

**Questions:**

**1. Modular Exponentiation**

- (a) Calculate  $17^{27} \bmod 23$ .
- (b) Consider the following two cases of raising a number to a certain exponent:
  - $a^{255} \bmod b$
  - $a^{257} \bmod b$

Using the square and multiply method, which one of these two exponentiations will be significantly more expensive? Why? Calculate the total number of modular multiplications required for each case (counting a squaring operation as a modular multiplication).

**2. Modular Inverse:**

- (a) Calculate the modular inverse of  $2019 \bmod 5285$  (Use the extended Euclidean Algorithm).
- (b) Without calculating anything, can you tell whether  $360 \bmod 555$  has a modular inverse? Explain why.

**3. Eulers Totient**

- 1) Calculate  $\phi(n)$  for the following values of  $n$ .

- (a)  $n = 83$
- (b)  $n = 1210$

- 2) Calculate  $39^{191} \bmod 47$

- 4. **RSA Encryption** Suppose we are using an RSA encryption scheme with  $n = pq$ , private key  $d$ , public key  $e$ , where the ciphertext is calculated as  $C = M^e \bmod n$  and can be decrypted by checking  $C^d \bmod n = M$ .

- (a) Can you show why RSA encryption works? Hint: Fermat's Little Theorem...
- (b) Can you encrypt  $M$  when it is larger than  $n$ ?