Morphological Operations

Outline

- Basic concepts:
    - Erode and dilate
    - Open and close.
- Granulometry
- Hit and miss transform
- Thinning and thickening
- Skeletonization and the medial axis transform
- Introduction to gray level morphology.

What Are Morphological Operators?

- Local pixel transformations for processing region shapes

- Most often used on binary images

- Logical transformations based on comparison of pixel neighbourhoods with a pattern.

Simple Operations - Examples

- Eight-neighbour erode
    - a.k.a. Minkowsky subtraction

- Erase any foreground pixel that has one eight-connected neighbour that is background.
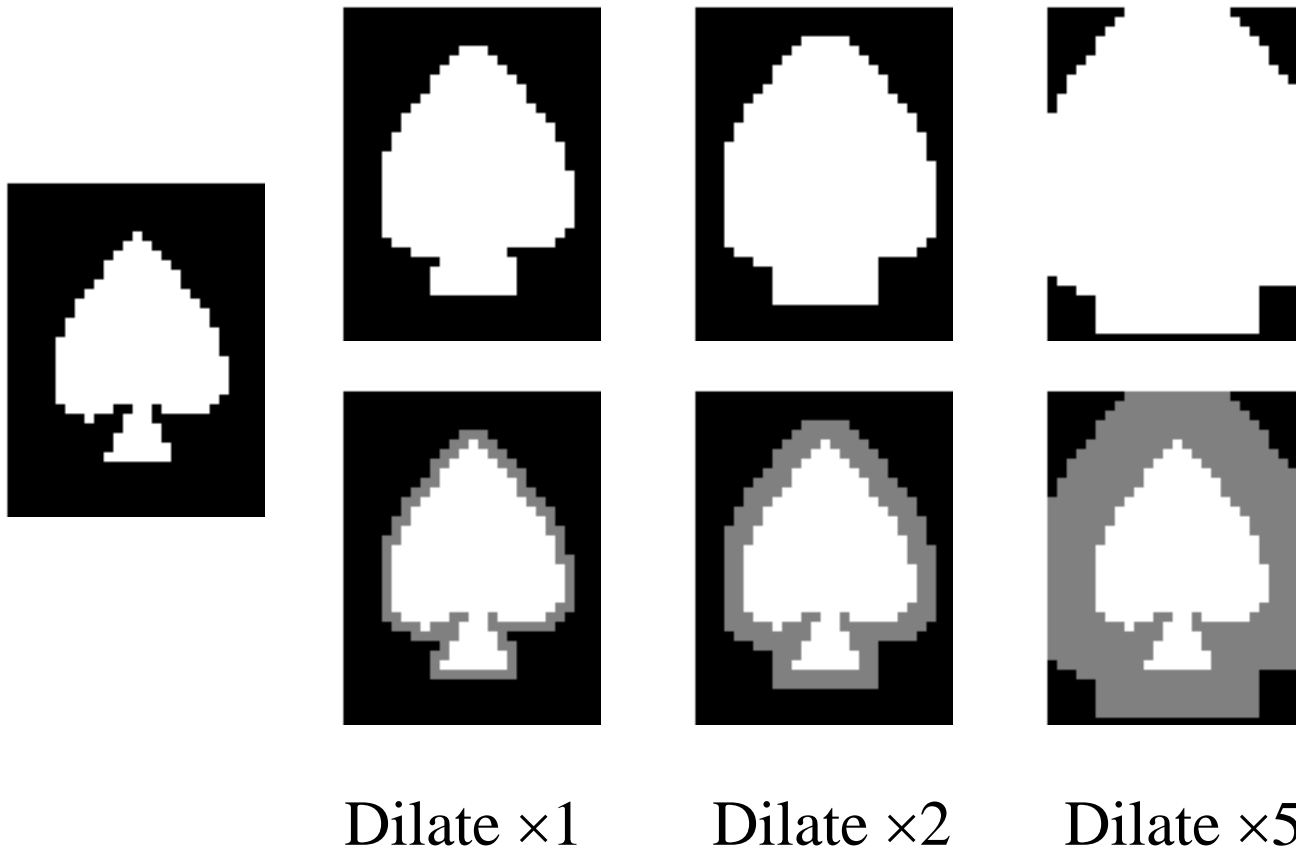
8-neighbour erode

Threshold

Erode ×1   Erode ×2   Erode ×5
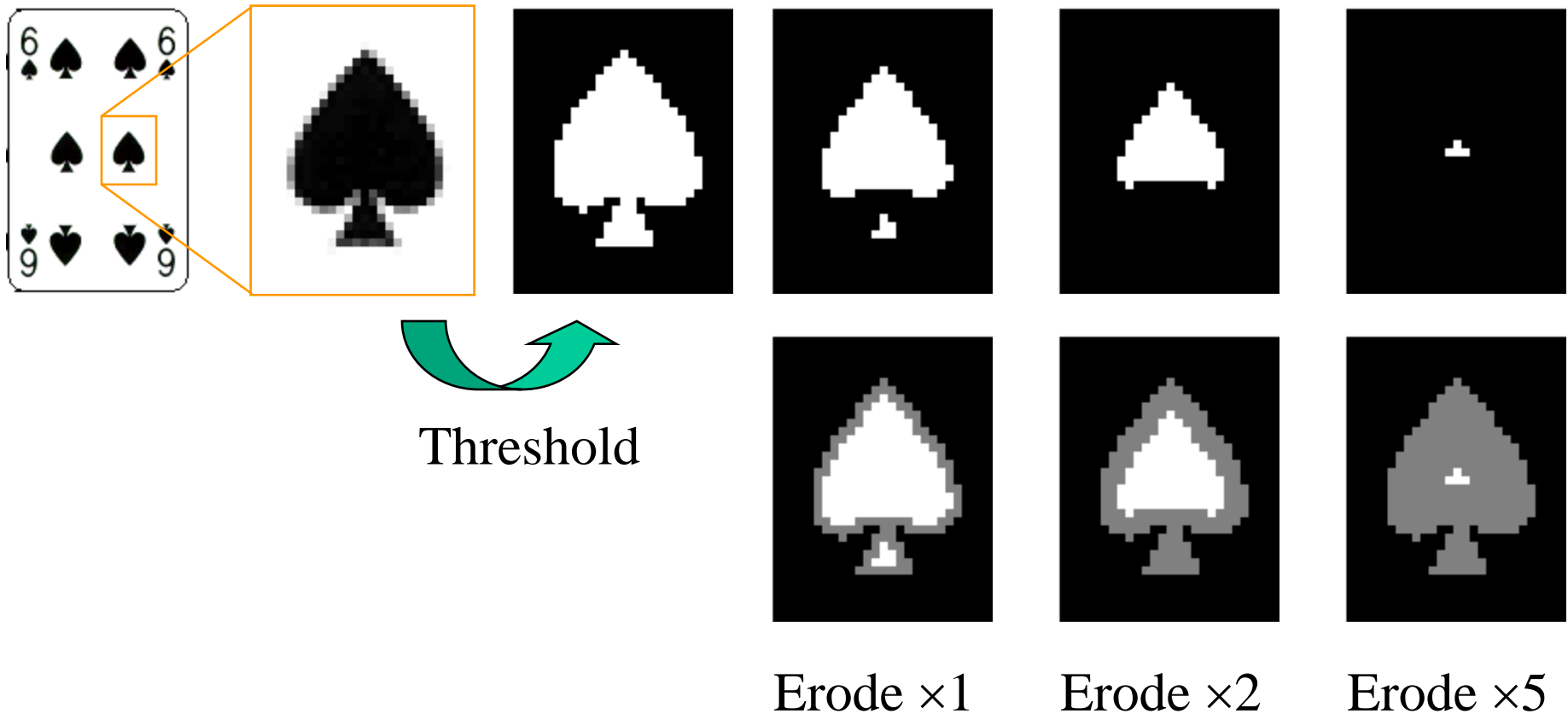
8-neighbour dilate

- Eight-neighbour dilate
  - a.k.a. Minkowsky addition

- Paint any background pixel that has one eight-connected neighbour that is foreground.

8-neighbour dilate

Dilate ×1　　　Dilate ×2　　　Dilate ×5

Why?

- Smooth region boundaries for shape analysis.

- Remove noise and artefacts from an imperfect segmentation.

- Match particular pixel configurations in an image for simple object recognition.

Structuring Elements

- Morphological operations take two arguments:
    - A binary image
    - A *structuring element.*

- Compare the structuring element to the neighbourhood of each pixel.

- This determines the output of the morphological operation.

Structuring elements

- The structuring element is also a binary array.

- A structuring element has an origin.

1	1	1
1	1	1
1	1	1

0	1	0
1	1	1
0	1	0

0	1	0	1
1	0	1	1
0	1	0	0

Binary images as sets

- We can think of the binary image and the structuring element as sets containing the pixels with value 1.

1	1	1	0	0
0	1	1	0	0
0	0	0	0	0
0	0	0	1	0
0	0	0	0	0

$I = \{(1,1), (2,1), (3,1),$
$(2,2), (3,2), (4,4)\}$

Some sets notation

- Union and intersection:

$$I_1 \cup I_2 = \{\underline{x} : \underline{x} \in I_1 \text{ or } \underline{x} \in I_2\}$$

$$I_1 \cap I_2 = \{\underline{x} : \underline{x} \in I_1 \text{ and } \underline{x} \in I_2\}$$

- Complement

$$I^C = \{\underline{x} : \underline{x} \notin I\}$$

- Difference

$$I_1 \setminus I_2 = \{\underline{x} : \underline{x} \in I_1 \text{ and } \underline{x} \notin I_2\}$$

- We use ϕ for the empty set .

More sets notation

- The *symmetrical set* of *S* with respect to point $\underline{o}$ is

$$\breve{S} = \{\underline{o} - \underline{x} : \underline{x} \in S\}$$

| 1 | 1 | $\longrightarrow$ | 1 | 1 |

1	1	1
0	1	1
0	0	0

$\longrightarrow$

0	0	0
1	1	0
1	1	1
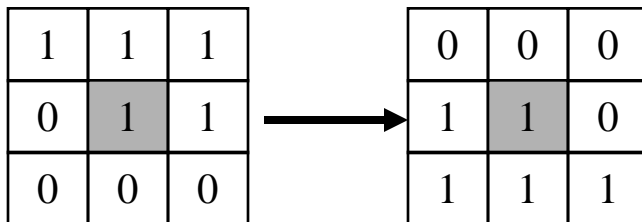
0	1	1	1	0
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
0	1	1	1	0

0	1	1	1	0
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
0	1	1	1	0

Fitting, Hitting and Missing

- S fits I at $\underline{x}$ if

$$\{\underline{y} : \underline{y} = \underline{x} + \underline{s}, \underline{s} \in S\} \subset I$$

- S hits I at $\underline{x}$ if

$$\{\underline{y} : \underline{y} = \underline{x} - \underline{s}, \underline{s} \in S\} \cap I \neq \phi$$

- S misses I at $\underline{x}$ if

$$\{\underline{y} : \underline{y} = \underline{x} - \underline{s}, \underline{s} \in S\} \cap I = \phi$$

Fitting, Hitting and Missing

Image

0	1	0	1	1	1	1	1
1	0	1	1	1	1	1	1
0	1	1	1	0	0	0	0
0	1	1	1	0	0	0	0
0	1	1	1	1	0	0	0
0	1	0	1	0	1	0	0
0	1	0	0	0	0	1	0
0	1	0	0	0	0	0	0

Structuring element

| 0	1	0
1	1	1
0	1	0

Erosion

- The image $E = I \ominus S$ is the *erosion* of image I by structuring element S.

$$E(\underline{x}) = \begin{cases} 1 \text{ if } S \text{ fits } I \text{ at } \underline{x} \\ 0 \text{ otherwise} \end{cases}$$

$$E = \{\underline{x} : \underline{x} + \underline{s} \in I \text{ for every } s \in S\}$$

Implementation (naïve)

```
% I is the input image, S is a structuring element
% with origin (ox, oy).  E is the output image.
function E = erode(I, S, ox, oy)
[X,Y] = size(I);
[SX, SY] = size(S);
E = ones(X, Y);
for x=1:X; for y=1:Y
    for i=1:SX; for j=1:SY
        if(S(i,j))
            E(x,y) = E(x,y) & ImageEntry(I, x+i-ox, y+j-oy);
        end
    end; end
end; end
```

Example
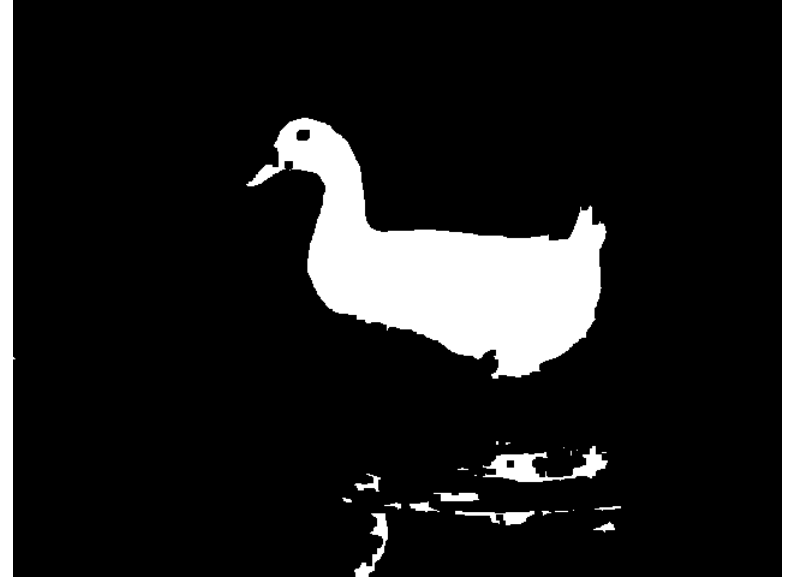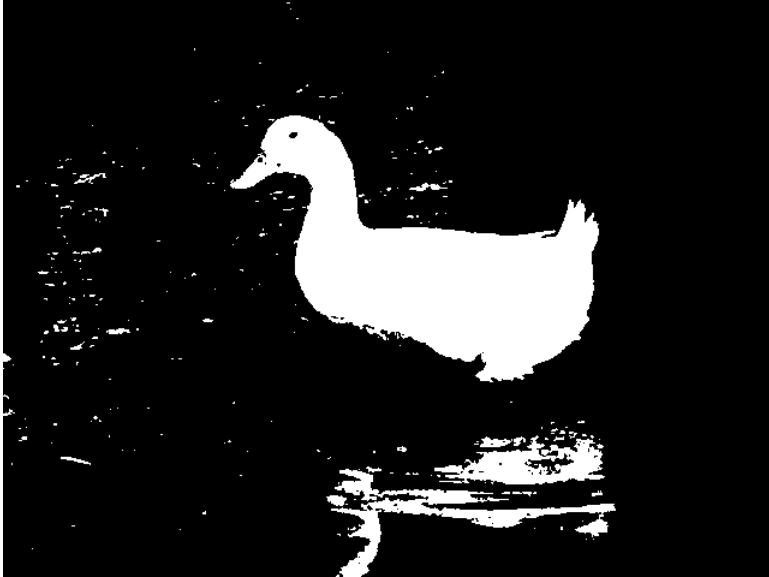
Structuring element
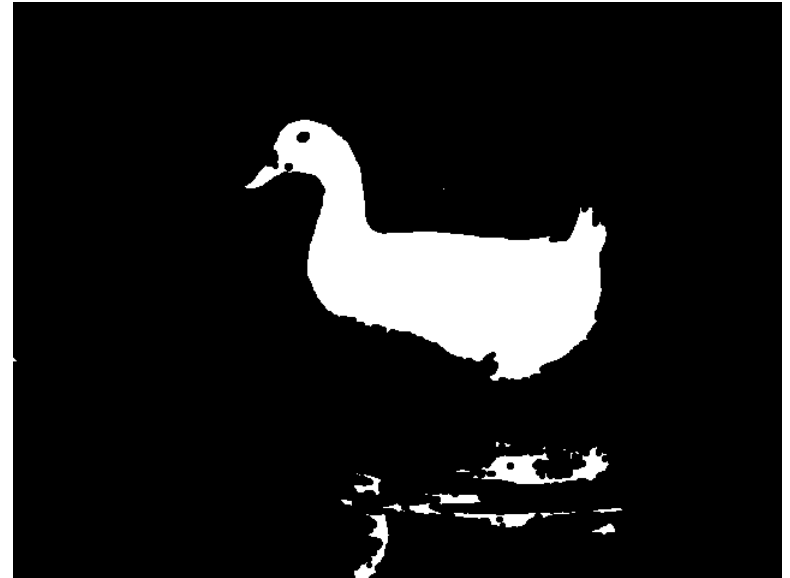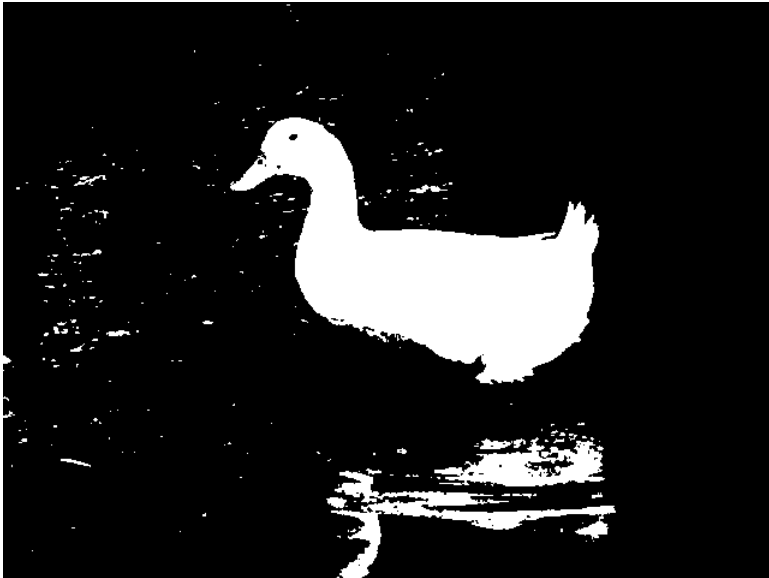
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

Example

Structuring element

0	1	1	1	0
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
0	1	1	1	0

Example

Structuring element

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

GV12/3072
Image Processing.
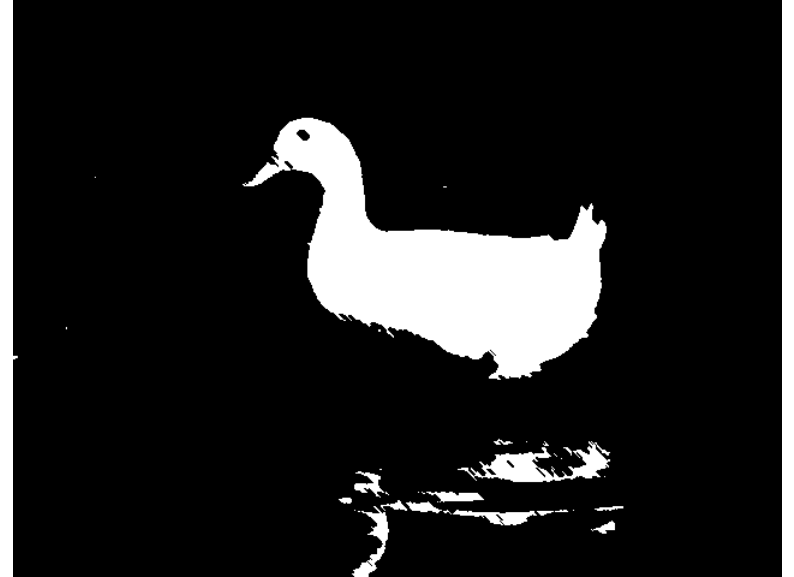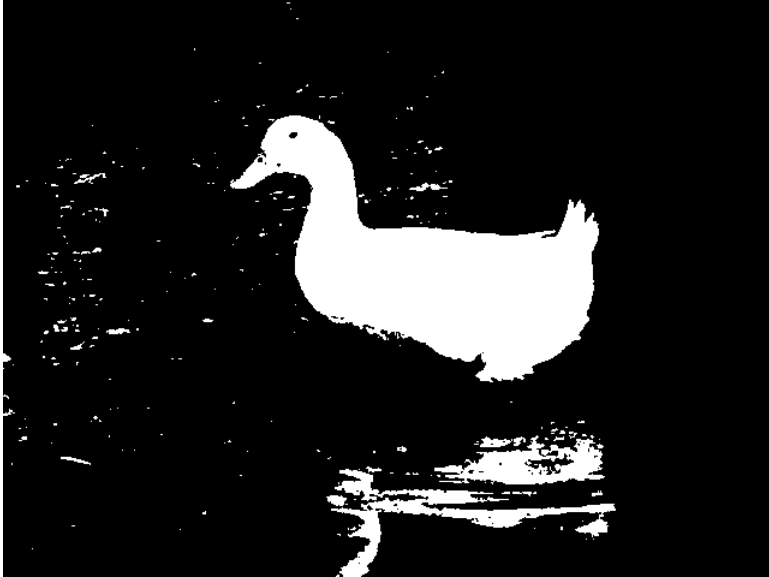
Dilation

- The image $D = I \oplus S$ is the *dilation* of image I by structuring element S.

$$D(\underline{x}) = \begin{cases} 1 \text{ if } S \text{ hits } I \text{ at } \underline{x} \\ 0 \text{ otherwise} \end{cases}$$

$$D = \left\{ \underline{x} : \underline{x} - \underline{s}, \underline{y} \in I \text{ and } \underline{s} \in S \right\}$$

Example
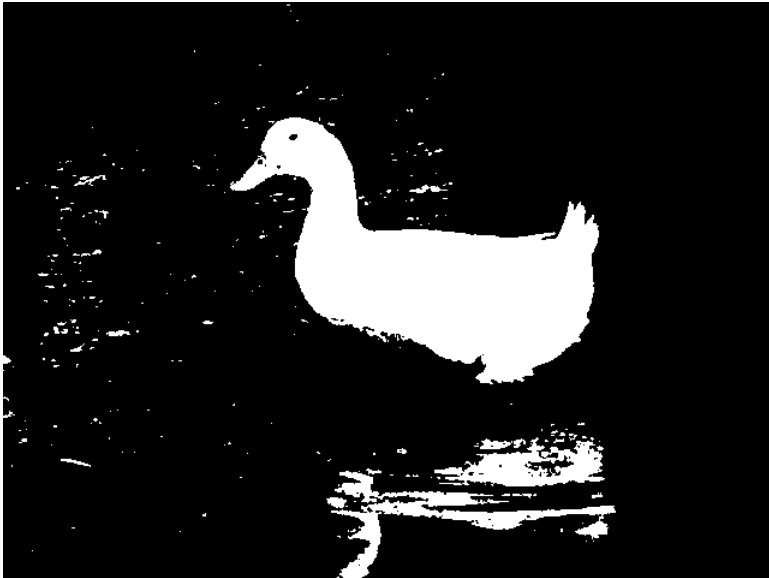
Structuring element
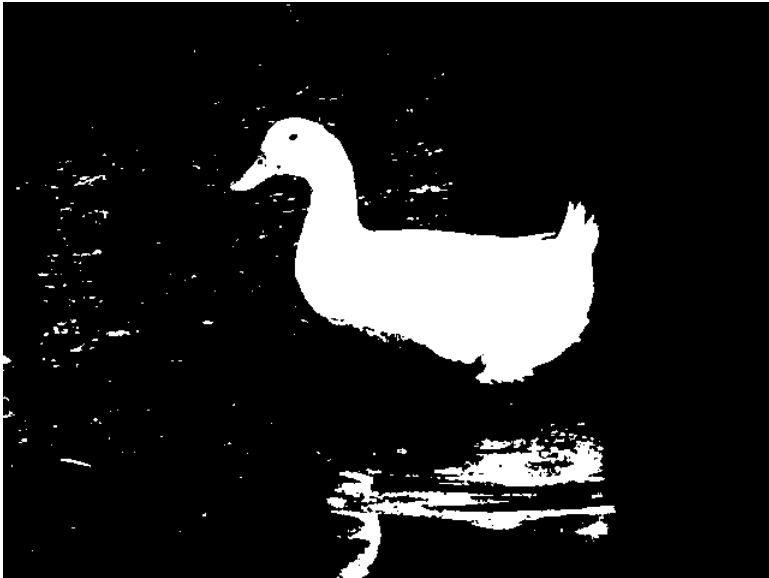
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

Example

Structuring element

0	1	1	1	0
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
0	1	1	1	0

Example
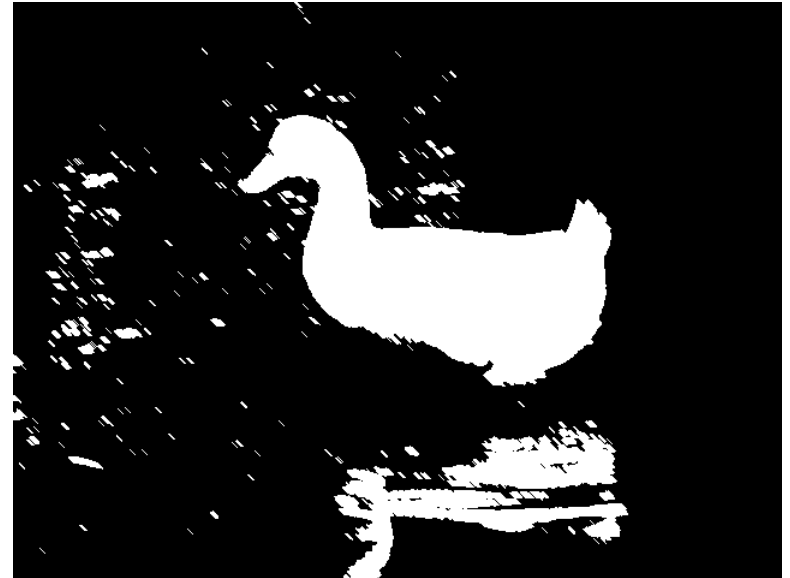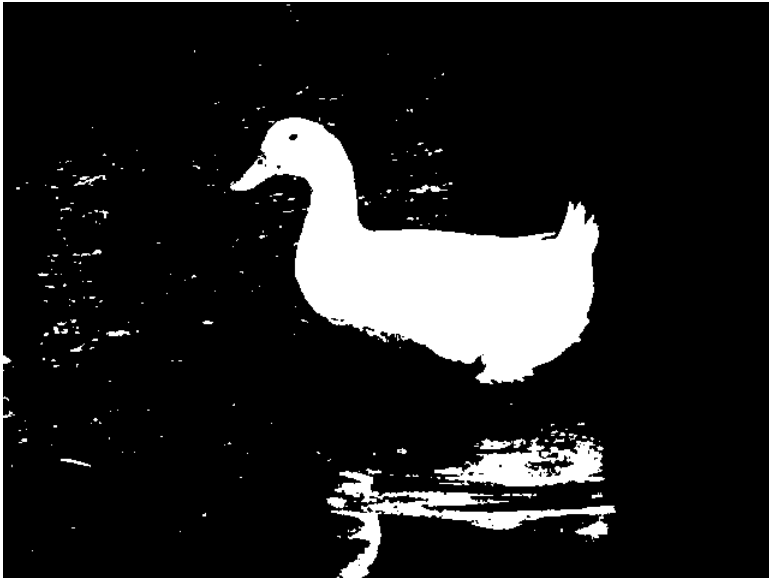
Structuring element

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

Erosion and dilation

- Erosion and dilation are dual operations:

$$(I \ominus S)^C = I^C \oplus \breve{S}$$

- Commutativity and associativity

$$I \oplus S = S \oplus I \qquad\qquad (I \oplus S) \oplus T = I \oplus (S \oplus T)$$

$$I \ominus S \neq S \ominus I \qquad\qquad (I \ominus S) \ominus T = I \ominus (S \oplus T)$$

Opening and Closing

- The *opening* of *I* by *S* is

$$I \circ S = (I \ominus S) \oplus S$$

- The *closing* of *I* by *S* is

$$I \bullet S = (I \oplus S) \ominus S$$

Opening and Closing

- Opening and closing are dual transformations:

$$(I \bullet S)^C = I^C \circ \breve{S}$$

- Opening and closing are *idempotent* operations:

$$I \circ S = (I \circ S) \circ S$$
$$I \bullet S = (I \bullet S) \bullet S$$

Example
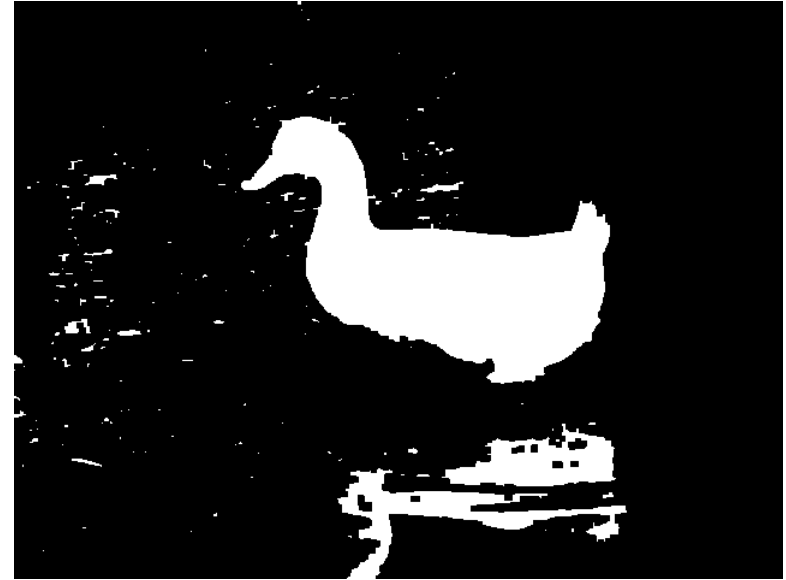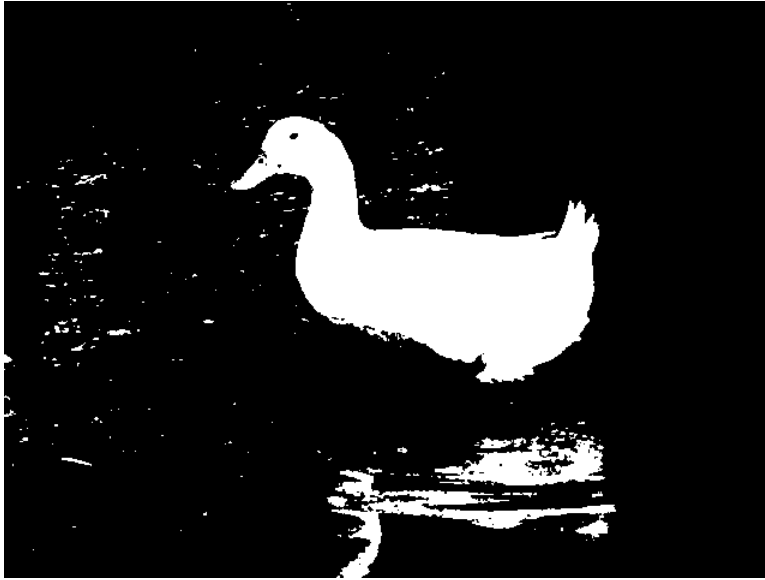
close
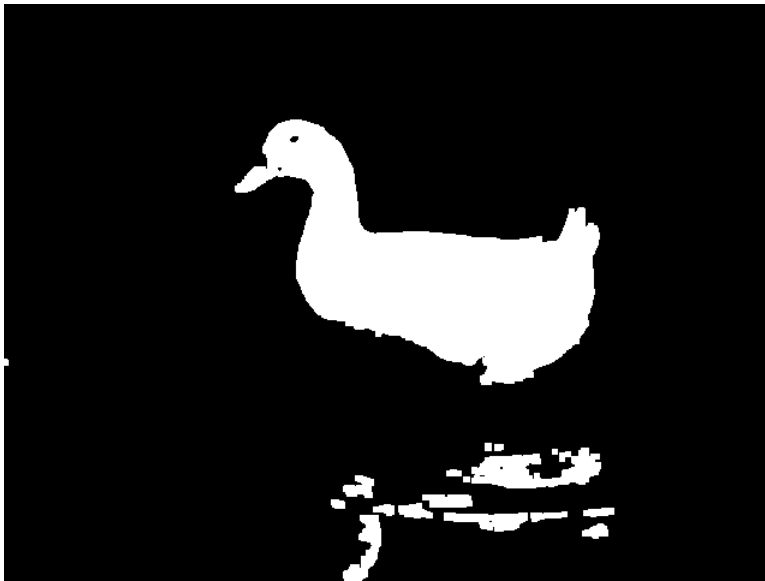
open

Structuring element

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

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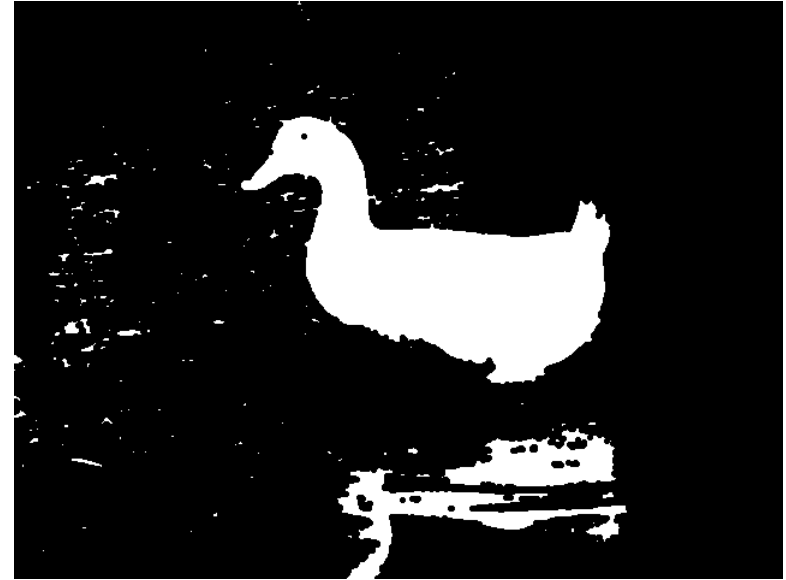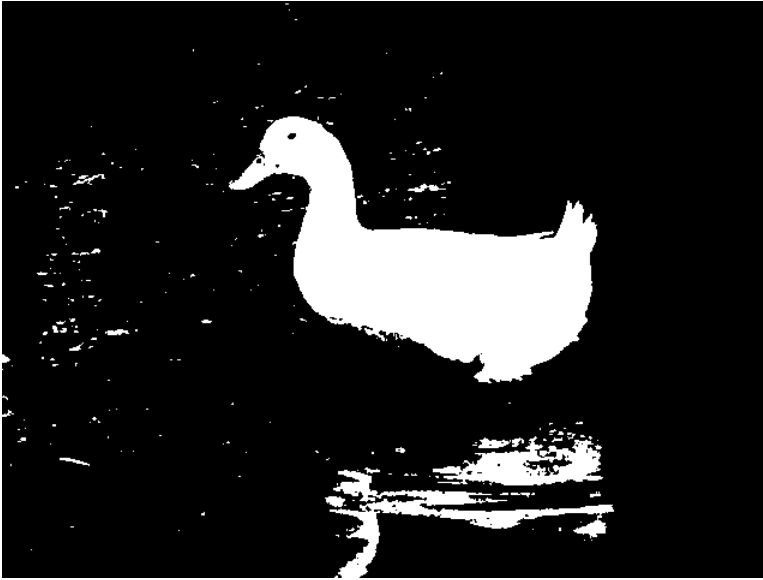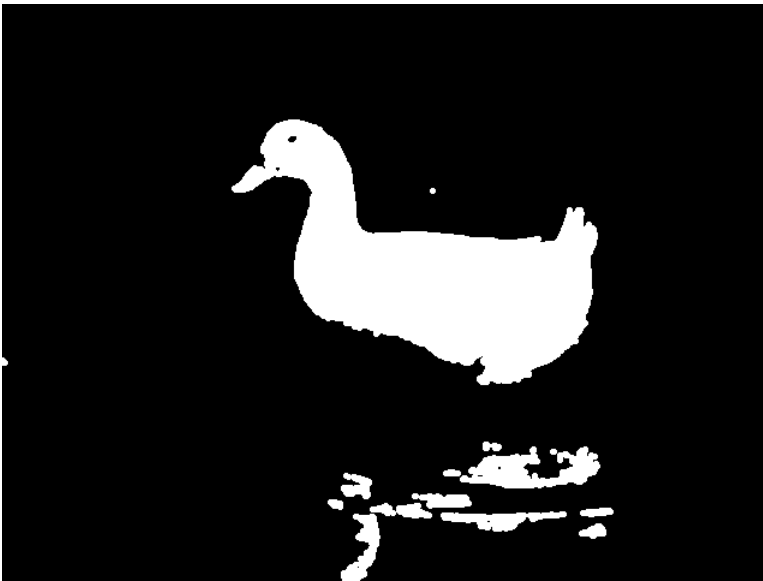
Example

close

open

Structuring element

0	1	1	1	0
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
0	1	1	1	0

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Example
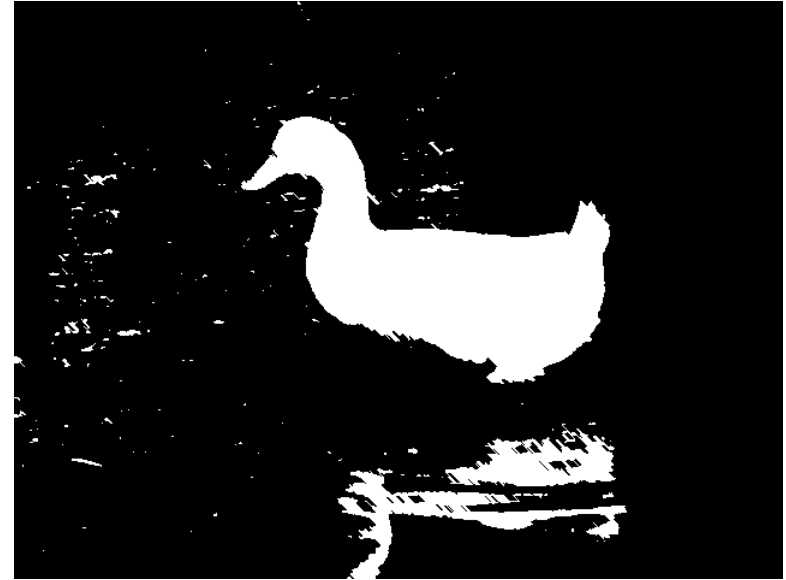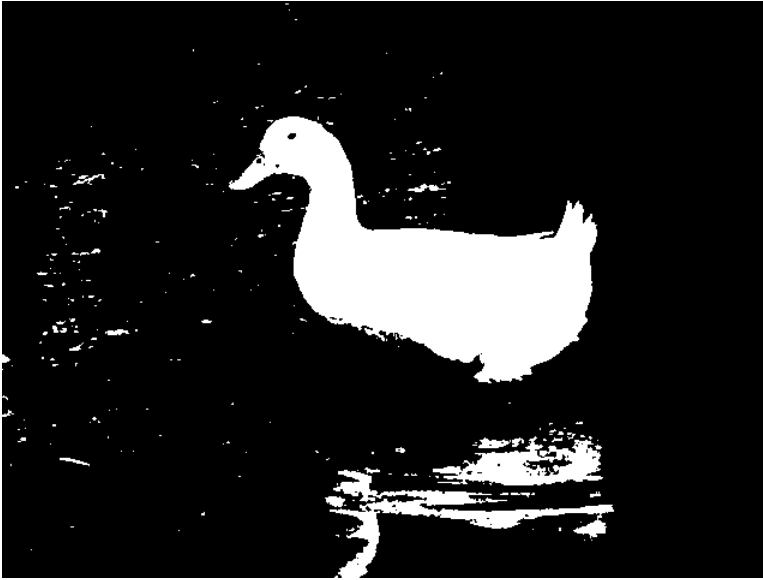
close
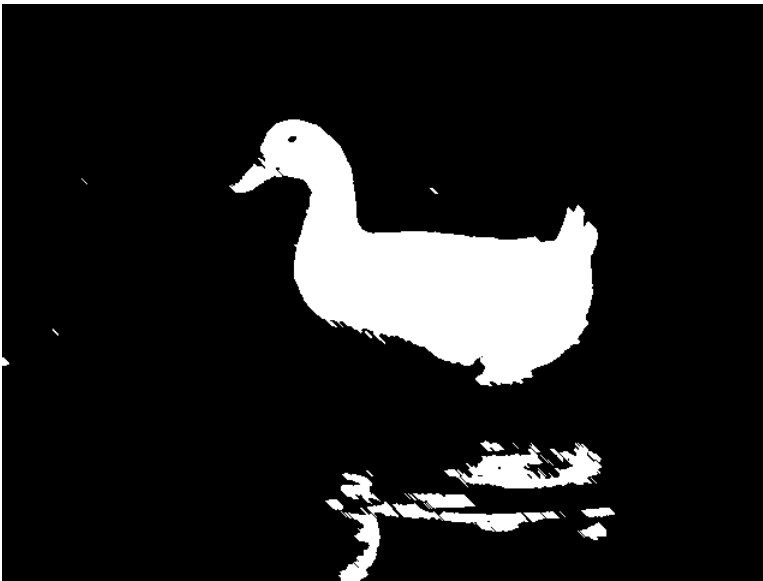
open

Structuring element

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

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Morphological filtering

- To remove holes in the foreground and islands in the background, do both opening and closing.

- The size and shape of the structuring element determine which features survive.

- In the absence of knowledge about the shape of features to remove, use a circular structuring element.

Example
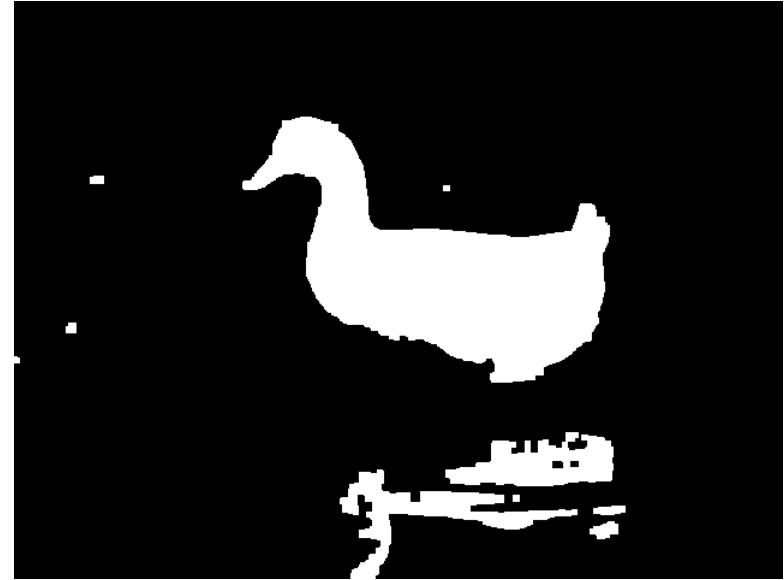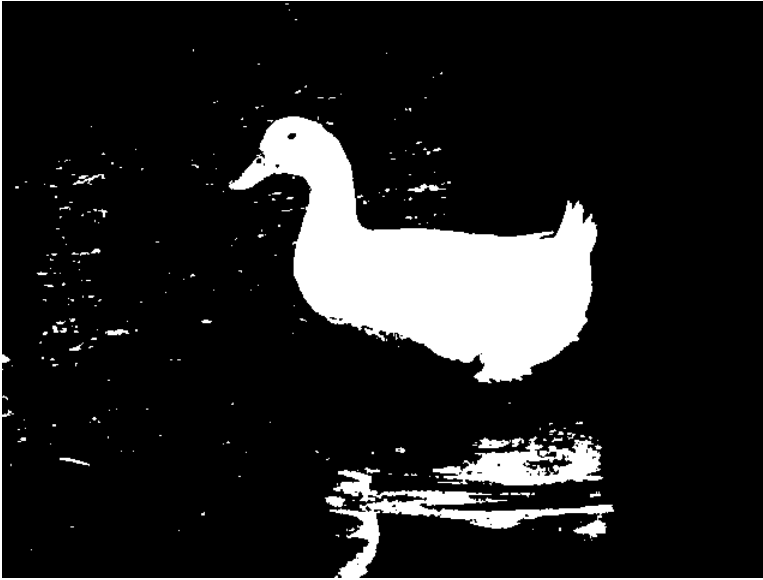
Close then open
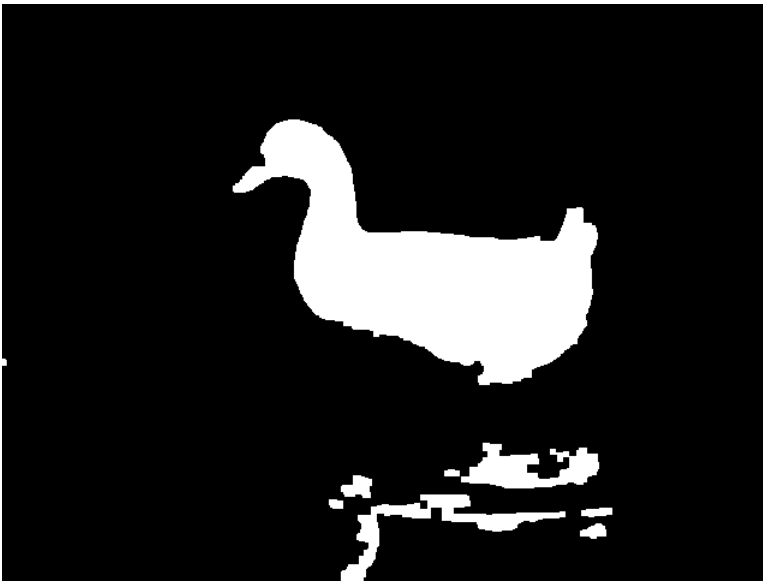
Open then close

Structuring element

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

33
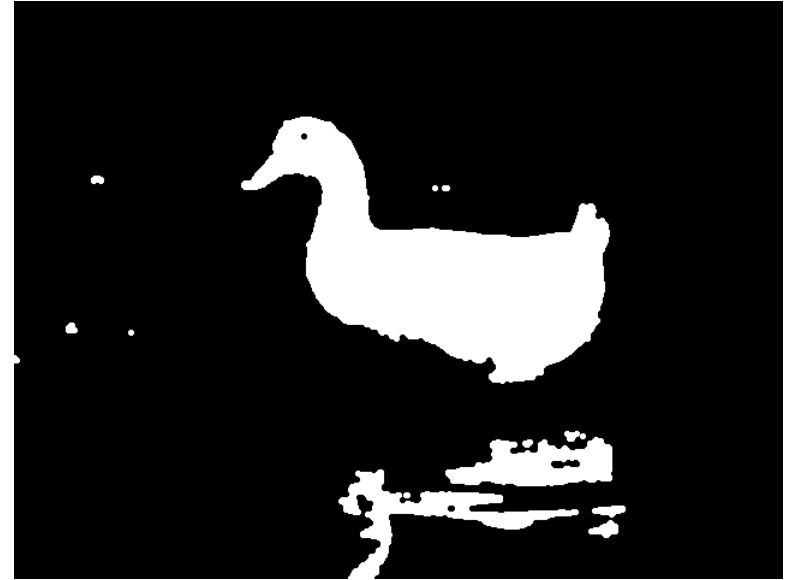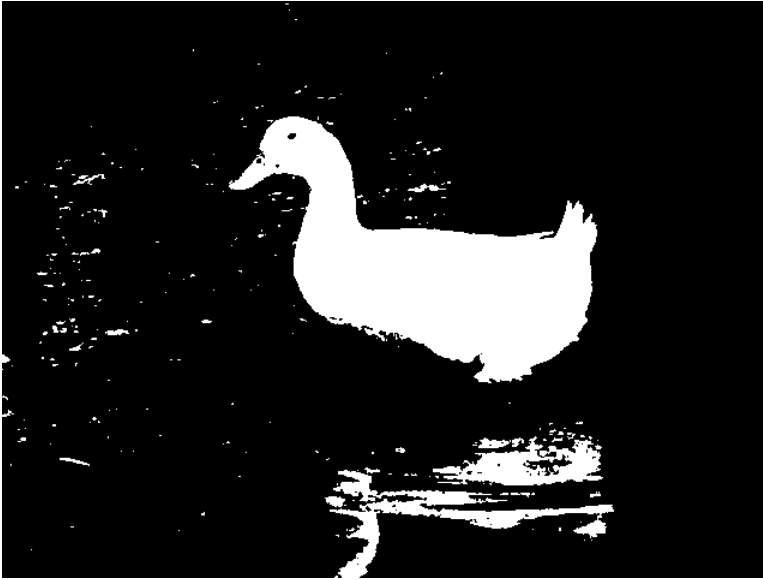
Example
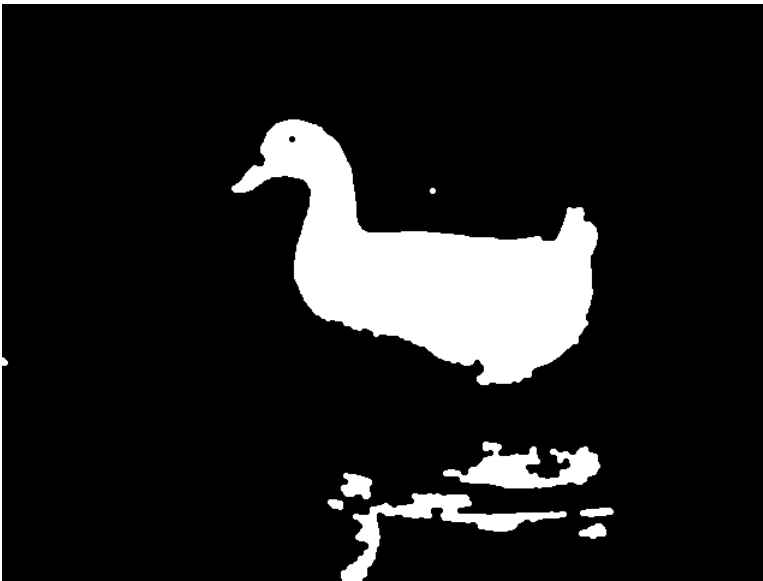
Close then open

Open then close

Structuring element

0	1	1	1	0
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
0	1	1	1	0

Example

Close then open

Open then close

Structuring element

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

35
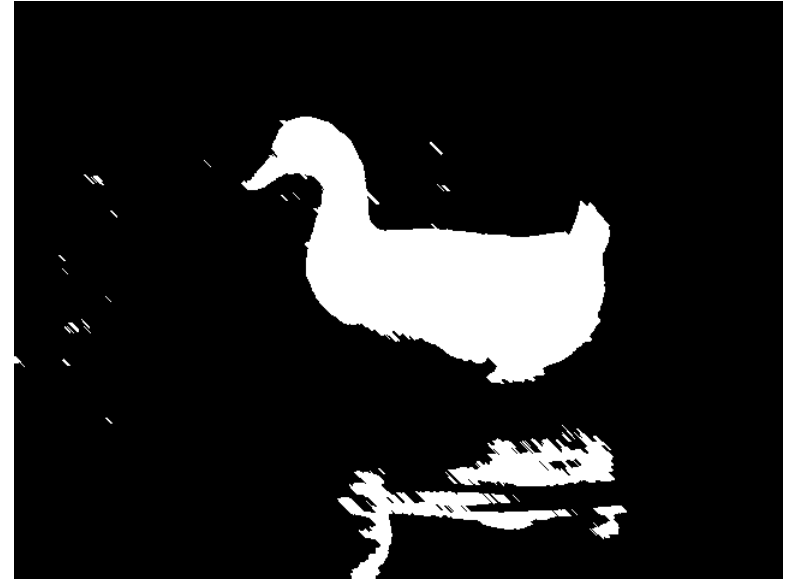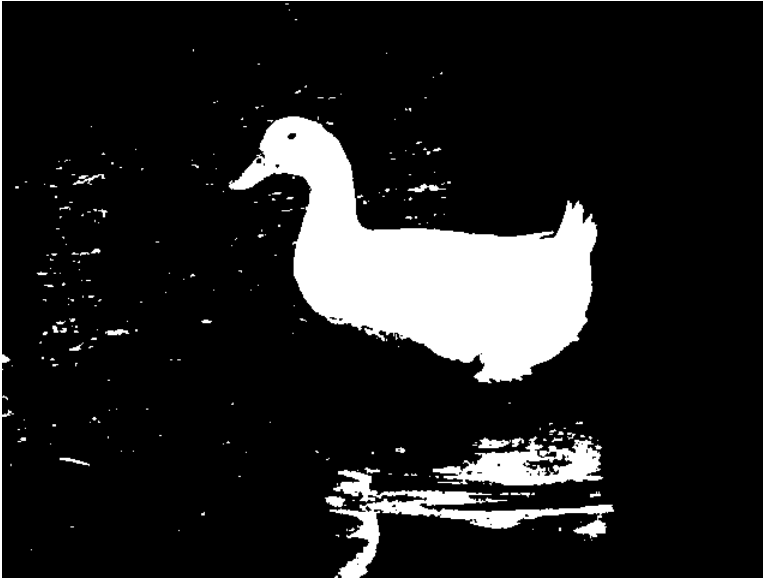
Count the Red Blood Cells

Granulometry

- Provides a size distribution of distinct regions or "granules" in the image.

- We open the image with increasing structuring element size and count the number of regions after each operation.

- Creates a "morphological sieve".

Granulometry

```
function gSpec = granulo(I, T, maxRad)

% Segment the image I

B = (I>T);

% Open the image at each structuring element size up to a

% maximum and count the remaining regions.

for x=1:maxRad

    O = imopen(B,strel('disk',x));

    numRegions(x) = max(max(connectedComponents(O)));

end

gSpec = diff(numRegions);
```

Count the Red Blood Cells
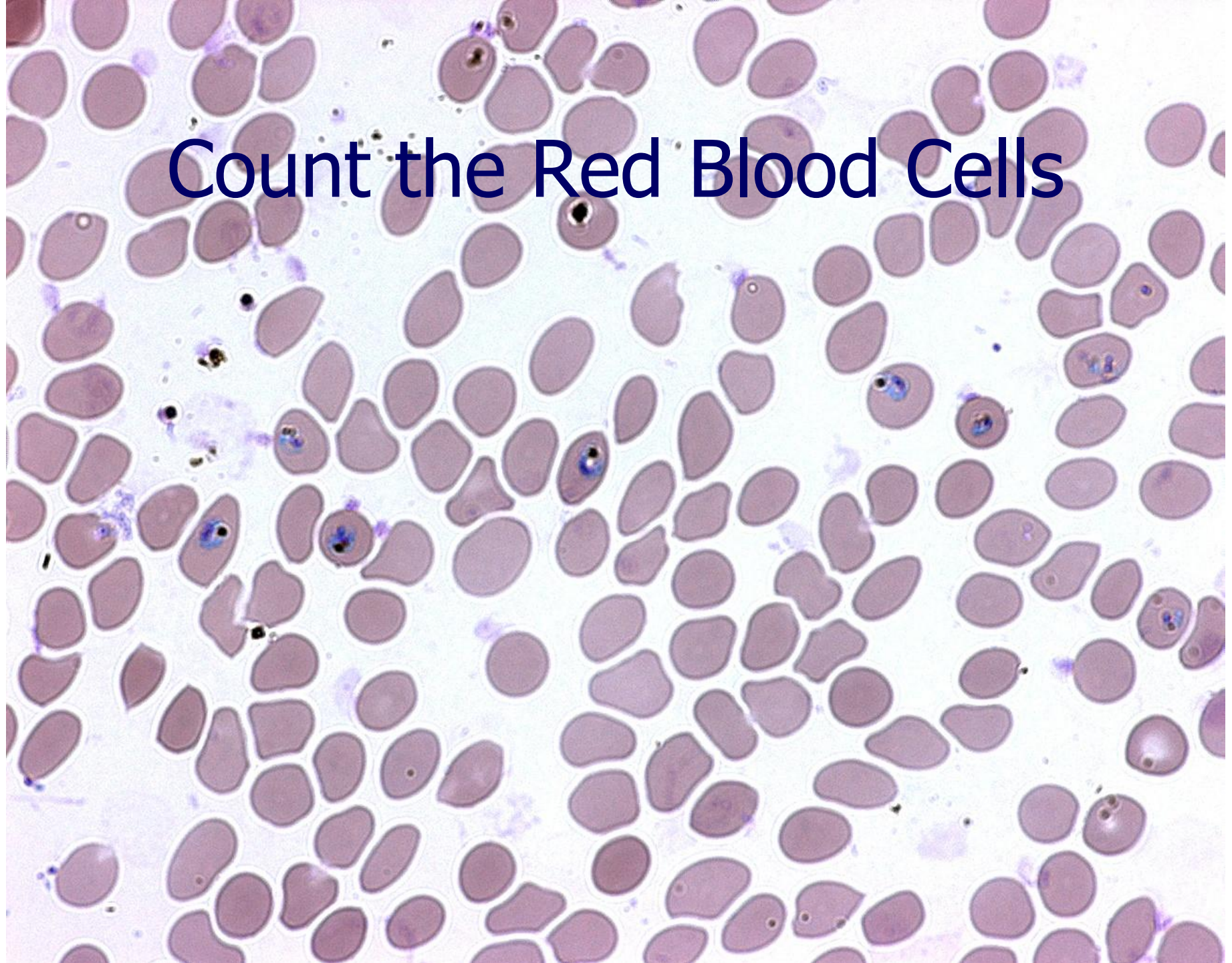
Threshold and Label

Disc(11)

Disc(19)

Disc(59)

Number of Regions

Granulometric *Pattern Spectrum*
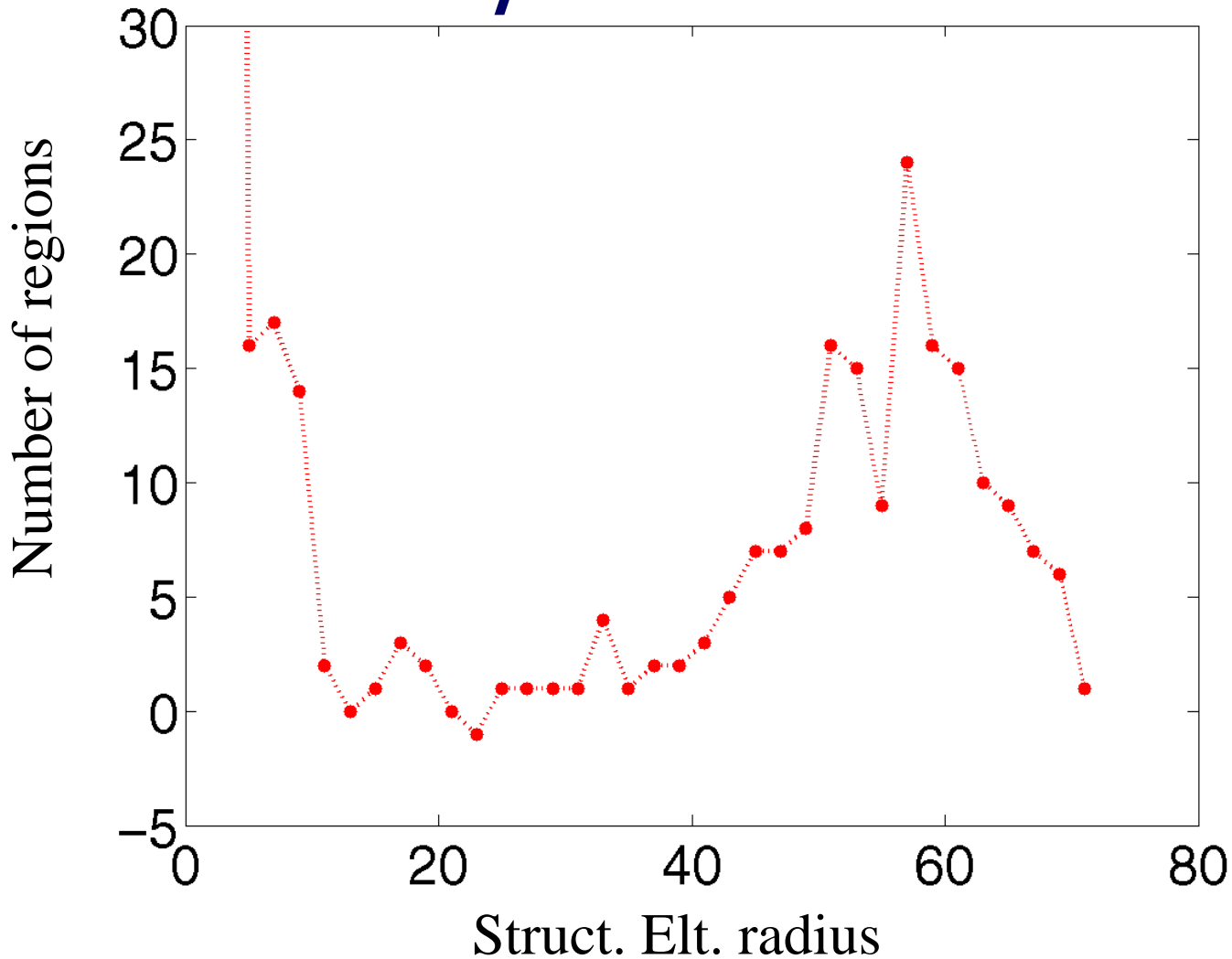
Hit-and-miss transform

- Searches for an exact match of the structuring element.

- $H = I \otimes S$ is the hit-and-miss transform of image I by structuring element S.

- Simple form of template matching.

Hit-and-miss transform

1	0	1	1	1
1	1	0	1	0
1	0	0	1	1
1	1	0	1	1
1	0	1	0	1

$\otimes$

1	0	1

$=$

0	1	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	1	0	0
0	1	0	1	0

1	0	1	1	1
1	1	0	1	0
1	0	0	1	1
1	1	0	1	1
1	0	1	0	1

$\otimes$

1	0
*	1

$=$

0	1	0	0	0
0	0	0	0	1
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0

Upper-Right Corner Detector

x	0	0
1	1	0
x	1	x

0	0	0
1	1	0
0	1	0

J

0	1	1
0	0	1
0	0	0

K

Thinning and Thickening

- Defined in terms of the hit-and-miss transform:

- The *thinning* of *I* by *S* is

$$I \oslash S = I \setminus (I \otimes S)$$
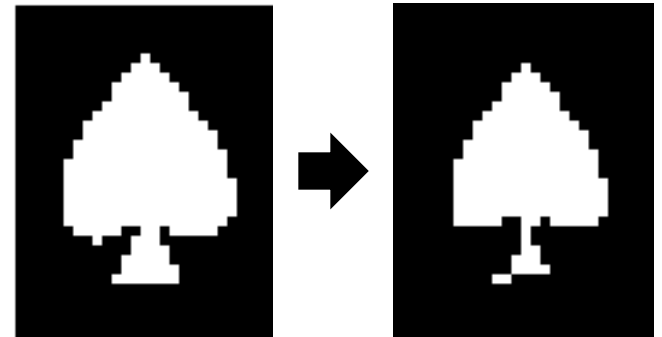
- The *thickening* of *I* by *S* is

$$I \odot S = I \cup (I \otimes S)$$

- Dual operations: $(I \odot S)^C = I^C \oslash S$

Sequential Thinning/Thickening

- These operations are often performed in sequence with a selection of structuring elements S_1, S_2, …, S_n.

- Sequential thinning:

$$I \oslash \{S_i : i = 1,...,n\} = (((I \oslash S_1) \oslash S_2)...\oslash S_n)$$

- Sequential thickening:

$$I \odot \{S_i : i = 1,...,n\} = (((I \odot S_1) \odot S_2)...\odot S_n)$$

Sequential Thinning/Thickening

- Several sequences of structuring elements are useful in practice

- These are usually the set of rotations of a single structuring element.

- Sometimes called the *Golay alphabet.*

Golay element L

0	0	0
*	**1**	*
1	1	1

L_1

*	0	0
1	**1**	0
*	1	*

L_2

1	*	0
1	**1**	0
1	*	0

L_3

*	1	*
1	**1**	0
*	0	0

L_4

1	1	1
*	**1**	*
0	0	0

L_5

*	1	*
0	**1**	1
0	0	*

L_6

0	*	1
0	**1**	1
0	*	1

L_7

0	0	*
0	**1**	1
*	1	*

L_8

Sequential Thinning

- See `bwmorph` in matlab.

0 iterations	1 iteration	2 iterations	5 iterations	Inf iterations
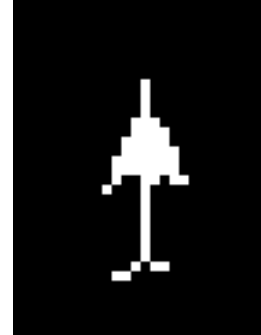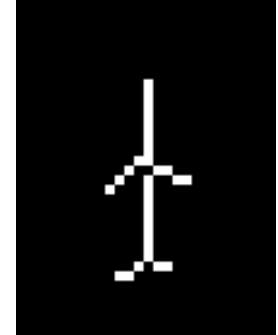
Sequential Thickening

1 iteration

2 iterations

5 iterations

Inf iterations
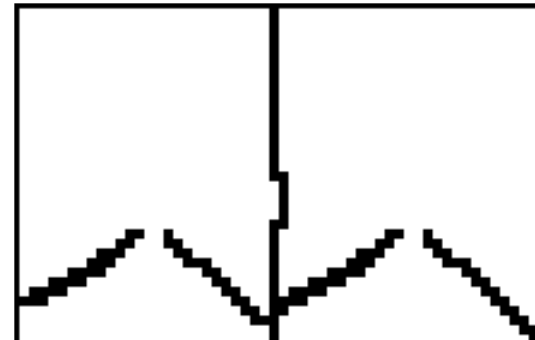
Skeletonization and the Medial Axis Transform

- The skeleton and *medial axis transform* (MAT) are stick-figure representations of a region $X \subset \Re^2$.

- Start a grassfire at the boundary of the region.

- The skeleton is the set of points at which two fire fronts meet.

Skeletons
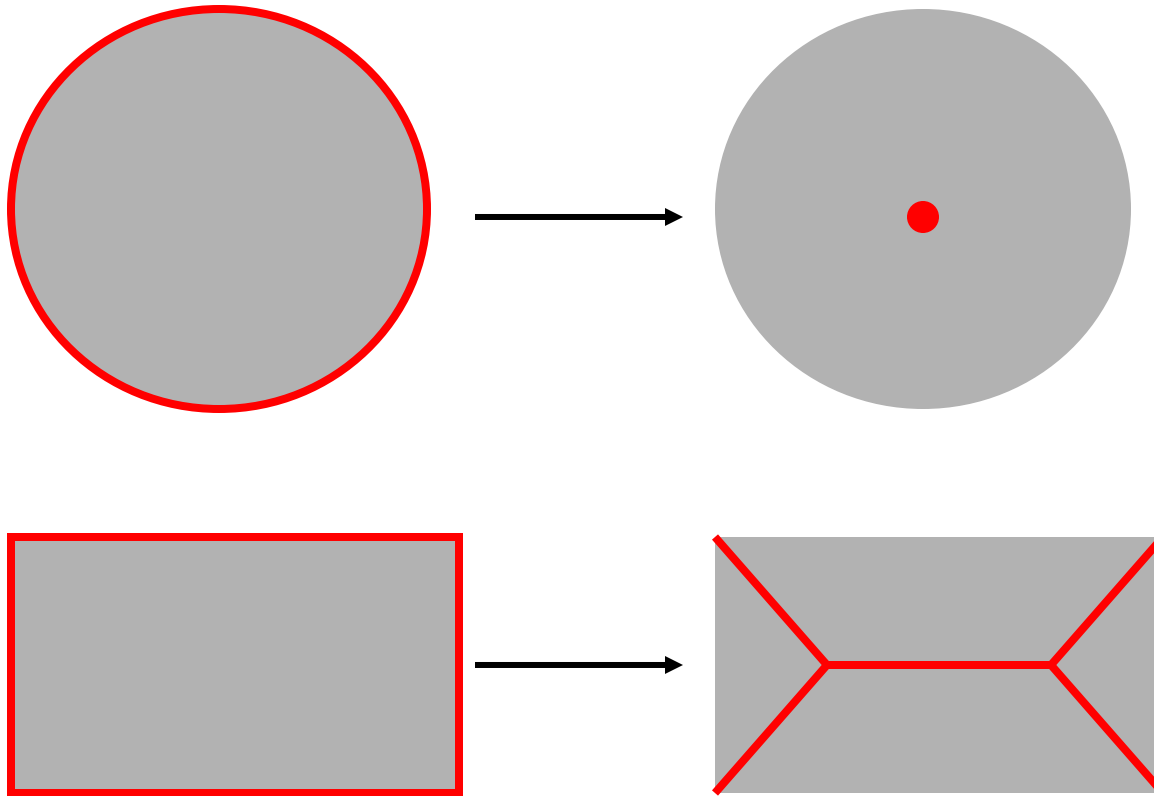
Medial axis transform

- Alternative skeleton definition:
  - The skeleton is the union of centres of maximal discs within *X*.
  - A *maximal* disc is a circular subset of *X* that touches the boundary in at least two places.

- The MAT is the skeleton with the maximal disc radius retained at each point.

Medial axis transform
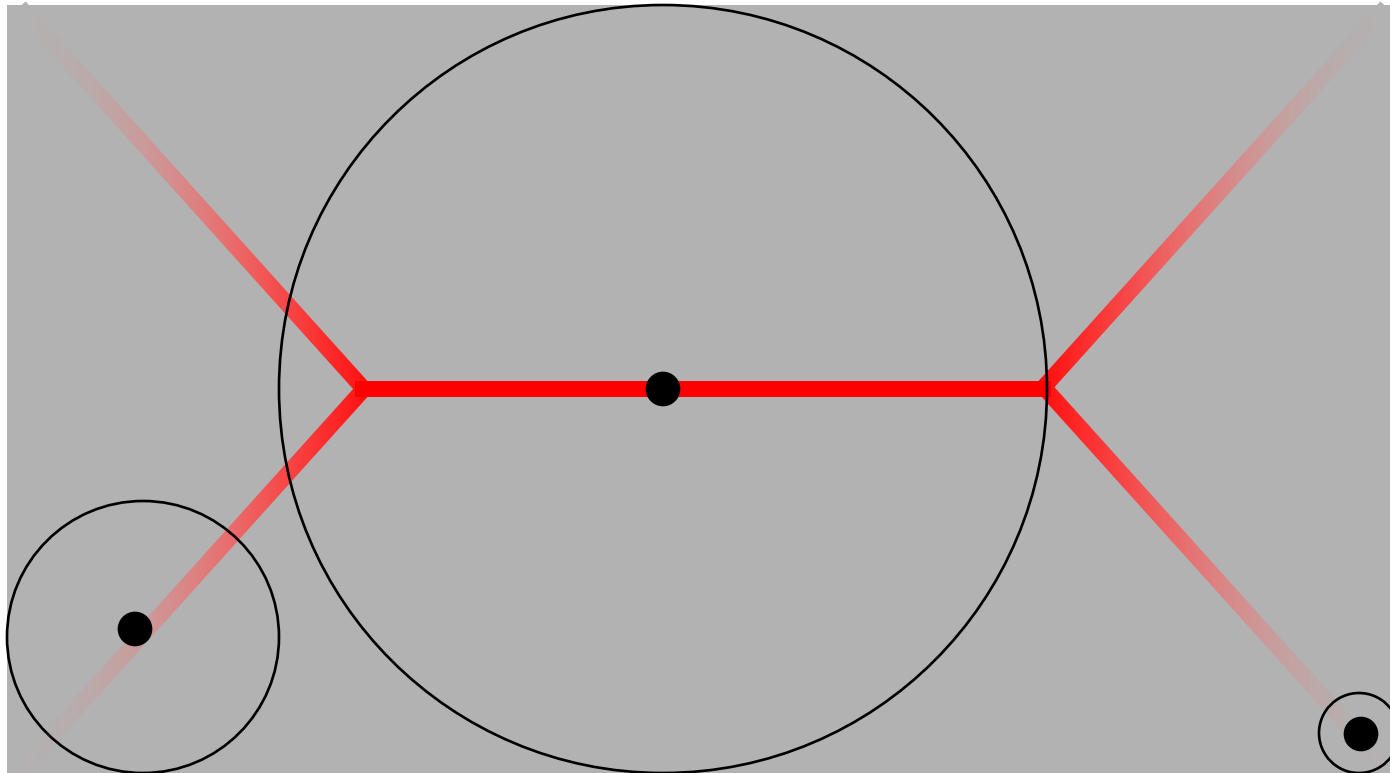
Skeletonization using morphology

- Use structuring element

$$B = \begin{array}{|c|c|c|} \hline 0 & 1 & 0 \\ \hline 1 & 1 & 1 \\ \hline 0 & 1 & 0 \\ \hline \end{array}$$

- The n-th skeleton subset is

$$S_n(X) = (X \ominus_n B) \setminus [(X \ominus_n B) \circ B]$$

$\ominus_n$ denotes n successive erosions.

- The skeleton is the union of all the skeleton subsets:

$$S(X) = \bigcup_{n=1}^{\infty} S_n(X)$$

Reconstruction

- We can reconstruct region *X* from its skeleton subsets:

$$X = \bigcup_{n=0}^{\infty} S_n(X) \oplus_n B$$

- We can reconstruct *X* from the MAT.

- We cannot reconstruct *X* from *S(X)*.
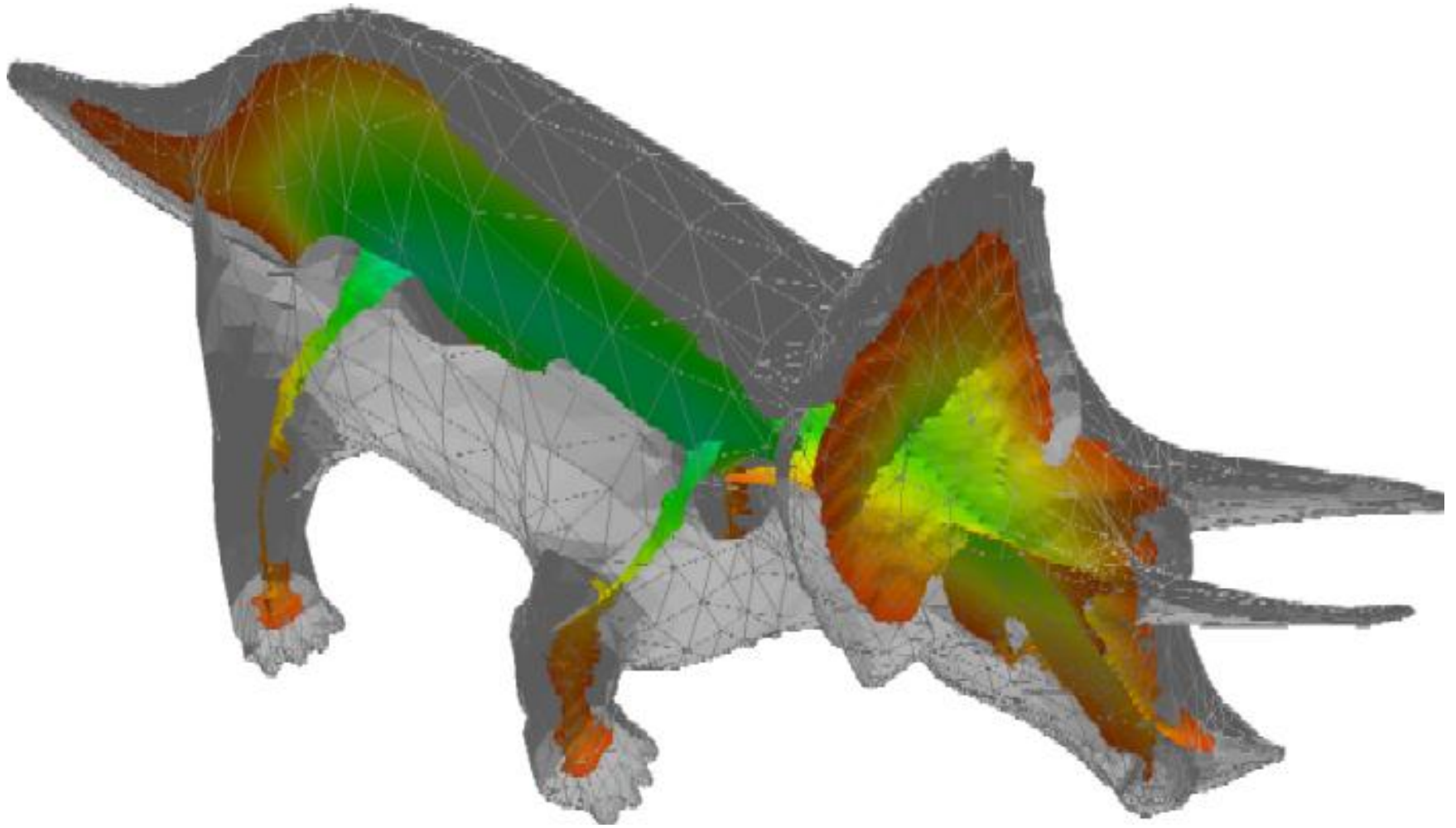
DiFi: Fast 3D Distance Field Computation Using Graphics Hardware
Sud, Otaduy, Manocha, Eurographics 2004

MAT in 3D
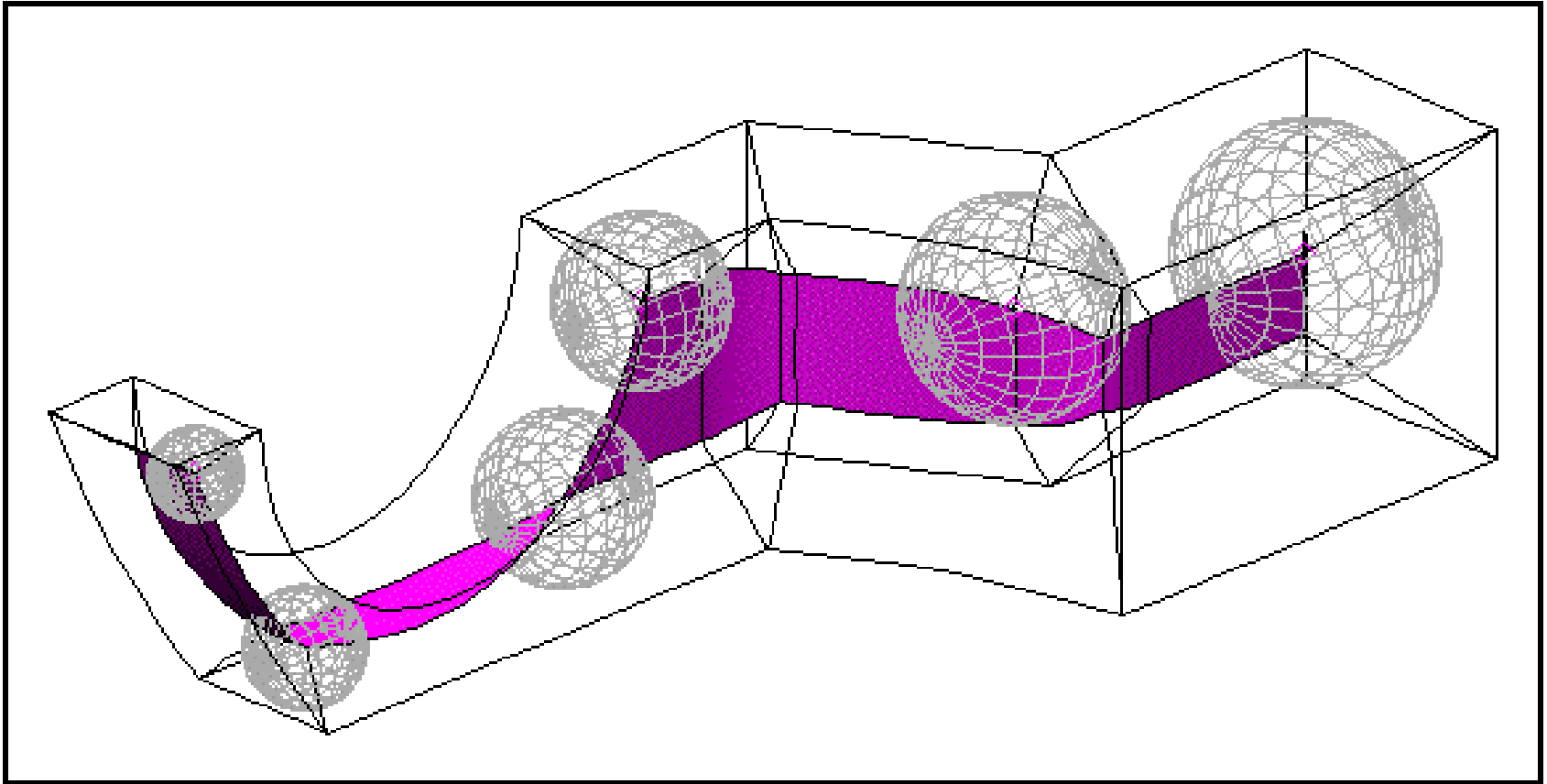
from Transcendata Europe Medial Object
Price, Stops, Butlin Transcendata Europe Ltd

Applications and Problems

- The skeleton/MAT provides a stick figure representing the region shape

- Used in object recognition, in particular, character recognition.

- Problems:
    - Definition of a maximal disc is poorly defined on a digital grid.
    - Sensitive to noise on the boundary.

- Sequential thinning output sometimes preferred to skeleton/MAT.

Example

Skeletons:
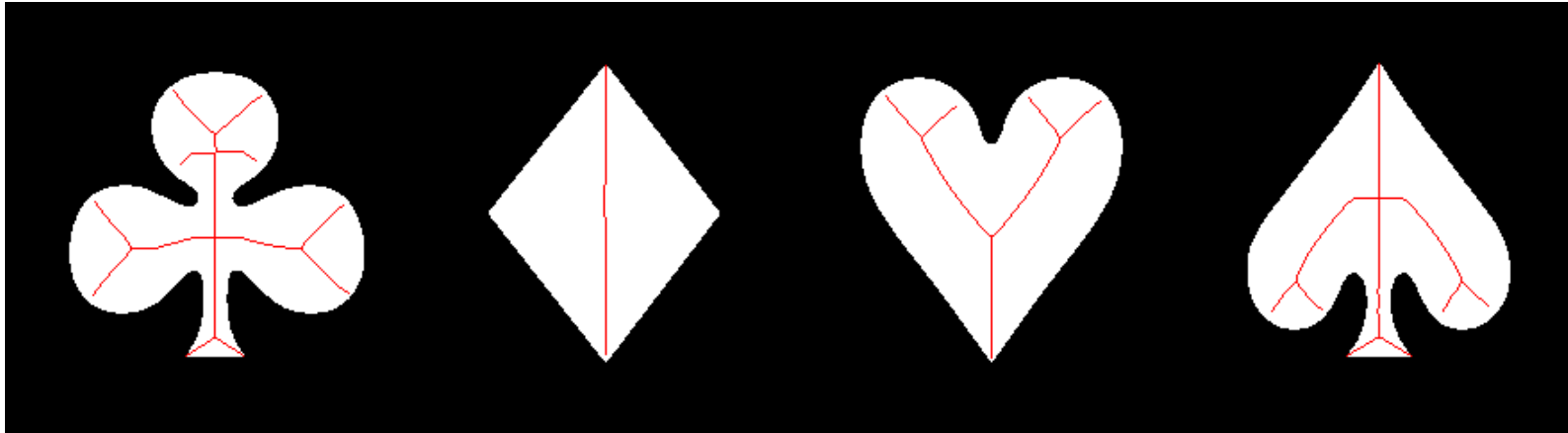
Thinned:
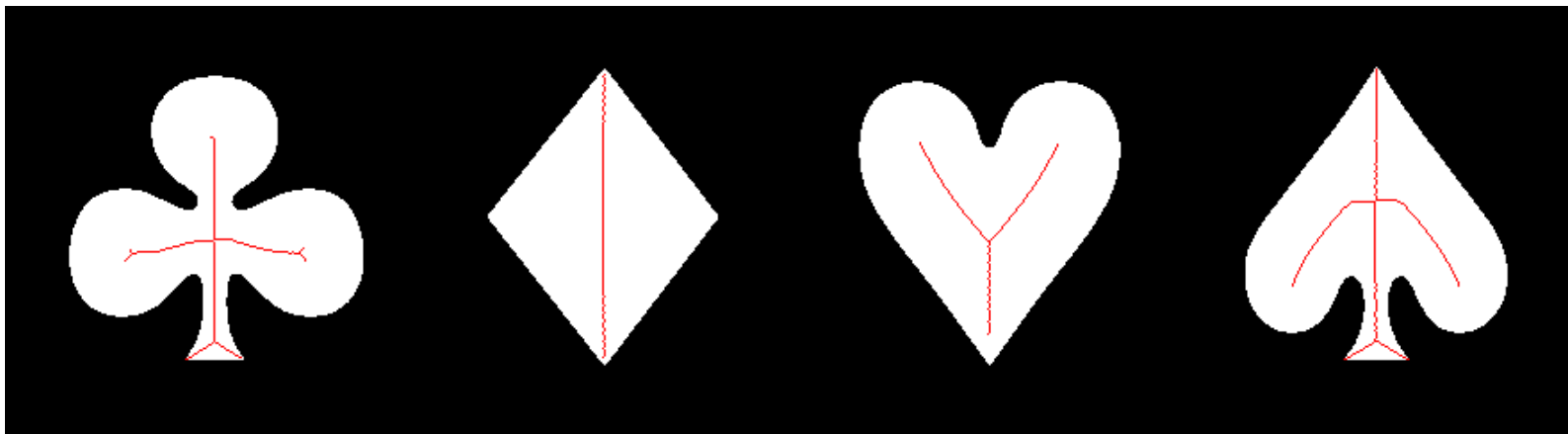
Gray-level Morphology

- Erosion, dilation

- Opening and closing

- The image and the structuring element are gray level arrays.

- Used to remove speckle noise.

- Also for smoothing, edge detection and segmentation.

Umbra

$$U(f) = \{(x, y) : y \le f(x)\}$$
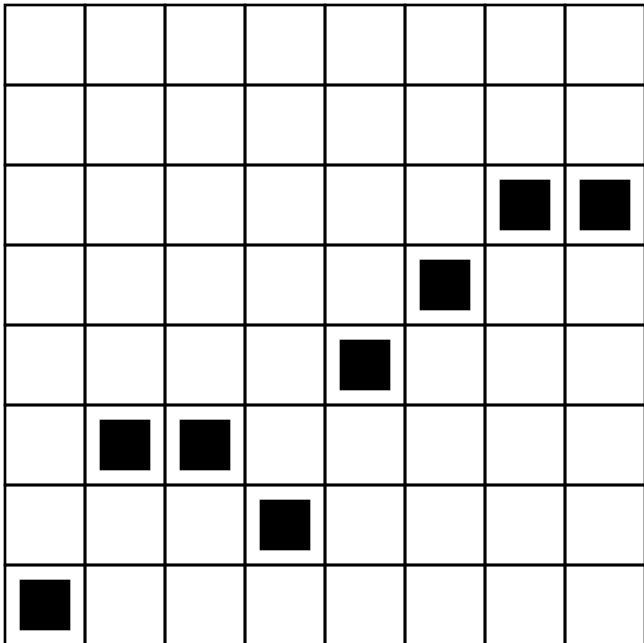
f
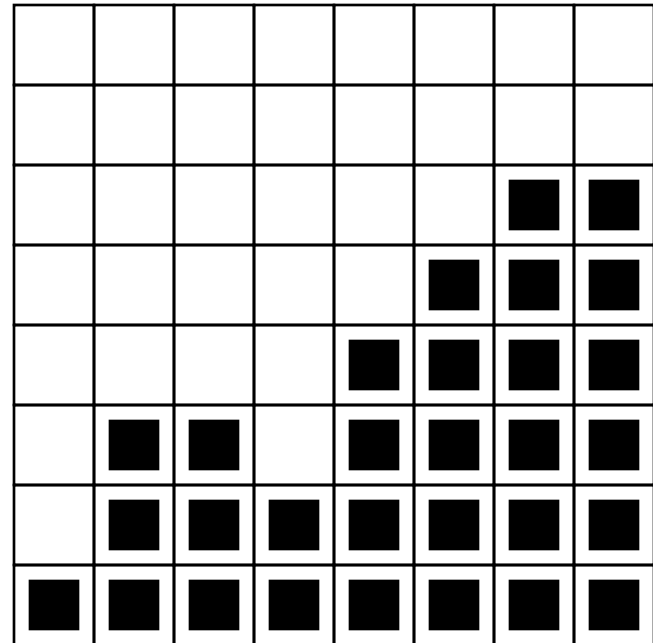
$U(f)$

Top surface

$$T(R) = \{(x, y) : y \geq z \text{ for all } (x, z) \in R\}$$
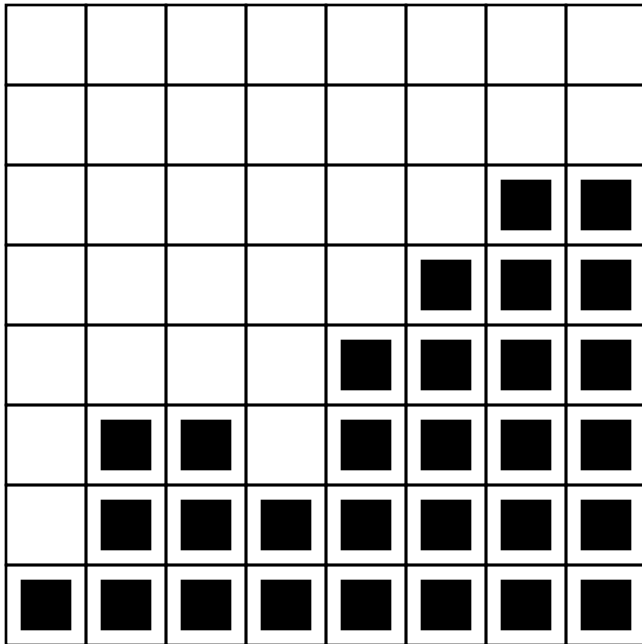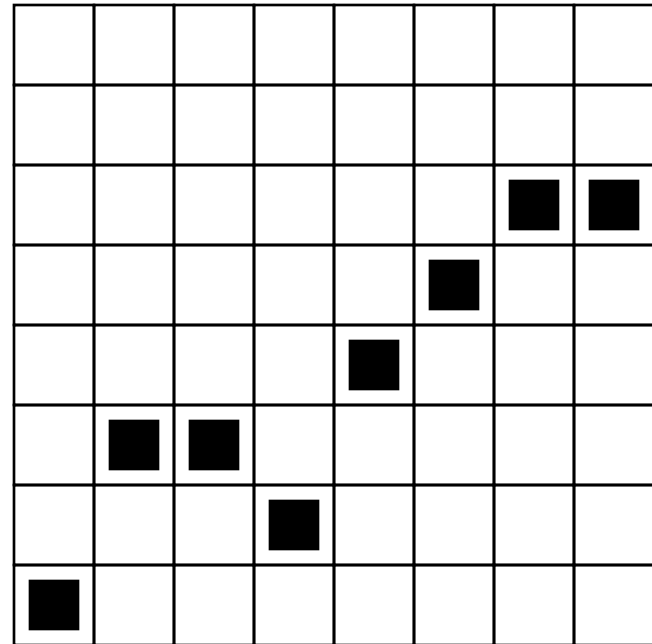
R $T(R)$

Gray level erode and dilate

- Erosion:

$$f \ominus k = T(U(f) \ominus U(k))$$

- Dilation:

$$f \oplus k = T(U(f) \oplus U(k))$$

- Open and close defined as before.

f

$U(f)$

k
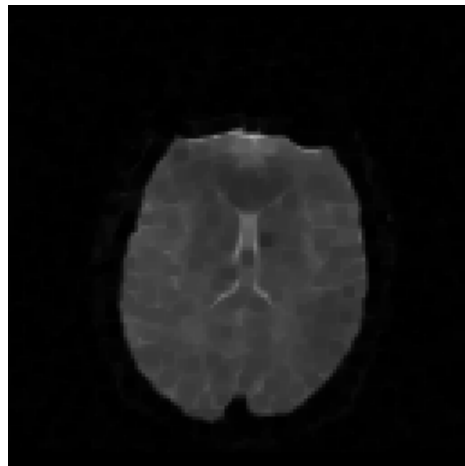
$U(k)$
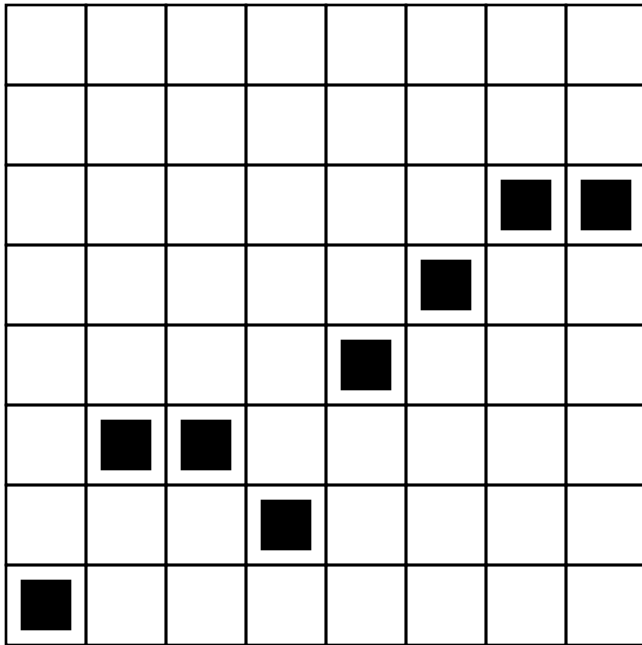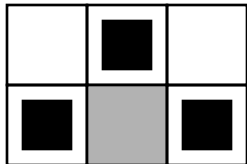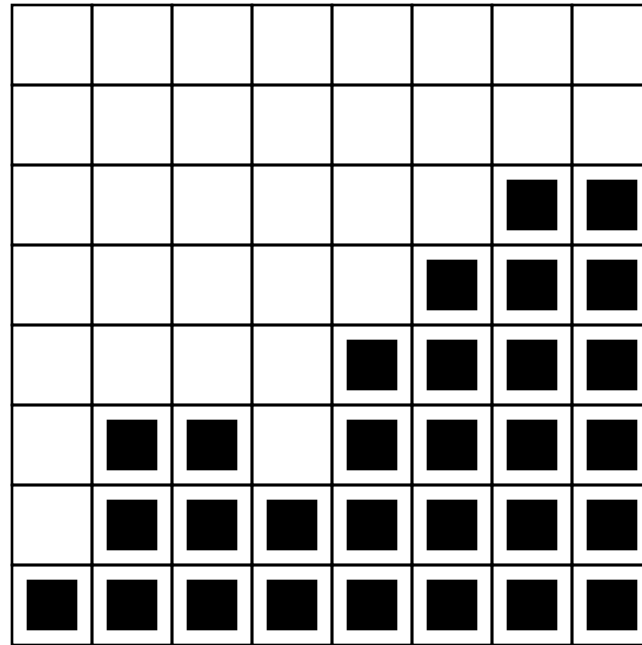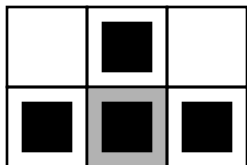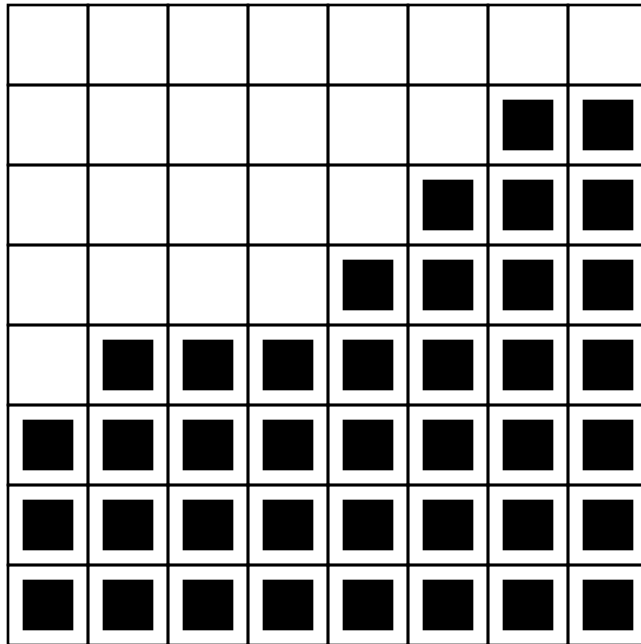
$U(f) \oplus U(k)$
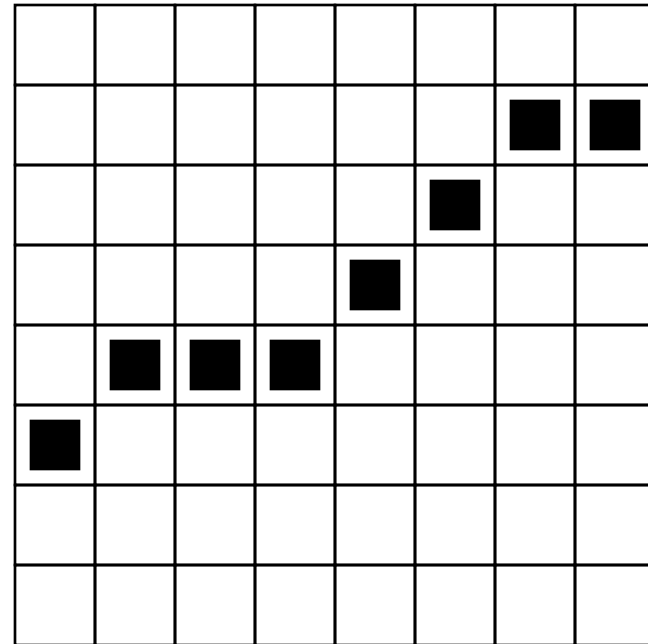
$T(U(f) \oplus U(k))$

Erosion
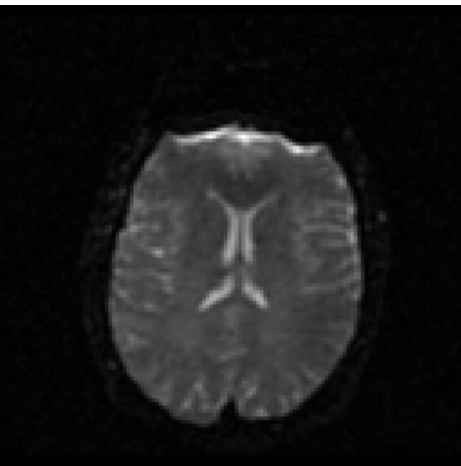
S=ones(3,3)   S=ones(5,5)   S=ones(7,7)

Dilation
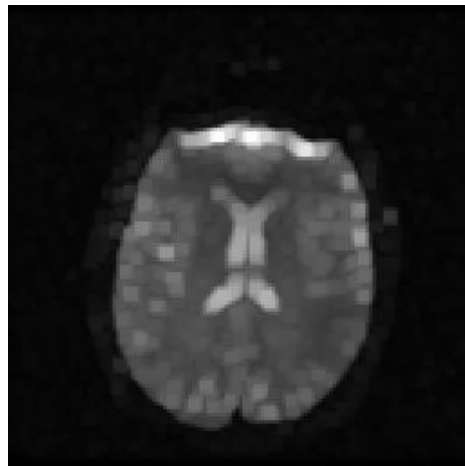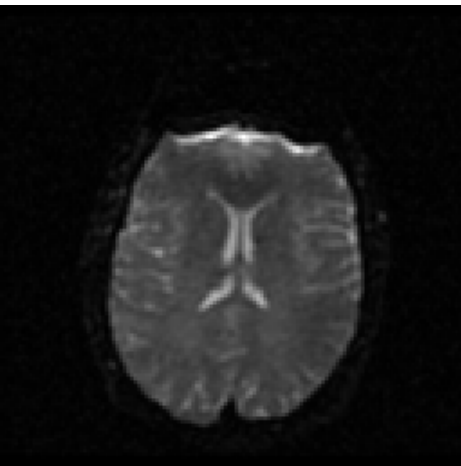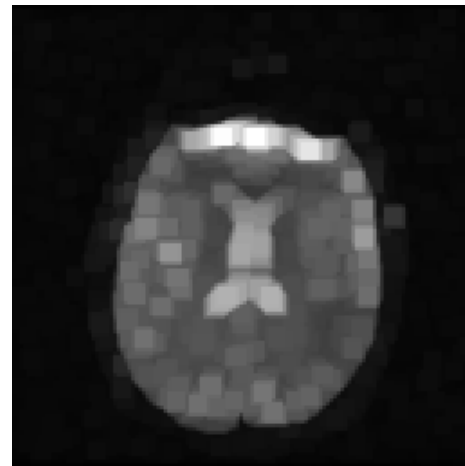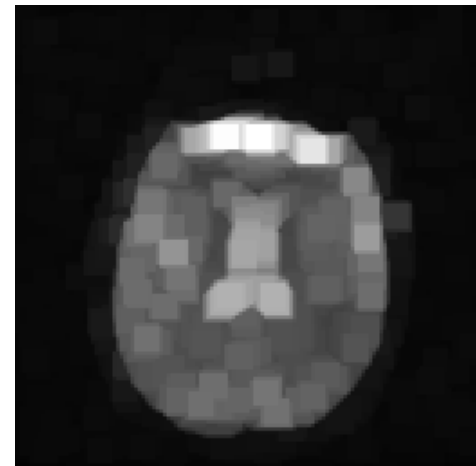
S=ones(3,3)   S=ones(5,5)   S=ones(7,7)

Speckle removal

Erode

Open

Salt and pepper noise
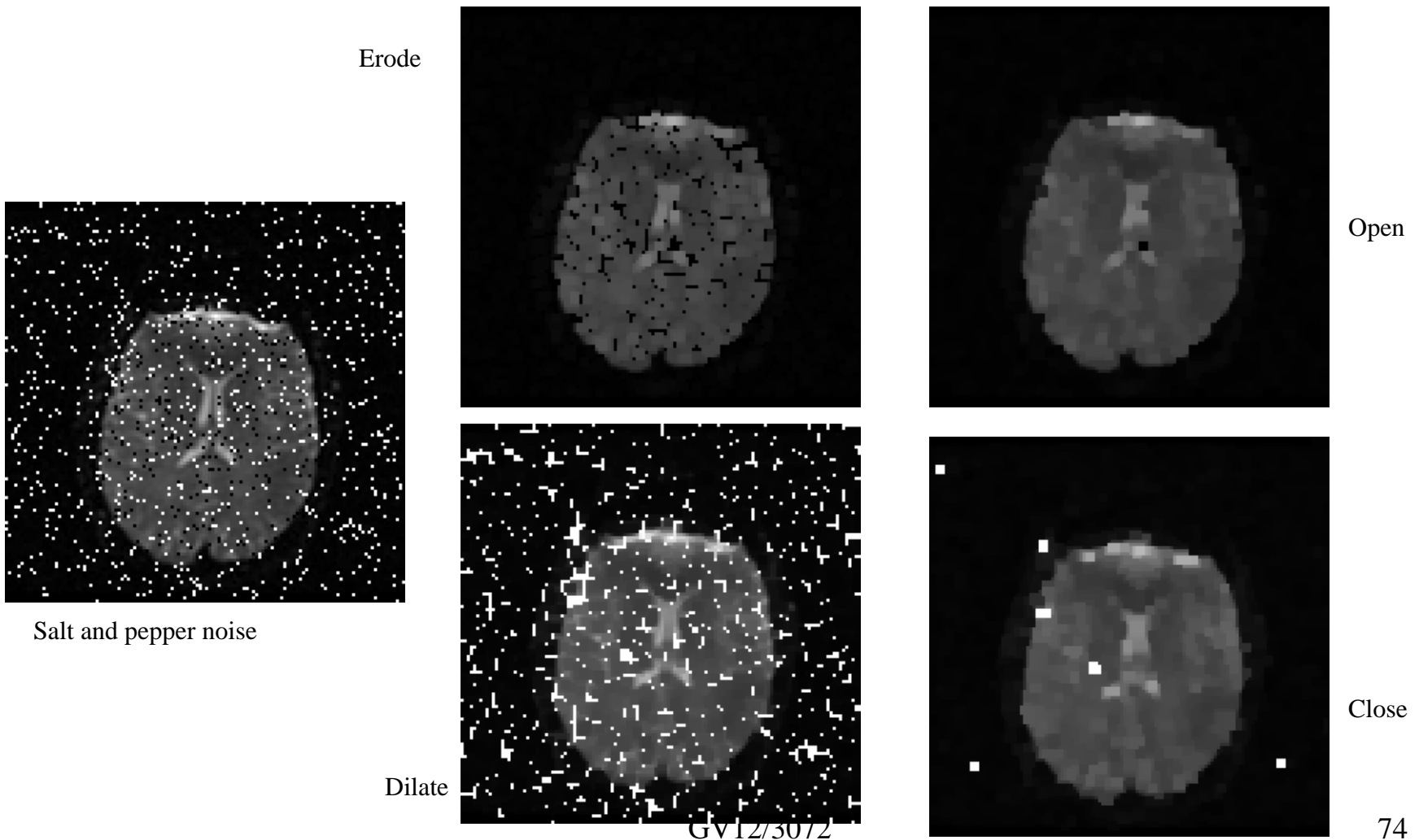
Dilate

Close

Favorites

- $E = \text{medfilt2}()$
- $[D,L] = \text{bwdist}()$

Summary

- Simple morphological operations
  - Erode and dilate
  - Open and close

- Applications:
  - Granulometry
  - Thinning and thickening
  - Skeletons and the medial axis transform

- Gray level morphology

Image Processing.

Find the Letter 'e'

Let freedom ring from every hill and molehill of Mississippi and every mountainside.

Let freedom ring from every hill and molehill of Mississippi and every mountainside.