Two people are good friends who are deciding to go hiking (H) or reading books (R) on weekends. Each of them has to make the decision simultaneously and independently(they cannot communicate). One of them is extroverted (player 1) who likes hiking very much but prefers to go with her friend. The other of them is introverted (player 2) enjoys reading books more than hiking, and sometimes prefers to be with her friend and sometimes prefers to be alone. Introverted person knows her preference today but extroverted person does not. The extroverted person thinks the probability that introverted person would prefer her company today is $\frac{3}{4}$. The situation is summarized in the following tables:

		Introverted	
		H	R
Extroverted	H	4,2	0,0
	R	0,0	1,4

Table 1: I person prefers E person's company, $p = \frac{3}{4}$

		Introverted	
		H	R
Extroverted	H	4,0	0,4
	R	0,2	2,0

Table 2: I person prefers to be alone, $1 - p = \frac{1}{4}$

1. Describe the situation as a Bayesian game.

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2. Find the Bayesian Nash equilibria.

Bayesian Nash equilibrium:
In a Bayesian game, a Bayesian Nash equilibrium for risk-neutral player is defined to be a strategy profile that maximizes the expected gains of all players depending on what each player believes about the strategies chosen by the other players

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1. Describe the situation as a Bayesian game.

   Solution: Two players: $N = \{1, 2\}$; Their types are $T_1 = \{c\}$, $T_2 = \{a, b\}$ where a represents the situation in table 1, b table 2. The sets of strategies are $S_1 = \{H, S\}$, $S_2 = \{HH, HR, RH, RR\}$. The beliefs of players are

   $$p_1(a|c) = \frac{3}{4}, p_1(b|c) = \frac{1}{4}, p_2(c|a) = p_2(c|b) = 1.$$

   Payoffs are in tables.

2. Find the Bayesian Nash equilibria.

   Solution: Note that $BR_2(H) = HR, BR_2(R) = RH$. And

   $$u_1(H, HR) = \frac{3}{4} \times 4 + \frac{1}{4} \times 0 = 3, u_1(R, HR) = \frac{3}{4} \times 0 + \frac{1}{4} \times 2 = \frac{1}{2}$$

   $$u_1(H, RH) = \frac{3}{4} \times 0 + \frac{1}{4} \times 4 = 1, u_1(R, RH) = \frac{3}{4} \times 1 + \frac{1}{4} \times 0 = \frac{3}{4}$$

   Hence, $BR_1(HR) = H, BR_1(RH) = H$. Therefore, BNE in pure strategy is only (H, HR).