CSC343 Assignment 3 Part 2

Q1:

(a):

First, compute LPR^+, LR^+, M^+, MR^+.

$LPR^+ = LPQRST$

$LR^+ = LRST$

$M^+ = LMO$

$MR^+ = LMNORST$

So, NONE of these FDs satisfy BCNF.

Therefore, $\boldsymbol{LPR \to Q, LR \to ST, M \to LO, MR \to N}$, all violate BCNF.

(b): **Employ BCNF decomposition:**

Step 1: Decompose R using FD $LPR \to Q$, $LPR^+ = LPQRST$, this yields 2 relations:

$R_1: (L, P, Q, R, S, T)$ and $R_2: (L, M, N, O, P, R)$

Project the FDs onto $R_1: (L, P, Q, R, S, T)$

L	P	Q	R	S	T	Closure	FDs
✓						$L^+ = L$	ϕ
	✓					$P^+ = P$	ϕ
		✓				$Q^+ = Q$	ϕ
			✓			$R^+ = R$	ϕ
				✓		$S^+ = S$	ϕ
					✓	$T^+ = T$	ϕ
✓	✓					$LP^+ = LP$	ϕ
✓		✓				$LQ^+ = LQ$	ϕ
✓			✓			$LR^+ = LRST$	$LR \to ST$: violates BCNF

We must decompose R_1 futher.

Step 2: Decompose R_1 using FD $LR \rightarrow ST$, $LR^+ = LRST$, this yields 2 relations:

$R_3: (L, R, S, T)$ $R_4: (L, P, Q, R)$

Project the FDs onto $R_3: (L, R, S, T)$

L	R	S	T	Closure	FDs
✓				$L^+ = L$	ϕ
	✓			$R^+ = R$	ϕ
		✓		$S^+ = S$	ϕ
			✓	$T^+ = T$	ϕ
✓	✓			$LR^+ = LRST$	$LR \rightarrow ST$: LR is a superkey
✓		✓		$LS^+ = LS$	ϕ
✓			✓	$LT^+ = LT$	ϕ
	✓	✓		$RS^+ = RS$	ϕ
	✓		✓	$RT^+ = RT$	ϕ
		✓	✓	$ST^+ = ST$	ϕ
✓		✓	✓	$LST^+ = LST$	ϕ
	✓	✓	✓	$RST^+ = RST$	ϕ
Subsets of LR				irrelevant	Can only generate weaker FDs than what we already have

R_3 satisfies BCNF.

Project the FDs onto $R_4: (L, P, Q, R)$

L	P	Q	R	Closure	FDs
✓				$L^+ = L$	ϕ
	✓			$P^+ = P$	ϕ
		✓		$Q^+ = Q$	ϕ
			✓	$R^+ = R$	ϕ
✓	✓			$LP^+ = LP$	ϕ
✓		✓		$LQ^+ = LQ$	ϕ
✓			✓	$LR^+ = LRST$	ϕ
	✓	✓		$PQ^+ = PQ$	ϕ
	✓		✓	$PR^+ = PR$	ϕ
		✓	✓	$QR^+ = QR$	ϕ
✓	✓	✓		$LPQ^+ = LPQ$	ϕ
✓	✓		✓	$LPR^+ = LPQR$	$LPR \to Q$: LPR is a superkey
✓		✓	✓	$LQR^+ = LQR$	ϕ
	✓	✓	✓	$PQR^+ = PQR$	ϕ
Subsets of LPR				irrelevant	Can only generate weaker FDs than what we already have

R_4 satisfies BCNF.

Step 3: Return to $R_2: (L, M, N, O, P, R)$ and project the FDs onto it.

L	M	N	O	P	R	Closure	FDs
✓						$L^+ = L$	ϕ
	✓					$M^+ = LMO$	$M \to LO$: violates BCNF

We must decompose R_2 futher.

Step 4: Decompose R_2 using FD $M \to LO$, $M^+ = LO$, this yields 2 relations:

$R_5: (L, M, O)$ $R_6: (M, N, P, R)$

Project the FDs onto $R_5: (L, M, O)$

L	M	O	Closure	FDs
✓			$L^+ = L$	ϕ
	✓		$M^+ = LM$O	$M \to LO$: M is a superkey
		✓	$O^+ = O$	ϕ
Subsets of M			irrelevant	Can only generate weaker FDs than what we already have
✓		✓	$LO^+ = LO$	ϕ

R_5 satisfies BCNF.

Project the FDs onto $R_6: (M, N, P, R)$

M	N	P	R	Closure	FDs
✓				$M^+ = LMO$	ϕ
	✓			$N^+ = N$	ϕ
		✓		$P^+ = P$	ϕ
			✓	$R^+ = R$	ϕ
✓	✓			$MN^+ = LMNO$	ϕ
✓		✓		$MP^+ = LMOP$	ϕ
✓			✓	$MR^+ = LMNORST$	$MR \rightarrow N$: violates BCNF

We must decompose R_6 futher.

Step 5: Decompose R_6 using FD $MR \rightarrow N$, $MR^+ = LMNORST$, this yields 2 relations: $R_7: (M, N, R)$ $R_8: (M, P, R)$

Project the FDs onto $R_7: (M, N, R)$

M	N	R	Closure	FDs
✓			$M^+ = LMO$	ϕ
	✓		$N^+ = N$	ϕ
		✓	$R^+ = R$	ϕ
✓	✓		$MN^+ = MN$	ϕ
✓		✓	$MR^+ = LMNORST$	$MR \rightarrow N$: MR is a superkey
	✓	✓	$NR^+ = NR$	ϕ
Subsets of MR			irrelevant	Can only generate weaker FDs than what we already have

R_7 satisfies BCNF.

Project the FDs onto $R_8: (M, P, R)$

M	P	R	Closure	FDs
✓			$M^+ = LMO$	ϕ
	✓		$P^+ = P$	ϕ
		✓	$R^+ = R$	ϕ
✓	✓		$MP^+ = MP$	ϕ
✓		✓	$MR^+ = LMNORST$, We cannot get P since it is not on a RHS	ϕ
	✓	✓	$PR^+ = PR$	ϕ

R_8 satisfies BCNF.

Final Decomposition:

1. $R_3 = LRST$ with FD $LR \rightarrow ST$,
2. $R_4 = LPQR$ with FD $LPR \rightarrow Q$,
3. $R_5 = LMO$ with FD $M \rightarrow LO$,
4. $R_7 = MNR$ with FD $MR \rightarrow N$,
5. $R_8 = MPR$ with no FDs.

Q2:

(a). Compute minimal basis:

Step 1: Split the RHS to get initial set of FDs, S1:

1. $AB \to C$,
2. $AB \to D$,
3. $ACDE \to B$,
4. $ACDE \to F$,
5. $B \to A$,
6. $B \to C$,
7. $B \to D$,
8. $CD \to A$,
9. $CD \to F$,
10. $CDE \to F$,
11. $CDE \to G$,
12. $EB \to D$.

Step 2: For each FD, try to reduce the LHS:

1. Since $AB^+ = ABCDF$, $B^+ = ABCDF$, we can reduce the LHS (AB) of this FD, yielding the FD: $B \to C$. This will be removed because $B \to C$ is already existed.
2. Same as #1 above, reduce $AB \to D$ to $B \to D$. This will be removed because $B \to D$ is already existed.
3. Since $CD \to A$, we can reduce $ACDE \to B$ to $CDE \to B$.
4. Since $CD \to F$, we can reduce the LHS of this FD, yielding the FD: $CD \to F$. This will be removed because $CD \to F$ is already existed.
5. We cannot reduce $B \to A$ since this is a singleton.
6. Same as #5.

7. Same as #5.
8. Since no singleton LHS yields anything. We cannot reduce the LHS of this FD.
9. Same as #8.
10. Since $CD \rightarrow F$, we can reduce the LHS (CDE) of this FD, yielding the FD: $CD \rightarrow F$. This will be removed because $CD \rightarrow F$ is already existed.
11. Since no singleton LHS yields anything. Only the RHS of $CDE \rightarrow G$ is G among all FDs. So, we cannot reduce $CDE \rightarrow G$.
12. Since $B \rightarrow D$, we can reduce the LHS (EB) of this FD, yielding the FD: $B \rightarrow D$. This will be removed because $B \rightarrow D$ is already existed.

Our new set of FDs, S2, is:

1. $CDE \rightarrow B$,
2. $B \rightarrow A$,
3. $B \rightarrow C$,
4. $B \rightarrow D$,
5. $CD \rightarrow A$,
6. $CD \rightarrow F$,
7. $CDE \rightarrow G$.

Step 3: Try to eliminate each FD.

FDs	Exclude these from S2 when computing closure	Closure	Decision
#1. $CDE \rightarrow B$	#1	$CDE^+ = ACDEFG$, B is not existed.	NEED
#2. $B \rightarrow A$	#2	$B^+ = ABCDF$, A is the right side of #2.	REMOVE
#3. $B \rightarrow C$	#2, #3	$B^+ = BD$, C is not existed.	NEED
#4. $B \rightarrow D$	#2, #4	$B^+ = BC$, D is not existed.	NEED
#5. $CD \rightarrow A$	#2, #5	$CD^+ = CDF$, A is not existed.	NEED
#6. $CD \rightarrow F$	#2, #6	$CD^+ = ACD$, F is not existed.	NEED
#7. $CDE \rightarrow G$	#2, #7	$CDE^+ = ABCDEF$, G is not existed.	NEED

Therefore, our final minimal basis set of FDs is:

1. $CDE \to B$,
2. $B \to C$,
3. $B \to D$,
4. $CD \to A$,
5. $CD \to F$,
6. $CDE \to G$.

After the combination, the minimal basis for T is $\{ CDE \to BG,\ B \to CD,\ CD \to AF\}$.

(b). Compute all the keys:

Attribute	LHS	RHS	Conclusion
H	-	-	Must be in every key
E	✓	-	Must be in every key
AFG	-	✓	Is not in any key
BCD	✓	✓	Must check

Have to consider all combinations of BCD with EH:

$BEH^+ = ABCDEFGH$. So BEH is a key.

And we do not need to consider $BCEH$, $BDEH$ and $BCDEH$ since BEH is a key.

$CEH^+ = CEH$. This is not a key.

$DEH^+ = DEH$. This is not a key.

$CDEH^+ = ABCDEFGH$. So $CDEH$ is a key.

Therefore, for P, BEH and $CDEH$ are keys.

(c). Apply 3NF synthesis:

By (a), we have the minimal basis for T is $\{ CDE \rightarrow BG,\ B \rightarrow CD,\ CD \rightarrow AF \}$.

For each FD in T, we set relations:

$R_1: (B, C, D, E, G)$ with FD $CDE \rightarrow BG$.

$R_2: (B, C, D)$ with FD $B \rightarrow CD$.

$R_3: (A, C, D, F)$ with FD $CD \rightarrow AF$.

Since the attributes BD occur within R1, we do not need to keep the relation R2.

Since there is no key in P (our relation), we need to add a relation contains a key.

For example: $R_4: (B, E, H)$.

Final set of relations is:

$R_1: (B, C, D, E, G)$

$R_3: (A, C, D, F)$

$R_4: (B, E, H)$

(d): **Yes. Our schema allow redundancy.**

Because we formed each relation from an FD, the LHS of those FDs are indeed superkeys for their relations. However, there may be other FDs that violate BCNF and therefore allow redundancy. The only way to find out is to project FDs onto each relation.

We can find a relation that violate BCNF without doing all the full projections:

$B \rightarrow CD$ will project onto the relation R1. And $B^+ = ABCDF$, so B is not a superkey of this relation.

So, this schema allows redundancy.