Tarea entregable

Disease mapping: Modelo logistico - INLA - Reproducibilidad

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Introduccion

En el contexto del Máster de Bioestadística de la Universitat de València, la reproducibilidad y la replicabilidad se reconocen como pilares fundamentales de la investigación científica. Este principio asegura que los análisis estadísticos puedan ser verificados por terceros y reutilizados en nuevos contextos, fortaleciendo la confianza en los resultados obtenidos y su validez empírica

En esta práctica, se aborda un análisis reproducible basado en el mapeo de enfermedades (disease mapping), utilizando dos enfoques metodológicos: WingBUGS e INLA. Ambos modelos fueron implementados para estimar tasas de mortalidad y probabilidades ajustadas espacialmente en la región de Aragón. La comparación entre los resultados busca evaluar la consistencia entre ambos métodos y destacar diferencias clave en sus predicciones, empleando herramientas de reproducibilidad como RMarkdown, proyectos de R y rutas relativas

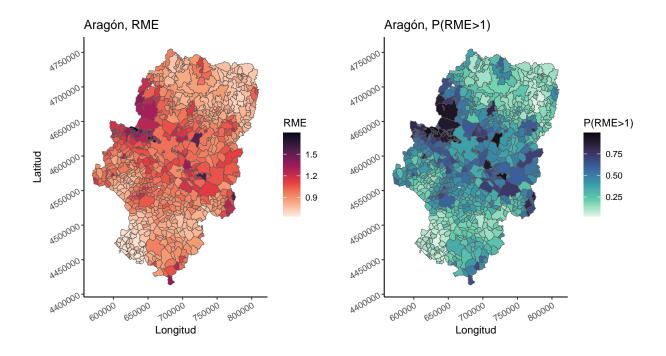
Carga de Datos

Para llevar a cabo este análisis reproducible, se utilizaron datos correspondientes a la región de Aragón, los cuales incluyen información geoespacial y de mortalidad. Los pasos de carga y preparación de los datos se llevaron a cabo utilizando rutas relativas y herramientas de R para garantizar la reproducibilidad del trabajo, independientemente de la máquina o el directorio de ejecución

```
# Programamos el modelo
mod1 <- function() {
    for (i in 1:n0bs) {
        O[i] ~ dpois(mu[i])
        log(mu[i]) <- log(E[i]) + m + het[i] + sp[i]
        het[i] ~ dnorm(0, prechet)
        R[i] <- exp(m + het[i] + sp[i])
    }
    sp[1:n0bs] ~ car.normal(adj[], w[], num[], precsp)
    m ~ dflat()
    prechet <- pow(sdhet, -2); precsp <- pow(sdsp, -2)
    sdhet ~ dunif(0, 10); sdsp ~ dunif(0, 10)

    for (j in 1:n0bs) {
        p.R[j] <- step(R[j] - 1) # Calcula la probabilidad de que RME sea mayor que 1</pre>
```

```
}
  # Ajustamos las distribuciones previas
  inits.mod1 <- function() {list(m = rnorm(1),</pre>
                                   sdhet = runif(1),
                                   sdsp = runif(1))}
  # Cargamos los datos
  datos.mod1 <- list(0 = aragon.spdf$0,</pre>
                      E = aragon.spdf$E,
                      adj = vecinos$adj,
                      num = vecinos$num,
                      w = vecinos$weights,
                      n0bs = length(aragon.spdf))
  # Pedimos los resultados
  params.mod1 <- c("R", "p.R")</pre>
  set.seed(123)
  # Lanzamos el modelo
  res.mod1 <- bugs(data = datos.mod1, model = mod1,
                    param = params.mod1, inits = inits.mod1)
Recuperamos el objeto sf para graficarlo mejor.
  # Añadimos al data.frame las columnas RME y p.RME
  aragon.sf$RME <- res.mod1$mean$R</pre>
  aragon.sf$p.RME <- res.mod1$mean$p.R</pre>
  rme.plot <- ggplot(aragon.sf) +</pre>
    geom_sf(aes(fill = RME)) +
    scale_fill_viridis_c(option = "rocket",direction = -1) +
    ggtitle("Aragón, RME") + xlab("Longitud") + ylab("Latitud") +
    theme classic() +
     theme(axis.text = element_text(hjust = 1, angle = 30))
  p.rme.plot <- ggplot(aragon.sf) +</pre>
    geom_sf(aes(fill = p.RME)) +
    scale_fill_viridis_c(option = "mako",direction=-1) +
    ggtitle("Aragón, P(RME>1)") + xlab("Longitud") + ylab("") +
    theme_classic() +
    labs(fill = "P(RME>1)") +
     theme(axis.text = element_text(hjust = 1, angle = 30))
  wrap_plots(plotlist = list(rme.plot, p.rme.plot))
```



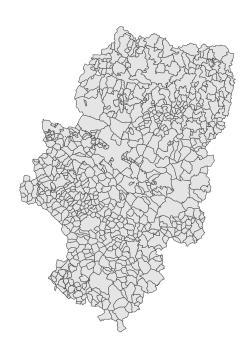
```
### --- O. Loading the packages --- ####
library(sf)
library(spdep)
library(lattice)
library(latticeExtra)
library(viridis)
library(gridExtra)
library(RColorBrewer)
library(INLA)
library(sf)
library(ggthemes)
#BiocManager::install(c("graph", "Rgraphviz"), dep=TRUE)
library(Rgraphviz)
library(graph)
library(leaflet)
library(ggplot2)
library(patchwork)
### --- 1. Loading the data --- ####
#Dataset
load("material/Aragon.Rdata") #datos
data <- Aragon.df
```

```
Nareas <- length(data[,1]) #número de áreas

# sf
aragon_shape <- read_sf("material/aragon.shp")

aragon_shape <- st_set_crs(aragon_shape, 27700)

ggplot(aragon_shape) +
  geom_sf() +
  theme_void()</pre>
```

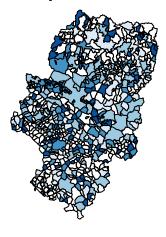


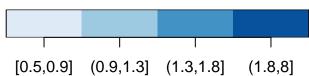
```
### --- 2. Checking if the data of the sp and the data.frame match --- ####
# The order of the areas needs to be the same between
# the data and the spatial polygon object obtained importing
# the shapefile, so we re-order the data.
data$CODMUNI == aragon_shape$CODMUNI
```

```
[13] FALSE F
```

```
[133] FALSE FALSE
[145] FALSE FALSE
[157] FALSE 
[169] FALSE 
[181] FALSE 
[193] FALSE 
[205] FALSE 
[217] FALSE 
[229] FALSE 
[241] FALSE 
[253] FALSE 
[265] FALSE 
[277] FALSE 
[289] FALSE 
[301] FALSE FALSE
[313] FALSE FALSE
[325] FALSE FALSE
[337] FALSE FALSE
[349] FALSE FALSE
[361] FALSE FALSE
[373] FALSE FALSE
[385] FALSE 
[397] FALSE FALSE
[409] FALSE 
[421] FALSE FALSE
[433] FALSE 
[445] FALSE 
[457] FALSE 
[469] FALSE 
[481] FALSE 
[493] FALSE 
[505] FALSE FALSE
[517] FALSE FALSE
[529] FALSE FALSE
[541] FALSE FALSE
[553] FALSE FALSE
[565] FALSE FALSE
[577] FALSE FALSE
[589] FALSE 
[601] FALSE FALSE
[613] FALSE FALSE
[625] FALSE 
[637] FALSE FALSE
[649] FALSE FALSE
[661] FALSE FALSE
[673] FALSE 
[685] FALSE 
[697] FALSE 
[709] FALSE 
[721] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
```

isq_raw_disc

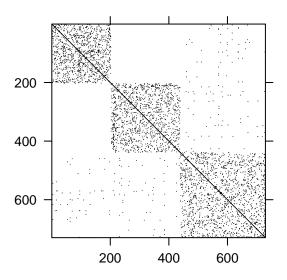




```
### --- 3. Defining neighbor relation --- ####
temp <- poly2nb(aragon_shape)

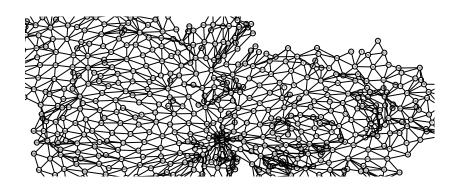
#This create a file called ``ARG.graph'' with the graph for INLA
nb2INLA("ARG.graph", temp)

### ---- 3.1. Plotting the generated graph --- ####
H <- inla.read.graph(filename="ARG.graph")
image(inla.graph2matrix(H),xlab="",ylab="") #identifica los vecinos de cada barrio, el barrio 1 tiene</pre>
```



Dimensions: 729 x 729

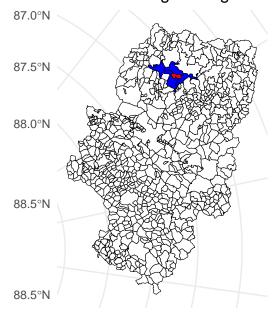
---- 3.2. More plotting --- #### plot(H)



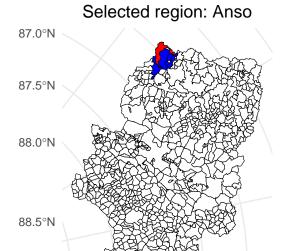
```
### ---- 3.3. Plotting the neighbors --- ####
plot_map_neig_ggplot <- function(neig, aragon_shape, temp) {</pre>
```

```
# Base map of London
  p <- ggplot() +</pre>
    geom_sf(data = aragon_shape, fill = "white", color = "black") +
    # Highlight the selected region in red
    geom_sf(data = aragon_shape[neig, ], fill = "red", color = "black") +
    # Highlight the neighbors in blue
    geom_sf(data = aragon_shape[temp[[neig]], ], fill = "blue", color = "black") +
    # Set the theme
    theme_minimal() +
    ggtitle(paste("Selected region:", aragon_shape$NOMBRE[neig])) +
    theme(plot.title = element_text(hjust = 0.5))
  # Print the plot
  print(p)
  # Print information about the selected region and its neighbors
  cat("You have selected", aragon_shape$NOMBRE[neig], "and its neighbors are:", "\n")
  cat(aragon_shape$NOMBRE[temp[[neig]]], "\n")
}
#esta funcion identifica los vecinos de cada vecindario en concreto
plot_map_neig_ggplot(neig = 30, aragon_shape, temp = temp)
```

Selected region: Arguis



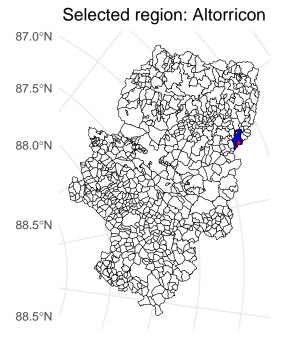
You have selected Arguis and its neighbors are: Caldearenas Nueno Las Penas de Riglos La Sotonera plot_map_neig_ggplot(neig = 25, aragon_shape, temp = temp)



You have selected Anso and its neighbors are: Aisa Aragues del Puerto Canal de Berdun Fago Valle de Hecho

88.5°N

plot_map_neig_ggplot(neig = 23, aragon_shape, temp = temp)

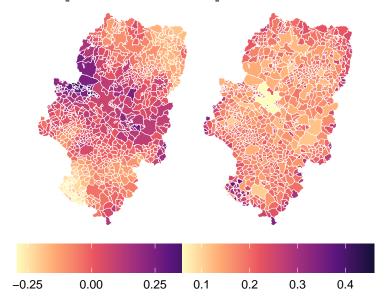


You have selected Altorricon and its neighbors are: Alcampell Tamarite de Litera

```
### --- 4. Fitting a model with bym effect --- ####
   ### ---- 4.0. Adding ids for the random effects --- ####
  S \leftarrow U \leftarrow seq(1,729) # crea dos veces el mismo índice, uno para el efecto aleatorio espacial y otro p
  data <- cbind(data, S, U)</pre>
   ### ---- 4.1. Formula --- ####
  formula <- 0 \sim 1 + f(S,
                                   = "besag",
                       model
                              = H,
                       graph
                       scale.model = TRUE.
                       hyper
                         list(prec = list(prior="loggamma",param = c(1,0.001)))) +
    f(U,
                 = "iid",
      model
      hyper
        list(prec = list(prior="loggamma",param = c(1,0.001))))
   ### ---- 4.2. Model --- ####
  set.seed(123)
  mod.isq <- inla(formula,</pre>
                                  = "poisson",
                  family
                  data
                                  = data,
                                  = E, #es el offset, se puede poner como offset o como esperados
                  control.compute = list(dic = TRUE, waic = TRUE, cpo = TRUE),
                  control.predictor = list(compute=TRUE, cdf=c(log(1)))) #cdf = cumulative density fund
   summary(mod.isq)
    Pre = 0.477, Running = 0.958, Post = 0.124, Total = 1.56
Fixed effects:
              mean
                      sd 0.025quant 0.5quant 0.975quant
(Intercept) -0.066 0.036
                           -0.139
                                    -0.066
                                                 0.003 -0.066
Random effects:
  Name Model
   S Besags ICAR model
   U IID model
Model hyperparameters:
                          sd 0.025quant 0.5quant 0.975quant mode
                 mean
Precision for S 19.02 11.38 6.37
                                          16.16
                                                       49.07 11.95
Precision for U 705.16 934.70
                                   13.17
                                          373.67
                                                     3199.56 18.41
Deviance Information Criterion (DIC) ..... 1897.94
```

```
Deviance Information Criterion (DIC, saturated) ....: 825.54
Effective number of parameters .....: 53.62
Watanabe-Akaike information criterion (WAIC) ...: 1902.53
Effective number of parameters ..... 52.37
Marginal log-Likelihood: -1246.57
CPO, PIT is computed
Posterior summaries for the linear predictor and the fitted values are computed
(Posterior marginals needs also 'control.compute=list(return.marginals.predictor=TRUE)')
  ### ---- 4.3. Posterior distribution of the random effects --- ####
  aragon shape$SPmean <- round(mod.isq$summary.random$S[["mean"]], 4)</pre>
  aragon shape$SPsd <- round(mod.isq$summary.random$S[["sd"]],5)</pre>
  #Mean posterior distribution
  a <- ggplot(data = aragon_shape) +</pre>
    geom_sf(aes(fill = SPmean), color = "white") +
    scale_fill_viridis_c(option = "magma",begin = 0.1, direction = -1) +
    theme_void() +
    theme(legend.position="bottom",
          plot.title = element_text(hjust = 0.5,
                                    color = "Gray40",
                                    size = 20,
                                    face = "bold"),
          legend.title = element_blank(),
          plot.subtitle = element_text(color = "blue"),
          plot.caption = element_text(color = "Gray60")) +
    guides(fill = guide colorbar(barwidth = 10, barheight = 1.5)) + # Adjusting the legend size
    ggtitle("Mean posterior of S")
  #Sd posterior distribution
  b <- ggplot(data = aragon_shape) +</pre>
    geom_sf(aes(fill = SPsd), color = "white") +
    scale_fill_viridis_c(option = "magma",begin = 0.1, direction = -1) +
    theme void() +
    theme(legend.position="bottom",
          plot.title = element_text(hjust = 0.5,
                                    color = "Gray40",
                                    size = 20,
                                    face = "bold"),
          legend.title = element blank(),
          plot.subtitle = element_text(color = "blue"),
          plot.caption = element text(color = "Gray60")) +
    guides(fill = guide_colorbar(barwidth = 10, barheight = 1.5)) + # Adjusting the legend size
    ggtitle("SD posterior of S")
  a | b
```

Mean posterios of S

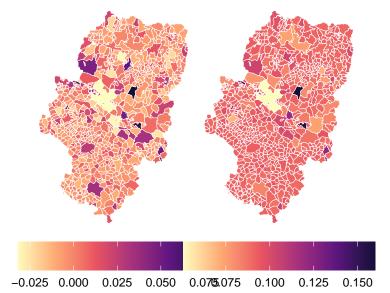


```
aragon_shape$UPmean <- round(mod.isq$summary.random$U[["mean"]], 4)</pre>
aragon_shape$UPsd <- round(mod.isq$summary.random$U[["sd"]],5)</pre>
#Mean posterior distribution
c <- ggplot(data = aragon_shape) +</pre>
  geom_sf(aes(fill = UPmean), color = "white") +
  scale_fill_viridis_c(option = "magma",begin = 0.1, direction = -1) +
  theme_void() +
  theme(legend.position="bottom",
        plot.title = element_text(hjust = 0.5,
                                   color = "Gray40",
                                   size = 20,
                                   face = "bold").
        legend.title = element_blank(),
        plot.subtitle = element_text(color = "blue"),
        plot.caption = element_text(color = "Gray60")) +
  guides(fill = guide_colorbar(barwidth = 10, barheight = 1.5)) + # Adjusting the legend size
  ggtitle("Mean posterior of U")
#Sd posterior distribution
d <- ggplot(data = aragon_shape) +</pre>
  geom_sf(aes(fill = UPsd), color = "white") +
  scale_fill_viridis_c(option = "magma",begin = 0.1, direction = -1) +
  theme_void() +
  theme(legend.position="bottom",
        plot.title = element_text(hjust = 0.5,
                                   color = "Gray40",
                                   size = 20,
                                   face = "bold"),
```

```
legend.title = element_blank(),
    plot.subtitle = element_text(color = "blue"),
    plot.caption = element_text(color = "Gray60")) +
guides(fill = guide_colorbar(barwidth = 10, barheight = 1.5)) + # Adjusting the legend size
ggtitle("SD posterior of U")

c | d
```

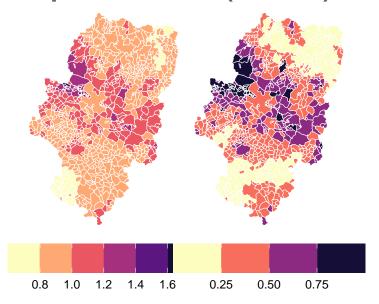
Mean posterio 6 of U



--- 4.4. Posterior distribution of suicides mortality --- #### aragon_shape\$isq_mean <- mod.isq\$summary.fitted.values\$mean # mean</pre> aragon_shape\$isq_sd <- mod.isq\$summary.fitted.values\$sd #s</pre> aragon_shape\$isq_median <- mod.isq\$summary.fitted.values\$`0.5quant` # median aragon_shape\$isq_q025 <- mod.isq\$summary.fitted.values\$`0.025quant` # quantile aragon_shape\$isq_q975 <- mod.isq\$summary.fitted.values\$`0.975quant` # quantile aragon_shape\$isq_p1 <- 1 - mod.isq\$summary.fitted.values\$`1cdf` # probability to be greater than 1 ### --- 4.5. Posterior distribution of suicides SMR with cutoff--- #### ## Also, the probability for SMR to be greater than 1. isq_disc1 <- ggplot(data = aragon_shape) +</pre> geom_sf(aes(fill = isq_mean), color = "white") + scale_fill_viridis_b(option = "magma", begin = 0.1, direction = -1) + theme void() + theme(legend.position = "bottom", plot.title = element_text(hjust = 0.5, color = "Grav40",

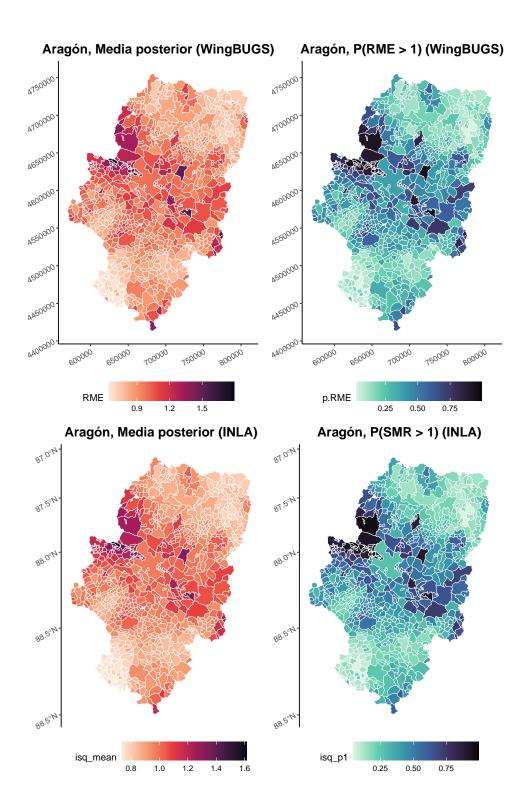
```
size = 20,
     face = "bold"
   ),
   legend.title = element_blank(),
   plot.subtitle = element_text(color = "blue"),
   plot.caption = element_text(color = "Gray60")
  guides(fill = guide_colourbar(barwidth = 10, barheight = 1.5)) +
  ggtitle("Mean posterior RME")
isq_disc2 <- ggplot(data = aragon_shape) +</pre>
  geom_sf(aes(fill = isq_p1), color = "white") +
  scale_fill_viridis_b(option = "magma", begin = 0.1, direction = -1) +
  theme_void() +
  theme(
   legend.position = "bottom",
   plot.title = element_text(
     hjust = 0.5,
     color = "Gray40",
     size = 20,
     face = "bold"
   ),
   legend.title = element_blank(),
   plot.subtitle = element_text(color = "blue"),
   plot.caption = element_text(color = "Gray60")
  ) +
  guides(fill = guide_colourbar(barwidth = 10, barheight = 1.5)) +
  ggtitle("P(RME>1)")
isq_disc1 | isq_disc2
```

Mean posterior RME(RME>1)



```
library(ggplot2)
library(patchwork)
# Gráfico INLA: Media posterior
inla_mean_plot <- ggplot(data = aragon_shape) +</pre>
  geom_sf(aes(fill = isq_mean), color = "white") +
  scale_fill_viridis_c(option = "rocket",direction = -1) +
  ggtitle("Aragón, Media posterior (INLA)") +
  theme_classic() +
  theme(
    legend.position = "bottom",
    axis.text = element_text(hjust = 1, angle = 30),
   plot.title = element_text(hjust = 0.5, size = 15, face = "bold")
  guides(fill = guide_colourbar(barwidth = 10, barheight = 1.5))
# Gráfico INLA: Probabilidad (SMR > 1)
inla_p_plot <- ggplot(data = aragon_shape) +</pre>
  geom_sf(aes(fill = isq_p1), color = "white") +
  scale_fill_viridis_c(option = "mako", direction=-1) +
  ggtitle("Aragón, P(SMR > 1) (INLA)") +
  theme_classic() +
  theme(
    legend.position = "bottom",
    axis.text = element_text(hjust = 1, angle = 30),
   plot.title = element_text(hjust = 0.5, size = 15, face = "bold")
  guides(fill = guide_colourbar(barwidth = 10, barheight = 1.5))
```

```
# Gráfico WingBUGS: Media (RME)
rme_plot <- ggplot(data = aragon.sf) +</pre>
  geom_sf(aes(fill = RME), color = "white") +
  scale_fill_viridis_c(option = "rocket",direction = -1) +
  ggtitle("Aragón, Media posterior (WingBUGS)") +
  theme_classic() +
  theme(
    legend.position = "bottom",
    axis.text = element_text(hjust = 1, angle = 30),
    plot.title = element_text(hjust = 0.5, size = 15, face = "bold")
  guides(fill = guide_colourbar(barwidth = 10, barheight = 1.5))
# Gráfico WingBUGS: Probabilidad (RME > 1)
p_rme_plot <- ggplot(data = aragon.sf) +</pre>
  geom sf(aes(fill = p.RME), color = "white") +
  scale_fill_viridis_c(option = "mako",direction = -1) +
  ggtitle("Aragón, P(RME > 1) (WingBUGS)") +
  theme_classic() +
  theme(
    legend.position = "bottom",
    axis.text = element_text(hjust = 1, angle = 30),
   plot.title = element_text(hjust = 0.5, size = 15, face = "bold")
  guides(fill = guide_colourbar(barwidth = 10, barheight = 1.5))
# Organización de los gráficos en una cuadrícula 2x2
final_plot <- (rme_plot | p_rme_plot) /</pre>
              (inla_mean_plot | inla_p_plot)
# Mostrar los gráficos
final_plot
```

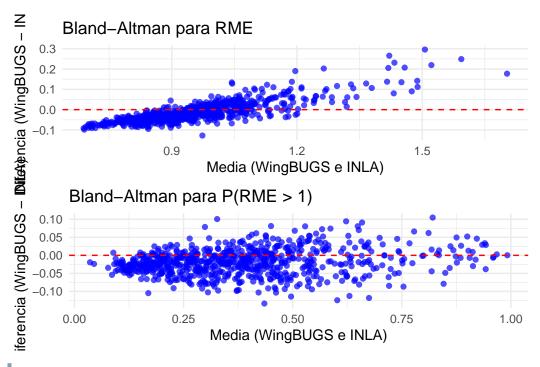


library(dplyr)
library(ggplot2)

```
library(gridExtra)
  ## 1. Resumen descriptivo
  descriptive stats <- aragon shape %>%
    summarise(
      mean_INLA_RME = mean(isq_mean, na.rm = TRUE),
      sd_INLA_RME = sd(isq_mean, na.rm = TRUE),
      mean_WingBUGS_RME = mean(aragon.sf$RME, na.rm = TRUE),
      sd_WingBUGS_RME = sd(aragon.sf$RME, na.rm = TRUE),
      mean_INLA_p = mean(isq_p1, na.rm = TRUE),
      sd_INLA_p = sd(isq_p1, na.rm = TRUE),
      mean_WingBUGS_p = mean(aragon.sf$p.RME, na.rm = TRUE),
      sd_WingBUGS_p = sd(aragon.sf$p.RME, na.rm = TRUE)
  descriptive_stats
Simple feature collection with 1 feature and 8 fields
Geometry type: POLYGON
Dimension:
               XY
Bounding box: xmin: 569301 ymin: 4413136 xmax: 810739 ymax: 4755089
Projected CRS: OSGB36 / British National Grid
# A tibble: 1 x 9
  mean_INLA_RME sd_INLA_RME mean_WingBUGS_RME sd_WingBUGS_RME mean_INLA_p
                      <dbl>
          <dbl>
                                         <dbl>
                                                         <dbl>
                                                                      <dbl>
                      0.123
                                         0.956
                                                                      0.399
          0.968
                                                         0.167
# i 4 more variables: sd_INLA_p <dbl>, mean_WingBUGS_p <dbl>,
  sd WingBUGS p <dbl>, geometry <POLYGON [m]>
  ## 2. Correlación
  cor_RME <- cor(aragon.sf$RME, aragon_shape$isq_mean, use = "complete.obs")</pre>
  cor_p <- cor(aragon.sf$p.RME, aragon_shape$isq_p1, use = "complete.obs")</pre>
  cat("Correlación entre RME (WingBUGS e INLA):", cor_RME, "\n")
Correlación entre RME (WingBUGS e INLA): 0.9786599
  cat("Correlación entre P(RME>1) (WingBUGS e INLA):", cor_p, "\n")
Correlación entre P(RME>1) (WingBUGS e INLA): 0.9830765
  ## 3. Test de diferencias estadísticas
  t_test_RME <- t.test(aragon.sf$RME, aragon_shape$isq_mean, paired = TRUE)</pre>
  t_test_p <- t.test(aragon.sf$p.RME, aragon_shape$isq_p1, paired = TRUE)</pre>
  t test RME
```

```
data: aragon.sf$RME and aragon_shape$isq_mean
t = -6.0426, df = 728, p-value = 2.42e-09
alternative hypothesis: true mean difference is not equal to 0
95 percent confidence interval:
 -0.015553115 -0.007925106
sample estimates:
mean difference
    -0.01173911
   t_test_p
    Paired t-test
data: aragon.sf$p.RME and aragon_shape$isq_p1
t = -14.01, df = 728, p-value < 2.2e-16
alternative hypothesis: true mean difference is not equal to 0
95 percent confidence interval:
 -0.02152283 -0.01623227
sample estimates:
mean difference
    -0.01887755
   ## 4. Bland-Altman
  bland_altman_data <- aragon.sf %>%
    mutate(
      mean_RME = (RME + aragon_shape$isq_mean) / 2,
      diff_RME = RME - aragon_shape$isq_mean,
      mean_p = (p.RME + aragon_shape$isq_p1) / 2,
      diff_p = p.RME - aragon_shape$isq_p1
    )
  bland_RME_plot <- ggplot(bland_altman_data, aes(x = mean_RME, y = diff_RME)) +</pre>
     geom_point(color = "blue", alpha = 0.7) +
     geom_hline(yintercept = 0, linetype = "dashed", color = "red") +
    ggtitle("Bland-Altman para RME") +
    xlab("Media (WingBUGS e INLA)") + ylab("Diferencia (WingBUGS - INLA)") +
     theme_minimal()
  bland_p_plot <- ggplot(bland_altman_data, aes(x = mean_p, y = diff_p)) +</pre>
     geom_point(color = "blue", alpha = 0.7) +
    geom_hline(yintercept = 0, linetype = "dashed", color = "red") +
    ggtitle("Bland-Altman para P(RME > 1)") +
    xlab("Media (WingBUGS e INLA)") + ylab("Diferencia (WingBUGS - INLA)") +
    theme_minimal()
   grid.arrange(bland RME plot, bland p plot, nrow = 2)
```

Paired t-test



```
## 5. Gráficos de dispersión
scatter_RME <- ggplot(data = NULL, aes(x = aragon.sf$RME, y = aragon_shape$isq_mean)) +
geom_point(color = "darkgreen", alpha = 0.6) +
geom_abline(slope = 1, intercept = 0, color = "red", linetype = "dashed") +
ggtitle("Dispersión RME: WingBUGS vs. INLA") +
xlab("WingBUGS (RME)") + ylab("INLA (RME)") +
theme_classic()

scatter_p <- ggplot(data = NULL, aes(x = aragon.sf$p.RME, y = aragon_shape$isq_p1)) +
geom_point(color = "darkblue", alpha = 0.6) +
geom_abline(slope = 1, intercept = 0, color = "red", linetype = "dashed") +
ggtitle("Dispersión P(RME>1): WingBUGS vs. INLA") +
xlab("WingBUGS (P(RME>1))") + ylab("INLA (P(RME>1))") +
theme_classic()

grid.arrange(scatter_RME, scatter_p, nrow = 2)
```

