CS6382 Assignment 2

Core

Consider the following game:

$$v(C) = \begin{cases} 0, & \text{if } C = \{0\} \\ 1, & \text{if } C = \{a\} \\ 5, & \text{if } C = \{b\} \\ 10, & \text{if } C = \{c\} \\ 10, & \text{if } C = \{a, b\} \\ 17, & \text{if } C = \{a, c\} \\ 13, & \text{if } C = \{b, c\} \\ 22, & \text{if } C = \{a, b, c\} \end{cases}$$

Q1 Is this game monotonic? Is it superadditive? Yes. No.

Q2 Find all the core of this game.
$\{17 - x, 5, x\}.x \in [10, 12]$

Q3 Now we change $v(\{b\})$ to 2, find all the core of this game.

Shapley Value

Consider the following game:

$$v(C) = \begin{cases} 3, & \text{if } C = \{1\} \\ 1, & \text{if } C = \{2\} \\ 3, & \text{if } C = \{3\} \\ 8, & \text{if } C = \{1, 2\} \\ 4, & \text{if } C = \{1, 3\} \\ 5, & \text{if } C = \{2, 3\} \\ 15, & \text{if } C = \{1, 2, 3\} \end{cases}$$

Q1. Is the game monotonic? Is the game superadditive?
It is monotonic. It is not superadditive.

Q2. Derive the Shapley value and show the steps.
The Shapley value is $\varphi = (\frac{17}{3}, \frac{31}{6}, \frac{25}{6})$. Here we give an example for calculating

$$\varphi_1 = \frac{2}{6} \cdot 3 + \frac{2}{6} \cdot (15 - 5) + \frac{1}{6} \cdot (8 - 1) + \frac{1}{6} \cdot (4 - 3) = \frac{34}{6} = \frac{17}{3}$$

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Q3. Suppose the Shapley value is ϕ. Is $(\{1, 2, 3\}, \phi)$ in the core?
Yes.