Home Assignment №4

Due on November 22, 2024, 11:59pm

Exercise 1

[2 points]. Prove that the eigenvalues of a real symmetric matrix $A \in \mathbb{R}^{N\times N}$, where $A = A^T$, are real-valued (*i.e.*, not complex-valued). Prove also that the eigenvectors of A corresponding to different eigenvalues are orthogonal to each other.

Exercise 2

[5 points]. Derive the K-th largest direction of variance in principal component analysis (PCA).

Exercise 3

[5 points]. 1) Suppose that a discrete-time linear system has outputs $y[n]$ for the given inputs $x[n]$, as shown in Fig. 1. Determine the response $y_4[n]$ when the input is as shown in Fig. 2.

a) [1 point]. Express $x_4[n]$ as a linear combination of $x_1[n]$, $x_2[n]$, and $x_3[n]$.

b) [1 point]. Using the fact that the system is linear, determine $y_4[n]$, the response to $x_4[n]$.

c) [1 point]. From the input-output pairs in Fig. 1, determine whether the system is time-invariant.

2) Determine the discrete-time convolution of $x[n]$ and $h[n]$ for the following two cases.

a) [1 point]. As shown in Fig. 3.

b) [1 point]. As shown in Fig. 4.

Exercise 4

[3 points]. From the lecture slides, we know that the convolution of discrete-time signals $x[n]$ and $y[n]$, for $n \in [-\infty, +\infty]$, is defined as

$$z[n] = (x * y)[n] = \sum_{m=-\infty}^{\infty} x[m]y[n-m] = \sum_{m=-\infty}^{\infty} x[n-m]y[m]. \tag{1}$$

Here we introduce the Discrete-time Fourier Transform (DTFT) of a discrete-time signal $x[n]$:

$$X(\omega) = \mathrm{DTFT}(x) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}. \tag{2}$$

Try to prove the Convolution Theorem:

$$Z(\omega) = \mathrm{DTFT}(x * y) = \mathrm{DTFT}(x) \cdot \mathrm{DTFT}(y) = X(\omega) \cdot Y(\omega). \tag{3}$$

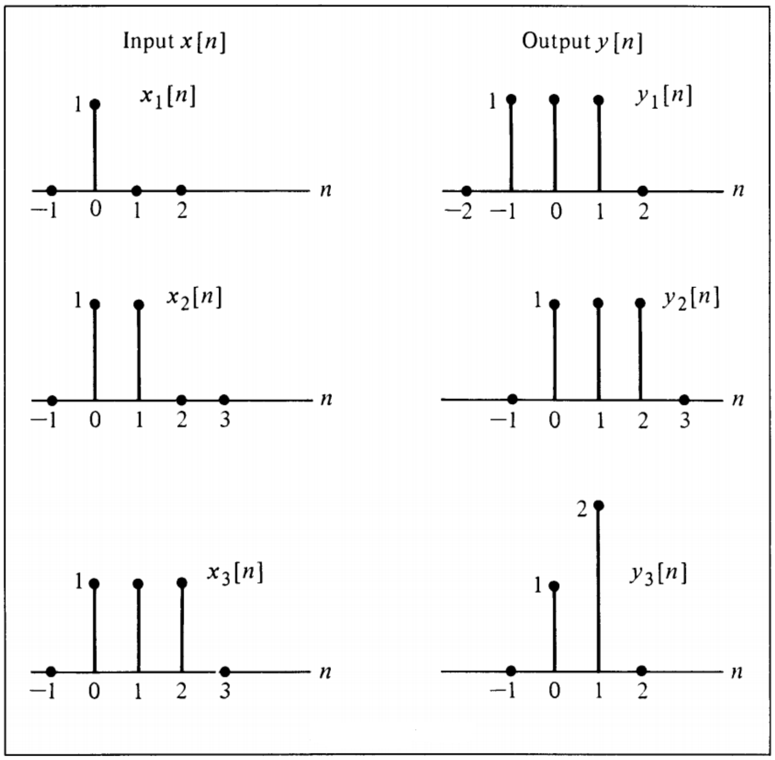

Figure 1:

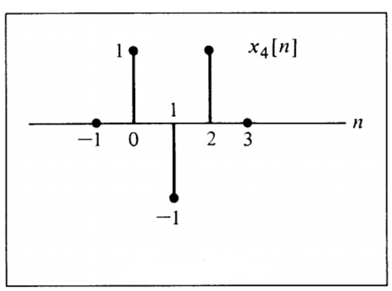

Figure 2:

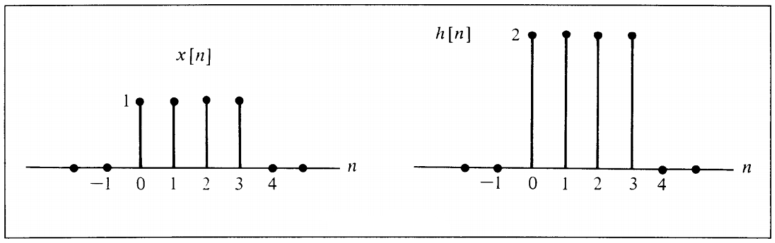

Figure 3:

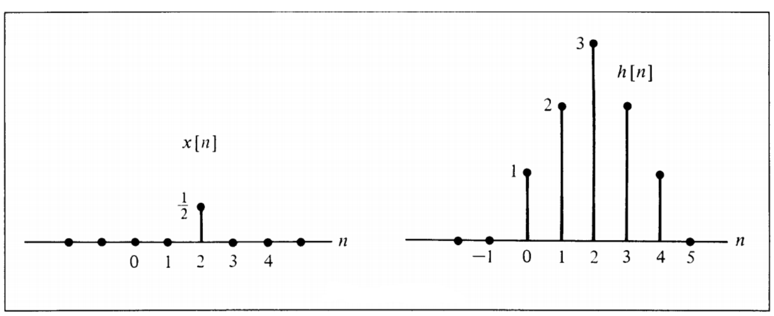

Figure 4: