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Submission: 12.04.22, until 12:15

Numerical Optimization - Sheet 1

If you are a student in mathematics please solve the exercises with no tag and the ones with the tag **Mathematics**. If you are a data science student please solve the problems with no tag and those with the tag **Data Science**. Submissions with tags other than your subject count as bonus points. The tag **Programming** marks programming exercises.

Ex 1 (4 Points)

Let $f : \mathbb{R}^n \to \mathbb{R}$ be convex, that means

$$f((1-\lambda)x + \lambda y) \leq (1-\lambda) \cdot f(x) + \lambda \cdot f(y), \quad \forall x, y \in \mathbb{R}^n, \ \forall \lambda \in [0,1].$$

Show that any local minimum of f is already a global minimum.

Ex 2 (4 Points)

Consider a linear mapping $f : \mathbb{R}^n \to \mathbb{R}^m, x \mapsto Cx$, defined by a matrix $C \in \mathbb{R}^{m \times n}$ with $m \leq n$. Show that f being surjective is equivalent to the matrix C having full rank.

Ex 3 (4 Points)

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x) = \frac{1}{2}\left[x_1^2 + 2x_1x_2 + 4x_2^2\right].$$

Find a scalar product such that $\nabla f(x) = x \ \forall x \in \mathbb{R}^2$.

Ex 4 Programming (3 Points)

Create a contour plot of the Rosenbrock function

$$f(x, y) = (a - x)^2 + b(y - x^2)^2,$$

for $a = 1$ and $b = 100$. Try to approximately find the region of its global minimum in the plot.
Hint:

- You can obtain a logarithmic scaling of the contour lines in `plt.contour()` via the option `locator=ticker.LogLocator()`.
- Please submit the solution as iPython Notebook. Read the information page on Olat if you need information about Python 3 or iPython Notebooks.