Production Scheduling Optimization

Set:

$P: Set\ of\ products, p \in P$

$T: Set\ of\ timeframes, t \in T$

Parameters:

$COT_{pq}: Changeover\ time\ between\ product\ p\ and\ q\ where\ p, q \in P$

$SS_{pt}: Safety\ stock\ of\ product\ p\ till\ time\ interval\ t\ where\ t \in T$

$LR_p: Line\ Rate\ of\ product\ p\ where\ p \in P$

$BINV_p: Begining\ Inventory\ of\ product\ p\ where\ p \in P$

$PC_p: Penalty\ cost\ of\ not\ meeting\ inventory\ level\ of\ product\ p\ where\ p \in P$

$IHC_p: Inventory\ holding\ cost\ of\ product\ p\ per\ unit\ of\ product\ per\ unit\ time\ where\ p \in P$

$PM_p: Profit\ margin\ of\ product\ p\ where\ p \in P$

$INV\_CAP_{pt}: Inventory\ Capacity\ (\sim mximum\ DOS)\ of\ product\ p\ till\ time\ interval\ t\ where\ t \in T$

$OC_p: Penalty\ cost\ for\ overflow\ (more\ than\ maximum\ inventory)\ of\ product\ p\ where\ p \in P$

Decision Variables:

$x_{pt} = 1\ if\ product\ p\ is\ produced\ during\ time\ interval\ t, else\ 0$

$w_{pqt} = 1\ if\ product\ p\ is\ changed\ over\ to\ product\ q\ after\ time\ interval\ t$

Auxiliary Variables:

$y_{pt}^{+}: Excess\ product\ produced\ above\ target\ inventory\ level\ of\ product\ p\ after\ time\ t$

$y_{pt}^{+}: Gap\ between\ required\ inventory\ level\ and\ current\ inventory\ level\ of\ product\ p\ after\ time\ t$

$of_{pt}^{+}: Excess\ product\ produced\ above\ maximim\ inventory\ level\ of\ product\ p\ after\ time\ t$

$of_{pt}^{-}: Gap\ between\ maximum\ inventory\ level\ and\ current\ inventory\ level\ of\ product\ p\ after\ time\ t$

Objective Function (Minimize Total Cost):

$$\min \sum_{p}\sum_{q}\sum_{t} COT_{pq} * w_{pqt} * LR_p * PM_p + \sum_{p}\sum_{t}(IHC_p * y_{pt}^{+} + PC_p * y_{pt}^{-} + OC * of_{pt}^{+})$$

Constraints:

Logical relationship among decision variables

$$x_{pt} * x_{q(t+1)} = w_{pqt}\ \forall\ p, q \in P\ and\ t \in T - \{T_n\} \quad (1)$$

To make constraint (1) linear,

$$w_{pqt} \geq x_{pt} + x_{q(t+1)} - 1 \; \forall p, q \in P \; and \; t \in T - \{T_n\} \; ----------(1a)$$

$$w_{pqt} \leq x_{pt} \; \forall p, q \in P \; and \; t \in T - \{T_n\} \; ----------(1b)$$

$$w_{pqt} \leq x_{q(t+1)} \; \forall p, q \in P \; and \; t \in T - \{T_n\} \; ----------(1c)$$

In a given time period only single product can run on a Line

$$\sum_{p} x_{pt} \leq 1 \forall t \in T \; ----------(2)$$

Relationship among production, demand, inventory, safety stock, excess inventory and inventory shortage

$$\sum_{t=T_1}^{T_t} x_{pt} * LR_p + BINV_p - SS_p - \sum_{t=T_1}^{T_t} D_{p,t} = y_{pt}^{+} - y_{pt}^{-} \; \forall p \in P \; and \; t \in T \; ----------(3)$$

Relationship among production, demand, inventory, overflow capacity & overflow inventory

$$\sum_{t=T_1}^{T_t} x_{pt} * LR_p + BINV_p - INV\_CAP_p - \sum_{t=T_1}^{T_t} D_{p,t} = of_{pt}^{+} - of_{pt}^{-} \; \forall p \in P \; and \; t \in T \; --------(4)$$

Alternate Solution

Set:

$P: Set\ of\ products, p \in P$

$T: Set\ of\ timeframes, t \in T$

Parameters:

$COT_{pq}: Changeover\ time\ between\ product\ p\ and\ q\ where\ p, q \in P$

$DOS_{pt}: Days\ of\ Supply\ of\ product\ p\ till\ time\ interval\ t\ where\ t \in T$

$OR_p: Outbound\ flow\ rate\ per\ day\ where\ p \in P$

$LR_p: Line\ Rate\ of\ product\ p\ where\ p \in P$

$IDOS_p: Line\ Rate\ of\ product\ p\ where\ p \in P$

Decision Variables:

$x_{pt} = 1\ if\ product\ p\ is\ produced\ during\ time\ interval\ t, else\ 0$

$w_{pqt} = 1\ if\ product\ p\ is\ changed\ over\ to\ product\ q\ after\ time\ interval\ t$

Objective Function (Minimize total Changeover time):

$$\min \sum_{p}\sum_{q}\sum_{t} COT_{pq} * w_{pqt}$$

Constraints:

Logical Relationship among decision variables

$$x_{pt} * x_{q(t+1)} = w_{pqt}\ \forall\ p, q \in P\ and\ t \in T - \{T_n\} - (1)$$

To make constraint (1) linear,

$$w_{pqt} \geq x_{pt} + x_{q(t+1)} - 1\ \forall p, q \in P\ and\ t \in T - \{T_n\} - (1a)$$

$$w_{pqt} \leq x_{pt}\ \forall p, q \in P\ and\ t \in T - \{T_n\} - (1b)$$

$$w_{pqt} \leq x_{q(t+1)}\ \forall p, q \in P\ and\ t \in T - \{T_n\} - (1c)$$

In a given time period only single product can run on a Line

$$\sum_{p} x_{pt} \leq 1 \forall t \in T - (2)$$

Days of supply to be maintained for all product in all time period

$$IDOS_p * OR_p + \sum_{t=T_1}^{t} x_{pt} * LR_p - t * OR_p \geq DOS_{pt} * OR_p\ \forall p, q \in P\ and\ t \in T - - - - - - - - (3)$$