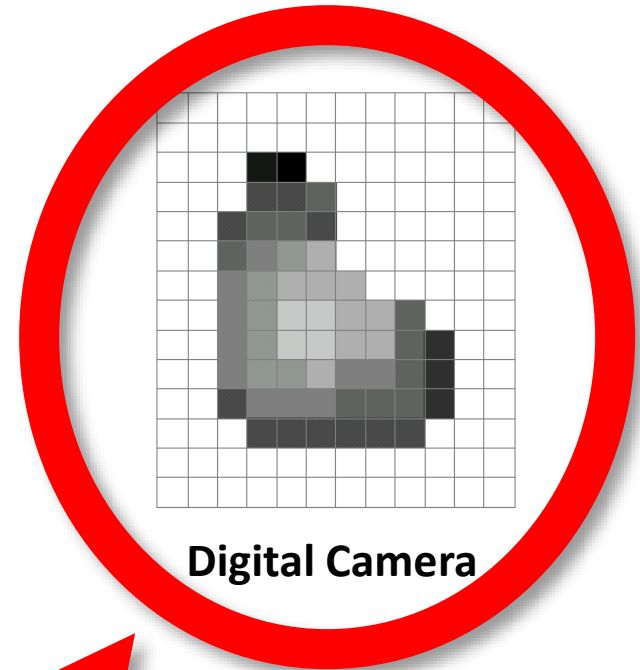
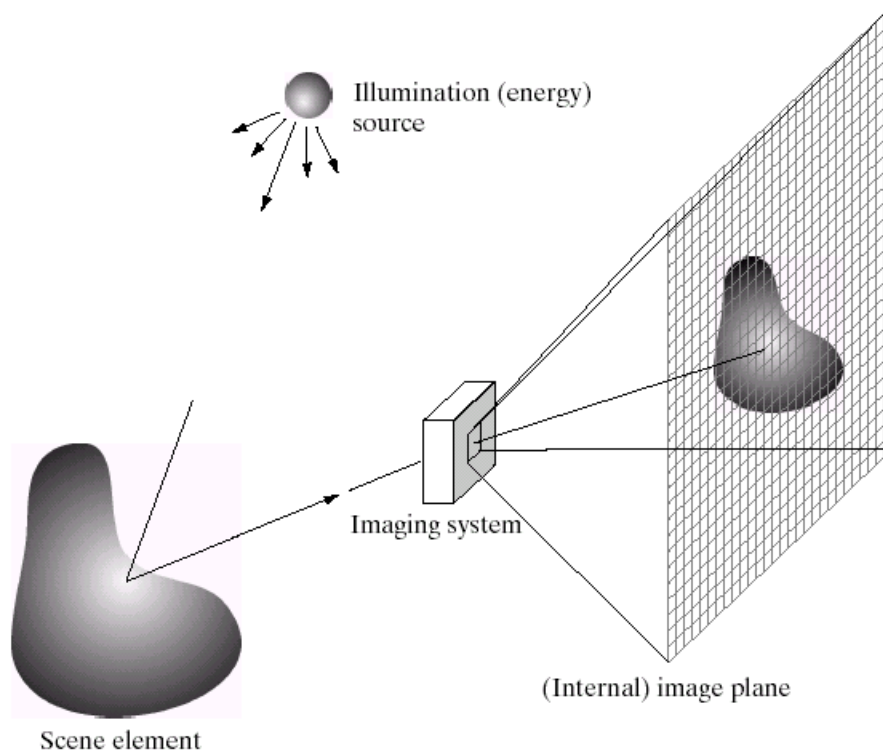


Lecture 1-Image filtering

What is an image?

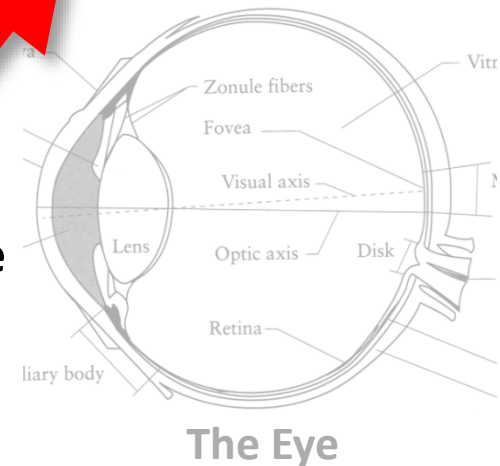


What is an image?



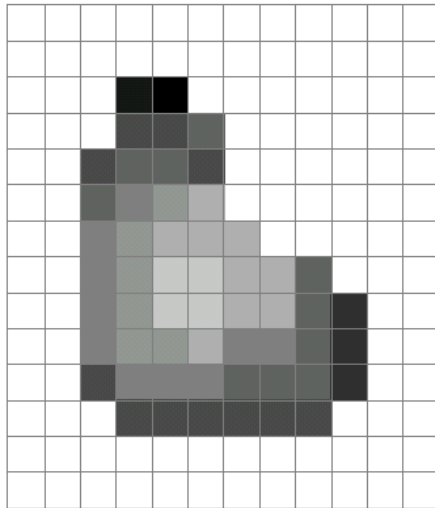
We'll focus on these in this course

Also image formation



What is an image?

- A grid (matrix) of intensity values



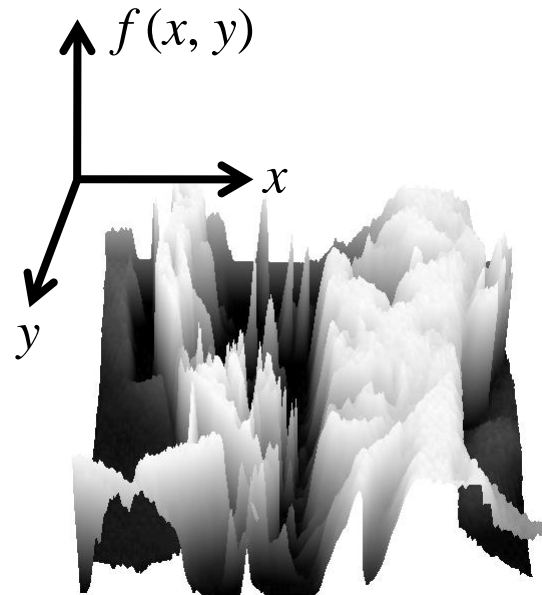
=

255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	20	0	255	255	255	255	255	255	255	255	255	255
255	255	255	75	75	75	255	255	255	255	255	255	255	255	255
255	255	75	95	95	75	255	255	255	255	255	255	255	255	255
255	255	96	127	145	175	255	255	255	255	255	255	255	255	255
255	255	127	145	175	175	175	255	255	255	255	255	255	255	255
255	255	127	145	200	200	175	175	95	255	255	255	255	255	255
255	255	127	145	200	200	175	175	95	47	255	255	255	255	255
255	255	127	145	145	175	127	127	95	47	255	255	255	255	255
255	255	74	127	127	127	95	95	95	47	255	255	255	255	255
255	255	255	74	74	74	74	74	74	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255	255	255	255

(common to use one byte per value: 0 = black, 255 = white)

What is an image?

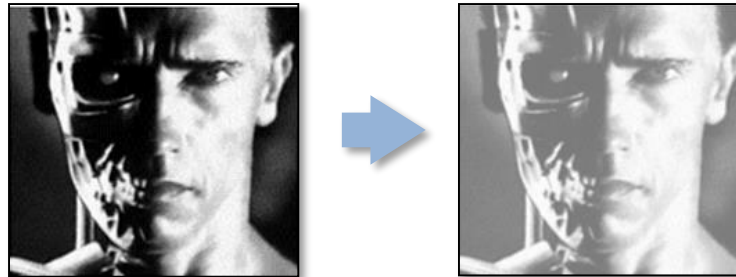
- We can think of a (grayscale) image as a **function**, f , from \mathbb{R}^2 to \mathbb{R} :
 - $f(x,y)$ gives the **intensity** at position (x,y)



- A **digital** image is a discrete (**sampled, quantized**) version of this function

Image transformations

- As with any function, we can apply operators to an image



$$g(x,y) = f(x,y) + 20$$

- We'll talk about a special kind of operator, *convolution* (linear filtering)

Filters

- Filtering
 - Form a new image whose pixels are a combination of the original pixels
- Why?
 - To get useful information from images
 - E.g., extract edges or contours (to understand shape)
 - To enhance the image
 - E.g., to remove noise
 - E.g., to sharpen or to “enhance image”

Image Processing problems

- Image Restoration
 - denoising
 - deblurring
- Image Compression
 - JPEG, JPEG2000, MPEG..
- Computing Field Properties
 - optical flow
 - disparity
- Locating Structural Features
 - corners
 - edges

Question: Noise reduction

- Given a camera and a still scene, how can you reduce noise?



Take lots of images and average them!

What's the next best thing?

Image filtering

- Modify the pixels in an image based on some function of a local neighborhood of each pixel

10	5	3
4	5	1
1	1	7

Local image data

Some function

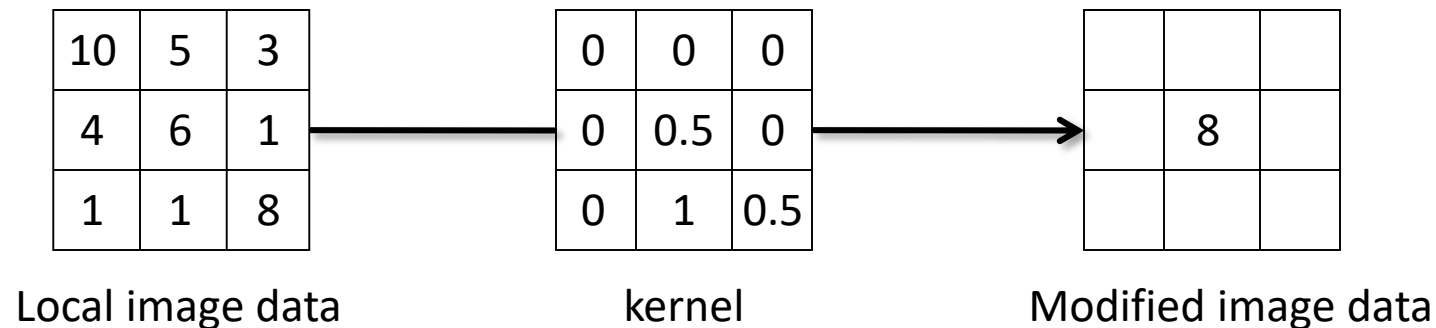


	7	

Modified image data

Linear filtering

- One simple version of filtering: linear filtering (cross-correlation, convolution)
 - Replace each pixel by a linear combination (a weighted sum) of its neighbors
- The prescription for the linear combination is called the “kernel” (or “mask”, “filter”)



Cross-correlation

Let F be the image, H be the kernel (of size $2k+1 \times 2k+1$), and G be the output image

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i + u, j + v]$$

This is called a **cross-correlation** operation:

$$G = H \otimes F$$

- Can think of as a “dot product” between local neighborhood and kernel for each pixel

Convolution

- Same as cross-correlation, except that the kernel is “flipped” (horizontally and vertically)

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i - u, j - v]$$

This is called a **convolution** operation:

$$G = H * F$$

Mean filtering

H



0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

F

=

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
10	20	30	30	30	30	20	10		
10	10	10	0	0	0	0	0		

G

Mean filtering/Moving average

$$F[x, y]$$
[illegible]
$$G[x, y]$$
A 10x10 grid with a red square in the top-left corner. The red square is located in the first row and first column, spanning from the first vertical line to the second vertical line and from the first horizontal line to the second horizontal line. The rest of the grid is empty.

Mean filtering/Moving average

$$F[x, y]$$

[illegible]

$$G[x, y]$$

[illegible]

Mean filtering/Moving average

$$F[x, y]$$

[illegible]

$$G[x, y]$$

[illegible]

Mean filtering/Moving average

$$F[x, y]$$

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0
0	0	0	90	90	90	90	90	0
0	0	0	90	90	90	90	90	0
0	0	0	90	0	90	90	90	0
0	0	0	90	90	90	90	90	0
0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

$$G[x, y]$$

[illegible]

Mean filtering/Moving average

$$F[x, y]$$

[illegible]

$$G[x, y]$$

[illegible]

Mean filtering/Moving average

$$F[x, y]$$

[illegible]

$$G[x, y]$$

[illegible]

Linear filters: examples



Original



0	0	0
0	1	0
0	0	0



Identical image

Linear filters: examples



Original



0	0	0
1	0	0
0	0	0



Shifted left
By 1 pixel

Linear filters: examples

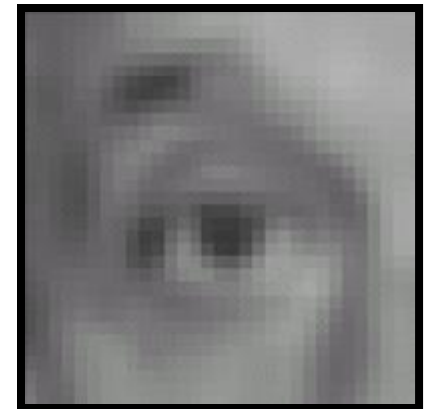


Original



$\frac{1}{9}$

1	1	1
1	1	1
1	1	1



Blur (with a mean filter)

Linear filters: examples



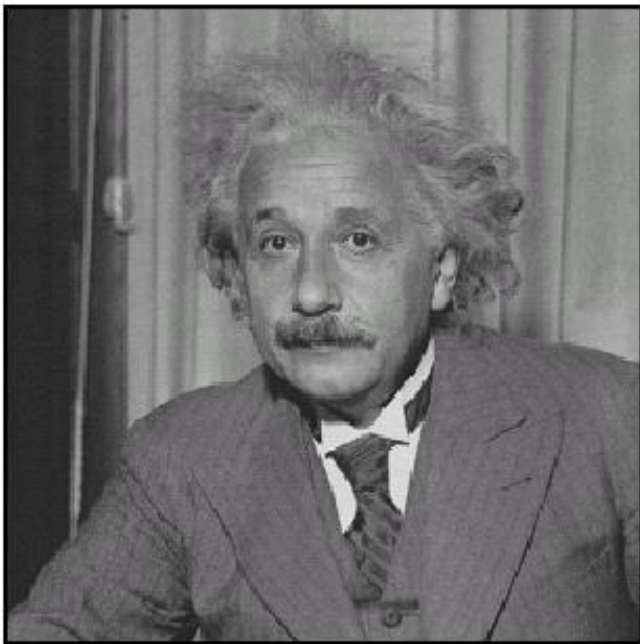
Original

$$* \left(\begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline 0 & 2 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} - \frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \right) =$$

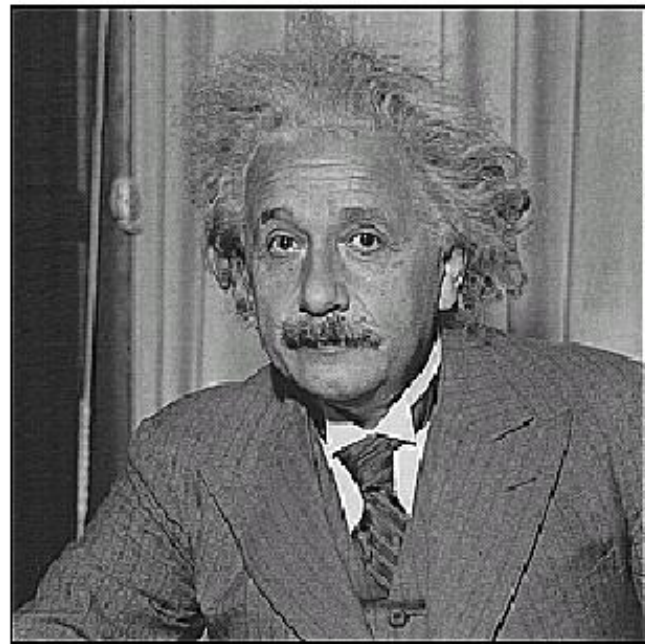


Sharpening filter

Sharpening



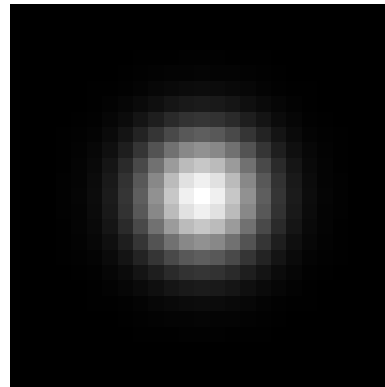
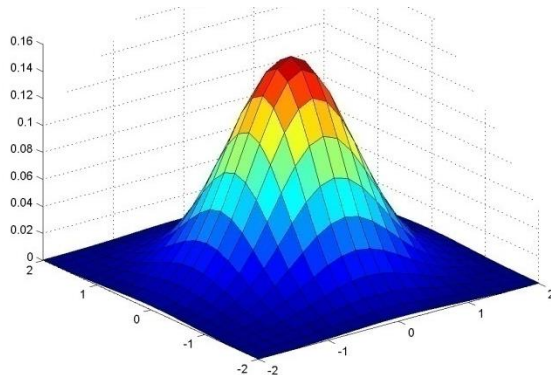
before



after

Gaussian Kernel

$$G_{\sigma} = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

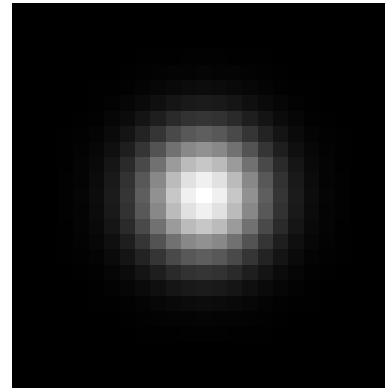
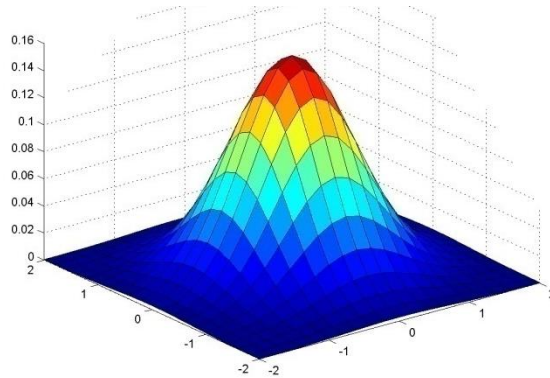


0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

5 x 5, $\sigma = 1$

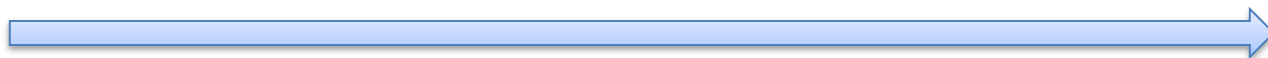
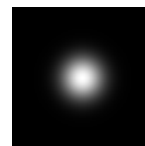
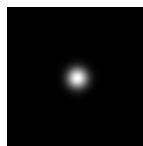
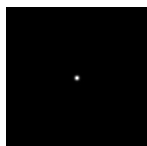
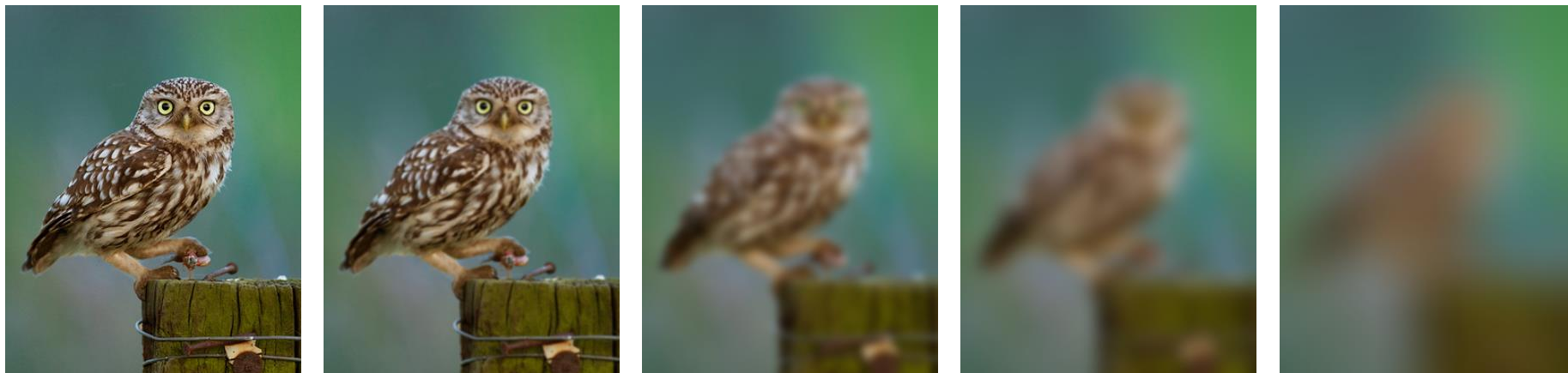
- Constant factor at front makes volume sum to 1 (can be ignored, as we should re-normalize weights to sum to 1 in any case)

Gaussian Kernel



$$G_{\sigma} = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

Gaussian filters

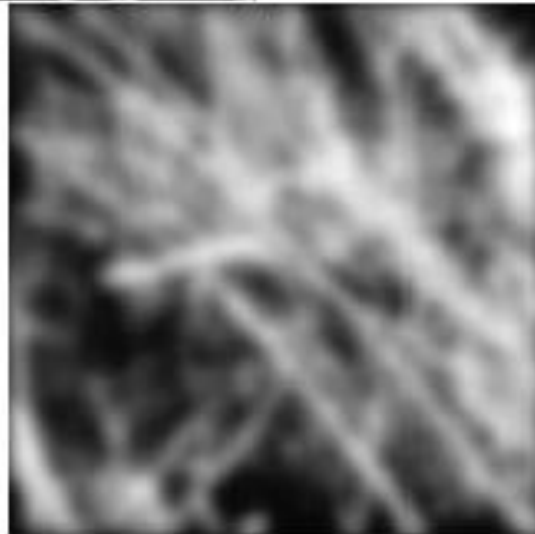
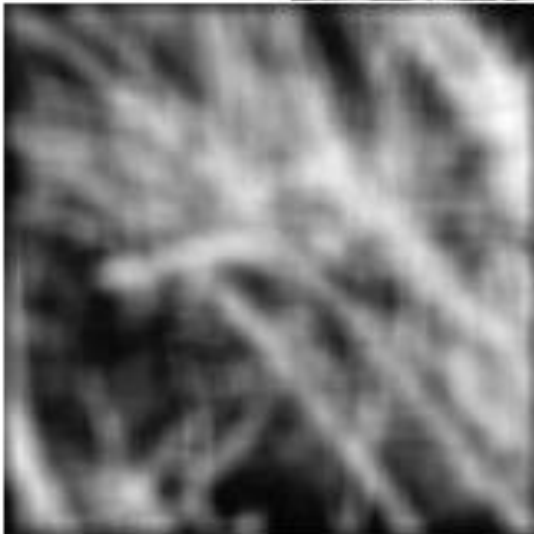


Increasing σ

Mean vs. Gaussian filtering

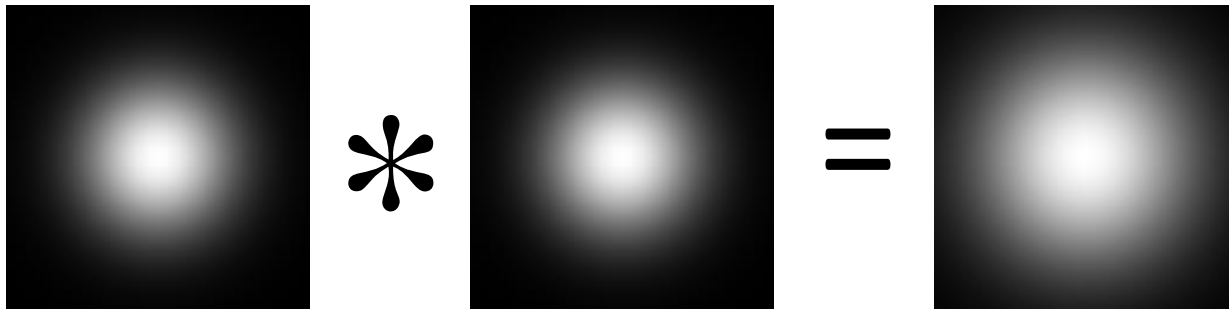


Mean filter may garble
high-frequency signal



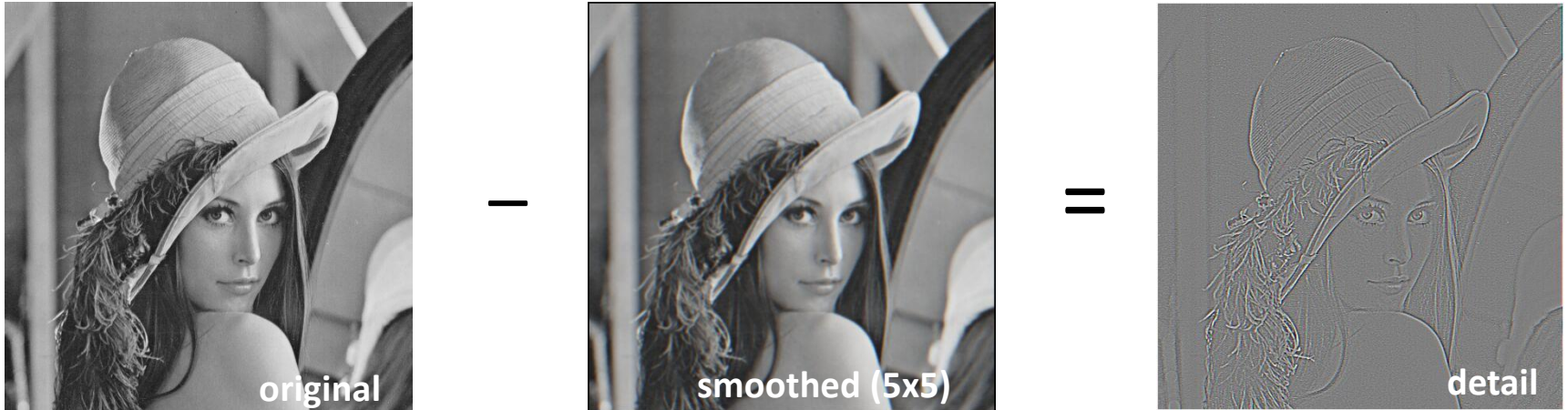
Gaussian filter

- Removes “high-frequency” components from the image (low-pass filter)
- Convolution with self is another Gaussian

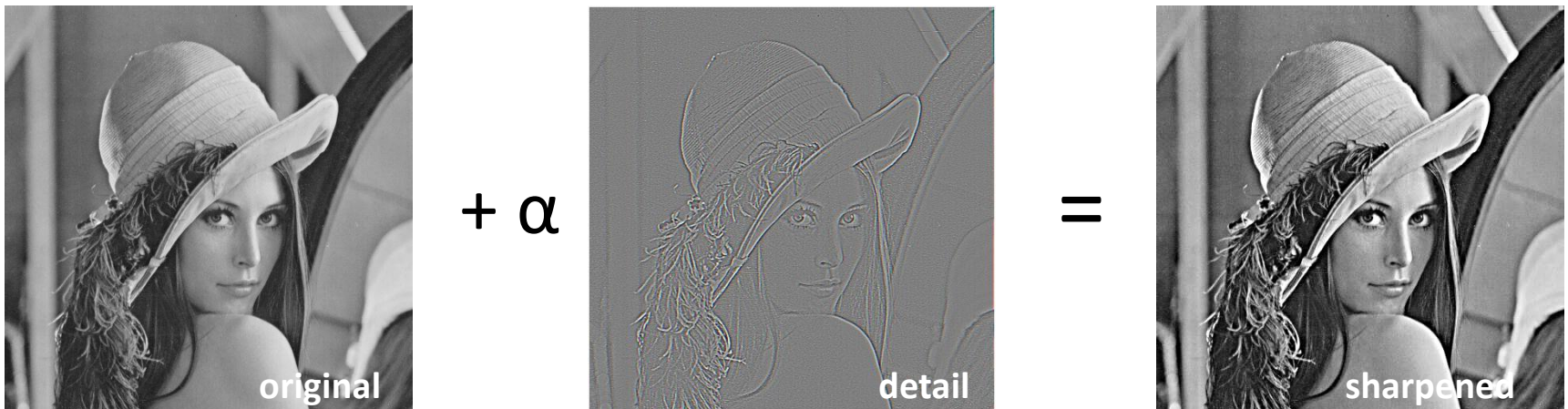


Sharpening revisited

- What does blurring take away?



Let's add it back:



Filters: Thresholding



$$g(m, n) = \begin{cases} 255, & f(m, n) > A \\ 0 & \text{otherwise} \end{cases}$$