Q1 BCNF

R: LMNOPQRST

$W = \{ LPR \to Q, \quad LR \to ST, \quad M \to LO, \quad MR \to N \}$

a) $LPR^+ = LPRQST$

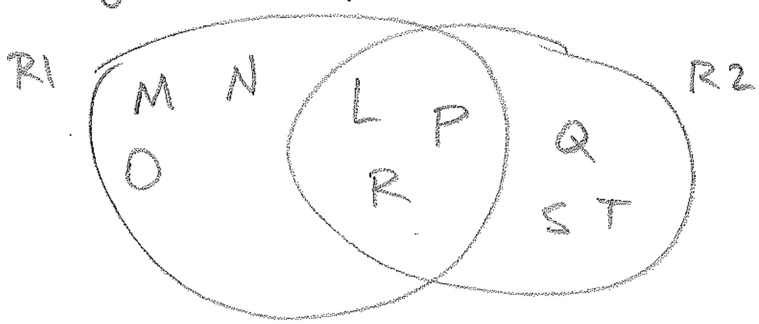
$LR^+ = LRST$

$M^+ = MLO$

$MR^+ = MRNLOST$

$\therefore$ all FDs violate BCNF

b) Choosing to decompose based on $LPR \to Q$



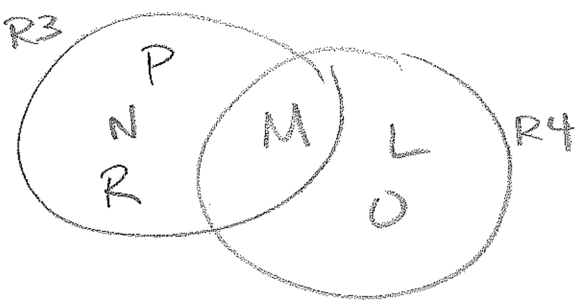
R1: LMNOPR

$L^+ = L$

$M^+ = MLO \quad \therefore M \to LO$

Not a superkey

$\therefore$ split again

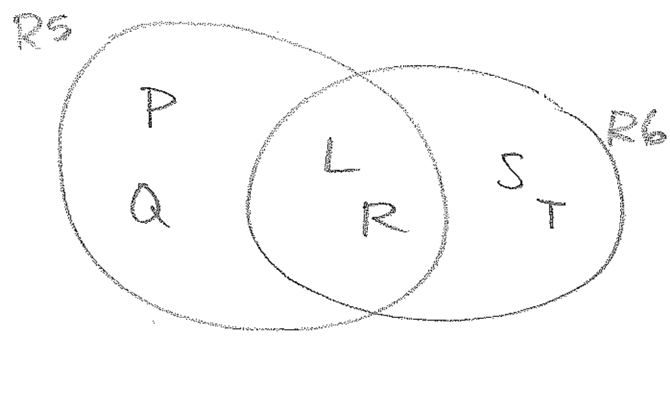


R2: LPQRST

$LR^+ = LRST \quad \therefore LR \to ST$

Not a superkey

$\therefore$ split again

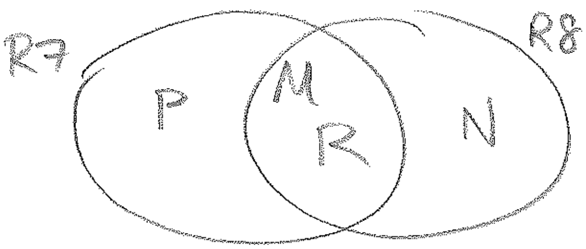


R3 : MNPR

$MR^+ = MRNLOST$

$\therefore MR \rightarrow N$

not a superkey
so split again!



R4 : LMO

$L^+ = L$

$M^+ = MLO \therefore M \rightarrow LO$

superkey for this relation ✓

$O^+ = O$

Ignore supersets of M since they can yield only weaker FDs

$LO^+ = LO$

R7 : MPR

$M^+ = MLO$

$P^+ = P$

$R^+ = R$

$MP^+ = MP$

$MR^+ = MRNLOST$

$PR^+ = PR$

$\therefore$ no FD's

R8 : MNR

$M^+ = MNO$

$N^+ = N$

$R^+ = R$

$MN^+ = MNLO$

$MR^+ = MRNLOST$

$\therefore MR \rightarrow N$

superkey ✓

$NR^+ = NR$

R5: LPQR

L, P, Q, and R yield nothing as singletons.

$LP^+ = LP$

$LQ^+ = LQ$

$LR^+ = LRST$

$PQ^+ = PQ$

$PR^+ = PR$

$LPQ^+ = LPQ$

$LPR^+ = LPRSTQ$

$\therefore LPR \rightarrow Q$

superkey ✓

$PQR^+ = PQR$

R6: LRST

L, R, S, + T yield nothing as singletons.

$LR^+ = LRST$

$\therefore LR \rightarrow ST$

superkey ✓

$LS^+ = LS$

$LT^+ = LT$

$RS^+ = RS$

$RT^+ = RT$

$ST^+ = ST$

Ignore supersets of LR

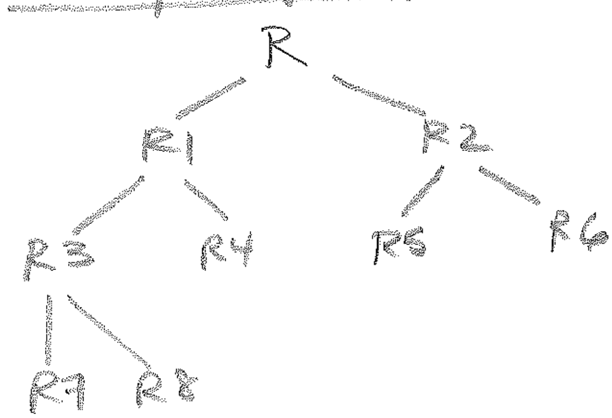
$LST^+ = LST$

$RST^+ = RST$

Final Decomposition

R4: LMO	$M \rightarrow LO$
R5: LPQR	$LPR \rightarrow Q$
R6: LRST	$LR \rightarrow ST$
R7: MPR	—
R8: MNR	$MR \rightarrow N$

History of Splits



Q2a Minimal Basis

First, split RHS's

$S_1 = \{$ 1. $AB \rightarrow C$

2. $AB \rightarrow D$
3. $ACDE \rightarrow B$
4. $ACDE \rightarrow F$
5. $B \rightarrow A$
6. $B \rightarrow C$
7. $B \rightarrow D$
8. $CD \rightarrow A$
9. $CD \rightarrow F$
10. $CDE \rightarrow F$
11. $CDE \rightarrow G$
12. $EB \rightarrow D$. $\}$

Attempt to siplify LHSs that have >1 attribute.

1+2) $A^+ = A$ ∴ can't remove B from LHS of $AB \rightarrow C$, or $AB \rightarrow D$

$B^+ = BACD$ ∴ can siplify $\cancel{A}B \rightarrow C$, $\cancel{A}B \rightarrow D$.

3+4) No LHS attribute has a closure larger than itself, so consider only larger subsets of the LHS attributes.
(The fact that B appears on the RHS of no other FD does not mean that we can't siplify the LHS of 3.)

$CD^+ = CDAF$. ∴ 4 can be siplified to $CD \rightarrow F$. $C^+ = C$ + $D^+ = D$ so we can't go further

$CDE^+ = CDEAFB$ ∴ 3 can be siplified to $CDE \rightarrow B$

$B \notin CD^+$, $DE^+ = DE$, $CE^+ = CE$, so we can't siplify 3 any further.

8+9) $C^+ = C$ and $D^+ = D$, so we can't siplify.

10) We already have $CD \to F$ + it can't be futher simplified as we showed.

11) $CD^+ = CDAF$. $DE^+ = DE$. $CE^+ = CE$.
∴ we can't siplify.

12) $E^+ = E$

$B^+ = BACD$ ∴ we can siplify to $B \to D$.

Our siplifications almost always produced FDs we already had. Removing duplicates, we are left with:

$S_2 = \{$
1. $B \to C$ $B^+_{S_2 - 1} = BDA$
2. $B \to D$ $B^+_{S_2 - 2} = BCA$
3. $CD \to F$ $CD^+_{S_2 - 3} = CDA$
4. $CDE \to B$ $CDE^+_{S_2 - 4} = CDEFAG$
5. ~~$B \to A$~~ $B^+_{S_2 - 5} = BCDA$ ∴ don't need
6. $CD \to A$ $CD^+_{S_2 - \{5,6\}} = CDF$
7. $CDE \to G$ $CDE^+_{S_2 - \{5,7\}} = CDEFBA$

$\}$

In this step, when there was no other way to get the RHS attribute, we could skip the closure + explain why we need the FD.
This is <u>not</u> the case when siplifying LHSs (see ★ above)

Final set of FDs in the minimal basis:

$B \rightarrow CD$

$CD \rightarrow AF$

$CDE \rightarrow BG$

We rejoined the RHSs here, but didn't have to.

Q2b keys

It would have been simpler if done based on the minimal basis.

attributes	On LHS?	on RHS?	conclusion
G F A	—	✓	In no key
D C B ~~A~~	✓	✓	Must check
E	✓	—	In every key
H	—	—	In every key

∴ Every key has EH, plus some (possibly empty) subset of DCB~~A~~.

We could have omitted A from this analysis

A	B	C	D	
✓				$EHA^+ = EHA$
	✓			$EHB^+ = EHBACDFG$ ← a key
		✓		$EHC^+ = EHC$
			✓	$EHD^+ = EHD$

Ignore supersets of B

A	B	C	D	
✓		✓		$EHAC^+ = EHAC$
✓			✓	$EHAD^+ = EHAD$
		✓	✓	$EHCD^+ = EHCDAFGB$ ← a key

Ignore supersets of CD

Nothing left!

∴ The keys are EHB and EHCD

Q2c 3NF

~~R1: BCD~~ This is a subset of R3, so can be removed.

R2: CDAF

R3: CDEBG

No relation is a superkey, since none includes H.
We must add another relation that is a key. (There are 2 options in this case.)

R4: EHB

Q2d) Redundancy

$B \to CD$ clearly holds in CDEBG (R3).

$B^+ = BCDAF$. It does not include E or G.

∴ we can have redundancy in R3.

Example:

C	D	E	B	G
2	3	4	1	5
2	3	6	1	7

These are constrained by the FD to these particular values, ∴ redundant.

But are we sure this is a valid instance of R3?
There could be other FDs it violates.
The only ones it *could* violate involve CD+B only, since only these attributes have the same values.
And if any further attributes are constrained, we would have found them in the closure of B.

∴ this is a valid instance + our schema allows redundancy.

Students did not have to reason through this.