Factor analysis

Outline

- Factor analysis in general
- Explorative factor analysis
  - Some basics
  - Fundamental theorem
  - Computational steps
  - Communality estimation
  - Extraction methods and criteria for the number
  - Methods of factor rotation
  - Factor interpretation
- Confirmatory factor analysis
  - Some basics
  - Proceeding
  - Model specification

Factor analysis in general

Basic concepts

- an ***item (**also **test item)*** is …

….. is part of a scale

….. a denotation that is used in assessment and exploratory factor analysis …

- for ***questions / statement / task*** *of a test and a questionnaire*
- for referring to data obtained by questions / statements / tasks

…. (sometimes) treated as equivalent to a manifest variable

Factor analysis in general

Basic concepts

- a ***factor*** is …

…... is a dimensions that characterizes a set of items

…... is considered as a latent variable

…. sometimes it is considered as *descriptive*
sometimes as an *underlying source*

Factor analysis in general

- Multivariate procedure

- *Purpose*: search for structure
- *Purpose*: data reduction
- *Purpose*: check of assumed structure
- *Purpose*: construction of questionnaire

- Two types:
  - explorative factor analysis
  - confirmatory factor analysis

Factor analysis in general

- Multivariate procedure
- *Purpose*: search for structure

  e.g. The researcher is interested in learning about ***humor*** (what is the structure? ...)

Factor analysis in general

- Multivariate procedure

- *Purpose*: search for structure
- *Purpose*: data reduction

  e.g. The researcher likes to investigate the effect of humor on health on the basis of 50 humor items

Factor analysis in general

- Multivariate procedure

- *Purpose*: search for structure
- *Purpose*: data reduction
- *Purpose*: check of assumed structure

e.g. The researcher has an idea of the structure of humor and likes to check whether it is correct

Factor analysis in general

- Multivariate procedure

- *Purpose*: search for structure
- *Purpose*: data reduction
- *Purpose*: check of assumed structure
- *Purpose*: construction of questionnaire

  e.g. The researcher likes to construct a humor questionnaire

Factor analysis in general

- Multivariate procedure

- *Purpose*: search for structure
- *Purpose*: data reduction
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- Two types:
  - explorative factor analysis
  - confirmatory factor analysis

Explorative/exploratory factor analysis

Some basics

What is exploratory factor analysis?

… it is the *search* for one or several dimensions that are underlying data collected in using a number of different measures / items.

… this search is expected to provide information on which measures / items aligns with which factor.

… there is **no** clear expectation as in confirmatory factor analysis

Some basics

Major outcomes of exploratory factor analysis?

… information on the number of underlying dimensions

… information regarding the grouping of measures / items

… information on participants‘ factor scores

Some basics

A remark:

Factor analysis is ***not*** one single method; it is a set of methods that are applied to data in order to achieve the final result.

It is necessary that the application of these methods follow a prescribed sequence of steps.

Some basics

A remark:

.....

It is necessary that the application of these methods follow a prescribed sequence of steps.

There are steps that require to choose between a number of alternative methods (e.g. extraction methods, rotation methods).

Fundamental theorem

This theorem formalizes basic assumptions of factor analysis.

It can be considered as a kind of precursor of the model of measurement.

Fundamental theorem

$$z_{mi} = \lambda_{i1}F_{m1} + \lambda_{i2}F_{m2} + \ldots + \lambda_{ip}F_{mp} + \varepsilon_{mi} = \sum_{k=1}^{p}\left(\lambda_{ik}F_{mk}\right) + \varepsilon_{mi}$$

A standardized score is described as a linear combination of factor scores, of their weights and an error component

<u>Explanation of symbols:</u>

m: Person

i : Item

λ: Factor loading

F: Factor score

ε: Error component

Fundamental theorem

$$z_{mi} = \lambda_{i1}F_{m1} + \lambda_{i2}F_{m2} + \ldots + \lambda_{ip}F_{mp} + \varepsilon_{mi} = \sum_{k=1}^{p}\left(\lambda_{ik}F_{mk}\right) + \varepsilon_{mi}$$

Example:

Assume z_{mi} is the score of Tina (participant) who completed item *i* that is an analogy problem:

$$\begin{aligned} z_{\text{Tina,item\_}i} = \; & \lambda_{\text{item\_}i\text{,analogical reasoning}} F_{\text{Tina,analogical reasoning}} + \cdots \\ & + \lambda_{\text{item\_}i\text{,figural reasoning}} F_{\text{Tina,figural reasoning}} + \cdots \\ & + \lambda_{\text{item\_}i\text{,verbal reasoning}} F_{\text{Tina,verbal reasoning}} + \varepsilon \end{aligned}$$

analogical reasoning figural reasoning verbal reasoning

(Abilities that possibly contribute to performance)

Fundamental theorem

$$z_{mi} = \lambda_{i1}F_{m1} + \lambda_{i2}F_{m2} + \ldots + \lambda_{ip}F_{mp} + \varepsilon_{mi} = \sum_{k=1}^{p}\left(\lambda_{ik}F_{mk}\right) + \varepsilon_{mi}$$

Example:

Assume z_{mi} is the score of Tina (participant) who completed item *i* that is an analogy problem:

$$z_{\text{Tina,item\_}i} = \lambda_{\text{item\_}i\text{,analogical reasoning}} F_{\text{Tina,analogical reasoning}} + \cdots$$
$$+ \lambda_{\text{item\_}i\text{,figural reasoning}} F_{\text{Tina,figural reasoning}} + \cdots$$
$$+ \lambda_{\text{item\_}i\text{,verbal reasoning}} F_{\text{Tina,verbal reasoning}} + \varepsilon$$

Information is regarding Tina and item!

Fundamental theorem

$$z_{mi} = \lambda_{i1}F_{m1} + \lambda_{i2}F_{m2} + \ldots + \lambda_{ip}F_{mp} + \varepsilon_{mi} = \sum_{k=1}^{p}\left(\lambda_{ik}F_{mk}\right) + \varepsilon_{mi}$$

Example:

Assume z_{mi} is the score of Tina (participant) who completed item *i* that is an analogy problem:

What do we know before conducting exploratory factor analysis?

$$\begin{aligned}\mathbf{z}_{\textbf{Tina,item\_}\boldsymbol{i}} = {} & \lambda_{\text{item\_}i,\text{analogical reasoning}}F_{\text{Tina,analogical reasoning}} + \cdots \\ & + \lambda_{\text{item\_}i,\text{figural reasoning}}F_{\text{Tina,figural reasoning}} + \cdots \\ & + \lambda_{\text{item\_}i,\text{verbal reasoning}}F_{\text{Tina,verbal reasoning}} + \varepsilon\end{aligned}$$

Fundamental theorem

What is achieved by factor analysis?

$$z_{mi} = \lambda_{i1}F_{m1} + \lambda_{i2}F_{m2} + \ldots + \lambda_{ip}F_{mp} + \varepsilon_{mi} = \sum_{k=1}^{p}\left(\lambda_{ik}F_{mk}\right) + \varepsilon_{mi}$$

Fundamental theorem

What is achieved by factor analysis?

$$z_{mi} = \lambda_{i1}F_{m1} + \lambda_{i2}F_{m2} + \ldots + \lambda_{ip}F_{mp} + \varepsilon_{mi} = \sum_{k=1}^{p}\left(\lambda_{ik}F_{mk}\right) + \varepsilon_{mi}$$

… is given

… are achieved as estimates

Fundamental theorem

What **is** achieved by factor analysis (from the *technical perspective*)?

- the factor loadings
- the number of substantial factors
- (event.) the factor scores
- (error estimates)

Procedural steps

The various methods contributing to explorative factor analysis have to be applied in successive steps.

Procedural steps

1. Computation of correlation matrix
2. Selection of extraction method
3. Estimation of communalities
4. Factor extraction
5. Determination of the number of factors
6. Factor rotation
7. Factor interpretation

Procedural steps

The research problem for illustrating factor analysis

There are six items *for assessing a person‘s mood* …..

… but it is *not known* whether these items represent *one attribute* that is in this case mood or different attributes.

1.Computation of correlation matix

Data

An example: *an item for assessing the persons‘ mood*

I am very angry	o
really angry	o
angry	o
somewhat angry	o
not angry	o

1.Computation of correlation matix

Data

An example: an item .... the coding

I am very angry	o	1
really angry	o	2
angry	o	3
somewhat angry	o	4
not angry	o	5

The following moods are considered …

angry, nervous, grieved, determined, strong, excited

1.Computation of correlation matix

An example:

	angry	nervous	grieved	determined	strong	excited
angry	1.000	.329	.346	.123	-.207	-.042
nervous	.329	1.000	.405	-.125	-.317	-.136
grieved	.346	.405	1.000	-.009	-.389	-.238
determine d	.123	-.125	-.009	1.000	.247	.310
strong	-.207	-.317	-.389	.247	1.000	.372
excited	-.042	-.136	-.238	.310	.372	1.000

Procedural steps

1. Computation of correlation matrix
2. *Selection of extraction method*
3. Estimation of communalities
4. Factor extraction
5. Determination of the number of factors
6. Factor rotation
7. Factor interpretation

2.Selection of extraction method

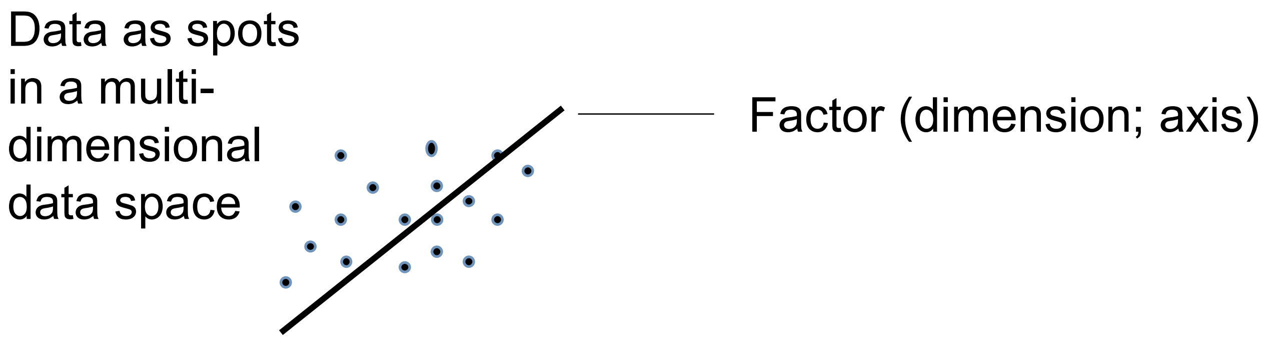
Extraction methods serve
… the **identification of the factors** (components) that are basic for describing / explaining the data (space)

(There is a tradition to consider the search for factors as seach for axes of a high-dimentional space)

2.Selection of extraction method

Extraction methods serve
… the **identification of the factors** (components) that are basic for describing / explaining the data

Illustration: spatial representation of data and factor



2.Selection of extraction method

Extraction methods serve
... the identification of the factors / components that are basic for describing / explaining the data (space)

(There is a tradition to consider the seearch for factors as seach for as axes of a high-dimentional space)

→ The mathematical method of factor extraction searches for the of axis in the data space that enables the factor *to account for as much **variance** as possible*

→ After one or several factor(s) have been detected, the search for the next factor follows the principle that this new factor has to account for the **successively largest amount of variance**

2.Selection of extraction method

There are two types of basic methods for factor extraction (or component extraction):

Component analysis

- observed data are without error
- identifies components
- theoretically the whole variance is explained

(PCA)

(actual) Factor analysis

- observed data include error
- identifies latent variables (reflecting constructs)
- explains only the true variance

(FA) Explorative factor analysis

2.Selection of extraction method

There are two types of basic methods for factor extraction:

Only (actual) factor analysis fits with the framework of *modeling latent variables*

Component analysis

- observed variables are without error
- theoretically the whole variance is explained

(actual) Factor analysis

- observed variable including error
- identifies latent

2.Selection of extraction method

There are two types of basic methods for factor extraction:

Only (actual) factor analysis fits with the framework of *modeling latent variables*

This means …

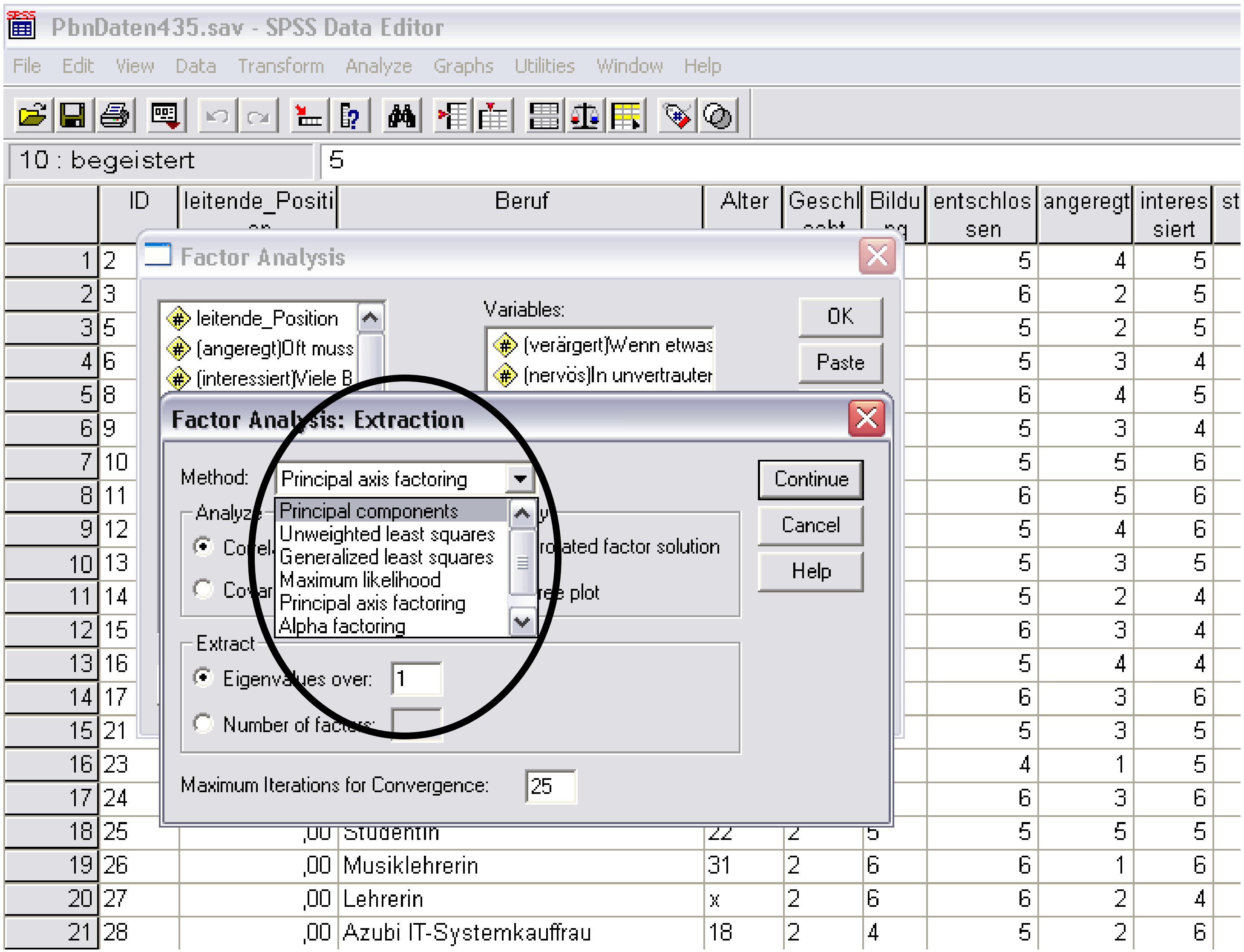
- **Factor extraction** has to be done by (**actual**) factor analysis
- Component analysis is only necessary for determining the number of factors (… is explained in another section)

2.Selection of extraction method

…. it means for the user to select one of the available methods for factor extraction.

Using SPSS it means …

2. … SPSS



Explorative factor analysis

Procedural steps

1. Computation of correlation matrix
2. Selection of extraction method
3. *Estimation of communalities*
4. Factor extraction
5. Determination of the number of factors
6. Factor rotation
7. Factor interpretation

3.Estimation of communalities

This step is necessary because the *true* (=*systematic*) *variance of the items* is **not known**!

- The true variance of the item is referred to as its

  communality
- A number of methods for estimating communality is available
- No one of them is considered perfect
- Research showed that the best estimates are obtainable in a iterative way

3.Estimation of communalities

Modern methods of factor analysis estimate communality in *iterative circles*:

- Initial estimates start the search for communalities
  → (mostly) squared multiple correlation
  of a variable with the other variables
  serve as initial estimates

3.SPSS Output

Example

Communalities

	Initial	Final
angry	.193	
nervous	.242	
grieved	.296	
determined	.165	
strong	.287	
excited	.207	

Extraction Method: Principal Axis Factoring.

3.Estimation of communalities

A *post-hoc* definition of communalities is given by ...

h_i膞 $\sum_{k=1}^{q} \lambda_{ik}$

h_i^2 : communality of item i
λ_{ik} : factor loading of item i on factor k
q : number of factors

- disadvantage: this formula is only applicable after the estimation of the factor loadings

3.Estimation of communalities

An example of using the equation:

Item number (variable)	Factor loading F_1	F_2	Squared Loading F_1	F_2	h^2
1	.30	.10	.09	.01	.10
2	.45	.05	.2025	.0025	.205
..	……				

3.Estimation of communalities

(Practice)

Find the communalities for

Item number	Factor loading F_1	F_2	h^2
1	.43	.09	.1848 + .0081 = .1929
2	.39	.05	.1521 + .0025 = .1546
3	.61	.13	.3721 + .0169 = .3890
4	.00	.35	....
5	.18	.41	

3.Estimation of communalities

A post-hoc definition of communalities is given by …

h_i膊 $\sum_{k=1}^{q} \lambda_{ik}$

h_i^2 : communality of item i
λ_{ik} : factor loading of item i on factor k
q : number of factors

- disadvantage: this formula is only applicable after the estimation of the factor loadings

- changes of factor loadings can lead to changes of communalities
- a search process including the modification of factor loadings can lead to larger communalities
- the aim of search is **the largest possible communalities**

Procedural steps

1. Computation of correlation matrix
2. Selection of extraction method
3. Estimation of communalities
4. *Factor extraction*
5. Determination of the number of factors
6. Factor rotation
7. Factor interpretation

4.Factor extraction

Factor extraction is a complex process

- *it includes*

(> ***the maximization of the communalities***)

> ***the estimation of factor loadings***

(> ***the determination of the number of factors***)

4.Factor extraction

Factor extraction is a complex process

- *it includes*

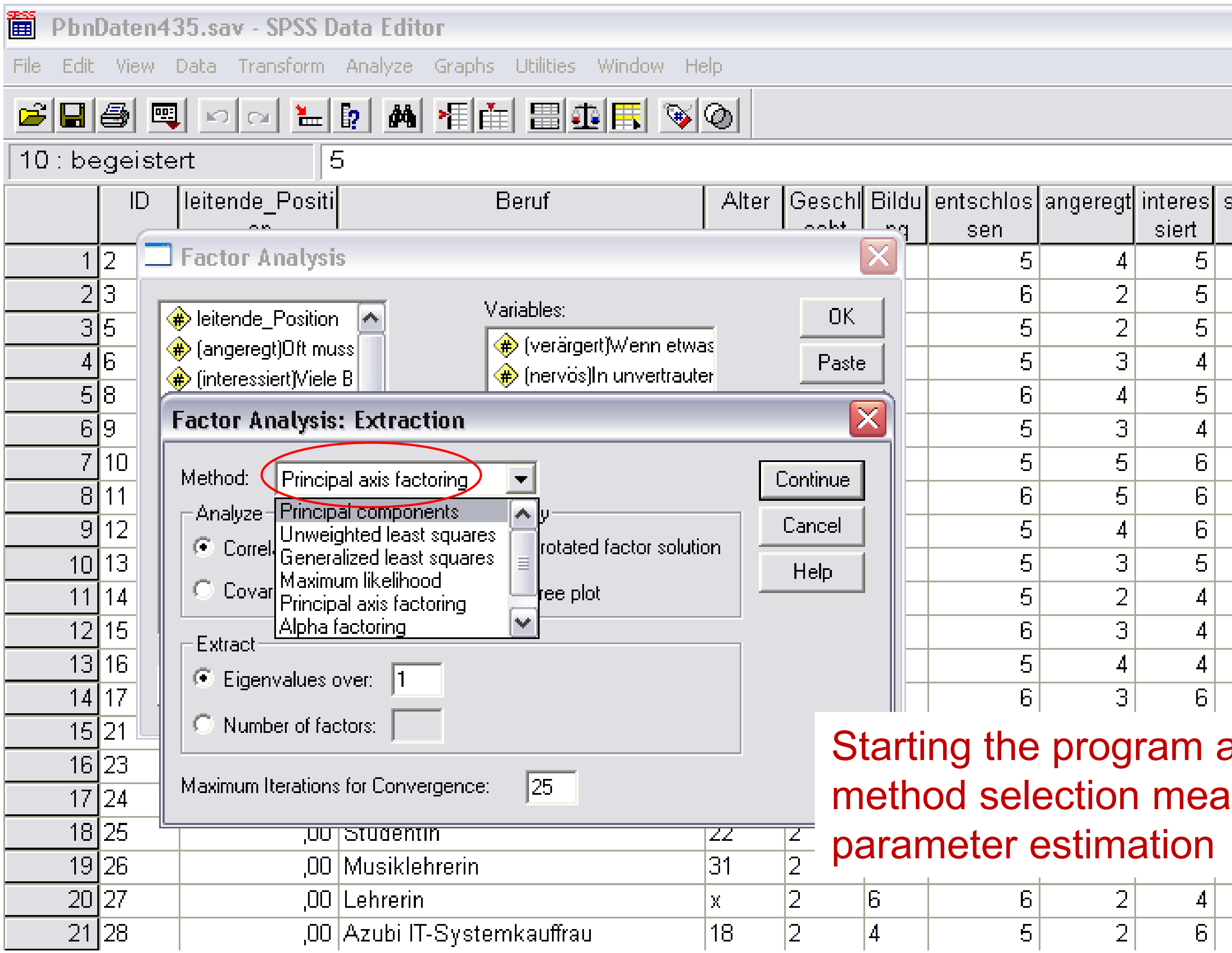
(> ***the maximization of the communalities***)

> ***the estimation of factor loadings***

(> ***the determination of the number of factors***)

It occurs iteratively (with the exception of the last one)

4. ... SPSS



Starting the program after the method selection means parameter estimation

4.Factor extraction

Factor extraction is designed as …

- *.. search for **estimates** of the parameters that are suited for reproducing* the correlation matrix **R**
- i.e. the parameters included in Λ (the factor loadings) have to be estimated
- i.e. reproduction means the calculation of the product of the factor loading matrix $\boxed{\Lambda\ (\Lambda\ \Lambda')}$ (plus the diagonal matrix of error components $\boldsymbol{\Theta}$) so that it is equal to the correlation matrix $\boxed{\mathrm{R}}$

$$\mathbf{R} = \Lambda\Lambda^{T} + \boldsymbol{\Theta}$$

4.Factor extraction

Factor extraction is designed as ...

- .. *reproduction* of the correlation matrix

e.g. assume the given correlation matrix:

$$R= \begin{bmatrix} 1.00 & .25 & .25 & .25 & .25 \\ .25 & 1.00 & .25 & .25 & .25 \\ .25 & .25 & 1.00 & .25 & .25 \\ .25 & .25 & .25 & 1.00 & .25 \\ .25 & .25 & .25 & .25 & 1.00 \end{bmatrix}$$

4.Factor extraction

Factor extraction is designed as …

- .. *reproduction* of the correlation matrix

e.g. assume that **0.25** as **communality** replaces the diagonal elements

$$R_h = \begin{bmatrix} .25 & .25 & .25 & .25 & .25 \\ .25 & .25 & .25 & .25 & .25 \\ .25 & .25 & .25 & .25 & .25 \\ .25 & .25 & .25 & .25 & .25 \\ .25 & .25 & .25 & .25 & .25 \end{bmatrix}$$

4.Factor extraction

Factor extraction is designed as …

- .. *reproduction* of the correlation matrix

… requires a vector $\underline{\boldsymbol{\lambda}}$ or a matrix $\underline{\boldsymbol{\Lambda}}$ so that …

$$R_h = \lambda \lambda^T \quad \text{or} \quad R_h = \Lambda \Lambda^T$$

T means transpose

(one factor case) (case of several factors)

4.Factor extraction

Factor extraction is designed as …

- .. *reproduction* of the correlation matrix

It is achieved by cross-multiplication:

$$\lambda = \begin{bmatrix} .5 \\ .5 \\ .5 \\ .5 \\ .5 \\ .5 \end{bmatrix} \times [\ .5\ .5\ .5\ .5\ .5\ ] = \lambda^T$$

.25

4.Factor extraction

Factor extraction is designed as …

- .. *reproduction* of the correlation matrix

e.g. reproduces

$$\lambda = \begin{bmatrix} .5 \\ .5 \\ .5 \\ .5 \\ .5 \\ .5 \end{bmatrix} \qquad R_h = \begin{bmatrix} .25 & .25 & .25 & .25 & .25 \\ .25 & .25 & .25 & .25 & .25 \\ .25 & .25 & .25 & .25 & .25 \\ .25 & .25 & .25 & .25 & .25 \\ .25 & .25 & .25 & .25 & .25 \end{bmatrix}$$

4.Factor extraction

Factor extraction is designed as ...

- .. *reproduction* of the correlation matrix

e.g. Communality Remainder

$$\lambda = \begin{bmatrix} .5 \\ .5 \\ .5 \\ .5 \\ .5 \\ .5 \end{bmatrix} \qquad R = \begin{bmatrix} .25+e & .25 & .25 & .25 & .25 \\ .25 & .25+e & .25 & .25 & .25 \\ .25 & .25 & .25+e & .25 & .25 \\ .25 & .25 & .25 & .25+e & .25 \\ .25 & .25 & .25 & .25 & .25+e \end{bmatrix}$$

4.Factor extraction

Factor extraction is designed as …

- .. *reproduction* of the correlation matrix

e.g. … for e=.75 (remainder):

$$\lambda = \begin{bmatrix} .5 \\ .5 \\ .5 \\ .5 \\ .5 \\ .5 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & .25 & .25 & .25 & .25 \\ .25 & 1 & .25 & .25 & .25 \\ .25 & .25 & 1 & .25 & .25 \\ .25 & .25 & .25 & 1 & .25 \\ .25 & .25 & .25 & .25 & 1 \end{bmatrix}$$

4.Factor extraction

Factor extraction is designed as …

- .. *reproduction* of the correlation matrix

It is achieved by **cross-multiplication**:

$$\times \quad [\ .5\ .5\ .5\ .5\ .5\ ] \quad = \lambda^T$$

$$\lambda = \begin{bmatrix} .5 \\ .5 \\ .5 \\ .5 \\ .5 \\ .5 \end{bmatrix}$$

.25

4.Factor extraction

Practice

Find the matrix that is reproduced by the following vector of factor loadings:

It is achieved by cross-multiplication:

$$\boldsymbol{\lambda} = \begin{bmatrix} .6 \\ .4 \\ .5 \end{bmatrix}$$

4.Factor extraction

Practice

Find the matrix that is repreduced by the following vector of factor loadings:

It is achieved by cross-multiplication:

$$\boldsymbol{\lambda} = \begin{bmatrix} .6 \\ .4 \\ .5 \end{bmatrix}$$

$$\lambda^{\mathrm{T}} = [.6\ .4\ .5]$$

$$\boldsymbol{\lambda} = \begin{bmatrix} .6 \\ .4 \\ .5 \end{bmatrix} \qquad \begin{matrix} .36 & .. & .. \\ .. & .. & .. \\ .. & .. & .. \end{matrix}$$

Practice

Find the matrix that is repreduced by the following vector of factor loadings:

Compute the missing numbers:

$$\boldsymbol{\lambda} = \begin{bmatrix} .6 \\ .4 \\ .5 \end{bmatrix}$$

$$\lambda^{T} = [.6\ .4\ .5]$$

$$\boldsymbol{\lambda} = \begin{bmatrix} .6 \\ .4 \\ .5 \end{bmatrix} \quad \boxed{\begin{matrix} .36 & .. & .. \\ .. & .. & \boxed{..} \\ \boxed{..} & .. & .. \end{matrix}}$$

4.Factor extraction

Factor extraction is designed as …

- … *reproduction* of the correlation matrix
- … with the re-estimation of communalities (and factor loadings) in an *iterative way*

4.Factor extraction

Factor extraction is designed as …

- … *reproduction* of the correlation matrix
- … with the re-estimation of communalities (and factor loadings) in an *iterative way*
- … repeated reproduction of the correlation matrix using modified parameters
- until a convergence criterion is reached.

4. Factor extraction

Factor extraction is designed as …

- …

A note.

- only in the case of several factors the communality shows major changes during iteration (see example with two factors).

4.SPSS Output

Communalities

	Initial	Final
angry	.193	.362
nervous	.242	.332
grieved	.296	.471
determined	.165	.336
strong	.287	.453
excited	.207	.348

Extraction Method: Principal Axis Factoring.

Before starting the iteration

After the end of the iteration

Enlargement of communality

4. Factor extraction

The final outcome of the factor extraction is the factor matrix

(here in the case of two factors):

Factor Matrix[a]

	Factor	
	1	2
angry	.426	.425
nervous	.553	.162
grieved	.649	.224
determined	-.256	.520
strong	-.651	.172
excited	-.454	.377

Extraction Method: Principal Axis Factoring.

a. 2 factors extracted. 10 iterations required.

4.SPSS Output

The matrix of factor loadings λ achieved in the end of factor extraction

High loadings

	Factor	
	1	2
angry	.426	.425
nervous	.553	.162
grieved	.649	.224
determined	-.256	.520
strong	-.651	.172
excited	-.454	.377

High loadings

Extraction Method: Principal Axis Factoring.

a. 2 factors extracted. 10 iterations required.

Procedural steps

1. Computation of correlation matrix
2. Selection of extraction method
3. Estimation of communalities
4. Factor extraction
5. *Determination of the number of factors*
6. Factor rotation
7. Factor interpretation

5. Determination of …

Number of factors

- Determination of the number of factors to be extracted
- The 3 most important methods
  1. Kaiser-Guttman criterion
  2. Scree test
  3. Parallel analysis

5. Determination of …

Number of factors

…………

Note.

eigen values play a major role;
"***eigen value***" is a term that is used to describe the amount of variance explained by a factor

5. Determination of …

Number of factors

…………

Note.

we distinguish between "***relevant eigen value***" and "***eigen value due to chance***"

5. Determination of …

Kaiser-Guttman criterion

- All factors with *eigen values* (*amount of explained variance*) larger 1 are retained
- If eigen value >1, then the corresponding factor explains more variance than a single variable

Problem:

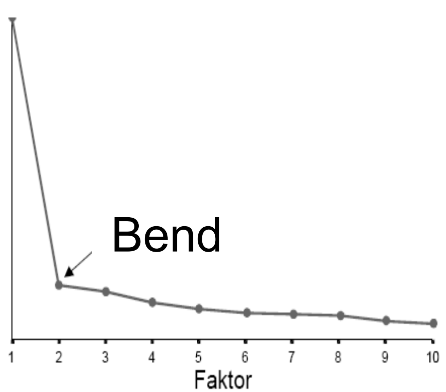
- Frequent over-estimation of number
- Factors at random can have eigen values >1

5. Determination of …

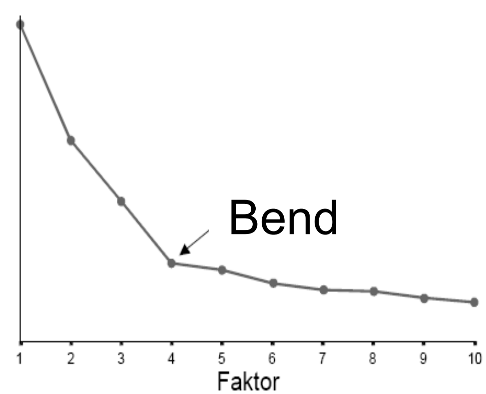
Scree test

- Sorting of factors according to eigen values
- Entry of eigenvalues in a diagram
- Relevant are all factors before the „bend“ point

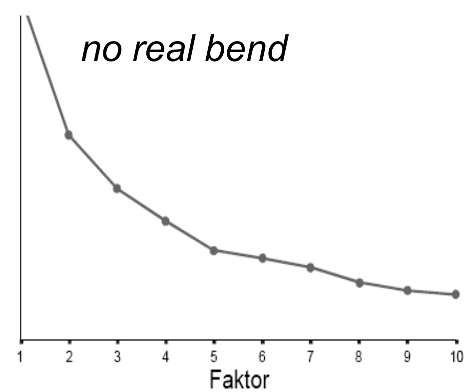
Ideal case



Clear case



Unclear case

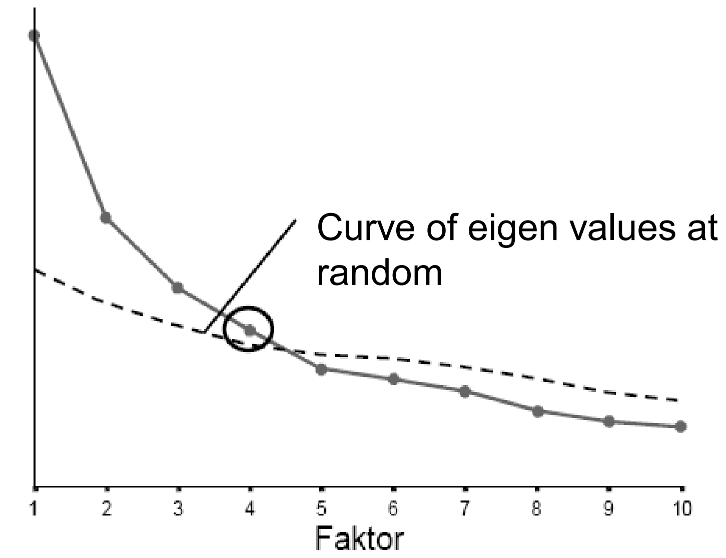


5. Determination of ...

Parallel analysis

- Best detection of eigen values due to chance
- (1) computation of eigen values at random, (2) entry into the diagram besides the observed eigen values

Determination of the number of factors: parallel analysis



Procedural steps

1. Computation of correlation matrix
2. Selection of extraction method
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6. *Factor rotation*
7. Factor interpretation

5.Rotation methods

Why is rotation necessary?

- The results of factor extraction usually leads to a ***bad interpretation***

- The literal **aim** of rotation is the achievment of *factors that enable a „good“ interpretation*

5.Rotation methods

- **Note.** If there are two or more factors virtually always the *reproduction* of the correlation matrix is possible by means of *different patterns of factor loadings*

- Rotation of factors in the factor space means transformation of factor loadings

5.Rotation methods

- **Note.** If there are two or more factors virtually always the *reproduction* of the correlation matrix is possible by means of *different patterns of factor loadings*

- Rotation of factors in the factor space means transformation of factor loadings

Formal goal of factor rotation (criterion)

- ***simple structure***

  of the pattern of factor loadings

5.Rotation methods

Principle of simple structure

… includes rules:

→each variable should show a **large** loading **on one** factor only and small loading on the other factors

→each factor should have **several** high loadings

5.Rotation methods

Principle of simple structure

This means .... (e.g.)

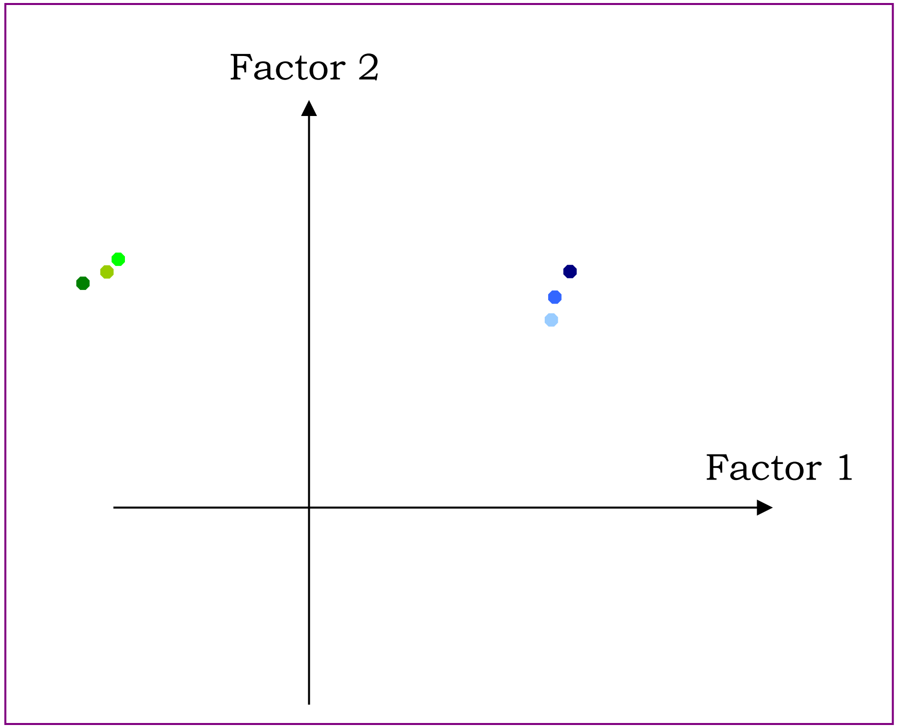
F1	F2	F3
xx	xx	xx
xx	xx	xx
xx	xx	xx
xx	xx	xx
xx	**xx**	xx
xx	**xx**	xx
xx	**xx**	xx
xx	xx	**xx**
xx	xx	**xx**
xx	xx	**xx**

5.Rotation methods

An example: illustration of how factor rotation changes the factors as axes of a two-dimensional space

5.Rotation methods

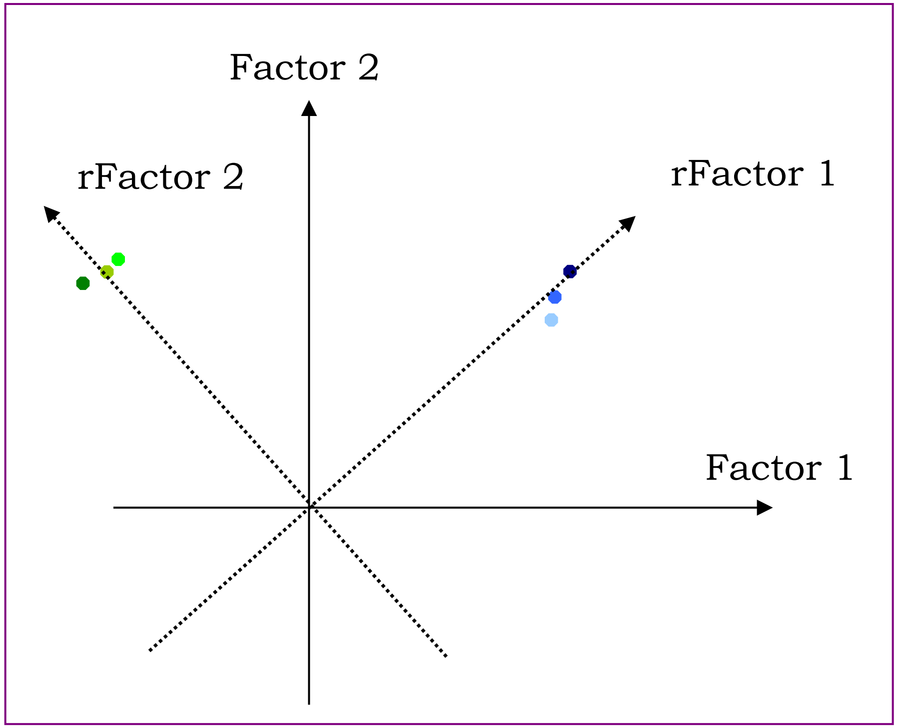
Situation after factor extraction



Each factor explains as much variance as possible.

5.Rotation methods

Situation after factor rotation



5.Rotation methods

Orthogonal rotation procedures

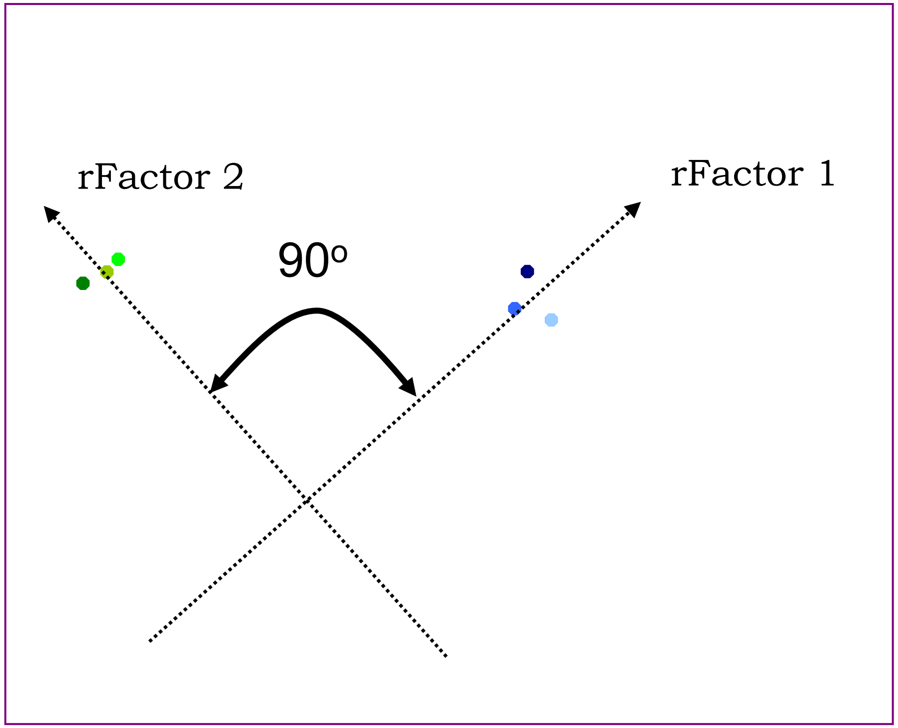
- Uncorrelated original factors
- Rotated factors are uncorrelated with each other

Varimax rotation (prominent example)

- Sum of the squared factor loadings λ^2 within each factor is maximized

5.Rotation methods

Situation after factor rotation



5.Rotation methods

Oblique rotation procedure

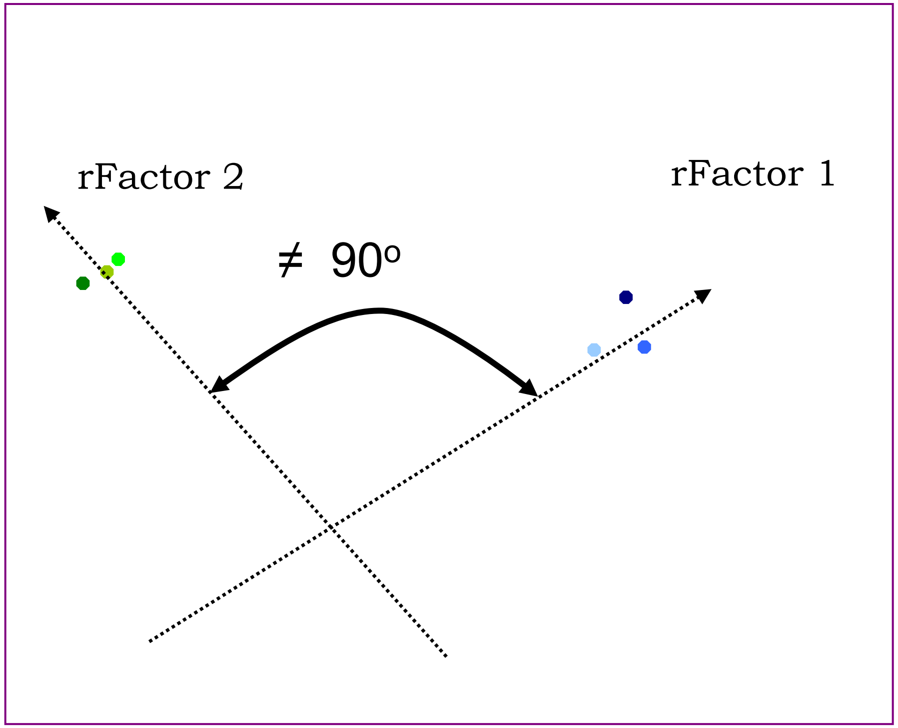
- After rotation factors are no more uncorrelated

Oblimin rotation (prominent example)

- Simultaneous optimization of an orthogonal and an oblique rotation criterion
- The influences of the two criteria can be modified by selecting weights; they influence the resulting correlation of factors

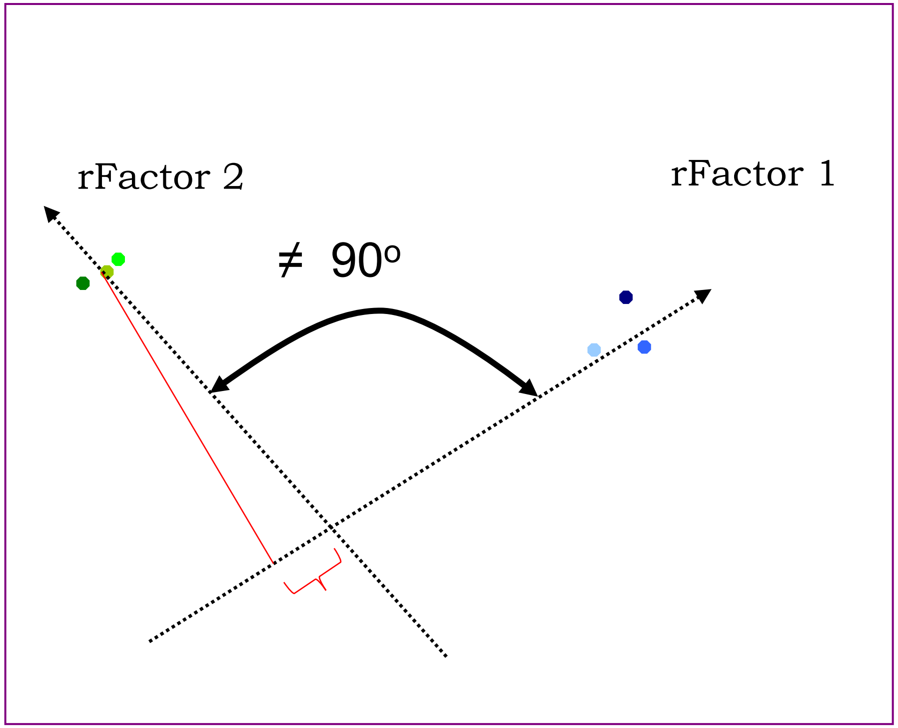
5.Rotation methods

Situation after factor rotation (Version 1)



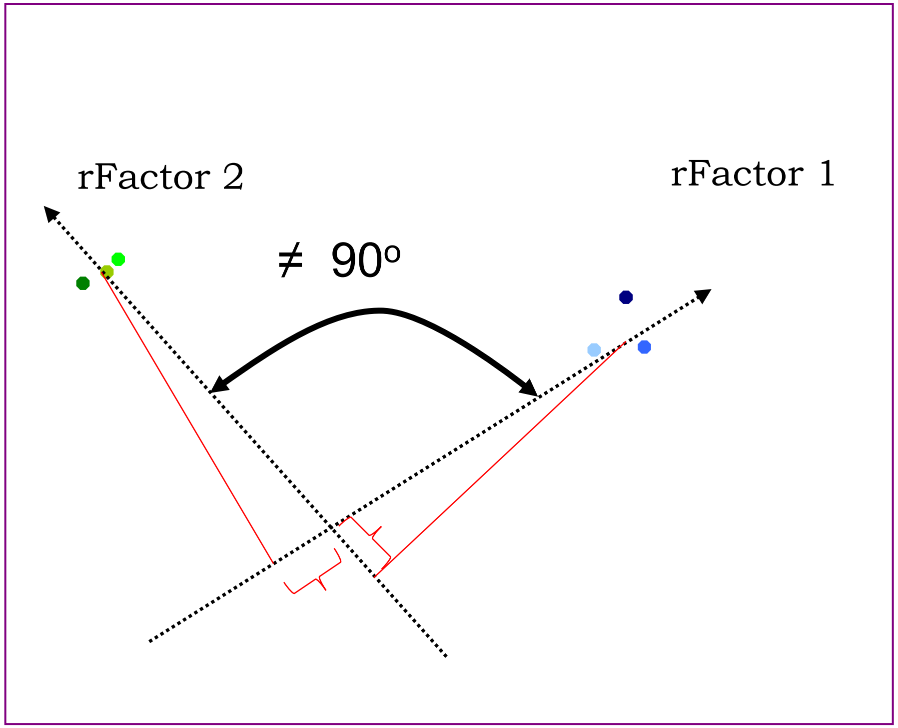
5.Rotation methods

Situation after factor rotation (Version 1)



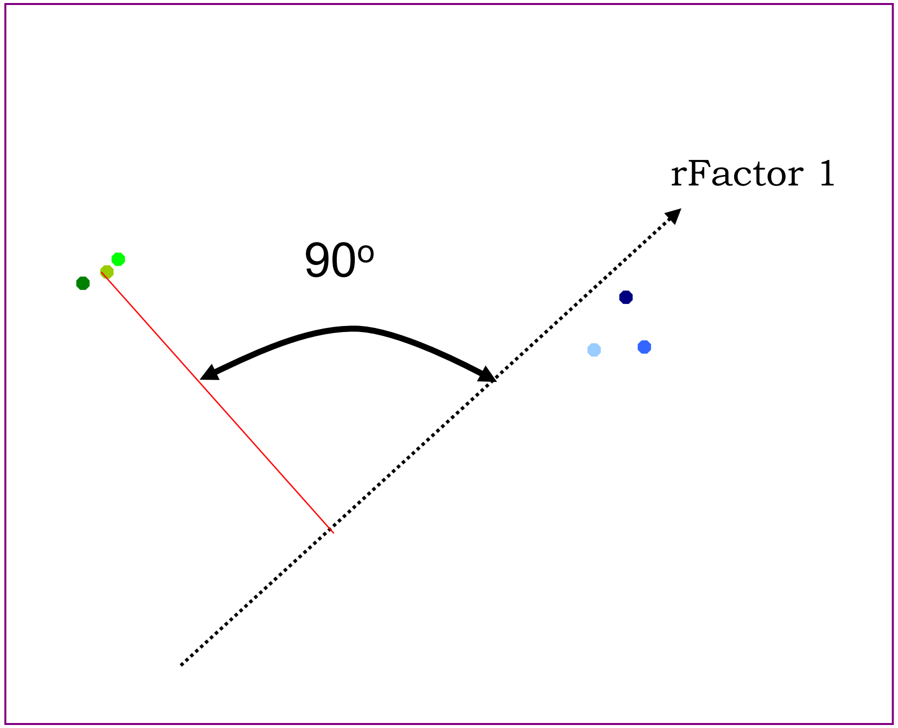
5.Rotation methods

Situation after factor rotation (Version 1)



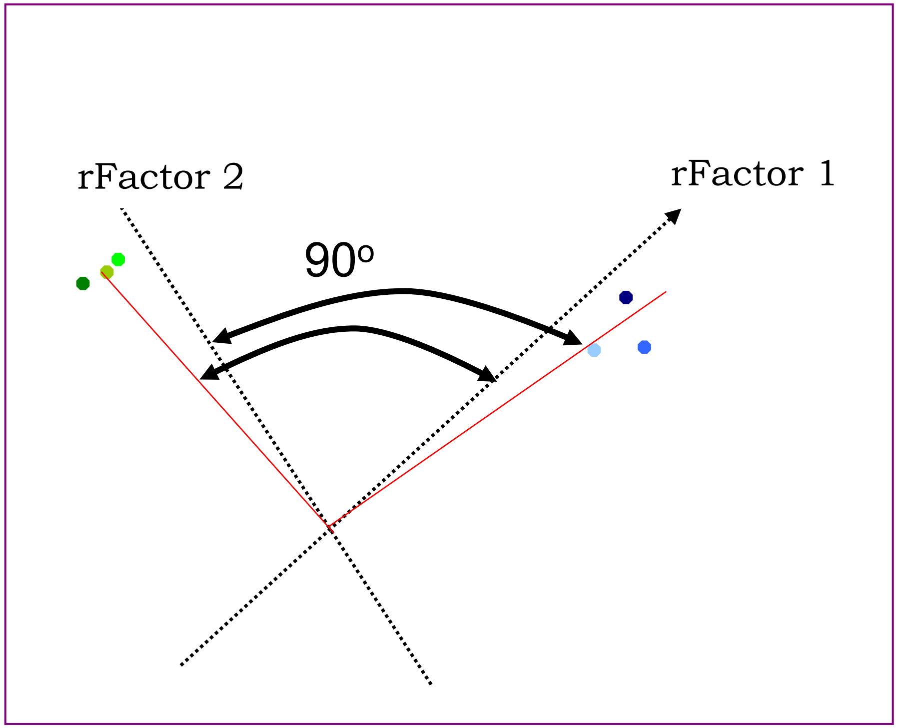
5.Rotation methods

Situation after factor rotation
(Version 2)



5.Rotation methods

Situation after factor rotation (Version 2)



5.SPSS Output

Evaluation regarding **cross-loadings**:

Factor Matrix[a]

	Factor	
	1	2
angry	.426	.425
nervous	.553	.162
grieved	.649	.224
determined	-.256	.520
strong	-.651	.172
excited	-.454	.377

Extraction Method: Principal Axis Factoring.

a. 2 factors extracted. 10 iterations required.

Rotated Factor Matrix[a]

	Factor	
	1	2
angry	.590	.115
nervous	.550	-.173
grieved	.663	-.175
determined	.077	.574
strong	-.445	.505
excited	-.167	.566

Extraction Method: Principal Axis Factoring.
Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 3 iterations.

5.SPSS Output

Evaluation regarding **highest loadings**:

Factor Matrix[a]

	Factor	
	1	2
angry	.426	.425
nervous	.553	.162
grieved	.649	.224
determined	-.256	.520
strong	-.651	.172
excited	-.454	.377

Extraction Method: Principal Axis Factoring.

a. 2 factors extracted. 10 iterations required.

Rotated Factor Matrix[a]

	Factor	
	1	2
angry	.590	.115
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Extraction Method: Principal Axis Factoring.
Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 3 iterations.

Procedural steps

1. Computation of correlation matrix
2. Selection of extraction method
3. Estimation of communalities
4. Factor extraction
5. Determination of the number of factors
6. Factor rotation
7. *Factor interpretation*

6.Factor interpretation

- Assigning meaning to factors
- Interpretion of the items sets
- A interpretation has to be based on the meanings of the items showing large loading on the factor

6.SPSS Output

Rotated Factor Matrix[a]

	Factor	
	1	2
angry	.590	.115
nervous	.550	-.173
grieved	.663	-.175
determined	.077	.574
strong	-.445	.505
excited	-.167	.566

Extraction Method: Principal Axis Factoring.
Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 3 iterations.

?

+

?

6.Factor interpretation

Factor 1:

- angry
- nervous
- grieved
- weak (= - strong)

Factor 2:

- determined
- strong
- excited

What are the common categories?

6.Factor interpretation

Factor 1:

- angry
- nervous
- grieved
- weak (= - strong)

Emotionality (?)

Factor 2:

- determined
- strong
- excited

Self-confidence (?)

A note.

$$z_{mi} = \lambda_{i1} F_{m1} + \lambda_{i2} F_{m2} + \ldots + \lambda_{ip} F_{mp} + \varepsilon_{mi} = \sum_{k=1}^{p} \left(\lambda_{ik} F_{mk} \right) + \varepsilon_{mi}$$

A standardized score is described as a linear combination of factor scores, of their weights and an error component

<u>Explanation of symbols:</u>

m: Person

i : Item

λ: Factor loading

F: Factor score

ε: Error component

A note

Furthermore, after the factors are identified there is the possibility to compute the factor scores of the participants (see fundamental equation).

$$\hat{F}_{mk} = \beta_{k1} z_{m1} + \ldots + \beta_{kp} z_{mp}$$

Explanation of symbols:

***F*: Factor score**

m: Person

k : Ability/Trait (Factor)

β: Regression weight

ε: Error component

z: Standardized score

A humorous characterization

of what a researcher does when he/she has no idea of what to do with many correlations between scales

"I'm not sure what is here and there are too many measures to make sense of, so let's do a factor analysis and reduce the measures to a more restricted set of variables and see what emerges"

Summary and brush up:

Explorative factor analysis

- Some basics What is the basic idea of explorative factor analysis?
  … search for (underlying) dimensions
- Fundamental theorem … formal representation of assumptions
- Computational steps … communality estimation , factor extraction, rotation, etc.
- Communality estimation ... estimation of explained variance
- Extraction methods … methods employed identifying factors
- Determination of number of factors … criteria for finding the number of to be extracted factors
- Method of factor rotation …orthogonal and oblique rotation methods, simple structure principle
- Factor interpretation … find the common categories for factors

QUESTIONS REGARDING COURSE UNIT 4

- What are the purposes for selecting FA?
- What type of variance is represented by the communality?
- Why is factor rotation conducted?
- What is the principle of factor rotation?

Literature

Brown, T. A. (2006) *Confirmatory factor analysis* (S. 1-11, 40-76). New York, NJ: The Guilford Press