

Tutorial 5

February 21, 2025

Question 1

Prove that $x^* = (1, 1/2, -1)$ is optimal for the optimization problem,

$$\begin{array}{ll}\text{minimize} & (1/2)x^T Px + q^T x + r \\ \text{subject to} & -1 \leq x_i \leq 1, \quad i = 1, 2, 3\end{array}$$

where,

$$P = \begin{bmatrix} 13 & 12 & -2 \\ 12 & 17 & 6 \\ -2 & 6 & 12 \end{bmatrix}, \quad q = \begin{bmatrix} -22.0 \\ -14.5 \\ 13.0 \end{bmatrix}, \quad r = 1.$$

Question 2

Give an explicit solution of each of the following LPs.

(a) Minimizing a linear function over an affine set.

$$\begin{array}{ll}\text{minimize} & c^T x \\ \text{subject to} & Ax = b\end{array}$$

hint: Suppose

(b) Minimizing a linear function over a halfspace.

$$\begin{array}{ll}\text{minimize} & c^T x \\ \text{subject to} & a^T x \leq b,\end{array}$$

where $a \neq 0$.

(c) Minimizing a linear function over a rectangle.

$$\begin{array}{ll}\text{minimize} & c^T x \\ \text{subject to} & l \preceq x \preceq u\end{array} \tag{1}$$

where l and u satisfy $l \preceq u$.