The Cross Product

- Given two nonzero vectors **a** = $\langle a_1, a_2, a_3 \rangle$ and $b = \langle b_1, b_2, b_3 \rangle$, it is very useful to be able to find a nonzero vector **c** that is perpendicular to both **a** and **b**.

- If **c** = $\langle c_1, c_2, c_3 \rangle$ is such a vector, then $\mathbf{a} \cdot \mathbf{c} = 0$ and $\mathbf{b} \cdot \mathbf{c} = 0$.

<u>**c = a × b**</u>

The **cross product a × b** of two vectors **a** and **b**, unlike the dot product, is a vector **c**.

The vector **c** is perpendicular to both **a** and **b.**

Example 1

- If **a** = ⟨1, 3, 4⟩ and **b** = ⟨2, 7, –5⟩, then

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & 4 \\ 2 & 7 & -5 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & 4 \\ 7 & -5 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & 4 \\ 2 & -5 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix} \mathbf{k}$$

$$= (-15 - 28)\mathbf{i} - (-5 - 8)\mathbf{j} + (7 - 6)\mathbf{k}$$

$$= -43\mathbf{i} + 13\mathbf{j} + \mathbf{k}$$

The Cross Product

- We can also represent the cross product of two vectors in matrix-vector multiplication:

- $$\mathbf{a}\times\mathbf{b} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = [a_\times]b$$