

Assignment: 01

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Question 1

$$M = \{\alpha, 3, \gamma\}, N = \{\blacksquare, \gamma, *, 0\}, K = \{\alpha, \{\alpha\}, M\}$$

$$1. M \cap N = \{\gamma\}$$

$$2. M \cap K = \{\alpha\}$$

$$3. P(M) \cap K$$

$$|M| = 3, P(M) = 2^{|M|} \Rightarrow 2^3 \Rightarrow 8$$

$$P(M) = \{\{\alpha, 3, \gamma\}, \{\alpha, 3\}, \{\alpha, \gamma\}, \{3, \gamma\}, \{\alpha\}, \{\gamma\}, \{3\}, \emptyset\}$$

$$P(M) \cap K = \{\{\alpha\}, M\}$$

$$4. M \setminus K = \{3, \gamma\}$$

$$5. N \cup k = \{\blacksquare, \gamma, *, 0, \alpha, \{\alpha\}, M\}$$

Question 2

$$\text{prove : } A \Delta B = (A \cup B) \cap \overline{(A \cap B)}$$

$$A \Delta B = (A \cup B) \cap \overline{(A \cap B)}$$

$$= (A \cap \overline{(A \cap B)}) \cup (B \cap \overline{(A \cap B)}) \quad \text{Distribution Low}$$

$$= (A \cap (\overline{A} \cup \overline{B})) \cup (B \cap (\overline{A} \cup \overline{B})) \quad \text{De Morgan's Low}$$

$$= ((A \cap \overline{A}) \cup (A \cap \overline{B}) \cup (B \cap \overline{A}) \cup (B \cap \overline{B}))$$

$$= (\emptyset \cup (A \cap \overline{B})) \cup ((B \cap \overline{A}) \cup \emptyset) \quad \text{Identity Low}$$

$$= (A \cap \overline{B}) \cup (B \cap \overline{A}) \quad [\text{Proved}]$$

Question 3

$$1. (4 + 5i)(4 - 5i)$$

$$= 16 - 20i + 20i - 25i^2$$

$$= 16 + 25$$

$$= 41 \quad (\text{Answer})$$

$$2. \frac{2+3i}{4+5i}$$

$$= \frac{(2+3i)(4-5i)}{(4+5i)(4-5i)}$$

$$= \frac{8-10i+12i-15i^2}{16-20i+20i-25i^2}$$

$$= \frac{8+2i+15}{16+25}$$

$$= \frac{23+2i}{41}$$

$$= \frac{23}{41} + \frac{2i}{41} \quad (\text{Answer})$$

$$3. \sqrt{16b} + \sqrt{3a - 12a}$$

$$= \sqrt{16b} + \sqrt{-9a}$$

$$= 4\sqrt{b} + \sqrt{-1 * 9a}$$

$$= 4\sqrt{b} + 3i\sqrt{a}$$

$$= 4b^{\frac{1}{2}} + 3ia^{\frac{1}{2}} \quad (\text{Answer})$$

Question 4

prove : $f(A) \cap f(B) = f(A \cap B)$

Lets take, $y \in f(A) \cap f(B)$

So : $y \in f(A)$ and $y \in f(B)$

Since, $y \in f(A) \exists a \in A$, Such that $y = f(a)$

$y \in f(B) \exists b \in B$, Such that $y = f(b)$

So, $f(a) = f(b)$

Since f is injective, $a = b$, $a \in A \cap B$

$$y = f(a) \in f(A \cap B)$$

This shows, $f(A) \cap f(B) \subseteq f(A \cap B)$

Therefor, $f(A) \cap f(B) = f(A \cap B)$ (Proved)

Question 5