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Numerical Optimization - Sheet 11

If you are a student in mathematics please solve the exercises with no tag and the ones with the tag **Mathematics**. If you are a data science student please solve the problems with no tag and those with the tag **Data Science**. Submissions with tags other than your subject count as bonus points. The tag **Programming** marks programming exercises.

Ex 1 (6 Points)

Apply the range space method to solve the problem

$$\min \ \frac{1}{2} x^\top \begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} x$$

$$\begin{aligned} \text{unter} \quad x_1 - 2x_2 + x_3 &= -3 \\ -x_1 + x_2 - x_3 &= 2 \end{aligned}$$

in the following steps.

(i) Derive C, B, b, c, and A corresponding to the problem as given in the lecture.

(ii) Solve $A\lambda = \alpha$.

(iii) Solve $Bx = -(C^\top \lambda + b)$.

Ex 2 (10 Points)

Consider the equality constrained QP

$$\min_{x \in \mathbb{R}^3} \frac{1}{6} x^\top \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} x + \begin{bmatrix} 0 & 0 & \frac{1}{3} \end{bmatrix} x$$

$$\text{s.t.} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} x = 0$$

and solve it using the null space method in the following steps.

(i) Define $C, B, \ldots$ and $x = (y, z)$ as given in the lecture.

(ii) Show that the kernel of C is given by $\ker(C) = \{\alpha\,(1,1,1)^\top \mid \alpha \in \mathbb{R}\}$.

(iii) Show that the null-space method is applicable, i.e. that B is positive definite on $\ker(C)$.

(iv) Determine the reduced Hessian and the corresponding right hand side as given in the lecture (see *S Schur complement = reduced Hessian*).

(v) Successively determine z and y.

Ex 3 (3 Points)

Consider the optimization problem

$$\begin{aligned} &\min_{x\in\mathbb{R}^2} \frac{1}{2} x^\top x - \begin{pmatrix}1\\1\end{pmatrix}^\top x \\ &\text{s.t. } x_2 = 0. \end{aligned} \tag{1}$$

(i) Determine the solution $x^* \in \mathbb{R}^2$ and the corresponding adjoint variable $\lambda^* \in \mathbb{R}$ for problem (1).

(ii) Is x^* also the solution for the constraint $x_2 \leq 0$ instead of $x_2 = 0$?

(iii) Is x^* also the solution for the constraint $x_2 \geq 0$ instead of $x_2 = 0$?

Hint: Have a look at the adjoint variable λ^*.

Ex 4 Programming (4 Points)

(i) Implement the Uzawa-iteration as described in the script.

(ii) Test your iteration with parameters $\tau = 0.05$, tol$= 0.001$ on the problem

$$\begin{aligned} &\min_x \frac{1}{2} x^T B x + x^T b \\ &\text{s.t. } Cx = c \end{aligned}$$

$$B = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 1 & -1 \end{bmatrix}, \quad b = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad c = \begin{pmatrix} -3 \\ 2 \end{pmatrix}.$$