PHILOSOPHY OF LOGIC AND LANGUAGE

WEEK 3: 'ÜBER SINN UND BEDEUTUNG'

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OVERVIEW

Frege's career can be divided into three periods: early, middle, and late.

The early period, pre-1890s:

- *Begriffsschrift*, published in 1879
- *Foundations*, 1884

The middle period, 1890s to 1903:

- 'Function and Concept', 1891
- 'On Sense and Reference', 1892
- 'On Concept and Object', 1892

- *Basic Laws*, Vol. I, 1893
- Letter from Russell, 1902
- *Basic Laws*, Vol. II, 1903

The late period, post-1903:

- 'Thought', 1918
- 'Negation', 1918
- 'Compound Thoughts', 1923

Last week, we looked at the innovations in logic of Frege's early period, and the and the philosophical ideas that led to them.

This week, we'll look at ways in which Frege's philosophy of logic developed in his middle period, particularly the distinction between sense (Sinn) and reference (Bedeutung).

Next week, we'll hopefully look at some aspects of his late period, particularly 'Thought'. (But I'll also say a bit more about this week's topics.)

'FUNCTION AND CONCEPT'

THE PROBLEM

The developments were announced in the three papers of the early 1890s, 'Function and Concept', 'On Sense and Reference', and 'On Concept and Object'.

In *Foundations*, Frege insists on a sharp distinction between **CONCEPTS** and **OBJECTS**, listing as one of three fundamental principles:

"The distinction between concept and object must be kept in mind... As concerns [this] point, it is a mere illusion to suppose that a concept can be made into an object without altering it." (*Foundations*, p. X)

The distinction plays an important role in *Foundations*. One of Frege's key moves is to argue that statements of number ascribe numbers to concepts rather than objects:

"If I say 'Venus has 0 moons', then there is no moon or aggregate of moons to assert anything of at all; but instead it is the concept 'moon of Venus' to which a property is ascribed, namely that of including nothing under it." (*Foundations*, §46)

But what are concepts? Frege seems to have wanted even early on to extend the function-argument analysis of sentences to their *contents*.

We might then suspect that concepts are a kind of function, one that yields a conceptual content given an object as argument.

One problem: if concepts *are* a kind of function, the sharp distinction between concepts and objects would seem to require a sharp distinction between functions and arguments...

... but there doesn't seem to be a sharp distinction between functions and arguments, at least in *Begriffsschrift*. Frege explains functions in terms of invariant patterns:

- 'Seven is divisible by seven'
- 'Nine is divisible by seven'
- 'Fourteen is divisible by seven'

Here we consider 'seven' the variable *argument*, and 'is divisible by seven' the invariant *function*. But can't we also regard 'is divisible by seven' as variable?

- 'Seven is divisible by seven'
- 'Seven is the sum of four and three'
- 'Seven divides fourteen'

So it seems we can regard the same thing as either function or argument in the sentence 'Seven is divisible by seven'.

THE SOLUTION

'Function and Concept' addresses this problem. Frege offers a new account of functions as essentially incomplete or **UNSATURATED**. Consider:

- '$2.1^3 + 1$'
- '$2.4^3 + 4$'
- '$2.5^3 + 5$'

These are complex expressions naming the values that a particular function yields for the arguments 1, 4, and 5.

It seems then that each of the complex expressions must therefore contain as a constituent somehow expression that somehow stands for this function.

But if so, the expression that stands for the function can't be '2.x^3 + x', as it is *not* a constituent of '2.1^3 + 1' etc.

Might the expression rather be the gappy one, '2.^{3} + '? No. For that expression is also a part of the following complex expressions:

- '2.1^3 + 1'
- '2.4^3 + 2'
- '2.5^3 + 7'

And these are complex expressions naming the values that a *different* function yields for the argument *pairs* 1 and 1, 4 and 2, 3 and 7.

It seems that there is no way of specifying the expression that stands for the function without writing it out in combination with expressions that stand for its arguments.

Though we might gesture at it with expressions like '2.x^3 + x', or '2.$(\ )^3$ + $(\ )$', the actual expression is essentially in need of completion, unsaturated.

Frege holds that it is not just expressions that stand for functions that are unsaturated in this way; so too are the functions that these expressions stand for.

Certain unsaturated expressions are **CONCEPT-EXPRESSIONS**: expressions whose completions are (declarative) *sentences*.

And **CONCEPTS** are then the special kind of function for which these unsaturated concept-expressions stand.

Does this solve the problem? Does it yield a sharp distinction between functions, and especially concepts, on the one hand, and objects on the other?

One might think that it does not. Consider the expression 'the concept *horse*'. One the one hand, it seems to stand for a concept — the concept *horse*...

On the other hand, it is an expression that can be specified without writing it out in combination with other expressions.

So if it stands for anything — and as a definite description, it seems that that is what is supposed to do — it stands for an **OBJECT**.

So 'the concept *horse*' seems to stand for something — the concept *horse* — which is both an object and a concept.

This objection was raised against Frege by the philosopher Benno Kerry, a student of Brentano's.

Frege's paper 'Concept and Object' is his response.

Frege accepts that the word 'concept' can be used in the way Kerry suggests, but insists that it is not how he intends to use it.

Frege doesn't deny that, as he intends to use the word 'concept', the concept *horse* is an object.

What he denies is rather that the concept *horse* is a concept: concepts are essentially unsaturated, and can only be what is designated by a predicate.

"By a kind of necessity of language, my expressions, taken literally, sometimes miss my thought; I mention an object, when what I intend is a concept. I fully realize that in such cases I was relying upon a reader who would be ready to meet me halfway — who does not begrudge a pinch of salt." ('Concept and Object', p. 204)

Another issue: what sort of function *are* concepts, exactly? Consider the concept-expression that is common to the following sentences:

- '7 is divisible by 7'
- '14 is divisible by 7'
- '9 is divisible by 7'

It is reasonable to assume that the *arguments* of the concept that this expression stands for are the numbers, 7, 14, and 9. But what are its *values*?

Presumably it is whatever the sentences stand for, and going by *Begriffsschrift*, we might take that to be their conceptual contents. But there's a problem.

Consider:

- '7 is divisible by 7'
- 'The positive square root of 49 is divisible by 7'

The expressions '7' and 'the positive square root of 49' both stand for the same thing, the number 7.

So if concepts yield conceptual contents as values, the conceptual contents of these sentences should be the same.

But it seems that the conceptual contents of these sentences are *not* the same, for they differ in inferential properties.

As Frege would have it, at any rate, the first entails on its own that something is divisible by itself, but the second does not.

Frege's solution is to draw a distinction between what he calls *Sinn* and *Bedeutung*, or **SENSE** and **REFERENCE**.

Frege briefly mentions this in 'Function and Concept', but gives it an extended treatment in 'On Sense and Reference'.

'ON SENSE AND REFERENCE'

IDENTITY STATEMENTS

In 'On Sense and Reference', Frege spells out the problem as it concerns identity statements. We looked at this very briefly last week.

Consider:

- 'Hesperus is Hesperus'
- 'Hesperus is Phosphorus'

(Frege actually uses the definite descriptions, 'the evening star' and 'the morning star', rather than the names 'Hesperus' and 'Phosphorus'.)

We saw last week that these sentences (or the judgements that they express) seem to differ in conceptual content.

Together with the judgement that Phosphorus is a planet, the second licenses the inference that Hesperus is a planet; the first does not.

In 'On Sense and Reference', Frege takes this to pose a problem for the view — one that he ultimately wants to endorse — that identity is a relation between objects.

The problem is that if identity is a relation between objects then it seems that the contents of the two sentences ought to be the same as well.

Why? First, **COMPOSITIONALITY** (of content): the content of a sentence is determined by the contents of its constituent parts and the way it is put together out of them.

Both sentences are put together in the same way. They differ only in that one contains 'Hesperus' where the other contains 'Phosphorus'.

It therefore follows that the sentences have the same content if 'Hesperus' and 'Phosphorus', as they occur in identity contexts at least, have the same content.

Second, however: if identity is a relation between objects then two names, as they occur in identity contexts, have the same content if they name the same object.

And of course, third: 'Hesperus' and 'Phosphorus' *do* name the same object, namely the planet Venus.

Moreover, since identity would then be a relation which each object stands in to itself and nothing else, each sentence ought to be *trivially true*.

But the contents of the two sentences, as we have seen, appear to be different. And only one of them, 'Hesperus is Hesperus', is obviously true.

METALINGUISTIC SOLUTION

Frege considers, but rejects, a **METALINGUISTIC** solution, according to which identity is not a relation between objects, but rather a relation between *names*.

This seems promising, as it allows us to trace the difference in contents between the two sentences to the difference between the names they contain.

We can therefore allow for the difference in contents without giving up compositionality, or the link between the content of a name and the object that it names.

Moreover, since identity would be a relation which each name stands in to itself and no other, only the first sentence will turn out to be trivially true.

But Frege thinks it won't work. The problem is that the use of a particular name to stand for a particular object is entirely arbitrary.

So if identity is a relation between names, identity statements are nothing more than statements about language.

But identity statements are *not* statements about language; they are statements about the world, e.g. astronomical objects.

There are two other problems with the metalinguistic solution worth mentioning, though Frege doesn't.

FIRST, a difference in the names used in an identity statement is not sufficient for the sort of puzzle that Frege has raised.

Consider:

- 'Dorothy is Dorothy'
- 'Dorothy is Dotty'

Only the second of these identity statements uses different names. But it's not clear they differ in content: 'Dorothy' and 'Dotty' are stylistic variants.

SECOND, and more importantly, the problem doesn't just arise for identity statements. We saw this earlier on.

Recall:

- '7 is divisible by 7'
- 'The positive square root of 49 is divisible by 7'

These also seem to differ in content. But if so, saying that identity is a relation between names does nothing to explain the fact.

FREGE'S SOLUTION

The problem with the metalinguistic solution is that it doesn't deliver the result that identity statements are about the objects that the names stand for.

Frege thinks that, to secure this result, we have to accept that identity *is* a relation between objects.

However, he thinks that we can accept this while allowing for the difference in logical status between 'Hesperus is Hesperus' and 'Hesperus is Phosphorus'.

He thinks that the logical properties of an identity statement depend not just on the objects that the names stand for.

It also depends on the *ways* in which they stand for these objects, or the *modes of presentation* or *designation* of them.

Here Frege is invoking his distinction. Identity is a relation between objects. So identity statements are about objects, the **REFERENCES** of names.

But differences in the logical properties of identity statements are differences at the level of **SENSE**, the modes by which names present their references.

Given the difference in logical status between the two sentences, all that compositionality demands is a difference in *sense* between the names 'Hesperus' and 'Phosphorus'.

That is to say, a difference between the ways in which they present their *reference*, the objects that they stand for.

But that they present their references in different ways is of course compatible with the references *that* they present being identical.

The original puzzle arose out of the combination of the view that identity is a relation between objects with three further assumptions. Put in terms of sense:

FIRST, compositionality: the sense of a sentence is determined by the sense of its constituent parts and the way it is put together.

SECOND: if identity is a relation between objects then two names, as they occur in identity contexts, have the same sense if they refer to the same object.

THIRD: 'Hesperus' and 'Phosphorus' refer to the same object, namely the planet Venus.

Once we distinguish between sense and reference, we can see that the fault lies with the second assumption.

Two expressions may have (or *express*) different senses even though they have (or *stand for*, *refer to*, *name*) the same reference.

SUMMARY

We've looked at the main developments in Frege's philosophy of logic of the 1890s.

First, his new characterisation of functions (and function-expressions) as essentially *unsaturated*.

Second, his distinction between sense (*Sinn*) and referrence (*Bedeutung*).

But there are a few loose threads. In particular, we haven't yet answered the question I raised a while back, if concepts are functions, what are their *values*?

Next week, I'll try to discuss some late period Frege...

But first we'll look at the answer to this question, Frege's controversial extension of the sense-reference distinction to sentences, and some criticisms of his distinction.