CS5285
Information Security for eCommerce

Prof. Gerhard Hancke

CS Department
City University of Hong Kong

Reminder of previous lecture

- Number theory
  - Basic number theory for cryptography
  - Familiar with terminology
  - Need to have functional knowledge

Today's Lecture

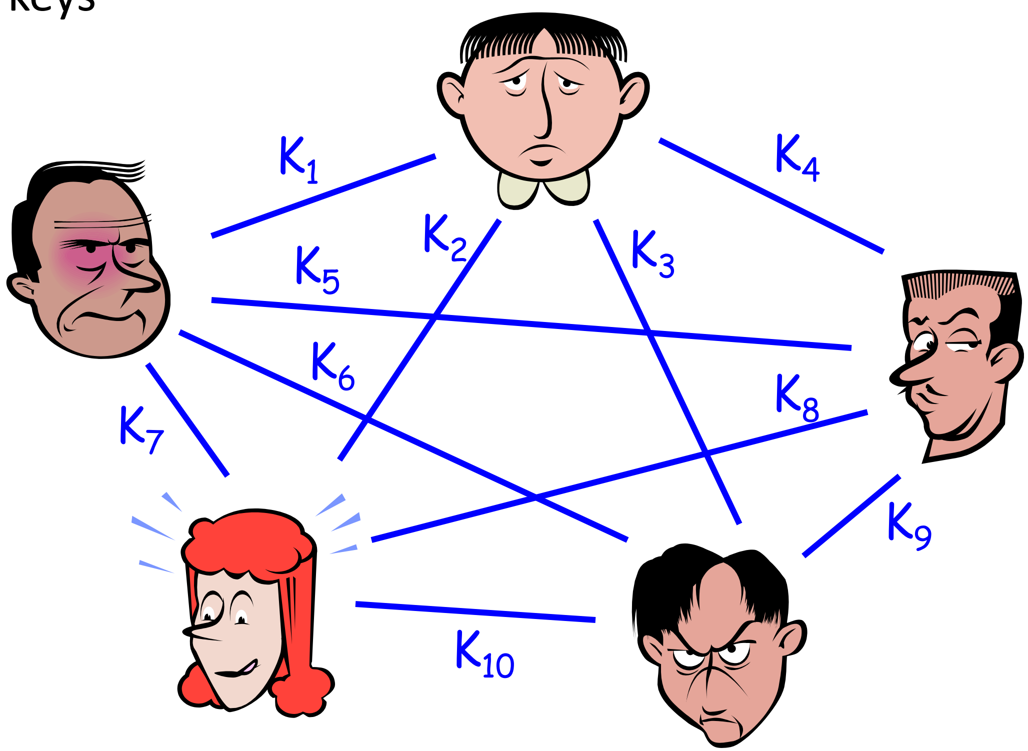
- Asymmetric encryption
  - Difference between symmetric and asymmetric
  - RSA and El-Gamal

- CILO2 and CILO5

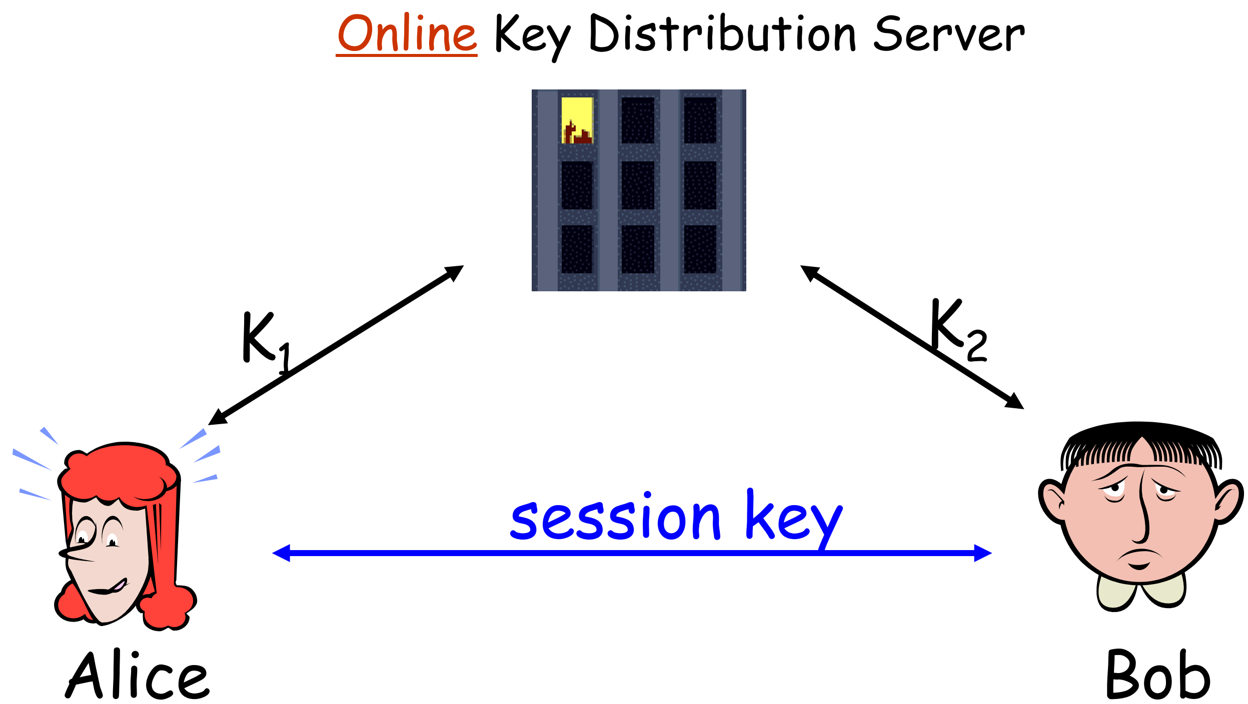
  (technology that impact systems, and security mechanisms)

Symmetric Key Management

- Each pair of communicating entities needs a shared key
- For an *n*-party system, there are $n(n-1)/2$ distinct keys in the system and each party needs to maintain n-1 distinct keys.
- How to reduce the number of shared keys in the system
  1. Centralized key management
  2. Public keys
- How to set up shared keys



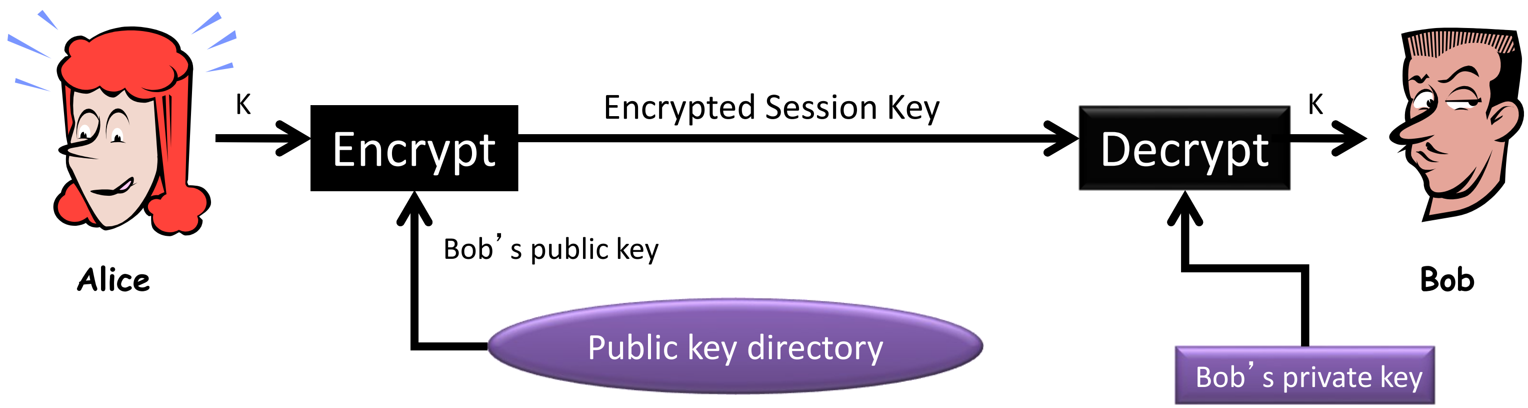
Centralized Key Management



- Only *n* long-term secret keys, instead of *n*(*n*-1)/2 in the system.
- Each user shares one long-term secret key with the Server.
- The Server may become the **single-point-of-failure** and the performance bottleneck.
- Secret keys are used only for the secure delivery of session keys.
- Real data are encrypted under session keys.

Public key Encryption

- Receiver Bob has a <u>key pair</u>: public and private
  - **publish** the public key such that the key is publicly known
  - Bob keeps the private key secret
- Other people use Bob's public key to encrypt messages for Bob
- Bob uses his private key to decrypt



- Security requirement 1: difficult to find private key or plaintext from ciphertext
- Security requirement 2: difficult to find private key from public key

Motivation of Public Key Cryptography (Summary)

- One problem with symmetric key algorithms is that the sender needs a secure method for telling the receiver about the encryption key.
- Plus, you need a separate key for everyone you might communicate with (scalability issue).

- Public key algorithms use a public-key and private-key pair to tackle the two problems
  - Each receiver has a public key pair.
  - The public key is publicly known (published).
  - A sender uses the receiver's public key to encrypt a message.
  - Only the receiver can decrypt it with the corresponding private key.

What is public key crypto based on?

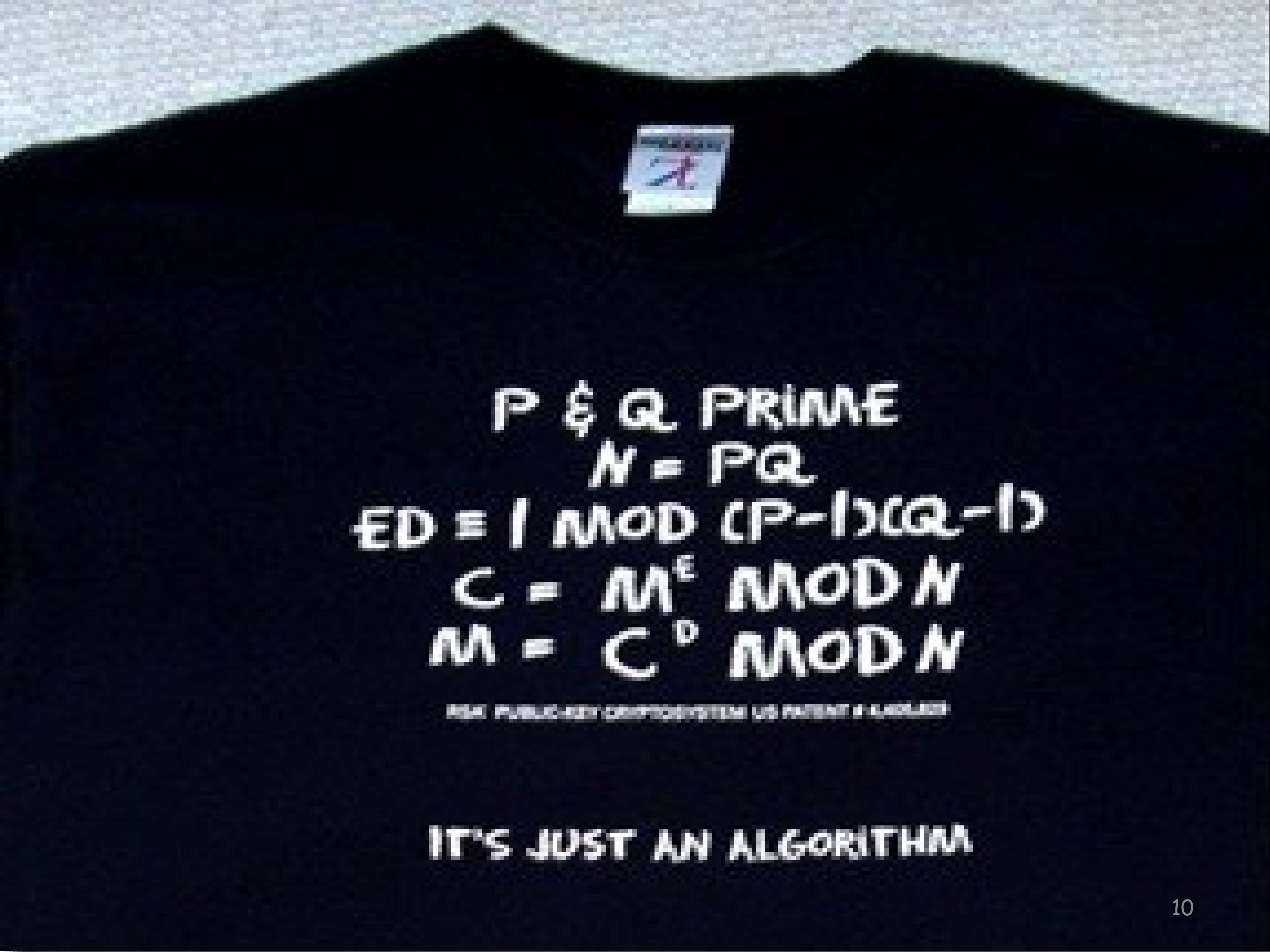
- Public key crypto is based on mathematical one way functions
  - Easy to compute output given the inputs
  - Difficult to compute input given the output
- Factorisation problem
  - Multiplying two prime numbers
  - Given prime x and y it is easy to compute x.y = z
  - Given z it is not easy to compute x and y
- Discrete logarithm problem
  - Exponentiation of a number
  - Given a, b and prime n is it easy to calculate z= a^b mod n
  - Given z, a and n it is not easy to compute b
- 'Not easy' means it is currently not computationally feasible…

Rivest, Shamir, and Adleman (RSA)

- Randomly choose two large and roughly equal-length prime numbers, p and q.
  - E.g. $|p| = |q| = 512$ bits
- Sets $n = pq$ (n is called the public modulus)
- Randomly choose e such that $\gcd(e, \phi(n)) = 1$.
  - e is called the public exponent.
  - $\phi(n) = \phi(pq) = (p\text{-}1)(q\text{-}1)$
- Compute d such that $de \equiv 1 \pmod{\phi(n)}$.
  - In other words, d is the modular inverse of e modular $\phi(n)$.
  - d is called the private exponent.

- Public Key: $PK = (n, e)$
- Private Key: $SK = d$

- Encryption: $C = M^e \bmod n$
- Decryption: $M = C^d \bmod n$


P & Q PRIME
N = PQ
ED ≡ 1 MOD (P-1)(Q-1)
C = M^E MOD N
M = C^D MOD N
IT'S JUST AN ALGORITHM

Your turn

- Given p=13, q=11 and choosing e=7. Use RSA to encrypt M=10

- n = p.q = 143; $\phi(n) = (p - 1)(q - 1) = 120$
- 120=17*7+1, so...
- 1=120.1-17*7 mod 120 so 1 = -17*7 mod 120…modulo inverse of 7 is -17
- d = -17 mod 120 = 103
- Public key (e=7,n=143), Private key (d=103)
- C = M^e mod n = 10^7 mod 143 = 10
- M = C^d mod n = 10^{103} mod 143 = 10

Example of RSA Encryption and Decryption

- Choose two primes p=47 and q=71 ⇨ $n = pq = 3337$.
- Choose e such that it is relatively prime to $\phi(n) = 46 \times 70 = 3220$.
  - e.g. $e = 79$.
- Compute $d = e^{-1} \bmod \phi(n)$ using extended Euclidean algorithm.
  - $d \equiv 79^{-1} \pmod{3220} = 1019$
- Public key PK = $(n, e) = (3337,79)$
- Private key SK = $d = 1019$

- Encrypt $M = 688$ ⇨ $688^{79} \bmod 3337 = 1570$
- Decrypt $C = 1570$ ⇨ $1570^{1019} \bmod 3337 = 688$

Security of RSA

- **Remember Factorization Problem (FACTORING)** : Given a positive integer n, find its prime factorization; that is, write $n = p_1^{e1} p_2^{e2} \ldots p_k^{ek}$ where the p_i are primes and each $e_i \geq 1$.
  - E.g. $72 = 2^3 \cdot 3^2$
- **RSA Problem (RSAP)** : Given
  - a positive integer n that is a product of two distinct equal-length primes p and q,
  - a positive integer e such that gcd(e, (p-1)(q-1)) = 1, and
  - an integer c chosen randomly from Z_n^*

  find an integer m such that $m^e \equiv c \pmod{n}$. Note: p and q are not given.
- The intractability of the RSAP forms the basis for the security of the RSA public-key cryptosystem.
  - **RSAP** is closely related to the Factorization Problem but not known to be equivalent.
  - If one can solve FACTORING, then one can solve RSAP.
  - Is FACTORING $\leq_P$ RSAP?
    - It is widely believed that it is true, although no proof of this is known

More about RSA Security Strength

- The strength of the RSA algorithm depends on the difficulty of doing prime factorization of large numbers:
  - Knowing the public key <*e*, *n*>, if the cryptanalyst could factor $n = pq$, then $\phi(n)$ ($= (p - 1)(q - 1)$) is obtained
  - Knowing *e* and $\phi(n)$, *d* can be obtained with a known algorithm (Euclid's algorithm) for finding multiplicative inverse ($de = 1 \bmod \phi(n)$)
- To break an RSA encryption (i.e., finding the decryption key) by brute force (i.e., by trying all possible keys) is not feasible given the relative large size of the keys
  - A better approach is to solve the prime factorization problem.
  - The best known factorization algorithms seem to indicate that the number of operations to factorize a number *n* is estimated by

$$\exp\left((\ln n)^{1/3}(\ln \ln n)\right)$$

More about RSA Security Strength

- First RSA challenge (for cracking) was posted by the inventors in 1978: a message was encrypted by a key of 430 bits (129 decimal digits). It was solved 16 years later.
- Over the years, computing power and factorization techniques have improved. The RSA challenges ended in 2007.
- Based on the above actual trial attacks, 512-bit RSA keys, which previously were considered as adequate for some commercial applications, are now in doubt. For high security requirements, 2048-bit keys may be considered

RSA SECURITY

Challenge Number	Key size (bits)	Prize ($US)	Factored
RSA-129	430	100	Apr 1994
RSA-155	512	9,383	Aug1999
RSA-576	576	10,000	Dec 2003
RSA-640	640	20,000	Nov 2005
RSA-704	704	30,000	July 2012 (prize retracted)
RSA-768	768	50,000	Dec 2009 (prize retracted)
RSA-1024	1024	100,000	
RSA-2048	2048	200,000	

2048-bit RSA keys are considered secure and widely used in e-commerce

Recommendations for RSA Key Sizes

- According to RSA Laboratories (year 2003)

http://www.rsa.com/rsalabs/node.asp?id=2004

Protection Lifetime of Data	Present – 2010	Present – 2030	Present – 2031 and Beyond
Minimum symmetric security level	80 bits	112 bits	128 bits
Minimum RSA key size	1024 bits	2048 bits	3072 bits

- Key size recommendations are continuously updated from time to time due to the advancement in technology (e.g. factorization techniques) and computing power.
- Schedule of key sizes should take into account **the lifetime of the data**, spanning the next several decades.
  - E.g. 80-bit security level (i.e. 1024-bit RSA keys) for protecting data through the year 2018 , and 112-bit security level through the year 2038.
  - Ref: NIST: Recommendation for Key Management. Part 1: General Guideline (http://csrc.nist.gov/groups/ST/toolkit/key_management.html)

ElGamal Encryption Scheme

- Let p be a large prime.
- Let $Z_p^* = \{ 1, 2, 3, \ldots, p\text{-}1 \}$
- Let $Z_{p-1} = \{ 0, 1, 2, \ldots, p\text{-}2 \}$
- $a \in_R S$ means that a is randomly chosen from the set S
- Let $g \in Z_p^*$ such that none of g^1 mod p, g^2 mod p, …, g^{p-2} mod p is equal to 1.

Public Key Pair:

- Private key: $x \in_R Z_{p-1}$
- Public key: $y = g^x \bmod p$

Encryption:

1. $r \in_R Z_{p-1}$
2. $A = g^r \bmod p$
3. $B = My^r \bmod p$ where $M \in Z_p^*$ is the message.

Ciphertext C = (A, B).

Decryption:

1. $K = A^x \bmod p$
2. $M = B\, K^{-1} \bmod p$

Your turn

- Let p =23, g =11 and x =6. Encrypt M =10 with r being 3
- Compute public y: $11^6 \bmod 23 = 9$
- Public key is 9 and private key is 6.
- Encrypt
  - $C1 = 11^3 \bmod 23 = 20$
  - $C2 = 10 \times 9^3 \bmod 23 = 10 \times 16 \bmod 23 = 22$
- Decrypt
  - $K = 20^6 \bmod 23 = 16$
  - $M = 22 \times 16^{-1} \bmod 23 = 22 \times 13 \bmod 23 = 10$

Example of ElGamal Encryption and Decryption

- Let $p = 2357$
  $g = 2$
  Private key: $x = 1751$
  Public key: $y = g^x = 2^{1751} = 1185 \pmod{2357}$
- Encryption:
  - say $M = 2035$
  1. Pick a random number $r = 1520$
  2. Computes A,B and C
     $A = g^r \equiv 2^{1520} \equiv 1430 \pmod{2357}$
     $B = My^r \equiv 2035 \times 1185^{1520} \equiv 697 \pmod{2357}$
     The ciphertext $C = (A, B) = (1430, 697)$
- Decryption:
  1. Computes $K \equiv A^x \equiv 1430^{1751} \equiv 2084 \pmod{2357}$
  2. $M \equiv B\, K^{-1} \equiv 697 \times 2084^{-1} \equiv 2035 \pmod{2357}$

Security of ElGamal Encryption Scheme

Encryption:

1. $r \in_R Z_{p-1}$
2. $A = g^r \bmod p$
3. $B = My^r \bmod p$ where $M \in Z_p^*$ is the message.

Ciphertext $C = (A, B)$.

- Given $C = (A, B)$ and public key $y = g^x \bmod p$, find M without knowing x.

1. If adversary can get r from $A=g^r \bmod p$, then the scheme is broken.
2. If adversary can get x from $y=g^x \bmod p$, then the scheme is broken.
3. From $A=g^r \bmod p$ and $y=g^x \bmod p$, if adversary can compute $g^{rx} \bmod p$, then the scheme is broken.

- First two correspond to DLP (Discrete Logarithm Problem)
- The last one corresponds to Diffie-Hellman Problem

Discrete Logarithm Problem (DLP)

- Let p be a prime number. Given two integers: g, y
  - g and y are integers chosen randomly in Z_p^*.
- Find a such that $g^a \bmod p = y$
- a is called the discrete log of y to the base g mod p.

DLP (Discrete Log Problem)

- Given a, g and p, compute $y \equiv g^a \bmod p$ is EASY
- However, given y, g and p, compute a is HARD

Factoring (revisit)

- Given p and q, compute n = pq is EASY
- However, given n, compute the prime factors p and q is HARD

DLP Example:

- For p=97, g = 5 and y= 35, compute a such that $g^a \bmod p = 35$.
  - We need to try all possibilities (from 1 to 96) to obtain such a

- When p is large, DLP is hard
- In practice, p should at least be 1024 bits long.
- Practical problems (not to be discussed in this course): How to generate and verify such a large prime number p? How to generate g?

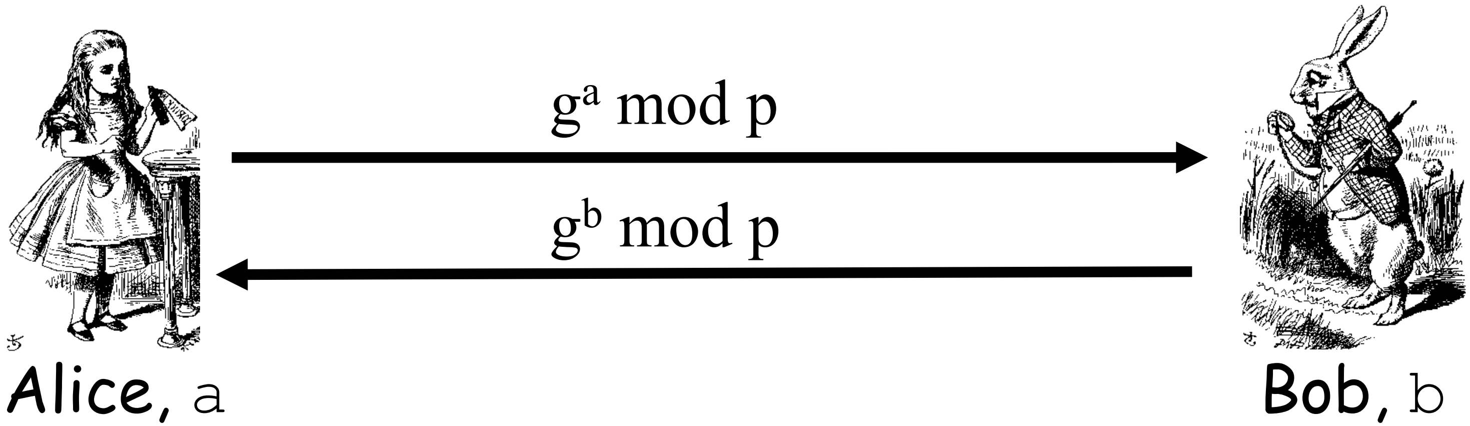
Diffie-Hellman Problem

- Given $A=g^x \bmod p$ and $B=g^y \bmod p$, find $C=g^{xy} \bmod p$.

- If DLP can be solved, then Diffie-Hellman Problem can be solved.

- Open Problem: If Diffie-Hellman Problem can be solved, can DLP be solved?

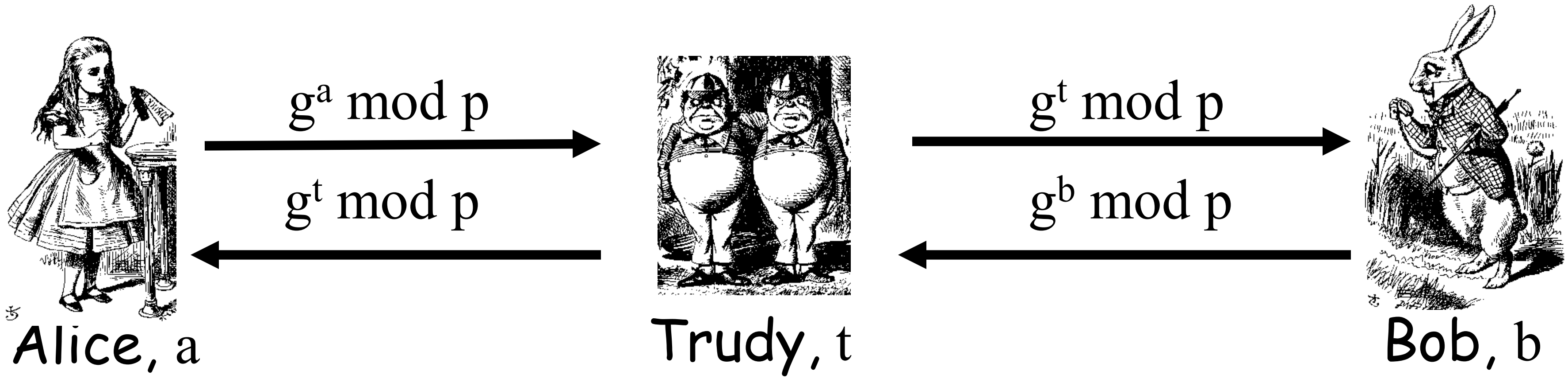
Diffie-Hellman Key Exchange



- Alice computes $(g^b)^a = g^{ba} = g^{ab}$ mod p
- Bob computes $(g^a)^b = g^{ab}$ mod p
- Could use $K = g^{ab}$ mod p as symmetric key

- This key exchange scheme is secure against eavesdroppers if Diffie-Hellman Problem is assumed to be hard to solve.
- However, it is insecure if the attacker in the network is active: man-in-the-middle attack. "Active" means that the attacker can intercept, modify, remove or insert messages into the network.

Man-in-the-Middle Attack (MITM)



- Trudy shares secret g^{at} mod p with Alice
- Trudy shares secret g^{bt} mod p with Bob
- Alice and Bob don't know Trudy exists!

Public key vs. Symmetric key

Symmetric key	Public key
Two parties MUST trust each other	Two parties DO NOT need to trust each other
Both share the same key (or one key is computable from the other)	Two separate keys: a public and a private key
Attack approach: bruteforce	Attack approach: solving mathematical problems (e.g. factorization, discrete log problem)
Faster	Slower (100-1000 times slower)
Smaller key size	Larger key size
Examples: DES, 3DES, DESX, RC6, AES, …	Examples: RSA, ElGamal, ECC,…

Post-Quantum Crypto

- The need for quantum-resistant cryptography
  - Schor’s algorithm
  - Implications for symmetric and asymmetric crypto?
- NIST competition (3rd round)
  - Key exchange and encryption (4 candidates)
  - Signatures (3 candidates)
  - Mostly lattice and code-based cryptography
  - Based on different NP-hard problems

The end!

?

Any questions…