Numerical Optimization - Sheet 7

If you are a student in mathematics please solve the exercises with no tag and the ones with the tag Mathematics. If you are a data science student please solve the problems with no tag and those with the tag Data Science. Submissions with tags other than your subject count as bonus points. The tag Programming marks programming exercises.

Ex 1   Data Science   (3 Points)

Show that for the problem

$$\min_{x \in \mathbb{R}^n} \frac{1}{2} x^\top Q x - x^\top b$$

for $Q \in \mathbb{R}^{n \times n}$ positive definite and symmetric and $b \in \mathbb{R}^n$, the Newton method converges after one step.

Ex 2   (3 Points)

Apply Theorem 3.9 and Corollary 3.10 to show that a Newton method with Wolfe line search converges if the spectra of the Hessians satisfy $\sigma(\mathrm{Hess}f(x^k)) \subset [m, L]$ with $0 < m \leq L$ independent of k.

Ex 3 Programming   (12 Points)

Implement the following quasi Newton and the Newton method. Your function has to provide the interface `minimization_alg(f, gf, Hessf, x0, callback)` where the first three arguments are callable objects which evaluate a function, its gradient and Hessian with respect to the Euclidean scalar product. The value `x0` is the starting value and `callback` is a callback function (see below).

  (i) **SR1: Broydens symmetric rank-1-update**
      Implement `broydensymm1(f, gf, Hessf, x0, callback)` and run `test(broydensymm1)`.

 (ii) **BFGS (Broyden-Fletcher-Goldfarb-Shanno)**
      Implement `bfgs(f, gf, Hessf, x0, callback)` and run `test(bfgs)`.

(iii) **Newton method**
      Implement `newton(f, gf, Hessf, x0, callback)` and run `test(newton)`.

Hints:

- You might want to set an error tolerance and a maximal number of iterations as optional parameters of your algorithm.

- Please call the function `callback` in your algorithm in the form `callback(xk)` where `xk` is the current iterate. This allows the function `test` in the module `newton_cg_test` to plot your iterates and the errors.

- You find the function `armijo` in `newton_cg_test`.

- The broyden and bfgs update fail under certain conditions (see lecture). If that happens simply restart the current Hessian approximation by `B = np.eye(n)`.