

Minimum Makespan Problem

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Agenda

1. **Makespan Problem Definition**
2. **Previous Work**
 1. **Online List** Scheduling Algorithm – Greedy
 2. **Online MR** Algorithm – Current Best Deterministic Algorithm
 3. **Online Rand** Algorithm – Current Best Randomized Algorithm for general case
 4. **Offline** Algorithm – NEAR Optimal
3. **Input Sequence Generation**
 1. **Uniformly** Distributed Randoms
 2. **Normally** Distributed Randoms
 3. **Zipfian** Distributed Randoms
4. **Report Table**

What is **Makespan** Problem?

Basic

Legendary

Classic

SCHEDULING!

What is Makespan Problem?

Basic

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SCHEDULING!

2 1 3 6 4 5 ... N jobs

2 ← Processing Time

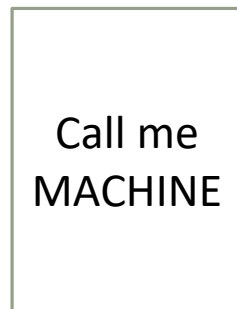
What is Makespan Problem?

Basic

Legendary

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SCHEDULING!



M machines

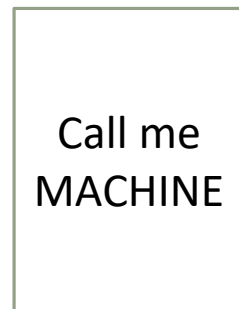
What is Makespan Problem?

Basic

Legendary

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SCHEDULING!



**Let's minimize the
max load
/makespan/**

What is Makespan Problem?

Basic

Legendary

Classic

SCHEDULING!



Call me
MACHINE

Call me
MACHINE

Let's minimize the
max load
/makespan/

**How Would You
Schedule, tho?**

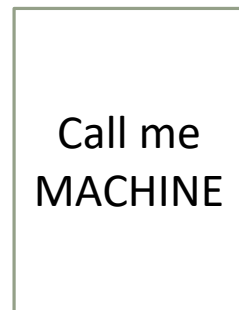
The Famous

Introduced by Graham in 1966

Online List Scheduling Algorithm

1 3 6 4 5 ... N jobs

Processing Time
2



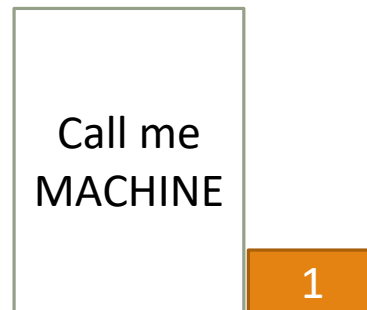
The Famous

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Online List Scheduling Algorithm

3 6 4 5 ... N jobs

Processing Time
2



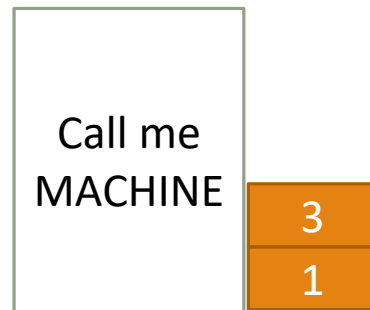
The Famous

Introduced by Graham in 1966

Online List Scheduling Algorithm

6 4 5 ... N jobs

Processing Time
2



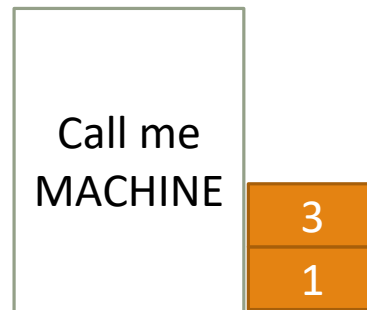
The Famous

Introduced by Graham in 1966

Online List Scheduling Algorithm

4 5 ... N jobs

Processing Time
2



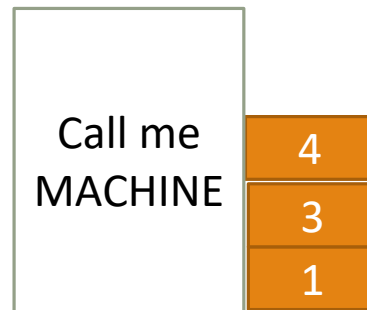
The Famous

Introduced by Graham in 1966

Online List Scheduling Algorithm

5 ... N jobs

2 ← Processing Time



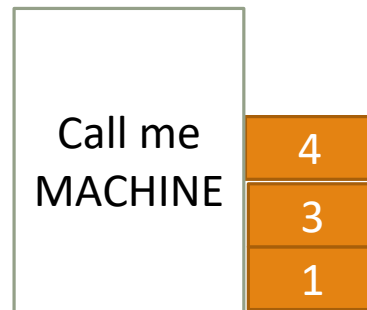
The Famous

Introduced by Graham in 1966

Online List Scheduling Algorithm

... N jobs

Processing Time
2

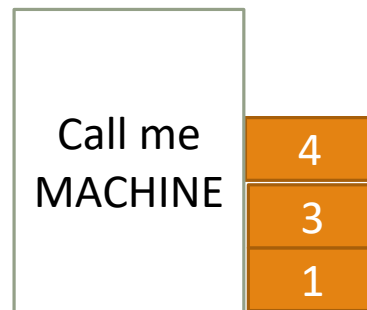
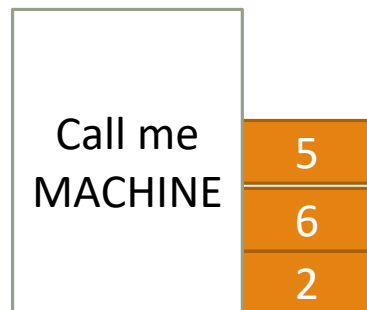


The Famous

Introduced by Graham in 1966

Online List Scheduling Algorithm

... N jobs



Makespan = 13.

However, OPT can do 11

Previous Work

Upper Bound

$(2 - 1/m)$ – List by Graham 1966

1.986 – Bartal et al.

1.945 – Karger et al

1.923 – M2 by Susan et al.

1.9201 – MR by Fleischer & Wahl 2000

Lower Bound /current best/

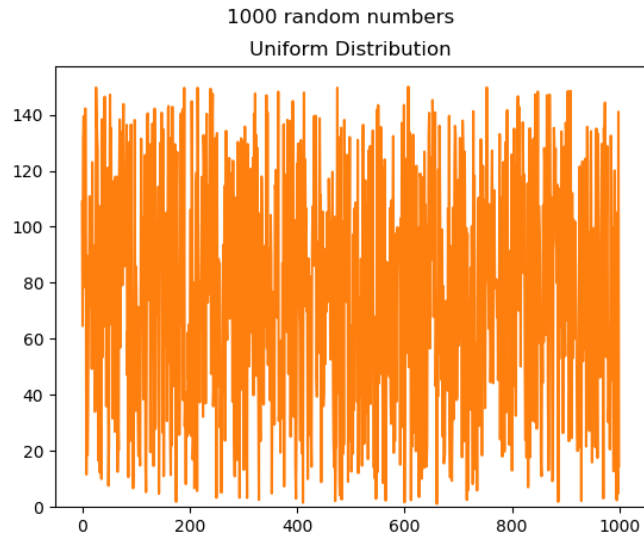
1.853 – Gormley et al.

Randomized Upper Bound /current best/

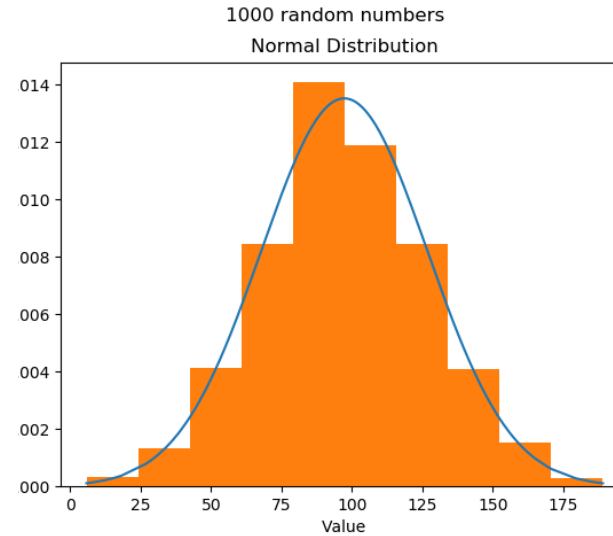
1.916 – Rand by Susan et al. 2002

Generating Input Sequence

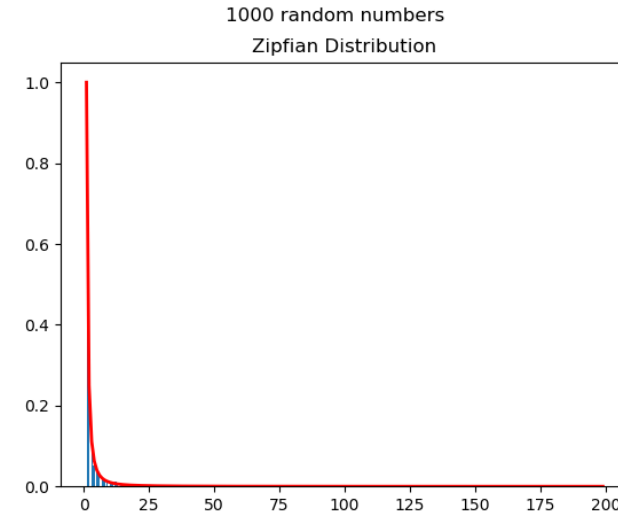
Uniform Distribution




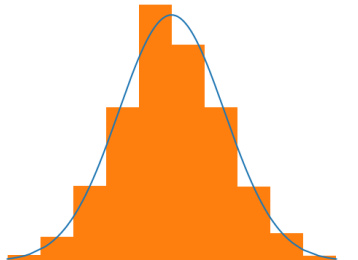

Normal Distribution



Zipfian Distribution



Report Table

Input sequence	Number of Jobs /n/	# of Machines /m/	Offline Approximation	Online Algorithm		
				Greedy	Current Best Deterministic	Current Best Random /m>7/
				List Scheduling	MR Algorithm	Rand Algorithm /avg of 10runs/
Uniform Distribution 	1000 (1<Tp<150)	5	15161.76	15191	15191	19401.7
		50	1518.65	1586.71	1586.71	1890.23
		150	512.93	588.86	588.86	682.64
	10000 (200<Tp<500)	5	697703.07	697835.85	697835.85	892607.39
		50	69770.44	70036.74	70036.74	83628.11
		150	23323.91	23538.27	23538.27	27806.87
Normal Distribution 	1000 (1<Tp<170)	5	19997.99	20015.61	20015.61	25600.01
		50	2014	2053.66	2291.78	2434.17
		150	689.89	750.39	750.39	868.49
	10000 (0<Tp<650)	5	600335.77	600366.77	600366.77	767888.58
		50	60044.81	60282.12	60282.12	72080.16
		150	20036.75	20332.37	21457.02	24019.45
Zipfian Distribution 	1000 (0<Tp<1034)	5	1034	1211	1211	1175
		50	1034	1046	1046	1043
		150	1034	1036	1036	1036
	10000	5	11361	11529	11529	14714
		50	3974	4068	4068	4038
		150	3974	3998	3998	3991

References

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