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Numerical Optimization - Sheet 4

If you are a student in mathematics please solve the exercises with no tag and the ones with the tag **Mathematics**. If you are a data science student please solve the problems with no tag and those with the tag **Data Science**. Submissions with tags other than your subject count as bonus points. The tag **Programming** marks programming exercises.

Ex 1 (2 Points)

Let $x \in \mathbb{R}^2$. Use the necessary and sufficient conditions for optimality to solve

$$\begin{aligned} &\min_x x_1 + x_2 \\ &\text{s.t. } x_1^2 + x_2^2 = 2. \end{aligned}$$

Ex 2 (4 Points)

Find a local minimum of the problems.

(i) Let $c > 0$ be fixed.

$$\begin{aligned} &\min_{r,h \in \mathbb{R}} r^2 + rh \\ &\text{s.t. } \pi r^2 h = c \\ &r \geq 0. \end{aligned}$$

(ii)

$$\begin{aligned} &\min_{x \in \mathbb{R}^2} \frac{1}{2} x^\top \begin{bmatrix} 4 & 6 \\ 6 & 9 \end{bmatrix} x - x^\top \begin{pmatrix} 14 \\ 21 \end{pmatrix} \\ &\text{s.t. } \begin{bmatrix} 1, -2 \end{bmatrix} x = 0. \end{aligned}$$

Ex 3 (6 Points)

Let $C > 0$. Compute the Wolfe-Dual of the optimization problem

$$\begin{aligned} &\min_{w,b,\xi} \frac{1}{2}\|w\|^2 + C \sum_{i=1}^{m} \xi_i \\ &\text{s.t.} \\ &y_i(w^T x_i + b) \geq 1 - \xi_i, \, \forall i = 1, \ldots, m \\ &\xi_i \geq 0, \, \forall i = 1, \ldots, m. \end{aligned}$$

Ex 4 Programming (3 Points)

You find the iPython notebook `numopt_version02` in the folder Lecture 6 on Olat. Use this notebook as basis for the following exercise.

(i) Consider the cell **chapter 2 SVM example code**. Uncomment the line which generates the *"not nicely separable points"* and fit a *linear* SVM using a hard and a soft margin. Fit a SVM with *rbf*-Kernel, again with hard and soft margin. Visualize your results with the code given in the notebook.

(ii) The cell **chapter 3 3D pic for SVM illustration** visualizes the feature map

$$(x_1, x_2) \mapsto (x_1^2, x_2^2, \sqrt{2}x_1x_2).$$

The function `svm.SVC` allows user defined kernels. Determine and implement the corresponding kernel function and fit and visualize the model to the data of (i).

(iii) Please comment on the result and the capabilites of the given kernel.