Tutorial 9

March 24, 2025

Question 1

Let f be strongly convex on $S = \{x|f(x) \leq f(x^0)\}$, i.e. there exists an $m > 0$ such that

$$\nabla^2 f(x) > mI$$

for all x in S. Then

$$f(x) - p^* \leq (1/2m)||\nabla f(x)||_2^2$$

Question 2

Let f be strongly convex on $S = \{x|f(x) \leq f(x^0)\}$ Suppose the exact line search is used in the gradient decent method, prove:

$$f(x^k) - p^* \leq c^k[f(x^0) - p^*]$$

Question 3

Let $f(x) = (1/2)\,(x_1^2 + \gamma x_2^2)$, with $\gamma > 0$. Suppose that the gradient descent method is used with exact line search, starting at $x^{(0)} = (\gamma, 1)$. Prove

$$x_1^{(k)} = \gamma \left(\frac{\gamma - 1}{\gamma + 1}\right)^k, \quad x_2^{(k)} = \left(-\frac{\gamma - 1}{\gamma + 1}\right)^k$$

How about Newton descent?