e. we can get the unique augmented training data for each epoch). However, if we use the first way (i.e. augmented images when loading the them), then even we randomly assign the augmentations, the images will stay the same as they loaded when training the data, which is not random enough. However, the performance is just better than before for a bit (0.783 compare to 0.779). I think it is because the double the data size is not enough. Larger data may provide even better performance.

```
69 def augment_options(image, index):
70
71     if index == 0:
72         image = cv2.flip(image, 1)
73         image = transform(image)
74
75     if index == 1:
76         image = np.uint8(np.clip((1.5 * image + 10), 0, 255))
77         image = transform(image)
78
79     if index == 2:
80         (h,w) = image.shape[:2]
81         center = (w / 2,h / 2)
82         M = cv2.getRotationMatrix2D(center,30,1)
83         image = cv2.warpAffine(image,M,(w,h))
84         image = transform(image)
85
86     if index == 3:
87         M = np.float32([[1,0,25],[0,1,50]])
88         image = cv2.warpAffine(image,M,(image.shape[1],image.shape[0]))
89         image = transform(image)
90
91     elif index != 0 or 1 or 2 or 3:
92         image = transform(image)
93
94     return image

```

```
117 def load_dataset(input_path, mask_path):
118     images = []
119     masks = []
120     size = 0
121
122     for imagepath in input_path:
123         size = size + 1
124         color_img = cv2.imread(str(imagepath))
125         images.append(color_img)
126
127     for imagepath in mask_path:
128         img = cv2.imread(str(imagepath))
129         mask_img = cv2.cvtColor(img, cv2.COLOR_BGR2GRAY)
130         masks.append(mask_img)
131
132     return images, masks, size

```

```
172 class MyAugmentDataset:
173
174     def __init__(self, inputs, masks, size):
175         self.inputs = inputs
176         self.masks = masks
177         self.len = size
178
179     def __len__(self):
180         return self.len
181
182     def __getitem__(self, idx):
183         image = self.inputs[idx]
184         mask = self.masks[idx]
185         seed = np.random.randint(0, 999999)
186         random_index = np.random.randint(0, 4)
187         # print("random index = ", random_index)
188         np.random.seed(seed)
189         image = augment_options(image, random_index)
190         np.random.seed(seed)
191         mask = augment_options(mask, random_index)
192         return image, mask.float()

```

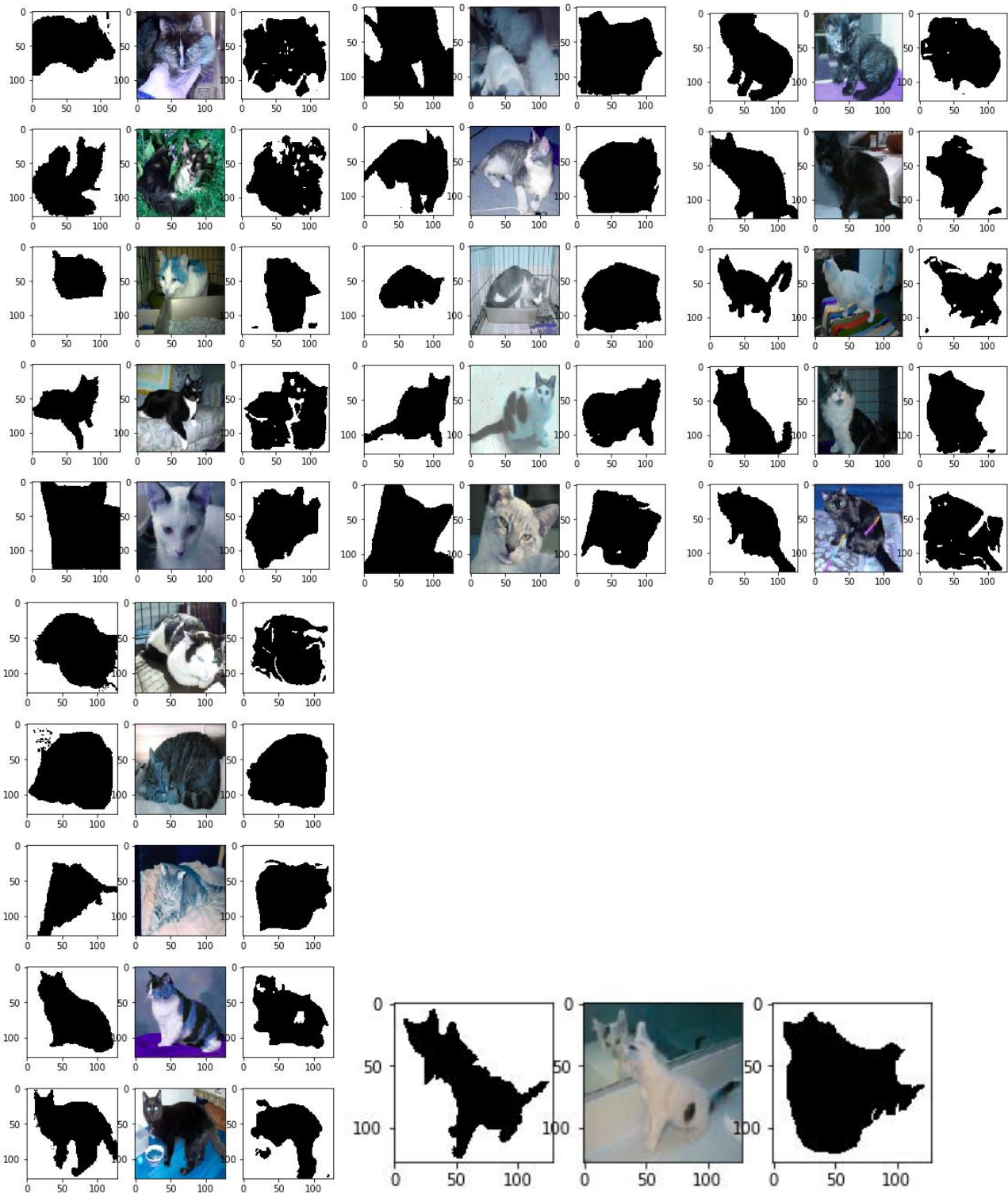
I doubled the data size:

```
1 train_inputs, train_thres, train_size = load_dataset(train_input_path, train_mask_path)
2 test_inputs, test_thres, test_size = load_transform_dataset(test_input_path, test_mask_path, transform)
3
4 train_data = MyAugmentDataset(train_inputs + train_inputs, train_thres + train_thres, train_size + train_size)
5 test_data = MyDataset(test_inputs, test_thres, test_size)
6
7 augment_trainloader = torch.utils.data.DataLoader(train_data, batch_size=5, shuffle=True)
8 testloader = torch.utils.data.DataLoader(test_data, batch_size=1)

```

Performance:

```
100% ██████████ 24/24 [00:07<00:00, 3.42it/s]
0% ██████████ 0/24 [00:00<?, ?it/s]Training loss: 0.3007693663239479
100% ██████████ 24/24 [00:07<00:00, 3.44it/s]
0% ██████████ 0/24 [00:00<?, ?it/s]Training loss: 0.25509018699328107
100% ██████████ 24/24 [00:06<00:00, 3.44it/s]
0% ██████████ 0/24 [00:00<?, ?it/s]Training loss: 0.23498573899269104
100% ██████████ 24/24 [00:07<00:00, 3.42it/s]
0% ██████████ 0/24 [00:00<?, ?it/s]Training loss: 0.2229666213194529
100% ██████████ 24/24 [00:07<00:00, 3.41it/s]
0% ██████████ 0/24 [00:00<?, ?it/s]Training loss: 0.22957989821831384
100% ██████████ 24/24 [00:07<00:00, 3.44it/s]
0% ██████████ 0/24 [00:00<?, ?it/s]Training loss: 0.2115496868888537
100% ██████████ 24/24 [00:07<00:00, 3.44it/s]
0% ██████████ 0/24 [00:00<?, ?it/s]Training loss: 0.20879298945267996
100% ██████████ 24/24 [00:07<00:00, 3.42it/s]
0% ██████████ 0/24 [00:00<?, ?it/s]Training loss: 0.20033962527910867
100% ██████████ 24/24 [00:07<00:00, 3.42it/s]
0% ██████████ 0/24 [00:00<?, ?it/s]Training loss: 0.2058428352077802
100% ██████████ 24/24 [00:07<00:00, 3.38it/s]
Training loss: 0.18970023343960443
Test Accuracy: 0.783
```





1.3. Transfer Learning (Oxford IIIT Pet Dataset: <https://www.robots.ox.ac.uk/~vgg/data/pets/>)

My trained net (the weight file): <https://drive.google.com/open?id=1my7gqC9yoBw9VP94AP84IJAqGZw6GNGr>

How I perform the transfer learning:

Step 0: Download the data from the website, and separate data reasonably.

Step 1. Find data path:

```
72 transfer_train_input_path = glob.glob("./images/*.jpg")
73 transfer_train_input_path.sort()
74 transfer_train_mask_path = glob.glob("./annotations/trimaps/*.png")
75 transfer_train_mask_path.sort()
```

Step 2. Load transfer data, transform them to tensor and handle corrupted data:

```
77 transform = transforms.Compose([transforms.ToPILImage(),
78                                 transforms.Resize((128, 128)),
79                                 transforms.ToTensor()])
163 def load_transfer_dataset(input_path, mask_path, transform):
164     images = []
165     masks = []
166     size = 0
167     corrupted_index = []
168     print("original number of images = ", len(input_path), ", original number of masks = ", len(mask_path))
169
170     for imagepath in input_path:
171         size = size + 1
172         color_img = cv2.imread(str(imagepath))
173         if color_img is None:
174             index = input_path.index(imagepath)
175             corrupted_index.append(index)
176             continue
177         color_img = transform(color_img)
178         images.append(color_img)
179
180     for imagepath in mask_path:
181         mask = Image.open(imagepath).convert("L")
182         mask = np.array(mask)
183         mask = (mask == 1)
184         mask = (mask * 255).astype("uint8")
185         mask_img = transform(mask)
186         masks.append(mask_img)
187
188     for index in sorted(corrupted_index, reverse=True):
189         del masks[index]
190
191     size = size - len(corrupted_index)
192
193     print("number of images = ", len(images), ", number of masks = ", len(masks), ", size = ", size)
194
195     return images, masks, size
```



### Step 3. Build basic dataset:

```
198 class MyDataset:
199
200     def __init__(self, inputs, masks, size):
201         self.inputs = inputs
202         self.masks = masks
203         self.len = size
204
205     def __len__(self):
206         return self.len
207
208     def __getitem__(self, idx):
209         return self.inputs[idx], self.masks[idx].float()
```

### Step 4. Use the same UNet, loss function and train/test function from 1.1 and 1.2. Execute and save the trained net:

```
439 if __name__ == '__main__':
440
441     torch.backends.cudnn.benchmark = True
442     torch.cuda.empty_cache()
443     # os.environ['CUDA_VISIBLE_DEVICES'] = '0, 1, 2'
444
445     # train_inputs, train_masks, train_size = load_transform_dataset(train_input_path, train_mask_path, transform)
446     # transfer_train_inputs, transfer_train_masks, transfer_train_size = load_transform_dataset(transfer_train_input_path, transfer_train_mask_path, transform)
447     transfer_train_inputs, transfer_train_masks, transfer_train_size = load_transfer_dataset(transfer_train_input_path,
448                                                                                           transfer_train_mask_path,
449                                                                                           transform)
450     test_inputs, test_masks, test_size = load_transform_dataset(test_input_path, test_mask_path, transform)
451
452     # train_inputs, train_masks, train_size = load_dataset(train_input_path, train_mask_path)
453     # train_data = MyAugmentDataset(train_inputs, train_masks, train_size, all_transform)
454
455     # train_data = MyDataset(train_inputs, train_masks, train_size)
456     train_data = MyDataset(transfer_train_inputs, transfer_train_masks, transfer_train_size)
457     test_data = MyDataset(test_inputs, test_masks, test_size)
458
459     trainloader = torch.utils.data.DataLoader(train_data, batch_size=64, shuffle=True, pin_memory=True)
460     testloader = torch.utils.data.DataLoader(test_data, batch_size=1, shuffle=True)
461
462
463     transfer_learning_net = UNet()
464     if torch.cuda.is_available():
465         transfer_learning_net = transfer_learning_net.cuda()
466     # transfer_learning_net.load_state_dict(torch.load("./tf_model_post.pt"))
467
468     # net = UNet()
469     # if torch.cuda.is_available():
470     #     net = net.cuda()
471
472     # train(net, trainloader)
473     # te_result = test(net, testloader)
474
475     train(transfer_learning_net, trainloader)
476     torch.save(transfer_learning_net.state_dict(), "./my_net.pt")
477     te_result = test(transfer_learning_net, testloader)
```

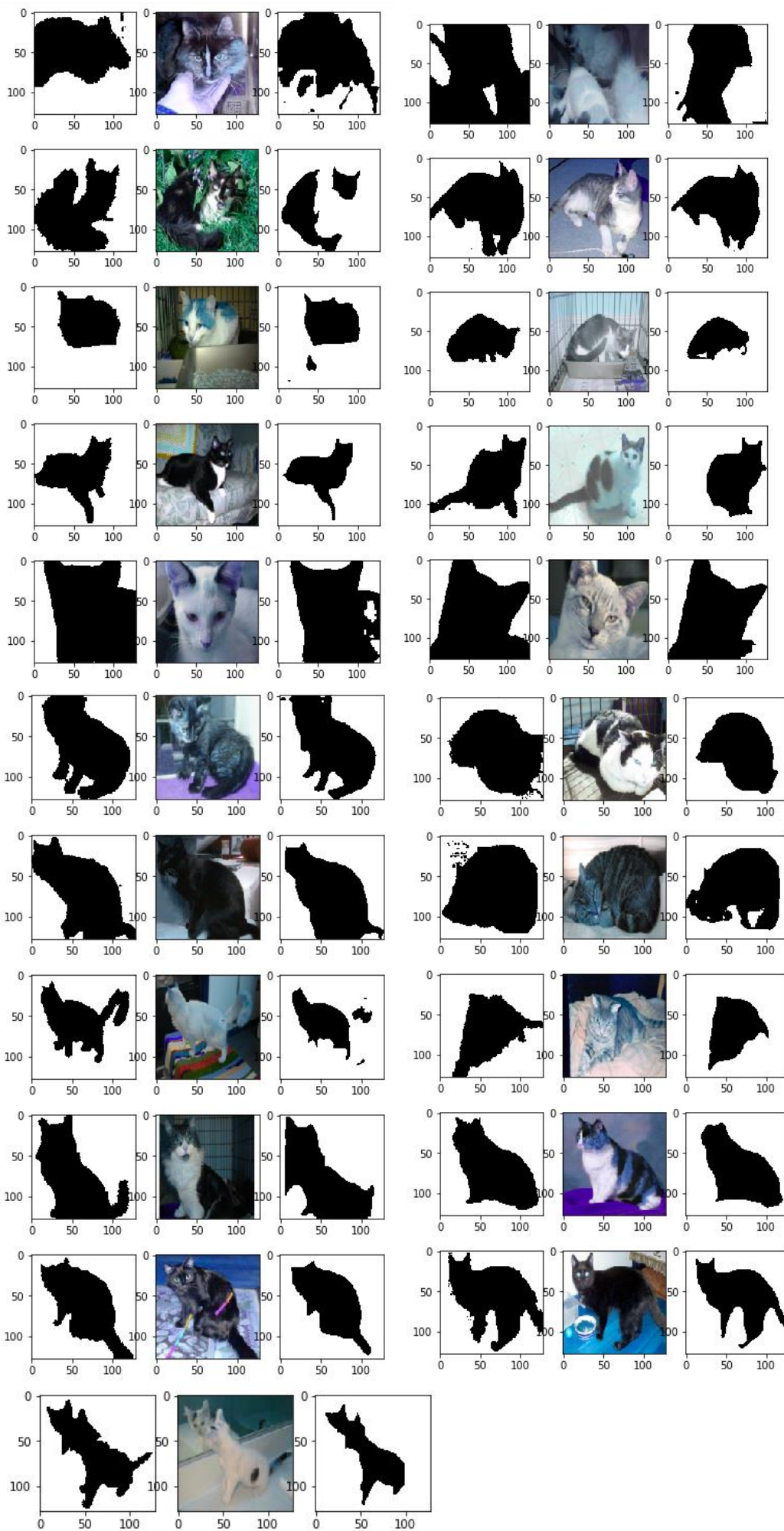
Step 5. Check the performance, the training set is the transfer data from Oxford, the test set is from a3 cat\_data

```
number of images = 7384 , number of masks = 7384 , size = 7384
original number of images = 21 , original number of masks = 21
number of images = 21 , number of masks = 21 , size = 21
99%|██████████| 115/116 [00:48<00:00, 2.82it/s]Training loss: 0.24215494712878918
100%|██████████| 116/116 [00:54<00:00, 2.14it/s]
99%|██████████| 115/116 [00:41<00:00, 2.65it/s]Training loss: 0.16631031087760267
100%|██████████| 116/116 [00:41<00:00, 2.78it/s]
100%|██████████| 116/116 [00:41<00:00, 2.81it/s]
0%|          | 0/116 [00:00<?, ?it/s]Training loss: 0.1374676489624484
100%|██████████| 116/116 [00:41<00:00, 2.80it/s]
Training loss: 0.1150483694569818
99%|██████████| 115/116 [00:41<00:00, 2.78it/s]Training loss: 0.10264479394616752
100%|██████████| 116/116 [00:41<00:00, 2.80it/s]
100%|██████████| 116/116 [00:41<00:00, 2.79it/s]
0%|          | 0/116 [00:00<?, ?it/s]Training loss: 0.09419669776127257
100%|██████████| 116/116 [00:41<00:00, 2.79it/s]
0%|          | 0/116 [00:00<?, ?it/s]Training loss: 0.087798109342312
99%|██████████| 115/116 [00:41<00:00, 2.77it/s]Training loss: 0.07970565300563286
100%|██████████| 116/116 [00:41<00:00, 2.79it/s]
99%|██████████| 115/116 [00:41<00:00, 2.71it/s]Training loss: 0.07418641139721048
100%|██████████| 116/116 [00:41<00:00, 2.77it/s]
99%|██████████| 115/116 [00:41<00:00, 2.77it/s]Training loss: 0.0730496783708704
100%|██████████| 116/116 [00:41<00:00, 2.78it/s]
100%|██████████| 116/116 [00:41<00:00, 2.78it/s]
0%|          | 0/116 [00:00<?, ?it/s]Training loss: 0.06982231705353178
99%|██████████| 115/116 [00:41<00:00, 2.77it/s]Training loss: 0.06446692604443123
100%|██████████| 116/116 [00:41<00:00, 2.78it/s]
100%|██████████| 116/116 [00:41<00:00, 2.78it/s]
0%|          | 0/116 [00:00<?, ?it/s]Training loss: 0.06139969517444742
100%|██████████| 116/116 [00:41<00:00, 2.78it/s]
Training loss: 0.05790502089878608
99%|██████████| 115/116 [00:42<00:00, 2.68it/s]Training loss: 0.05404536426067352
100%|██████████| 116/116 [00:42<00:00, 2.71it/s]
Test Accuracy: 0.867
```

```
1 transfer_learning_net = UNet()
2 if torch.cuda.is_available():
3     transfer_learning_net = transfer_learning_net.cuda()
4
5 transfer_learning_net.load_state_dict(torch.load("../drive/My Drive/csc420/my_net_3.pt"))
6 transfer_te_result = test(transfer_learning_net, testloader)
```

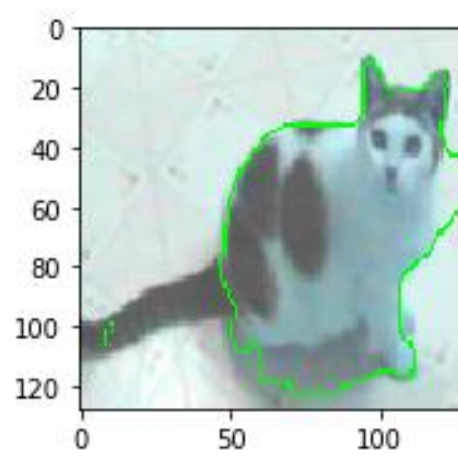
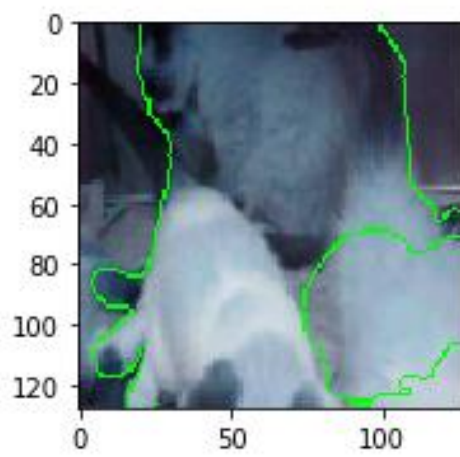
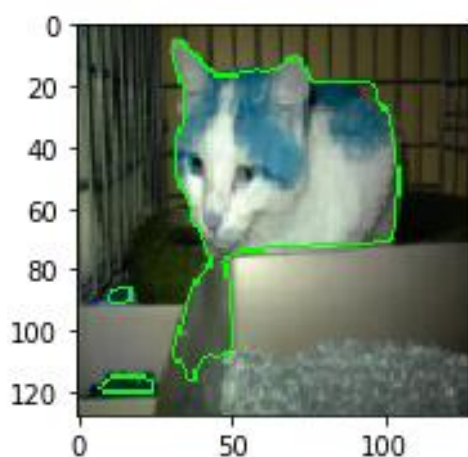
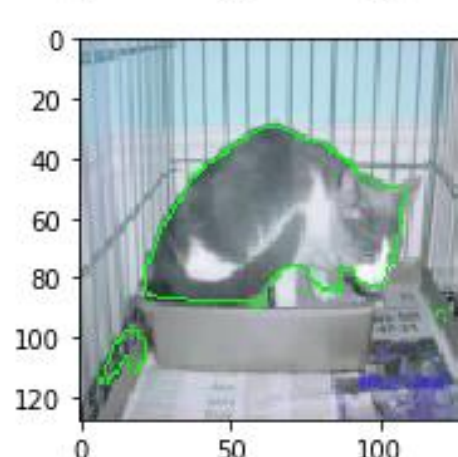
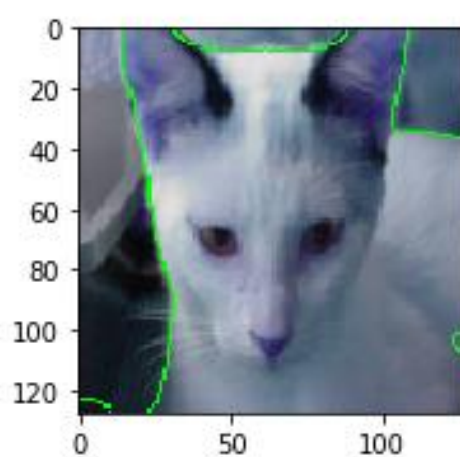
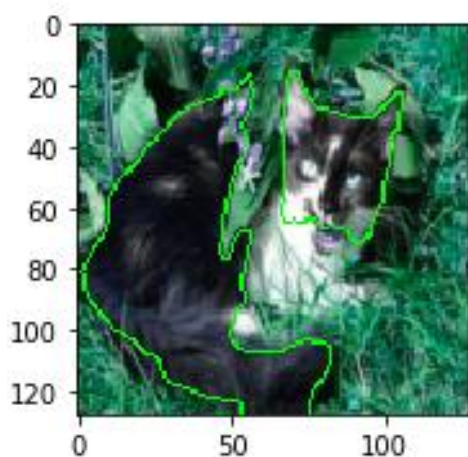
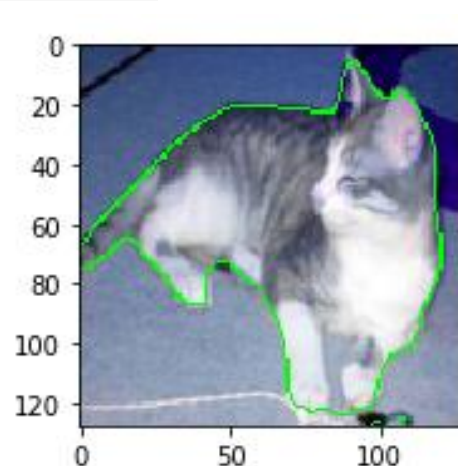
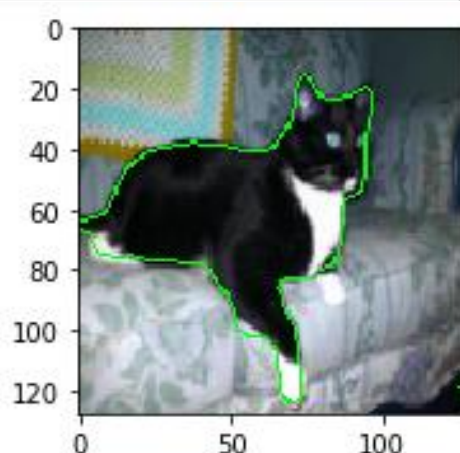
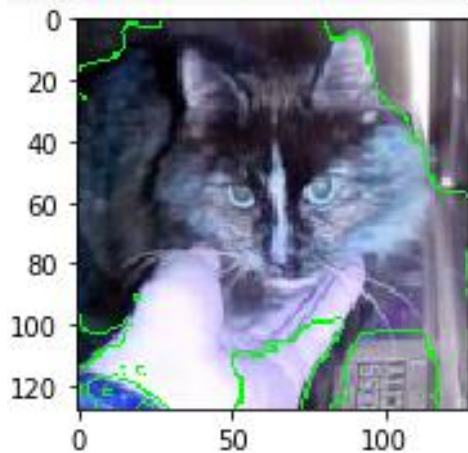
Test Accuracy: 0.911



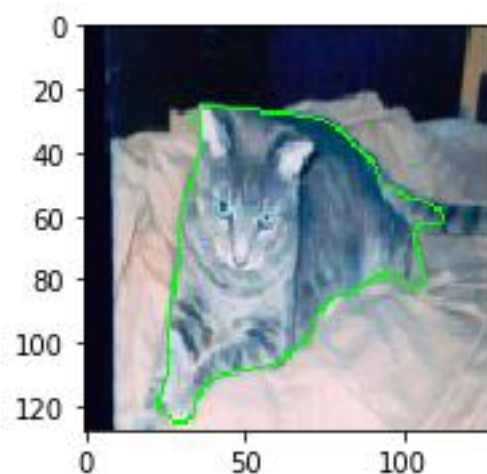
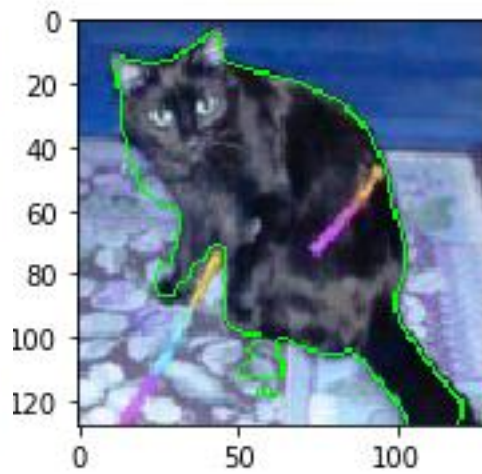
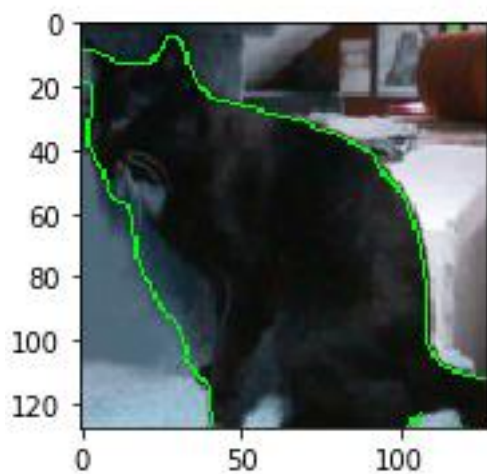
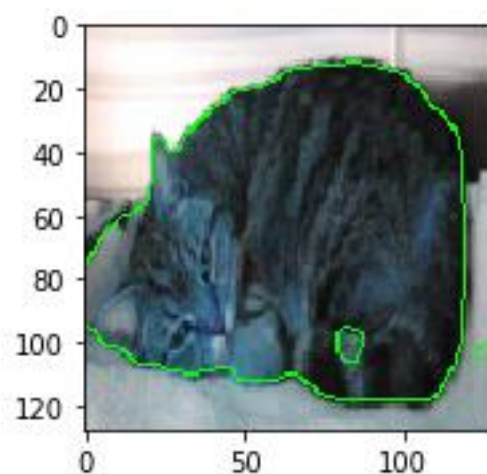
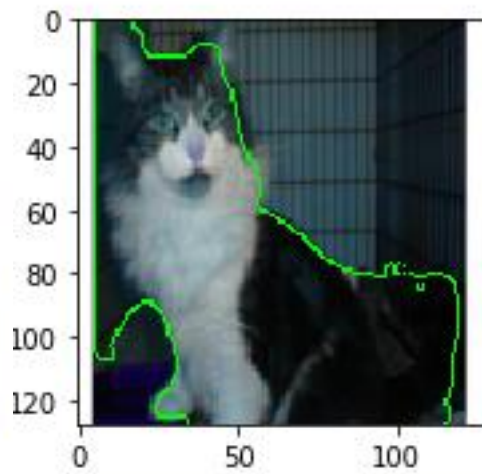
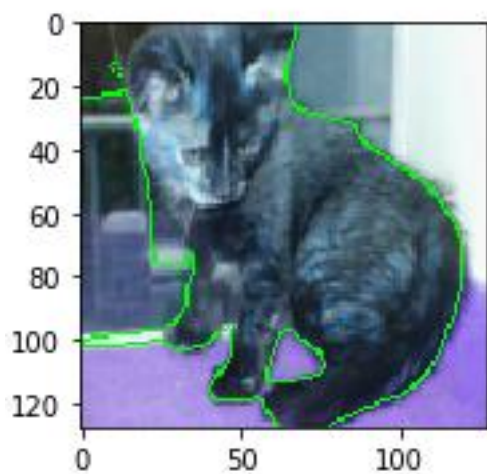
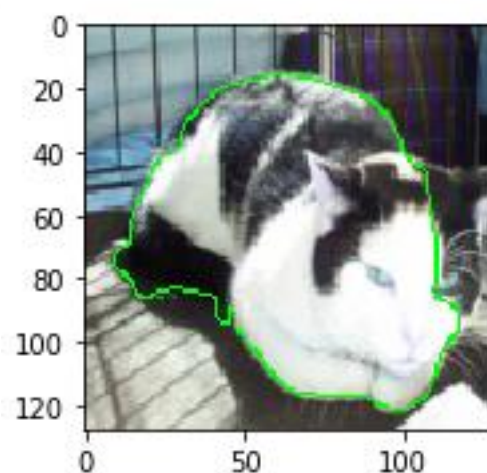
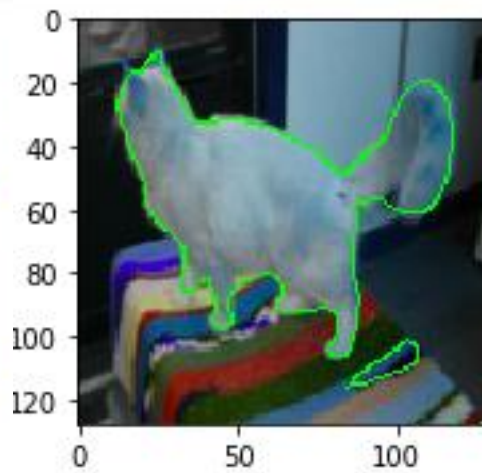
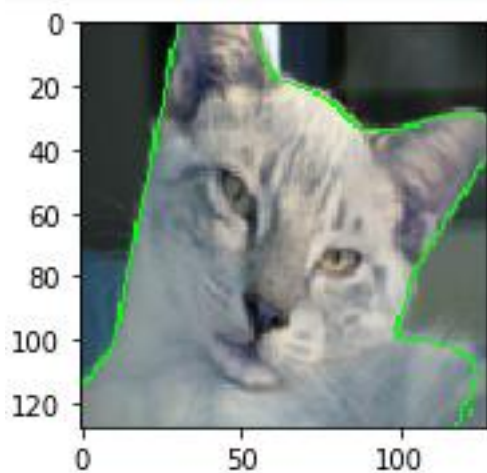


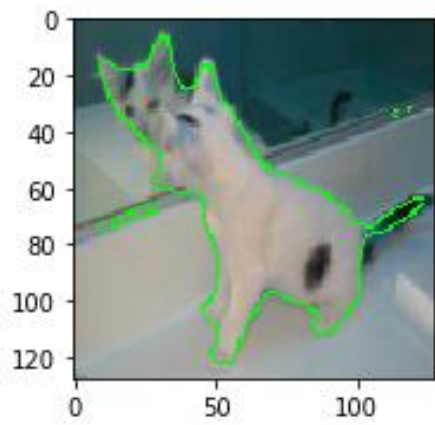
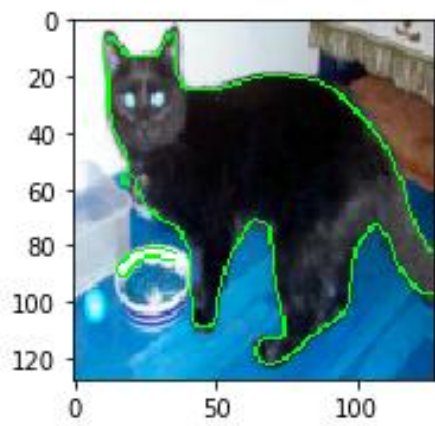
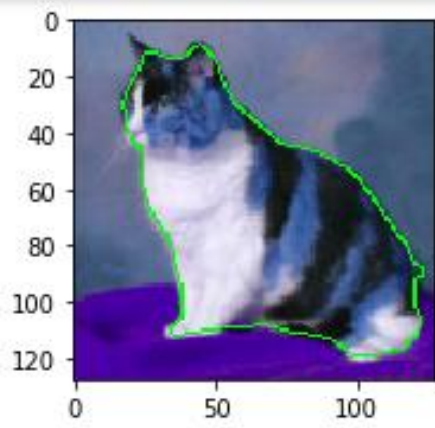
#### 1.4. Visualizing segmentation predictions:

```
1 def visualize(result, index):
2     ret, binary_pred = cv2.threshold(result[index][0].cpu().squeeze().detach().numpy(), 0.05, 1, 0)
3     outputs = np.array(binary_pred)*255
4     images = np.array(result[index][1].cpu().squeeze().permute(1,2,0))
5
6     result = np.copy(images)
7     outputs_canny = cv2.Canny(np.uint8(outputs), 20, 250)
8     result[outputs_canny == 255] = [0,1,0]
9     plt.subplot(1, 2, 1)
10    plt.imshow(result)
11    plt.tight_layout()
12
13 visualize(transfer_te_result, 18)
```









## 2.1. Problem definition

Input and output of my neural network:

Input: The original image generated by function `noisy_circle` from `main.py`.

Output: The prediction image of the circle.

```
def train(net, trainloader, epochs=20, loss_fun='iou'):
    if torch.cuda.is_available():
        net.cuda()

    criterion = loss_function(loss_fun)
    optimizer = torch.optim.Adam(net.parameters())

    for e in range(epochs):
        net.train()
        running_loss = 0
        for images, labels in tqdm(trainloader):
            if torch.cuda.is_available():
                images = images.cuda()
                labels = labels.cuda()

            log_ps = net(images)
            loss = criterion(log_ps, labels)

            optimizer.zero_grad()
            loss.backward()
            optimizer.step()

            running_loss += loss.item()
        else:
            print(f"Training loss: {running_loss / len(trainloader)}")
```

Loss function: Dice Loss

```
class DiceLoss(nn.Module):
    def __init__(self):
        super(DiceLoss, self).__init__()

    def forward(self, input, target):
        smooth = 1.

        iflat = input.view(-1)
        tflat = target.view(-1)
        intersection = (iflat * tflat).sum()

        return 1 - ((2. * intersection + smooth) /
                    ((iflat * iflat).sum() + (tflat * tflat).sum() + smooth))
```

Why I represent the problem this way:

I want to use the similar process from question 1 to question 2 (i.e. using UNet for question 2). UNet is a good neural network for image segmentation. For question 2, when we generating the noisy image with the circle, we can also generate the “mask” image synchronously, which is the original generated circle image without the noise. Then, we can apply the similar technique from Q1 to Q2: using the noisy circle images and the corresponding “masks” (i.e. clear circle images) to train the net and predict the “masks” (i.e. clear circle images) for random inputs.

## 2.2. Implementation

My neural network: UNet with less layers

```
class DoubleConv(nn.Module):
    def __init__(self, in_ch, out_ch):
        super(DoubleConv, self).__init__()
        self.conv = nn.Sequential(
            nn.Conv2d(in_ch, out_ch, 3, padding=1),
            nn.ReLU(),
            nn.BatchNorm2d(out_ch),
            nn.Conv2d(out_ch, out_ch, 3, padding=1),
            nn.ReLU(),
            nn.BatchNorm2d(out_ch)
        )

    def forward(self, x):
        return self.conv(x)
```

```
class UNet(nn.Module):
    def __init__(self, in_ch=1, out_ch=1):
        super(UNet, self).__init__()

        self.conv1 = DoubleConv(in_ch, 64)
        self.pool1 = nn.MaxPool2d(2)

        self.conv2 = DoubleConv(64, 128)
        self.pool2 = nn.MaxPool2d(2)

        # self.conv3 = DoubleConv(128, 256)
        # self.pool3 = nn.MaxPool2d(2)

        # self.conv4 = DoubleConv(256, 512)
        # self.pool4 = nn.MaxPool2d(2)

        self.conv5 = DoubleConv(128, 256)

        # self.up6 = nn.ConvTranspose2d(1024, 512, 2, stride=2)
        # self.conv6 = DoubleConv(1024, 512)

        # self.up7 = nn.ConvTranspose2d(512, 256, 2, stride=2)
        # self.conv7 = DoubleConv(512, 256)

        self.up8 = nn.ConvTranspose2d(256, 128, 2, stride=2)
        self.conv8 = DoubleConv(256, 128)

        self.up9 = nn.ConvTranspose2d(128, 64, 2, stride=2)
        self.conv9 = DoubleConv(128, 64)

        self.conv10 = nn.Conv2d(64, out_ch, 1)
```

```
    def forward(self, x):
        c1 = self.conv1(x)
        p1 = self.pool1(c1)
        c2 = self.conv2(p1)
        p2 = self.pool2(c2)

        # c3 = self.conv3(p2)
        # p3 = self.pool3(c3)
        # c4 = self.conv4(p3)
        # p4 = self.pool4(c4)

        c5 = self.conv5(p2)

        # up_6 = self.up6(c5)
        # merge6 = torch.cat([up_6, c4], dim=1)
        # c6 = self.conv6(merge6)
        # up_7 = self.up7(c5)
        # merge7 = torch.cat([up_7, c3], dim=1)
        # c7 = self.conv7(merge7)

        up_8 = self.up8(c5)
        merge8 = torch.cat([up_8, c2], dim=1)
        c8 = self.conv8(merge8)
        up_9 = self.up9(c8)
        merge9 = torch.cat([up_9, c1], dim=1)
        c9 = self.conv9(merge9)

        c10 = self.conv10(c9)

        out = nn.Sigmoid()(c10)
        return out
```



## Visualization of network (includes Parameters):

```
114
115 model = UNet()
116 if torch.cuda.is_available():
117     model.cuda()
118 summary(model, (1, 200, 200))
```

Layer (type)	Output Shape	Param #		
Conv2d-1	[-1, 64, 200, 200]	640		
ReLU-2	[-1, 64, 200, 200]	0		
BatchNorm2d-3	[-1, 64, 200, 200]	128		
Conv2d-4	[-1, 64, 200, 200]	36,928		
ReLU-5	[-1, 64, 200, 200]	0		
BatchNorm2d-6	[-1, 64, 200, 200]	128		
DoubleConv-7	[-1, 64, 200, 200]	0		
MaxPool2d-8	[-1, 64, 100, 100]	0		
Conv2d-9	[-1, 128, 100, 100]	73,856		
ReLU-10	[-1, 128, 100, 100]	0		
BatchNorm2d-11	[-1, 128, 100, 100]	256		
Conv2d-12	[-1, 128, 100, 100]	147,584		
ReLU-13	[-1, 128, 100, 100]	0		
BatchNorm2d-14	[-1, 128, 100, 100]	256		
DoubleConv-15	[-1, 128, 100, 100]	0	BatchNorm2d-30	[-1, 128, 100, 100] 256
MaxPool2d-16	[-1, 128, 50, 50]	0	DoubleConv-31	[-1, 128, 100, 100] 0
Conv2d-17	[-1, 256, 50, 50]	295,168	ConvTranspose2d-32	[-1, 64, 200, 200] 32,832
ReLU-18	[-1, 256, 50, 50]	0	Conv2d-33	[-1, 64, 200, 200] 73,792
BatchNorm2d-19	[-1, 256, 50, 50]	512	ReLU-34	[-1, 64, 200, 200] 0
Conv2d-20	[-1, 256, 50, 50]	590,080	BatchNorm2d-35	[-1, 64, 200, 200] 128
ReLU-21	[-1, 256, 50, 50]	0	Conv2d-36	[-1, 64, 200, 200] 36,928
BatchNorm2d-22	[-1, 256, 50, 50]	512	ReLU-37	[-1, 64, 200, 200] 0
DoubleConv-23	[-1, 256, 50, 50]	0	BatchNorm2d-38	[-1, 64, 200, 200] 128
ConvTranspose2d-24	[-1, 128, 100, 100]	131,200	DoubleConv-39	[-1, 64, 200, 200] 0
Conv2d-25	[-1, 128, 100, 100]	295,040	Conv2d-40	[-1, 1, 200, 200] 65
ReLU-26	[-1, 128, 100, 100]	0		
BatchNorm2d-27	[-1, 128, 100, 100]	256		
Conv2d-28	[-1, 128, 100, 100]	147,584		
ReLU-29	[-1, 128, 100, 100]	0		
BatchNorm2d-30	[-1, 128, 100, 100]	256		
DoubleConv-31	[-1, 128, 100, 100]	0		
ConvTranspose2d-32	[-1, 64, 200, 200]	32,832		
Conv2d-33	[-1, 64, 200, 200]	73,792		
ReLU-34	[-1, 64, 200, 200]	0		
BatchNorm2d-35	[-1, 64, 200, 200]	128		
Conv2d-36	[-1, 64, 200, 200]	36,928		
ReLU-37	[-1, 64, 200, 200]	0		

```

Total params: 1,864,257
Trainable params: 1,864,257
Non-trainable params: 0

Input size (MB): 0.15
Forward/backward pass size (MB): 481.26
Params size (MB): 7.11
Estimated Total Size (MB): 488.53

```

## Explain of architecture:

As we already know from Q1, U-Net is a convolutional neural network for biomedical image segmentation. The network is based on the fully convolutional network and its architecture was modified and extended to work with fewer training images and to yield more precise segmentations. The main idea is to supplement a usual contracting network by successive layers, where pooling operations are replaced by up-sampling operators. Hence these layers increase the resolution of the output. What's more, a successive convolutional layer can then learn to assemble a precise output based on this information. U-Net has a large number of feature channels in the up-sampling part, which allow the network to propagate context information to higher resolution layers.

As I said before, I want to use the similar process from question 1 to question 2 (i.e. using U-Net for question 2). U-Net is a good neural network for image segmentation. For question 2, when we generating the noisy image with the circle, we can also generate the “mask” image synchronously, which is the original generated circle image without the noise. Then, we can apply the similar technique from Q1 to Q2: using the noisy circle images and the corresponding “masks” (i.e. clear circle images) to train the net and predict the “masks” (i.e. clear circle images) for random inputs.

Please notice that I changed the “noisy\_circle” function from main.py slightly, by returning an extra result “img” as the “mask” (i.e. clear circle image without noise). The original “img” is called “noisy\_img” now.

```
def noisy_circle(size, radius, noise):
    img = np.zeros((size, size), dtype=np.float)

    # Circle
    row = np.random.randint(size)
    col = np.random.randint(size)
    rad = np.random.randint(10, max(10, radius))
    draw_circle(img, row, col, rad)

    noisy_img = img.copy()

    # Noise
    noisy_img += noise * np.random.rand(*img.shape)

    return (row, col, rad), noisy_img, img
```

Training set:

```
141     transform = transforms.Compose([transforms.ToTensor()])
142     |
143     def load_transform_train_dataset(transform):
144         images = []
145         masks = []
146         size = 0
147         while size < 100:
148             params, noisy_img, mask_img = noisy_circle(200, 50, 2)
149             noisy_img = transform(noisy_img)
150             mask_img = transform(mask_img)
151             images.append(noisy_img)
152             masks.append(mask_img)
153             size = size + 1
154         return images, masks, size
```

Testing set:

```
156     def load_transform_test_dataset(transform):
157         images = []
158         masks = []
159         size = 0
160         while size < 10:
161             params, noisy_img, mask_img = noisy_circle(200, 50, 2)
162             noisy_img = transform(noisy_img)
163             mask_img = transform(mask_img)
164             images.append(noisy_img)
165             masks.append(mask_img)
166             size = size + 1
167         return images, masks, size
```

Find Circle:

```
def find_circle(img, mask):
    # Fill in this function
    noisy_img = [transform(img)]
    mask_img = [transform(mask)]
    size = 1

    test_data = MyDataset(noisy_img, mask_img, size)
    testloader = torch.utils.data.DataLoader(test_data, batch_size=1)

    find_circle_net = UNet()
    if torch.cuda.is_available():
        find_circle_net = find_circle_net.cuda()
    find_circle_net.load_state_dict(torch.load("./q2_net_iou_loss.pt"))

    test_result = test(find_circle_net, testloader)
    center, radius = detect_circle(test_result[0][0].cpu().squeeze().detach().numpy())
    print("prediction: ", center, radius)
    print("=====")
    return center[1], center[0], radius
```

## Detect Circle:

```
def detect_circle(predict):
    # print(np.size(predict,0), np.size(predict,1))
    predict = predict[1:(np.size(predict,0)-1),1:(np.size(predict,1)-1)]
    # print(np.size(predict,0), np.size(predict,1))
    ret, binary = cv2.threshold(predict,0.04,1,0)
    # print(binary)

    circle_indexes = np.where(binary==np.max(binary))
    # print(circle_indexes)

    min_x = circle_indexes[0][0]
    y_index_for_min_x = np.where(circle_indexes[0]==min_x)[0]
    y_for_min_x_array = circle_indexes[1][y_index_for_min_x]
    y_for_min_x = np.mean(y_for_min_x_array)
    # print('min row = ', min_x, y_for_min_x)

    max_x = circle_indexes[0][-1]
    y_index_for_max_x = np.where(circle_indexes[0]==max_x)[0]
    y_for_max_x_array = circle_indexes[1][y_index_for_max_x]
    y_for_max_x = np.mean(y_for_max_x_array)
    # print('max row = ', max_x, y_for_max_x)

    min_y = min(circle_indexes[1])
    x_index_for_min_y = np.where(circle_indexes[1]==min_y)[0]
    x_for_min_y_array = circle_indexes[0][x_index_for_min_y]
    x_for_min_y = np.mean(x_for_min_y_array)
    # print('min col = ', min_y, x_for_min_y)

    max_y = max(circle_indexes[1])
    x_index_for_max_y = np.where(circle_indexes[1]==max_y)[0]
    x_for_max_y_array = circle_indexes[0][x_index_for_max_y]
    x_for_max_y = np.mean(x_for_max_y_array)
    # print('max col = ', max_y, x_for_max_y)
```

```
point_1 = (y_for_min_x, min_x)
point_2 = (y_for_max_x, max_x)
point_3 = (min_y, x_for_min_y)
point_4 = (max_y, x_for_max_y)
# print(point_1, point_2, point_3, point_4)

d_x_1 = point_1[0] - point_2[0]
d_y_1 = point_1[1] - point_2[1]
d_1 = math.sqrt(d_x_1**2 + d_y_1**2)
# print("d1 = ", d_1)

d_x_2 = point_3[0] - point_4[0]
d_y_2 = point_3[1] - point_4[1]
d_2 = math.sqrt(d_x_2**2 + d_y_2**2)
# print("d2 = ", d_2)

diameter = max(d_1, d_2)
# print("diameter = ", diameter)
radius = int(round(max(d_1, d_2)/2))
# print("radius = ", radius)

if diameter == d_1:
    x = int(round((point_1[0] + point_2[0] + 2) / 2))
    y = int(round((point_1[1] + point_2[1] + 2) / 2))
    center = (x, y)

elif diameter == d_2:
    x = int(round((point_3[0] + point_4[0] + 2) / 2))
    y = int(round((point_3[1] + point_4[1] + 2) / 2))
    center = (x, y)

# print("center = ", center)

return center, radius
```



Dice performance of my test (train data size = 1000, epoch = 15):

```
97%|██████████| 31/32 [00:11<00:00, 3.04it/s]Training loss: 0.958402244374156
100%|██████████| 32/32 [00:11<00:00, 2.84it/s]
100%|██████████| 32/32 [00:10<00:00, 3.10it/s]
 0%|          | 0/32 [00:00<?, ?it/s]Training loss: 0.9005990009754896
97%|██████████| 31/32 [00:10<00:00, 3.04it/s]Training loss: 0.8042261581867933
100%|██████████| 32/32 [00:10<00:00, 3.11it/s]
97%|██████████| 31/32 [00:10<00:00, 3.03it/s]Training loss: 0.6240626201033592
100%|██████████| 32/32 [00:10<00:00, 3.10it/s]
97%|██████████| 31/32 [00:10<00:00, 3.04it/s]Training loss: 0.4704882763326168
100%|██████████| 32/32 [00:10<00:00, 3.10it/s]
100%|██████████| 32/32 [00:10<00:00, 3.10it/s]
 0%|          | 0/32 [00:00<?, ?it/s]Training loss: 0.23173785209655762
100%|██████████| 32/32 [00:10<00:00, 3.10it/s]
 0%|          | 0/32 [00:00<?, ?it/s]Training loss: 0.1427176669239998
100%|██████████| 32/32 [00:10<00:00, 3.09it/s]
 0%|          | 0/32 [00:00<?, ?it/s]Training loss: 0.126893674954772
100%|██████████| 32/32 [00:10<00:00, 3.09it/s]
Training loss: 0.10626217536628246
100%|██████████| 32/32 [00:10<00:00, 3.09it/s]
 0%|          | 0/32 [00:00<?, ?it/s]Training loss: 0.13154328987002373
97%|██████████| 31/32 [00:10<00:00, 3.03it/s]Training loss: 0.08447357639670372
100%|██████████| 32/32 [00:10<00:00, 3.09it/s]
100%|██████████| 32/32 [00:10<00:00, 3.09it/s]
 0%|          | 0/32 [00:00<?, ?it/s]Training loss: 0.06870537996292114
100%|██████████| 32/32 [00:10<00:00, 3.09it/s]
 0%|          | 0/32 [00:00<?, ?it/s]Training loss: 0.07320396602153778
100%|██████████| 32/32 [00:10<00:00, 3.09it/s]
Training loss: 0.05407358519732952
97%|██████████| 31/32 [00:10<00:00, 3.01it/s]Training loss: 0.0493210032582283
100%|██████████| 32/32 [00:10<00:00, 3.09it/s]
Test Accuracy: 0.919
```



Performance of main.py test:

```
Test Accuracy: 0.965
prediction: (68, 139) 19
=====
True circle: (105, 98) 25
Test Accuracy: 0.940
prediction: (106, 98) 26
=====
True circle: (60, 55) 38
Test Accuracy: 0.936
prediction: (60, 53) 39
=====
True circle: (175, 185) 48
Test Accuracy: 0.918
prediction: (162, 171) 46
=====
True circle: (40, 118) 45
Test Accuracy: 0.953
prediction: (40, 118) 46
=====
True circle: (106, 195) 29
Test Accuracy: 0.881
prediction: (104, 197) 32
=====
True circle: (97, 33) 37
Test Accuracy: 0.964
prediction: (98, 32) 38
=====
True circle: (155, 192) 16
Test Accuracy: 0.897
prediction: (154, 191) 17
=====
True circle: (85, 6) 34
Test Accuracy: 0.950
prediction: (85, 4) 35
=====
True circle: (101, 101) 38
Test Accuracy: 0.973
prediction: (100, 102) 39
=====
iou mean = 0.904
```

My trained net (the weight file):

<https://drive.google.com/open?id=188nbv9hnsKAVfPUKU-SC9te4uhMotTkm>

### 2.3. IOU Optimization

```
100% ██████████ 2/2 [00:00<00:00, 2.59it/s]
  0% |          | 0/2 [00:00<?, ?it/s]Training loss: 0.9478632211685181
100% ██████████ 2/2 [00:00<00:00, 2.61it/s]
  0% |          | 0/2 [00:00<?, ?it/s]Training loss: 0.9453335106372833
100% ██████████ 2/2 [00:00<00:00, 2.64it/s]
  0% |          | 0/2 [00:00<?, ?it/s]Training loss: 0.9424173831939697
100% ██████████ 2/2 [00:00<00:00, 2.59it/s]
Training loss: 0.9396699368953705
Test Accuracy: 0.001
```

The result was really bad. The loss decreased very slow (barely moved).  
This is because the IOU is not differentiable.

## 2.4. Visualization and error analysis

Visualization Code: (Please notice that I changed the “noisy\_circle” function from main.py slightly, by returning an extra result “img” as the “mask” (i.e. clear circle image without noise). The original “img” is called “noisy\_img” now.)

```
27 def vis(detected, noisy_img, params):
28     true_row, true_col, true_radius = params
29     row, col, radius = detected
30     img_seg = np.dstack([noisy_img*255, noisy_img*255, noisy_img*255]).astype("uint8")
31     mask = np.zeros(noisy_img.shape, dtype=np.float)
32     draw_circle(mask, row, col, radius)
33     draw_circle(mask, true_row, true_col, true_radius)
34     img_seg[mask > 0.5] = [255, 255, 0]
35     return img_seg
36
20 def noisy_circle(size, radius, noise):
21     img = np.zeros((size, size), dtype=np.float)
22
23     # Circle
24     row = np.random.randint(size)
25     col = np.random.randint(size)
26     rad = np.random.randint(10, max(10, radius))
27     draw_circle(img, row, col, rad)
28
29     noisy_img = img.copy()
30
31     # Noise
32     noisy_img += noise * np.random.rand(*img.shape)
33     return (row, col, rad), noisy_img, img
34
36 def find_circle(img, mask):
37     # Fill in this function
38     noisy_img = [transform(img)]
39     mask_img = [transform(mask)]
40     size = 1
41
42     test_data = MyDataset(noisy_img, mask_img, size)
43     testloader = torch.utils.data.DataLoader(test_data, batch_size=1)
44
45     find_circle_net = UNet()
46     if torch.cuda.is_available():
47         find_circle_net = find_circle_net.cuda()
48     find_circle_net.load_state_dict(torch.load("./drive/My Drive/csc420/q2_net_dice_1000_15.pt"))
49
50     test_result = test([find_circle_net, testloader])
51     center, radius = detect_circle(test_result[0][0].cpu().squeeze().detach().numpy())
52     print("prediction: ", center, radius)
53     # print("=====")
54     return center[1], center[0], radius
55
70 def main():
71     results = []
72     for _ in range(10):
73         params, noisy_img, mask = noisy_circle(200, 50, 2)
74         print("=====")
75         print("True circle: ", (params[1], params[0]), params[2])
76         detected = find_circle(noisy_img, mask)
77         seg = vis(detected, noisy_img, params)
78         plt.imshow(seg, cmap="gray")
79         plt.show()
80         print("iou = ", iou(params, detected))
81         print("=====")
82         results.append(iou(params, detected))
83     results = np.array(results)
84     print("iou mean = ", (results > 0.7).mean())
85
86 main()
```



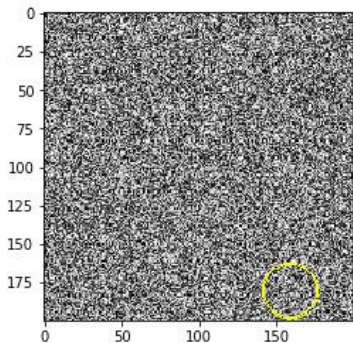
Results:

Notes:

1.The (0,0) point is at the top-left corner.

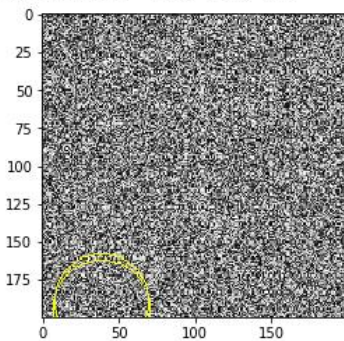
2.I printed the centers and radiuses of true and predicted circles as reference.

True circle: (159, 180) 17  
prediction: (159, 180) 18



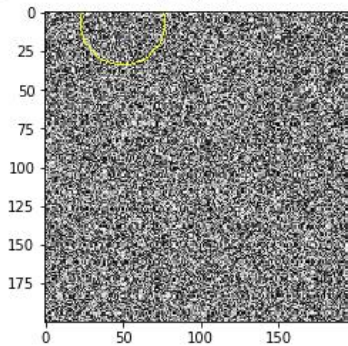
iou = 0.8919753086419752

True circle: (39, 189) 31  
prediction: (39, 194) 32



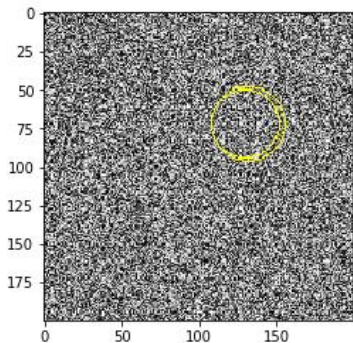
iou = 0.8131989065575698

True circle: (50, 7) 27  
prediction: (50, 7) 27



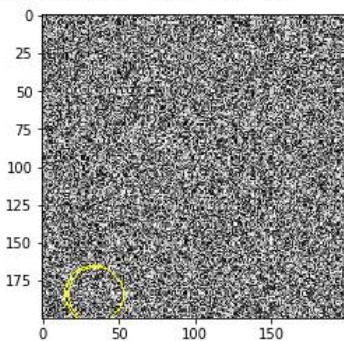
iou = 1.0

True circle: (130, 72) 22  
prediction: (132, 72) 24



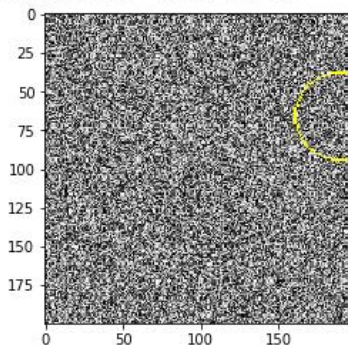
iou = 0.8402777777777778

True circle: (35, 184) 18  
prediction: (34, 184) 19



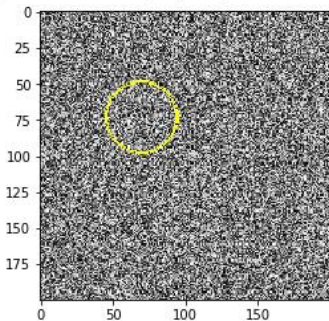
iou = 0.8975069252077565

True circle: (189, 66) 28  
prediction: (190, 66) 28



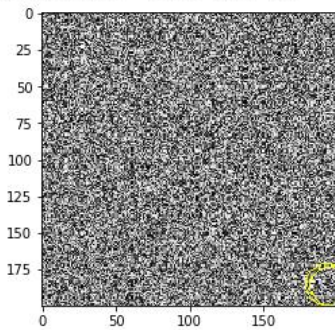
iou = 0.955487251646524

True circle: (70, 73) 24  
prediction: (70, 73) 25



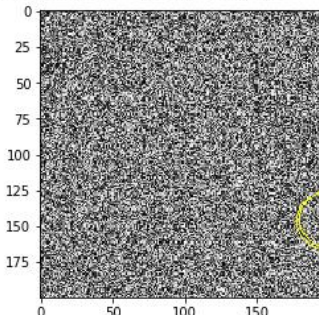
iou = 0.9215999999999998

True circle: (195, 186) 13  
prediction: (193, 185) 14



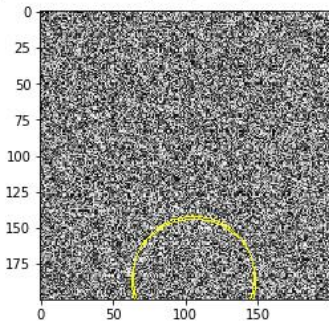
iou = 0.7921017625069104

True circle: (199, 145) 19  
prediction: (198, 146) 20



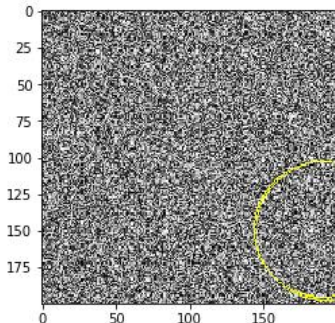
iou = 0.8898558326044305

True circle: (106, 184) 42  
prediction: (106, 187) 43



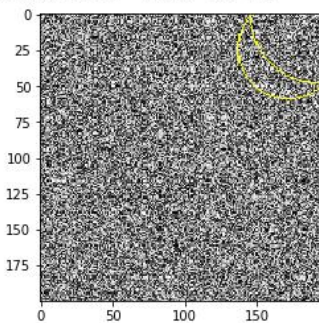
iou = 0.9093599389412192

True circle: (190, 149) 46  
prediction: (192, 150) 47



iou = 0.9347081309046523

True circle: (193, 1) 47  
prediction: (172, 24) 35

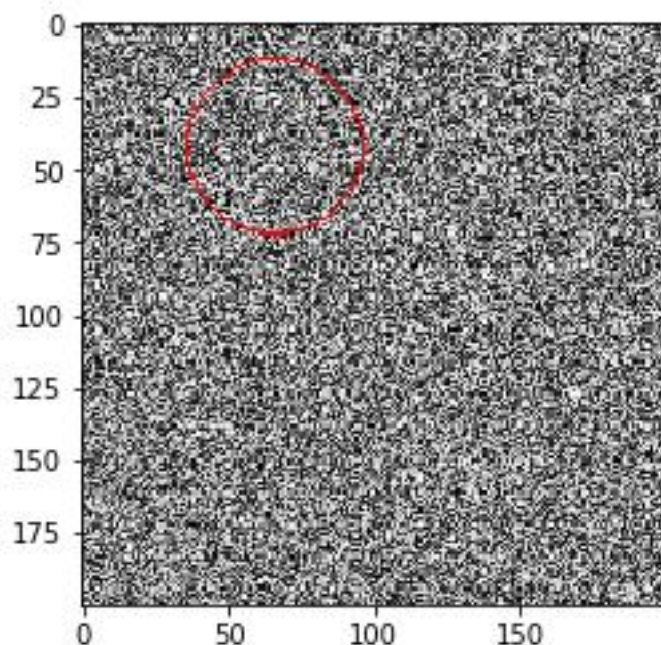


iou = 0.33314843194337573



This is an example of perfect detection.

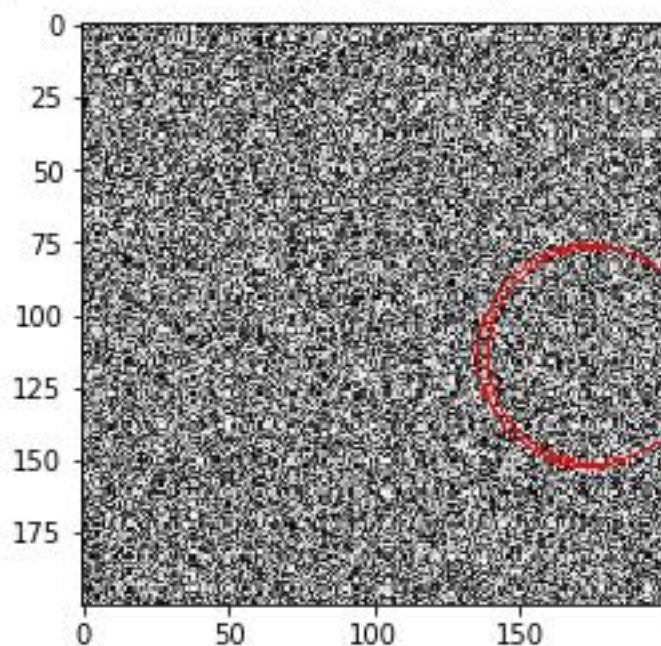
```
=====
True circle:  (66, 42) 30
prediction:    (66, 42) 30
```



```
iou = 1.0
```

This is an example of almost perfect detection. Even though the mismatch is little, you can see the predicted center is not perfectly match the true center, and the predicted radius is not equal to the real radius too. This is because when I was implementing my "detect\_circle" function, I applied the "round()" and "int()" to center points and radius. Doing this can help me return a reasonable integer result. But sometime it may lead to a mismatch.

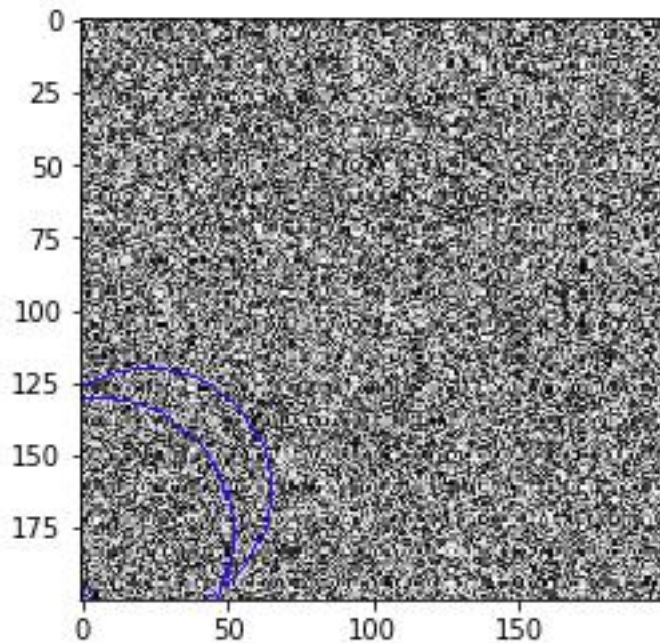
```
=====
True circle:  (176, 114) 37
prediction:    (174, 114) 38
```



```
iou = 0.9262091897853147
```

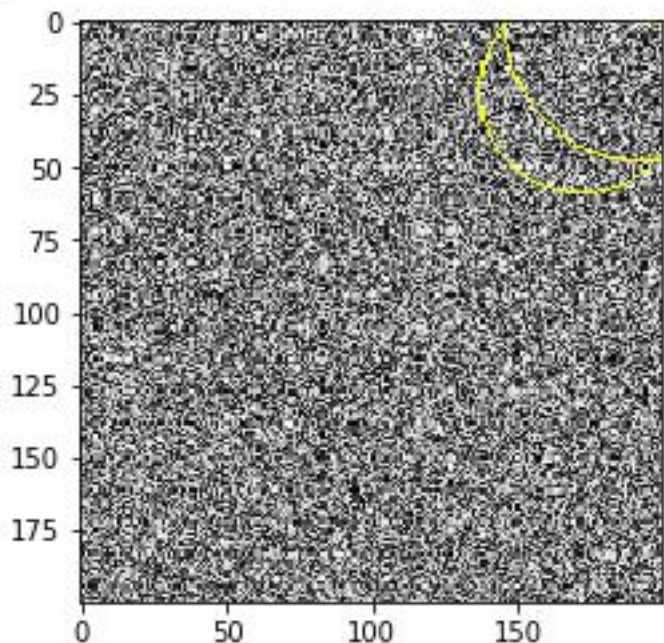
These are examples of bad detection. My detection method is hard to detect the circle which is not completed (i.e. semi-circles that were cut by edges). My “detect\_circle” is designed to find the extreme points of the circle to determine the diameter and the center. For these cases, the extreme points on the circles cannot find the correct diameters, which leads to the wrong centers too.

```
=====
True circle:  (5, 177) 47
prediction:    (23, 162) 42
```



```
iou = 0.49432918534393133
```

```
=====
True circle:  (193, 1) 47
prediction:    (172, 24) 35
```



```
iou = 0.33314843194337573
=====
```

### 3. Hot Dog or Not Hot Dog

The output (i.e. a single scalar  $c$ ) is a boundary (i.e. threshold) of the given binary classifier.

After the training, the model will generate a single scalar  $c$ . Let's assume that “larger or equal to  $c$ ” means “it is a hot dog”, “less than  $c$ ” means “it is not a hot dog” (this is designed by the programmer of the net). When input your picture to the model, if the output of your image is larger or equal to  $c$ , then it will be classified to “hot dog” by the classifier. If your output is less than  $c$ , then it will be classified to “not hot dog”.

So the value of  $c$  doesn't matter. The key is we need to know the input will be larger, equal to, or less than  $c$ .