

# Important concepts and definitions:

Notation, vectors, matrices and structure of models

# Outline

- 0. Introductory remarks
- 1. Formation of hypotheses
- 2. Path diagram and the specification of models
- 3. Identification of the structure of models

# 0.Introductory remarks

**This course unit is on the *formal foundation* of structural equation modeling and on the *preparation for the statistical investigation***

- **it reaches out to the first stage of an investigation**
- **it assures that the statistical investigation is in line with the plan that so far guided the research**

# 0.Introductory remarks

## **Research means**


- **creating new knowledge**
- **providing answers to research questions**
- **providing empirical evidence in favor or against hypotheses**
- **( comparing controversial ideas / theories )**

# 0. Introductory remarks

## Research means

- creating new knowledge
- providing answers to research questions
- providing empirical evidence in favor or against hypotheses
- ( comparing controversial ideas / theories )

## Conducting empirical research means ...

- selecting a research question / hypothesis
  - designing a research plan
  - data collection
  - statistical investigation
  - drawing conclusions
- 
- Structural equation modeling*

# 0.Introductory remarks

**These steps of empirical research must show *consistency*!**

# 0. Introductory remarks

These steps of empirical research must show *consistency*!

- formation of research question (hypothesis)



(prepares for)

- designing a research plan



(prepares for)

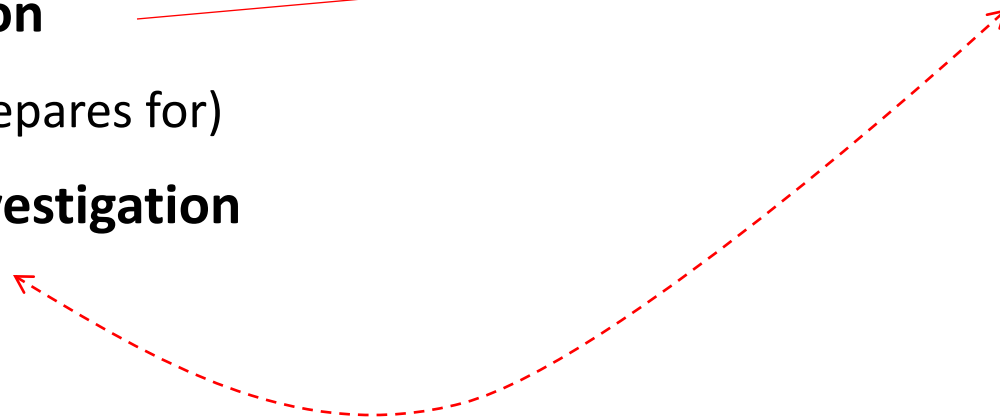
- data collection



(prepares for)

- statistical investigation

***Structural equation modeling***



# 0.Introductory remarks

**These steps of empirical research must show *consistency*!**

*Otherwise:* structural equation modeling may *not be* possible!

*For example:* there may not be enough indicator variables  
(there must be at least two of them)



# 0.Introductory remarks

**Variables are important ingredients of the formal foundation**

- there are different types
- these types have different roles
- they should not be confused

# 0.Introductory remarks

- Manifest variables** - variables that refer to data observed by means of measurement scales applied to a sample ( directly observable)
- Latent variables** - variables that are hypothetical in nature and are not directly observable
- access to these variables is possible by means of indicators (that are mostly manifest variables)
- Endogenous variables** - to be explained variables of the statistical model (criterion variables; dependent variables)
- Exogenous variables** - variables used for explaining the criterion in the statistical model ( predictors  $x_1....x_p$  ; independent variables)

# 1. Formation of hypotheses

- 1. Formation of hypotheses**
2. Path diagram and specification of model
3. Identification of the structure of the model
4. Estimation of parameters
5. Evaluation of the results
6. (modification of the structure of the model)

# 1. Formation of hypotheses

The formation requires being concerned with ...

- research questions
- hypotheses
- constructs
- indicators
- associations of constructs and indicators

# 1. Formation of hypotheses

The formation usually starts with a research question!

- the researcher selects a topic that he/she perceived as interesting/important ...

# 1. Formation of hypotheses

**Typical research questions that can be investigated in the factor-analytic framework of SEM ...**

... are questions regarding a relationship:

*Is extraversion related to professional success?*

... are questions concerning a differential construct:

*Is extraversion an empirical (= measurable) attribute?*

# 1. Formation of hypotheses

**Typical research questions that can be investigated in the factor-analytic framework of SEM ...**

... are questions regarding a relationship:

*Is extraversion related to professional success?*

... are questions concerning a differential construct:

*Is extraversion an empirical (= measurable) construct?*

**But questions regarding effects are excluded:**

*Does noise impair performance in completing cognitive tests?*

# 1. Formation of hypotheses: examples

## Research question 1:

Is the conscientiousness of a person related to the professional success of this person?

Conscientiousness *means* being careful in working / completing something / etc.

## Research question 2:

Is the work motivation of a person related to the professional reputation of this person?

Work motivation *means* how the person feels pressured to do a good job out of itself.



# 1. Formation of hypotheses: examples

There is a tradition to conduct research on the basis of hypotheses instead of research questions.<sup>1,2,3</sup>

Therefore, the research question has to be transformed into a hypothesis in the first step.

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Reasons:

- (1) the research question reflects the motivation of the researcher that has to be eliminated
- (2) The question can be answered by a Yes or No response; the hypothesis may demand a more differentiated outcome
- (3) Knowledge has the structure of a statement but not a question.

# 1. Formation of hypotheses: examples

- **H1:** The more conscientious a person is, the larger the professional success of this person.

Is the conscientiousness of a person related to the professional success of this person?

- **H2:** The larger the work motivation of a person is, the larger the professional reputation of this person.

Is the work motivation of a person related to the professional reputation of this person?

# 1. Formation of hypotheses: examples

## **A note.**

In psychological research we have a new tend regarding the research strategy.

- old strategy: investigation of a single hypothesis
- new strategy: consideration of several hypotheses and comparing them

# 1. Formation of hypotheses: examples

- **H1:** The more conscientious a person is, the larger the professional success of this person.  
(**Alternative hypothesis 1:** there is a linear relationship between conscientiousness and professional success)  
(**Alternative hypothesis 2:** there is a logistic relationship between conscientiousness and professional success)  
(**Alternative hypothesis 3:** there is no relationship between conscientiousness and professional success)
- **H2:** The larger the work motivation of a person is, the larger the professional reputation of this person.  
(**Alternative hypothesis:** there is no relationship between work motivation and reputation)

# 1. Formation of hypotheses: examples

## The identification of the relevant constructs:

- **H1:** The more conscientious a person is, the larger the professional success of this person is.
- **H2:** The larger the work motivation of a person is, the larger the professional reputation of this person.

... meaning conscientiousness

The constructs have to be represented by latent variables.

# 1. Formation of hypotheses: examples

## **The identification of the relationships:**

- **H1:** The more conscientious a person is, the larger the professional success of this person is.
- **H2:** The larger the work motivation of a person is, the larger the professional reputation of this person.

The relationships have to be identified  
... and the direction.

# 1. Formation of hypotheses: examples

## The selection of indicators:

- Conscientiousness is, for example, measured by NEO PI-R and IPIP NEO.
- ...

Revised NEO Personality  
Inventory

International Personality  
Item Pool

Questionnaires requiring self-description

# 1. Formation of hypotheses: examples

**The state:** what is available ...

- hypotheses
- constructs ...
- structure of model
- indicators



## 2.Path diagram and specification of model

1. Formation of hypothesis
- 2. Path diagram and specification of models**
3. Identification of the structure of the model
4. Estimation of parameters
5. Evaluation of the results
6. (modification of the structure of the model)

## 2.Path diagram and specification of model

A path diagram is an illustration of a model that prepares for the mathematical description

Such a model shows a complex structure

## 2. Path diagram and specification of model

### Components of a complete structural equation model (three parts):

- **Structural model:**  
... includes the theoretically assumed relationships between latent variables  
→ Regression analysis / Path analysis
- **Models of measurement** of the latent variables:
  - includes the indicators originating from the operationalization of the constructs
  - is specified according to the assumed relationships between the indicators and the latent variables  
→ Confirmatory factor analysis

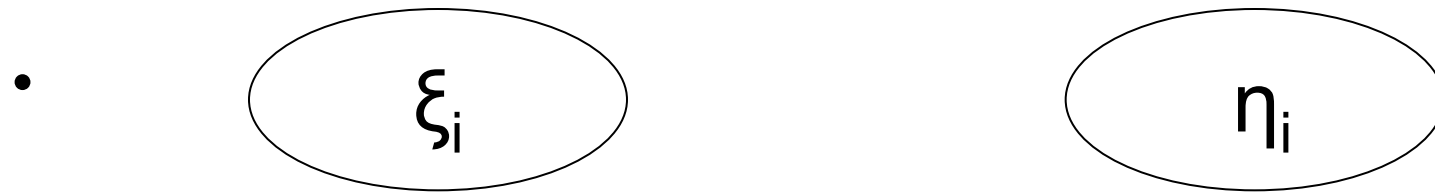
## 2. Path diagram and specification of model

**A complete structural equation model includes three parts:**

- **Structural model:**  
... includes the theoretically assumed relationships between latent variables  
→ Regression analysis / Path analysis
- **Models of measurement** of the latent variables:
  - one for the *endogenous part* of the complete structural equation model
  - one for the *exogenous part* of the complete structural equation model

## > Composition of path diagram (1)

- The latent variables are depicted as ellipses and accompanied by lower case greek letters.



- The manifest variables are depicted as rectangles and accompanied by lower case latin letters.



## > Composition of path diagram (2)

- Directed relationships are depicted by arrows:



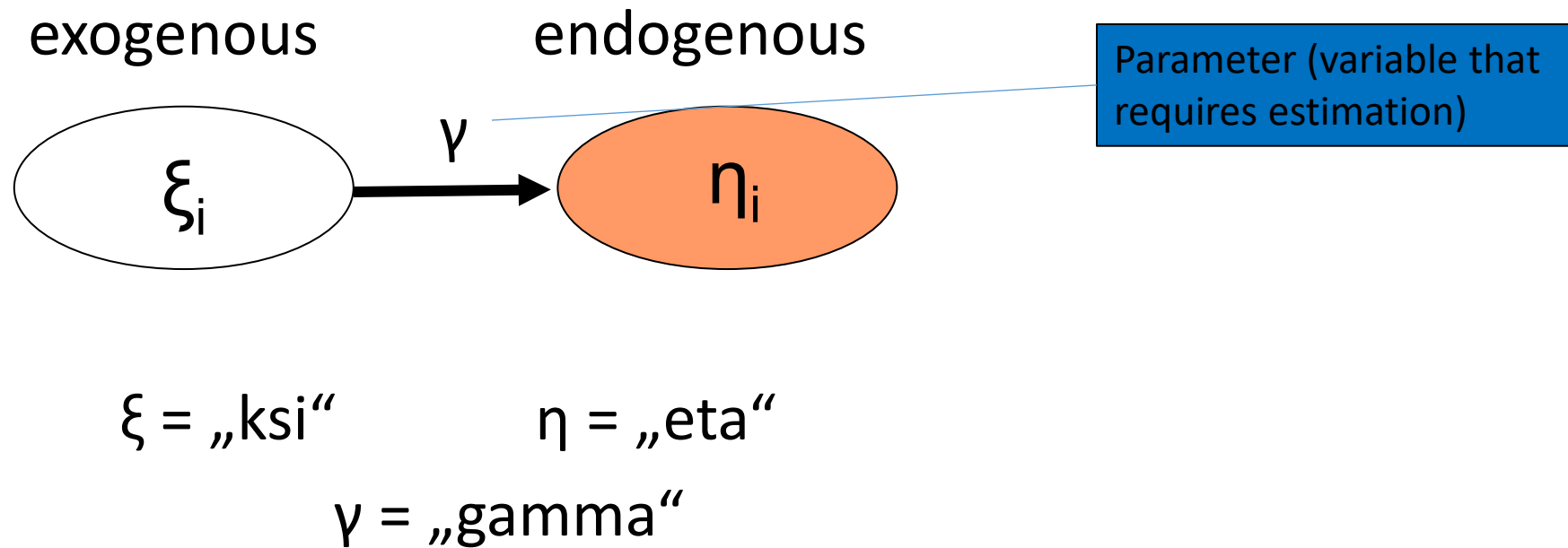
The arrow head points to the variable that is influenced.

- Non-directed relationships are depicted by bended double-headed arrows:



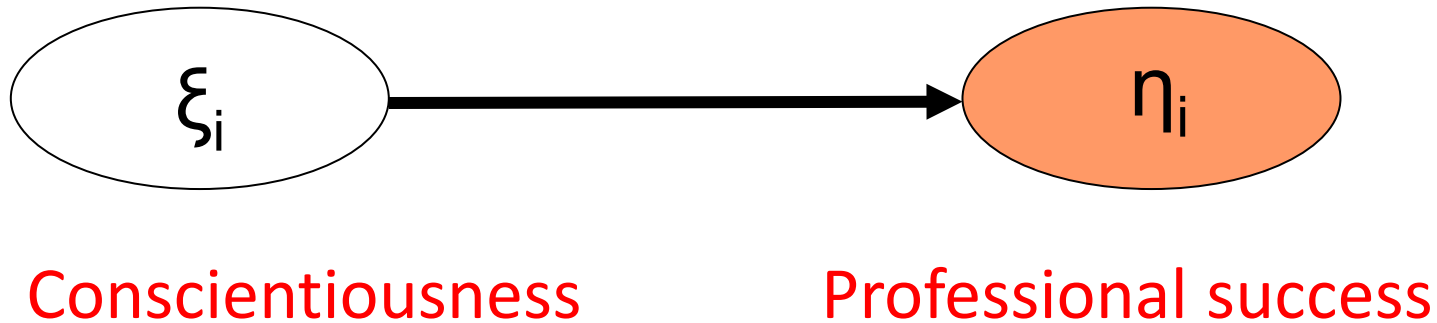
# > Structural model

Illustration of the relationships between the latent variables:



## > Structural model

Illustration of the relationships between the latent variables with example (H1):





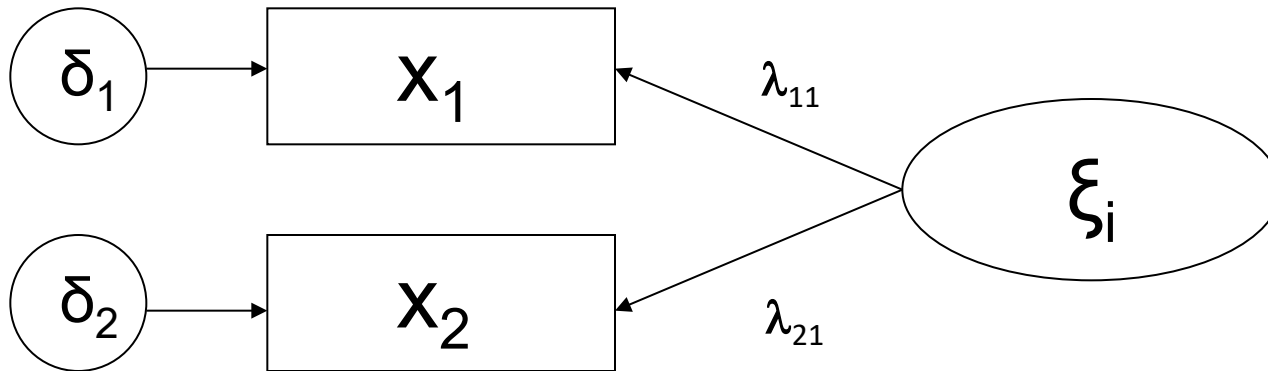
## > Structural model

Illustration of the relationships between the latent variables with example (H1):



## > Model of measurement

Operationalization of latent variable (exogenous part):

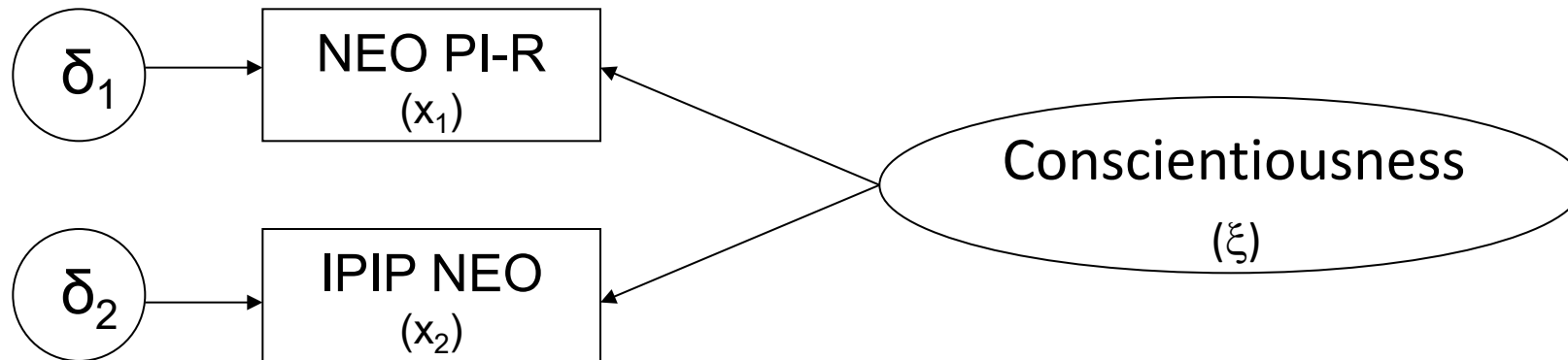


$\delta$  = „delta“

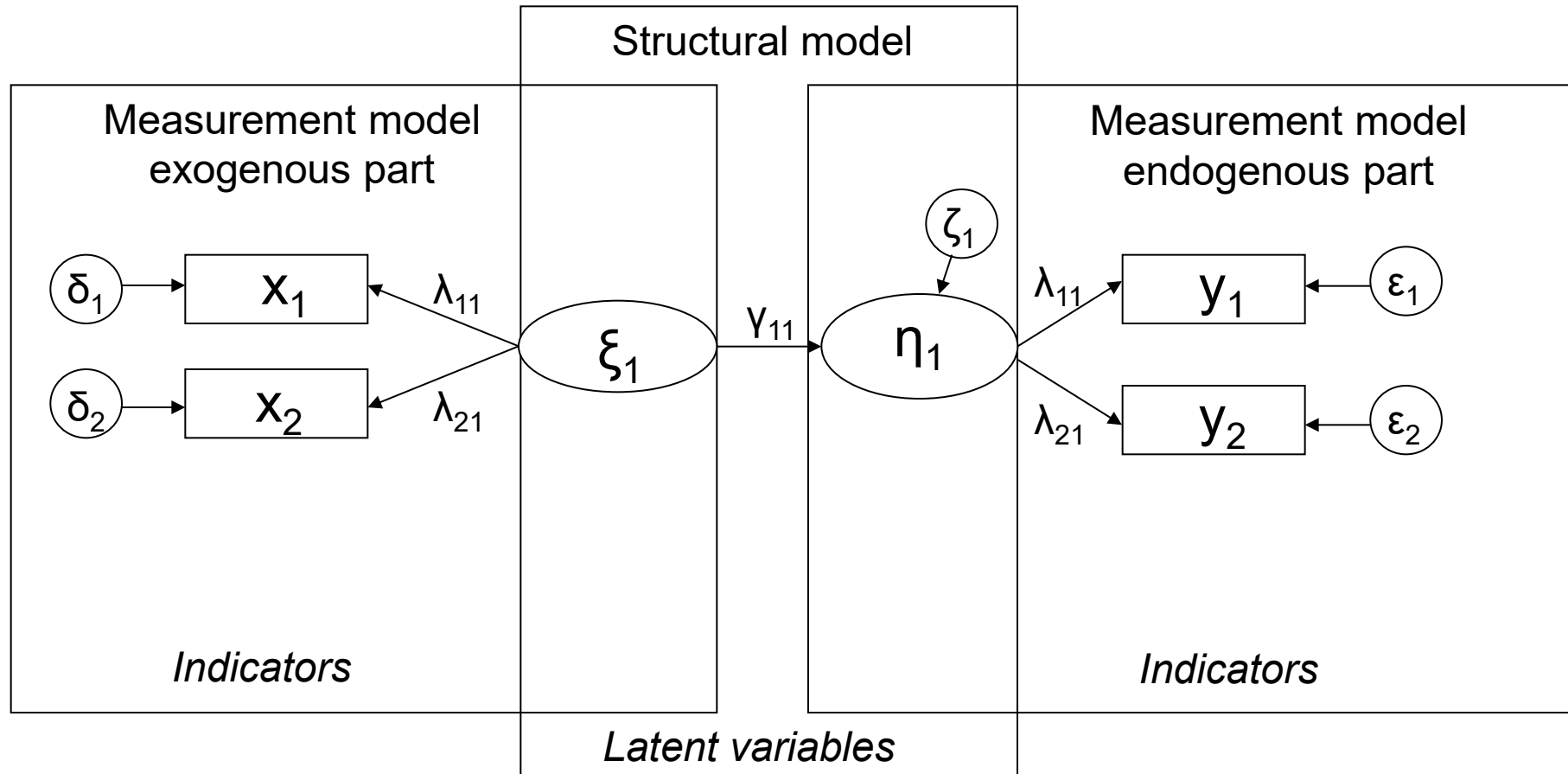
$\lambda$  = „lambda“

## > Model of measurement

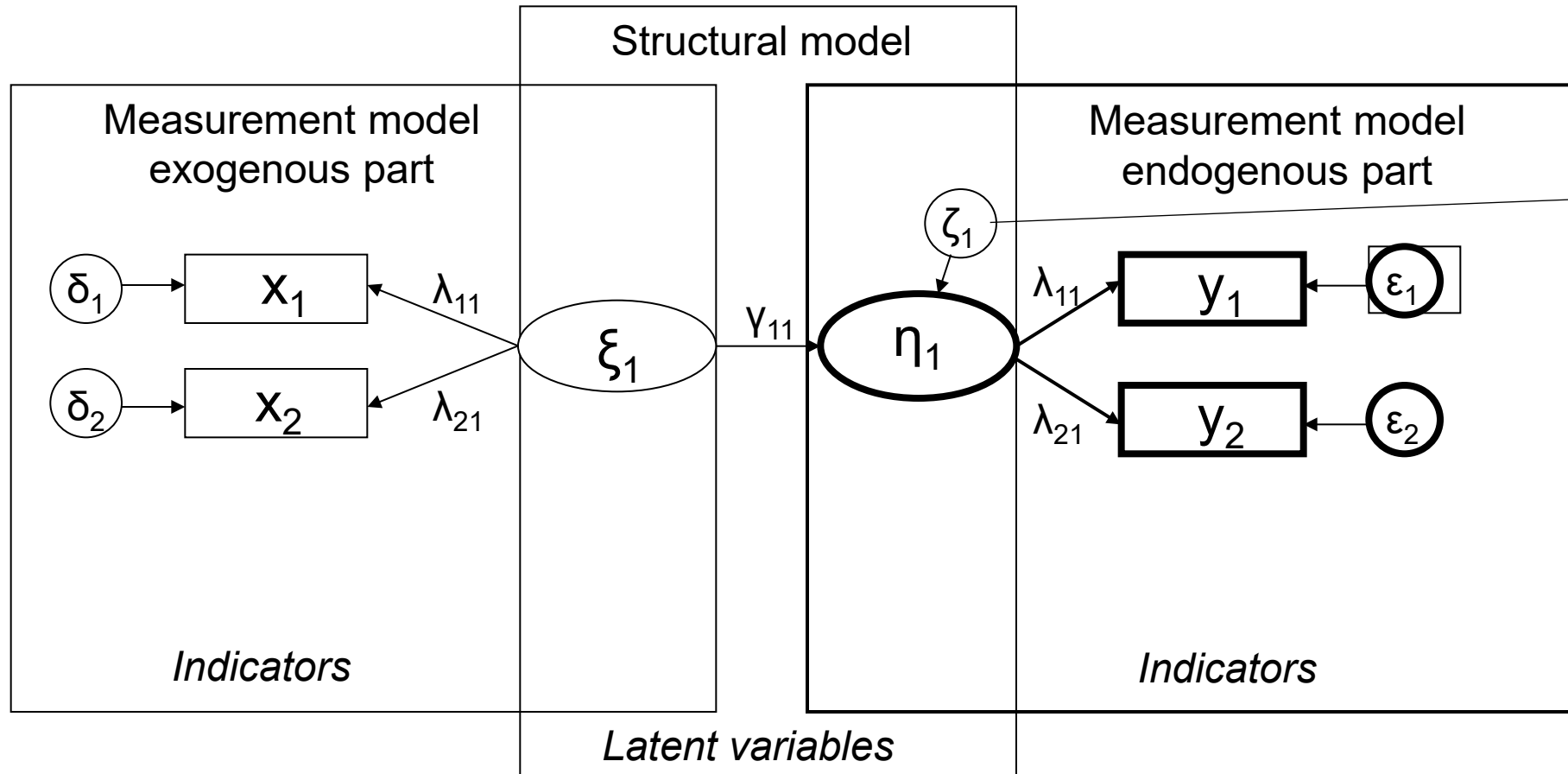
Operationalization of latent variable with example (exogenous part):



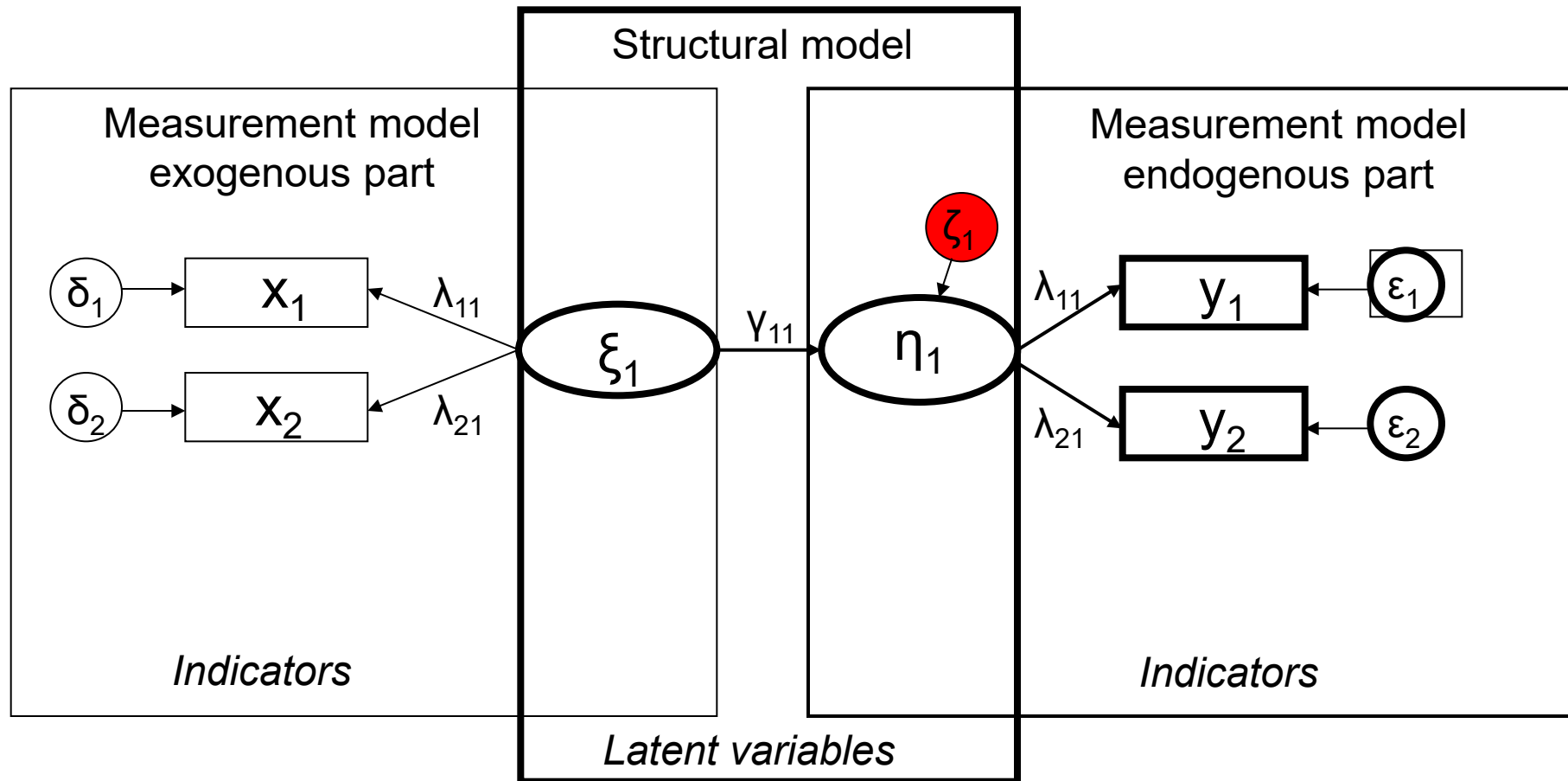
# > Complete structural equation model



# > Complete structural equation model



# > Complete structural equation model



Since  $\xi_1$  does normally not completely predict  $\eta_1$  there is a remainder that is represented by  $\zeta$

# > Notation

$\xi_i$	Ksi	Latent <i>exogenous</i> variable
$\eta_i$	Eta	Latent <i>endogenous</i> variable
$\zeta_i$	Zeta	Residual variable associated with latent <i>endogenous</i> variable
$\gamma_i$	Gamma	Path coefficient regarding the relationship of <i>exogenous and endogenous</i> latent variables
$\beta_{ij}$	Beta	Path coefficient regarding two <i>endogenous</i> latent variables

Alternative denotation „**regression weight**“

$x_i$		Indicator variable for <i>exogenous</i> variable
$\lambda_{ij}$	Lambda	Path coefficient between observed indicators $x_i$ and latent variable $\xi_j$
$\delta_i$	Delta	Residual variable for an <i>exogenous</i> Indikator variable

$y_i$		Indicator variable for <i>endogenous</i> variable
$\lambda_{ij}$	Lambda	Path coefficient between observed indicators $y_i$ and latent variablen $\eta_j$
$\varepsilon_i$	Epsilon	Residual variable for an <i>endogenous</i> Indikator variable

Alternative denotation „**factor loading**“

# > Notation

$\xi_i$	Ksi	Latent <i>exogenous</i> variable
$\eta_i$	Eta	Latent <i>endogenous</i> variable
$\zeta_i$	Zeta	Residual variable associated with latent <i>endogenous</i> variable
$\gamma_i$	Gamma	Path coefficient regarding the relationship of <i>exogenous and endogenous</i> latent variables
$\beta_{ij}$	Beta	Path coefficient regarding two <i>endogenous</i> latent variables

Parameters

$x_i$		Indicator variable for <i>exogenous</i> variable	$y_i$		Indicator variable for <i>endogenous</i> variable
$\lambda_{ij}$	Lambda	Path coefficient between observed indicators $x_i$ and latent variable $\xi_i$	$\lambda_{ij}$	Lambda	Path coefficient between observed indicators $y_i$ and latent variable $\eta_i$
$\delta_i$	Delta	Residual variable for an <i>exogenous</i> Indicator variable	$\varepsilon_i$	Epsilon	Residual variable for an <i>endogenous</i> Indicator variable



## > Notation regarding matrices and vectors

More complex structural equation models require the use of vectors and matrices instead of individual variables and parameters!

# > Notation regarding matrices and vectors

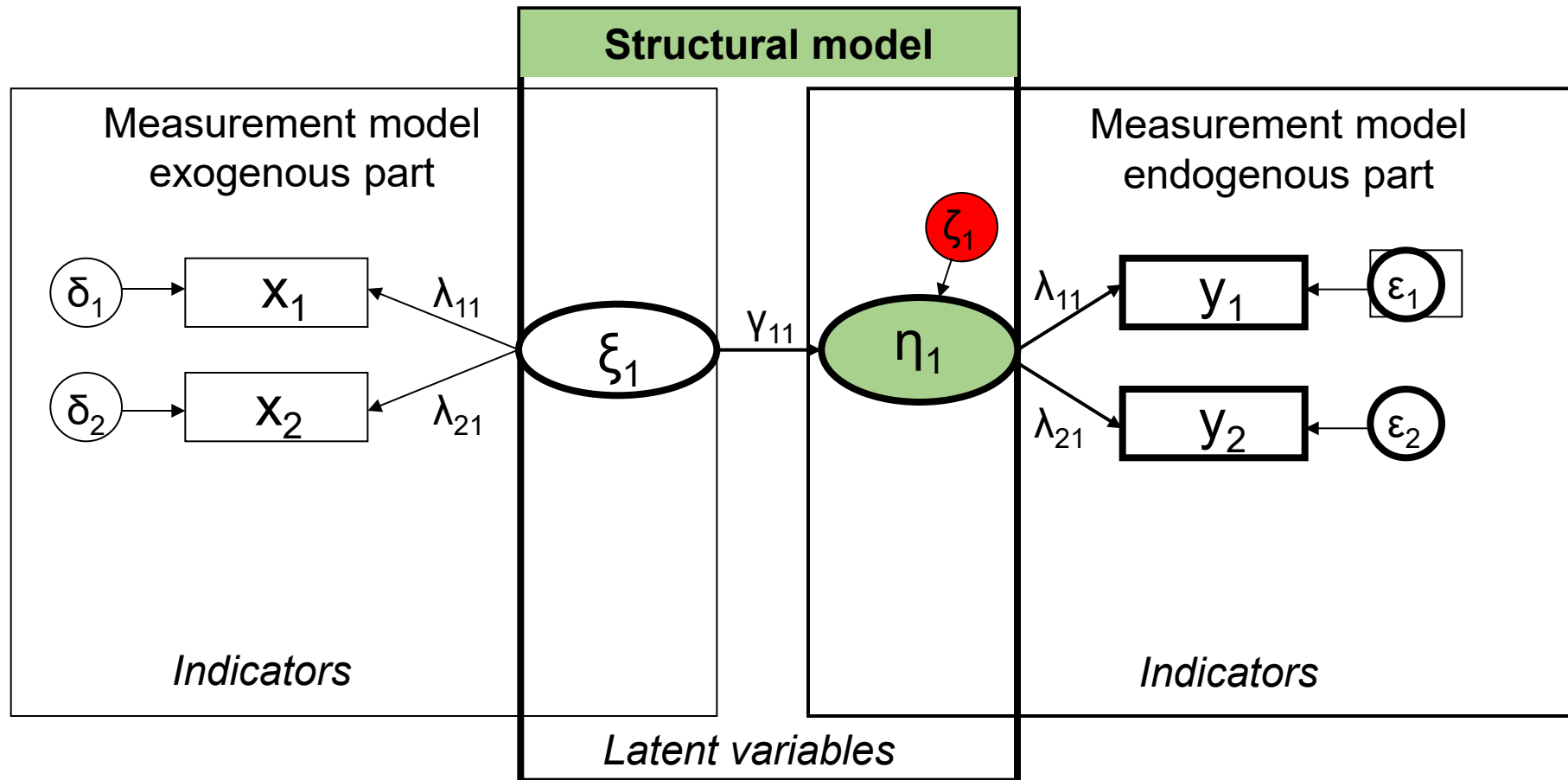
$$\mathbf{v}_{(p \times 1)} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_p \end{bmatrix}$$

$$\mathbf{M}_{(p \times q)} = \begin{bmatrix} m_{11} & m_{1q} \\ m_{21} & \\ & \\ m_{p1} & m_{pq} \end{bmatrix}$$

*Printed bold!*

> Switch to the formal (mathematical) model

> Switch to the formal (mathematical) model



## > Switch to the formal (mathematical) model

- Equation of measurement model of latent *exogenous* variables:

$$\eta_1 = \gamma_{11} \times \xi_1 + \zeta_1$$

Generalization



# > Switch to the formal (mathematical) model

Equation of structural model:

$$\eta = \Gamma \times \xi + \zeta$$

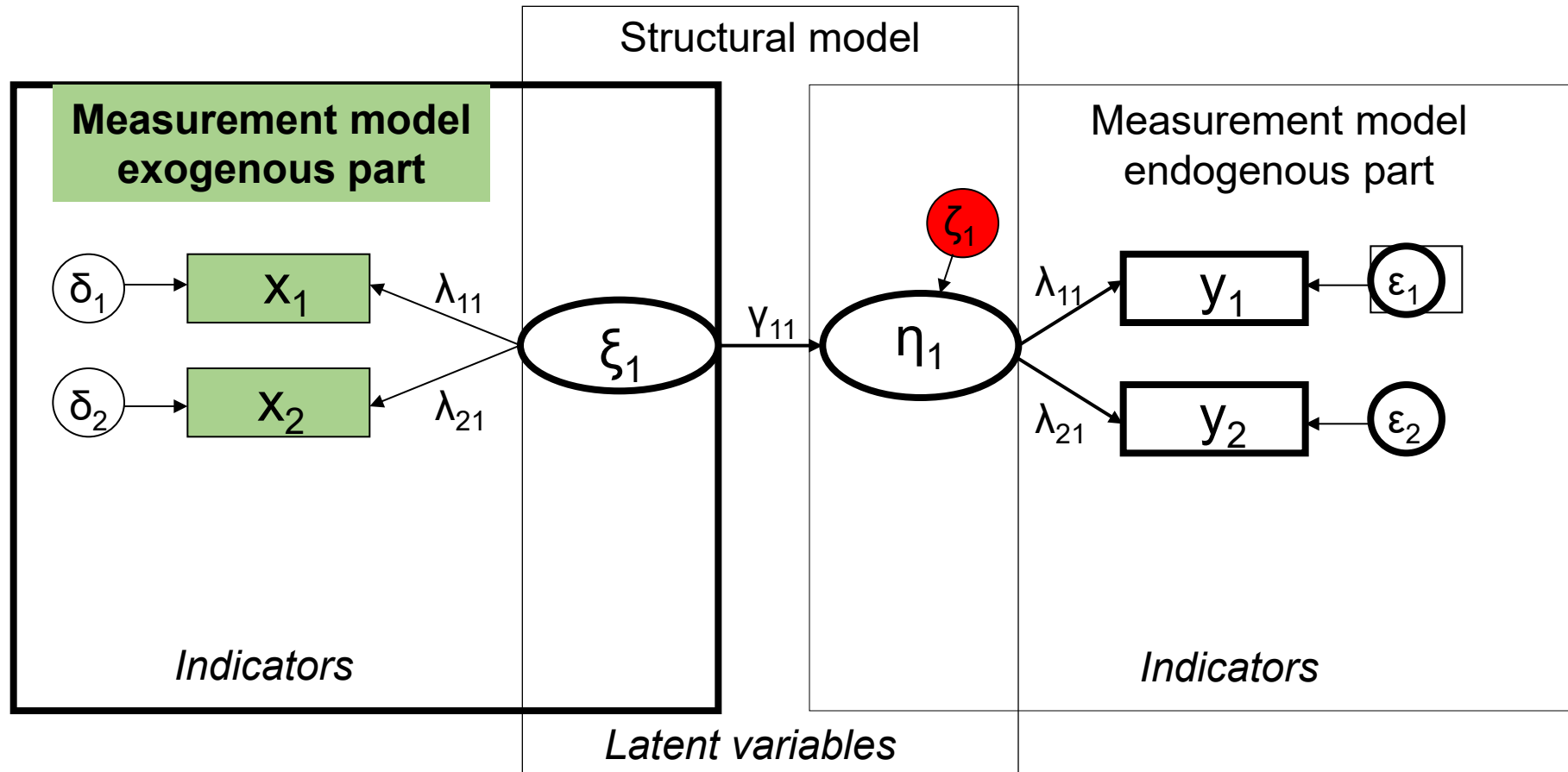
Vector of  
*endogenous*  
latent  
variables  $\eta_i$   
(m x 1)

Gamma  
coefficients  
of  $\xi_i$  and  $\eta_j$   
variables  
(m x n)

Vector of  
*exogenous*  
latent  $\xi_i$   
variables  
(n x 1)

Vector of  
residual variables of  
*latent endogenous*  
variables  $\eta_i$   
(m x 1)

> Switch to the formal (mathematical) model



## > Switch to the formal (mathematical) model

- Equation of measurement model of latent *exogenous* variables:

$$x_1 = \lambda_{11} \times \xi_1 + \delta_1$$

Generalization





# > Switch to the formal (mathematical) model

- Equation of measurement model of latent *exogenous* variables:

$$x = \Lambda \xi + \delta \quad (\text{new } X = \mu + \Lambda \xi + \delta)$$

$$\mathbf{x} = \mathbf{\Lambda} \times \boldsymbol{\xi} + \boldsymbol{\delta}$$

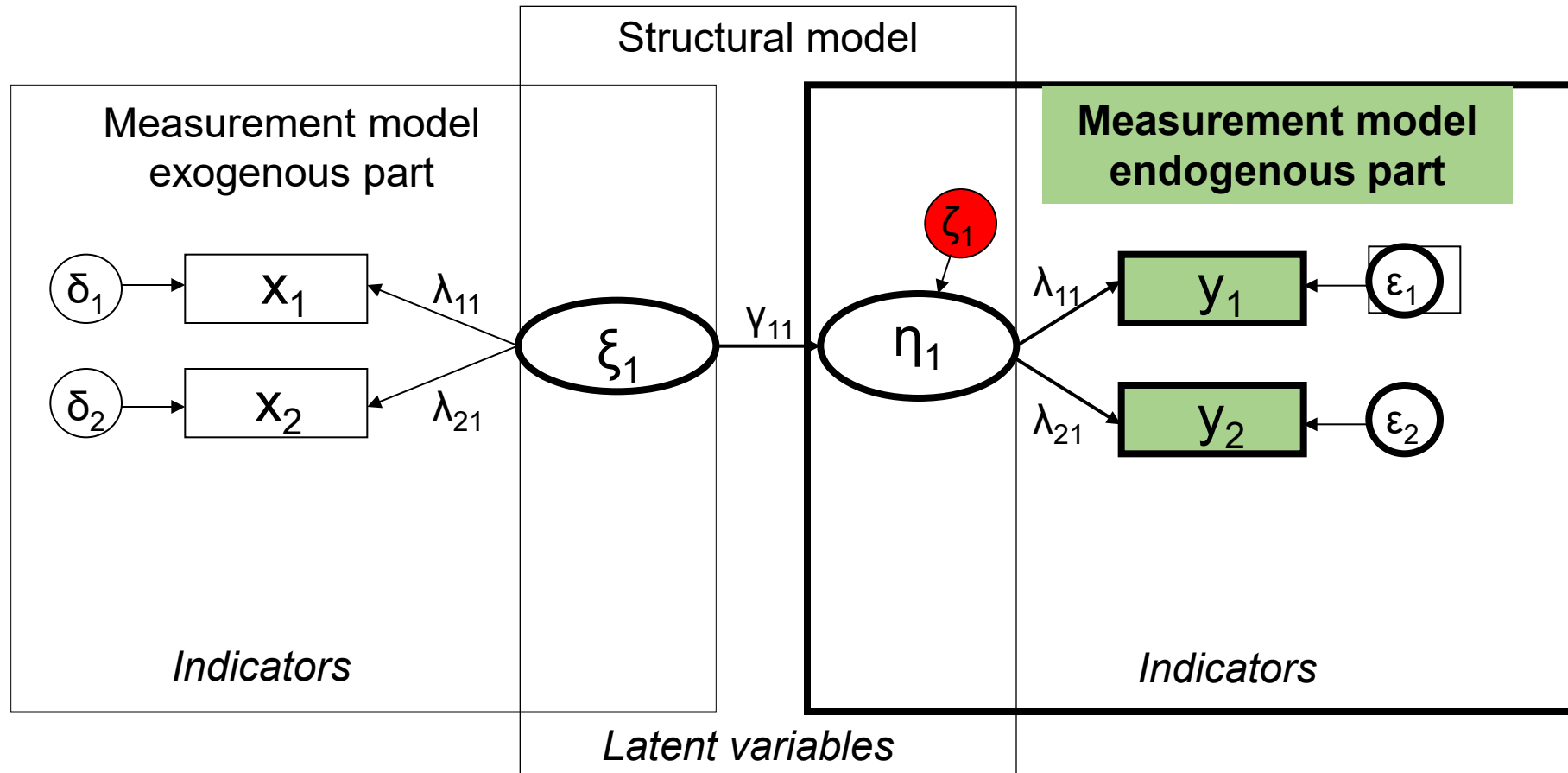
Vector of  
*exogenous*  
indicators  
( $p \times 1$ )

Matrix of (path)  
coefficients  
(factor loadings)  
of the model of  
measurement of  
the *exogenous*  
variables  
( $p \times n$ )

Vector of  
*exogenous*  
latent  
variables  
( $n \times 1$ )

Vector of  
residuals of  
*exogenous*  
indicators  
( $p \times 1$ )

# > Switch to the formal (mathematical) model



# > Switch to the formal (mathematical) model

- Equation of measurement model of latent *endogenous* variables:

$$y = \Lambda \eta + \varepsilon \quad (\text{new } Y = \mu + \Lambda \eta + \varepsilon)$$

$$\mathbf{y} = \mathbf{\Lambda} \times \boldsymbol{\eta} + \boldsymbol{\varepsilon}$$

Vector of  
*endogenous*  
indicators  
(manifest  
variables)  
(q x 1)

Matrix of (path)  
coefficients  
(factor loadings)  
of the model of  
measurement of  
the *endogenous*  
variables  
(q x m)

Vector of  
*endogenous*  
latent  
variables  
(m x 1)

Vector of  
residuals of  
*endogenous*  
indicators  
(q x 1)

# > Assumptions of structural equation models

## Basic assumptions:

- latent and residual variables do ***not*** correlate with each other
- different residual variables do ***not*** correlate with each other

## > Assumptions of structural equation models

Further, basic assumptions in detail:

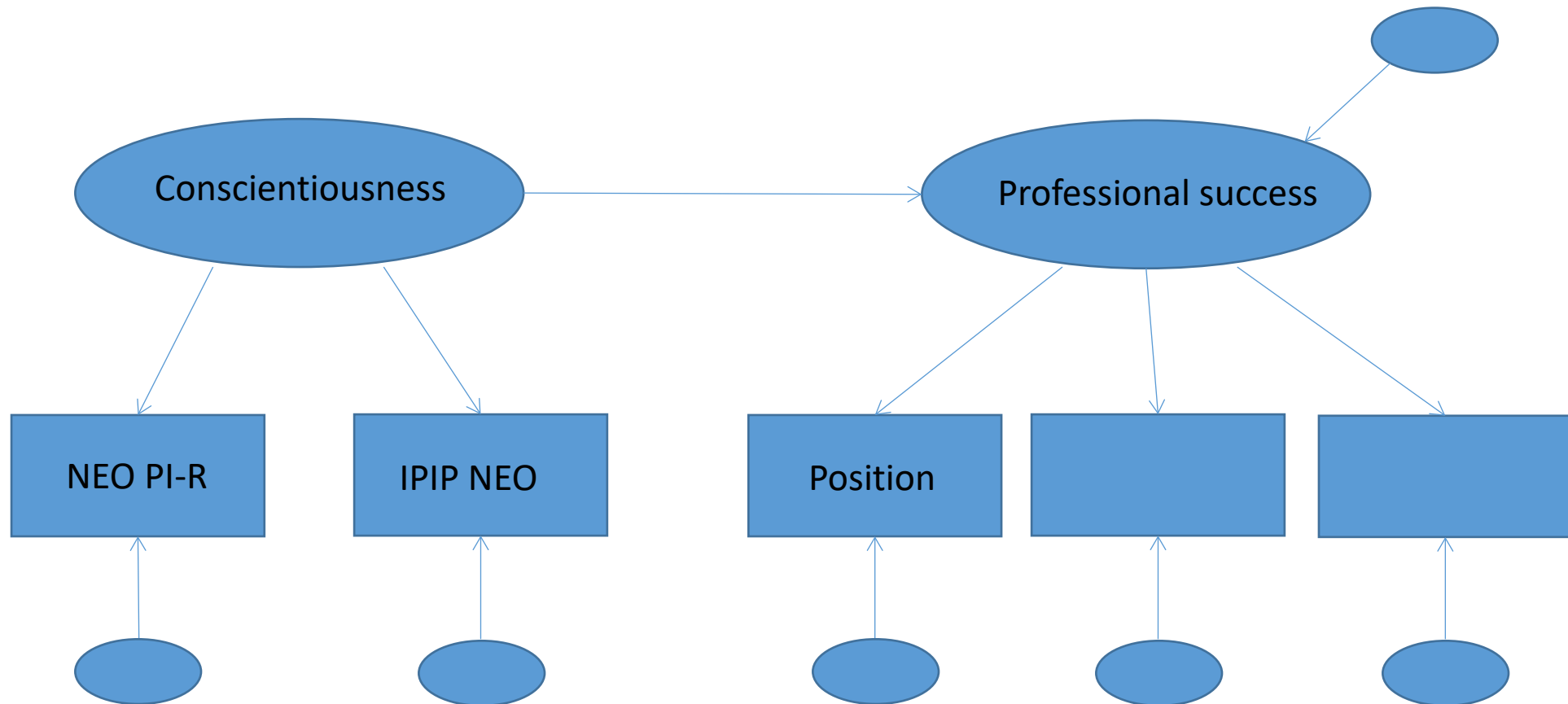
- $\zeta$  does not correlate with  $\xi$
- $\varepsilon$  does not correlate with  $\eta$
- $\delta$  does not correlate with  $\xi$
- $\delta$ ,  $\varepsilon$  and  $\zeta$  do not correlate with each other

### **Note.**

- there are two types of residuals:
  - residuals as characteristics of individual manifest variables
  - residuals as random error (= remainders)

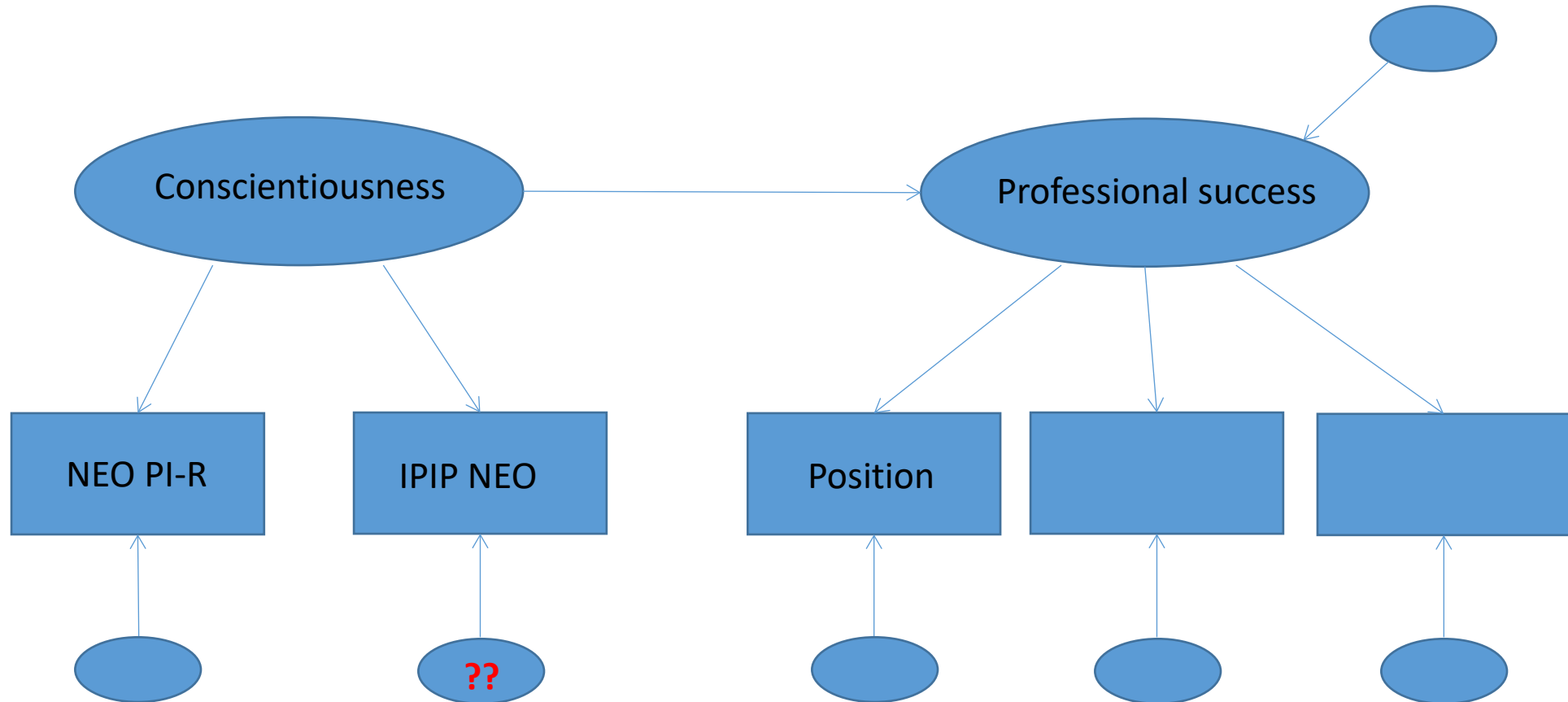
# A path diagram for H1

**H1:** The more conscientious a person is, the larger the professional success of this person.



# Practice B

Which symbol has to replace ?? in the diagram?



# Practice B

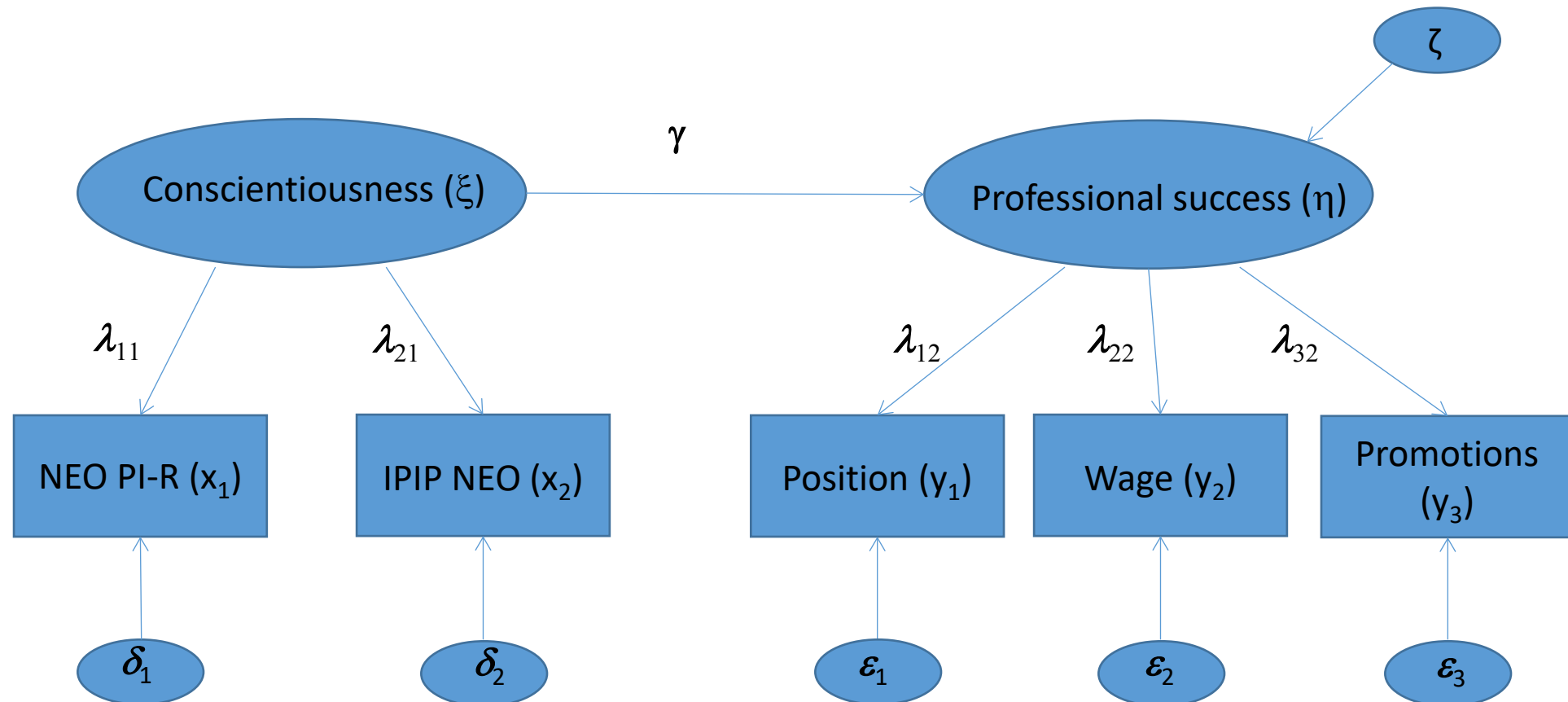
Which symbol has to replace ?? in the diagram?

- $\xi$
- $\varepsilon$
- $\eta$
- $\delta$
- $\pi$
- $\phi$



# A path diagram for H1

**H1:** The more conscientious a person is, the larger the professional success of this person is.



# > Hypotheses for a complex structural model: *Examples*

**In the following three other hypotheses are combined with the first one to achieve a *complex* model.**

# > Hypotheses for a complex structural model:

## *Examples*

- **H1:** The more conscientious a person is, the larger the professional success of this person (original H1).
- **H2:** The larger the work motivation of a person is, the larger the professional success of this person.
- **H3:** With more conscientiousness, the professional reputation is increasing.
- **H4:** Larger work motivation means an increasing professional reputation.

# > Hypotheses for a complex structural model:

## *Examples*

- **M1:** Conscientiousness is indicated by scales of two personality inventories: NEO PI-R and IPIP NEO.
- **M2:** Work motivation is measured by means of a concentration test (C test) and self evaluation.
- **M3:** Professional success is reflected by the professional position, wage and number of promotions.
- **M4:** Professional reputation is operationalized by the result of questioning colleagues.

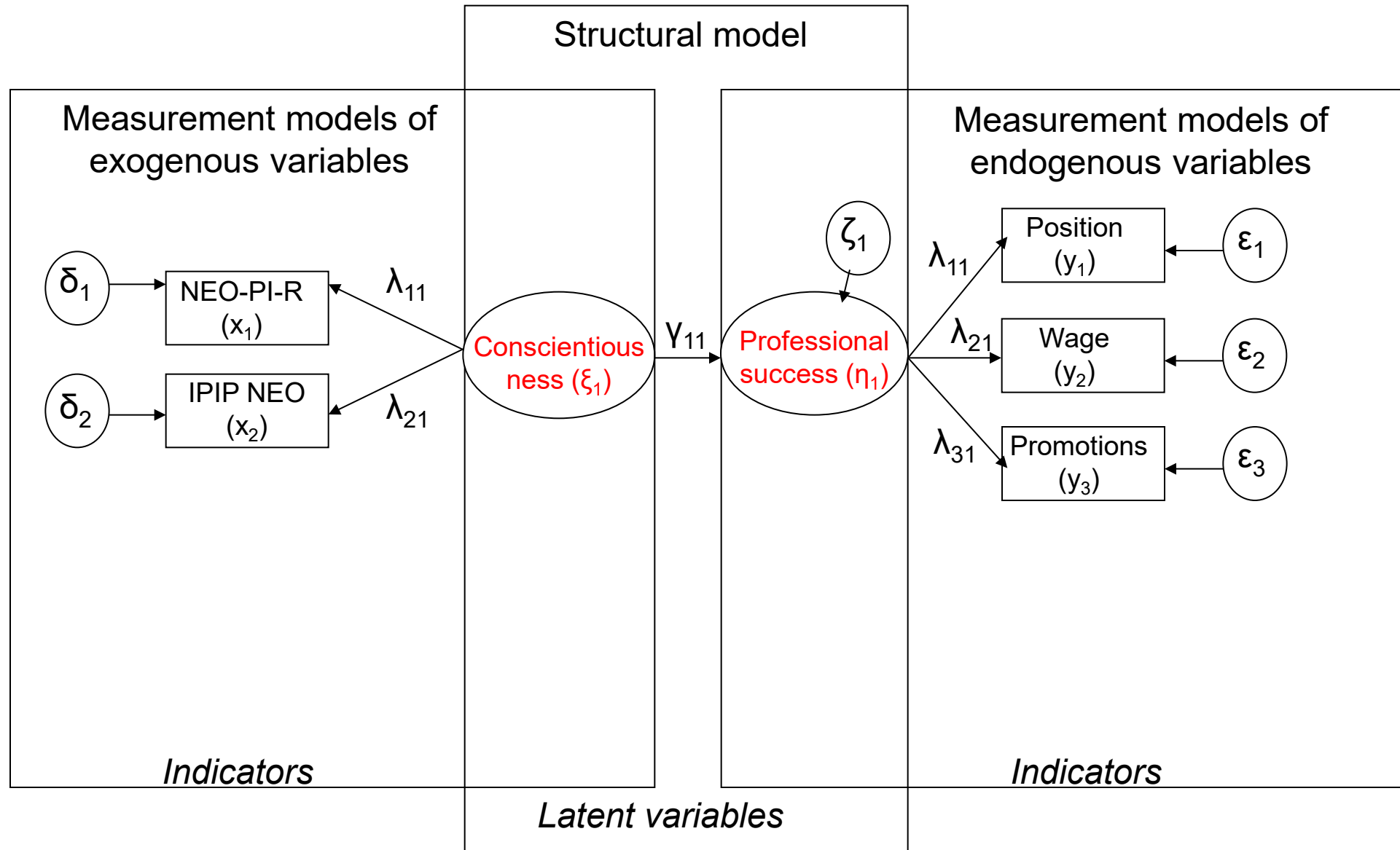
# > Hypotheses for a complex structural model:

## *Examples*

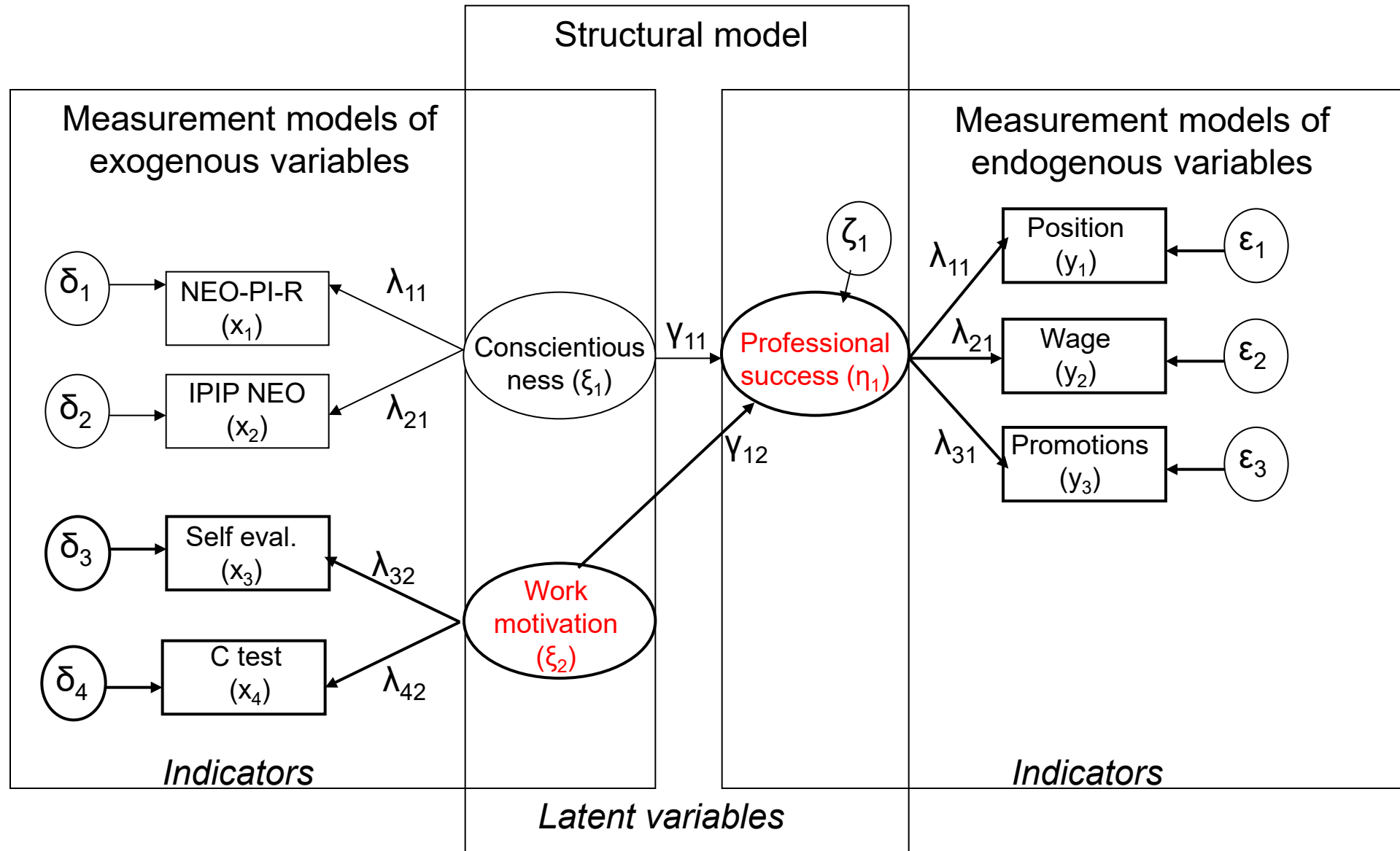
### **The outset:**

- **H1:** The more conscientious a person is, the larger the professional success of this person is.
- **H2:** The larger the work motivation of a person is, the larger the professional success of this person.
- **H3:** With more conscientiousness the professional reputation is increasing.
- **H4:** Larger work motivation means an increasing professional reputation.
  
- **M1:** Conscientiousness is indicated by scales of two personality inventories: NEO PI-R and IPIP NEO.
- **M2:** Work motivation is measured by means of a concentration test (C test) and self evaluation.
- **M3:** Professional success is reflected by the professional position, wage and number of promotions.
- **M4:** Professional reputation is operationalized by the result of questioning colleagues.

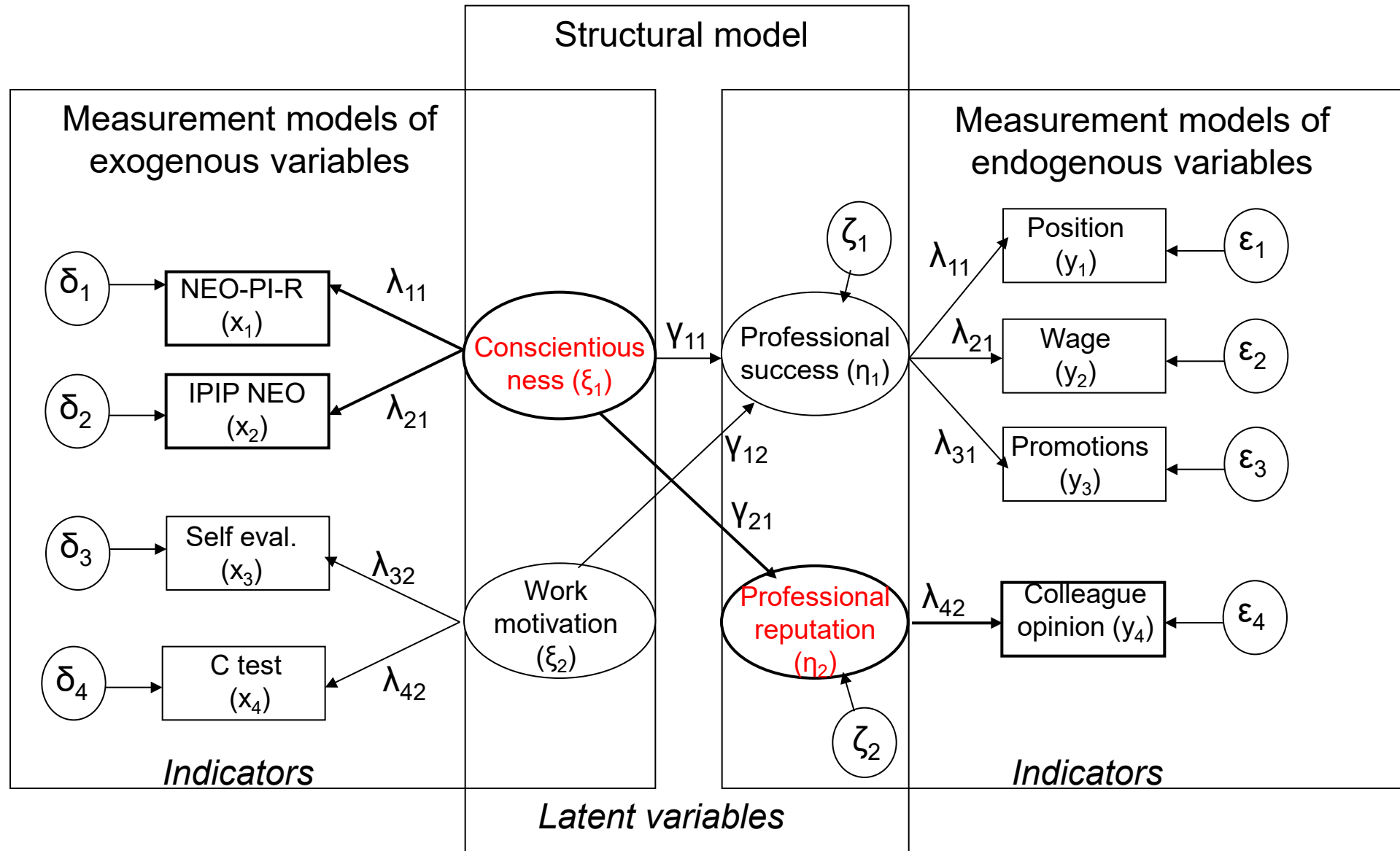
# > Complex structural equation model



# > Complex structural equation model

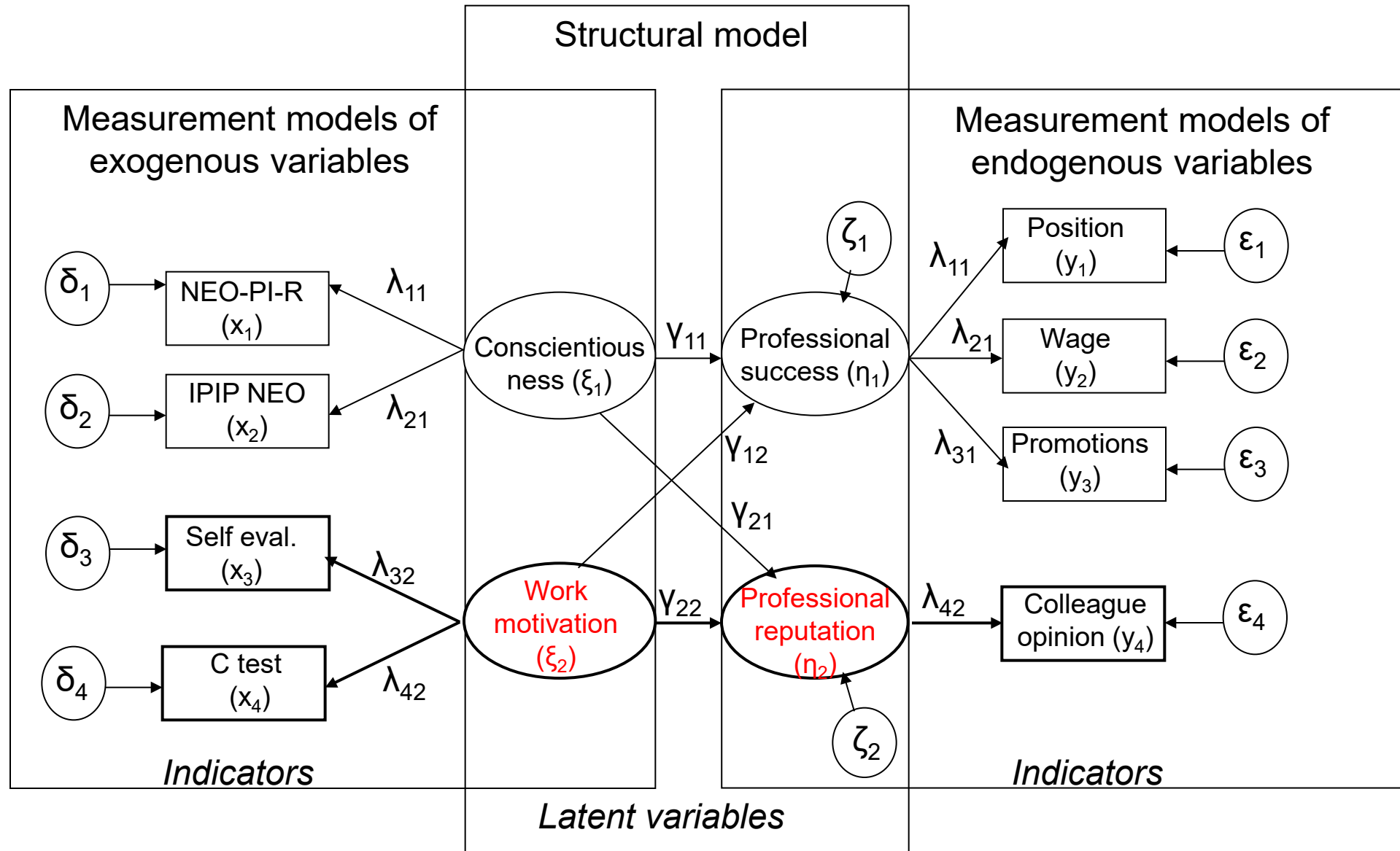


# > Complex structural equation model





# > Complex structural equation model



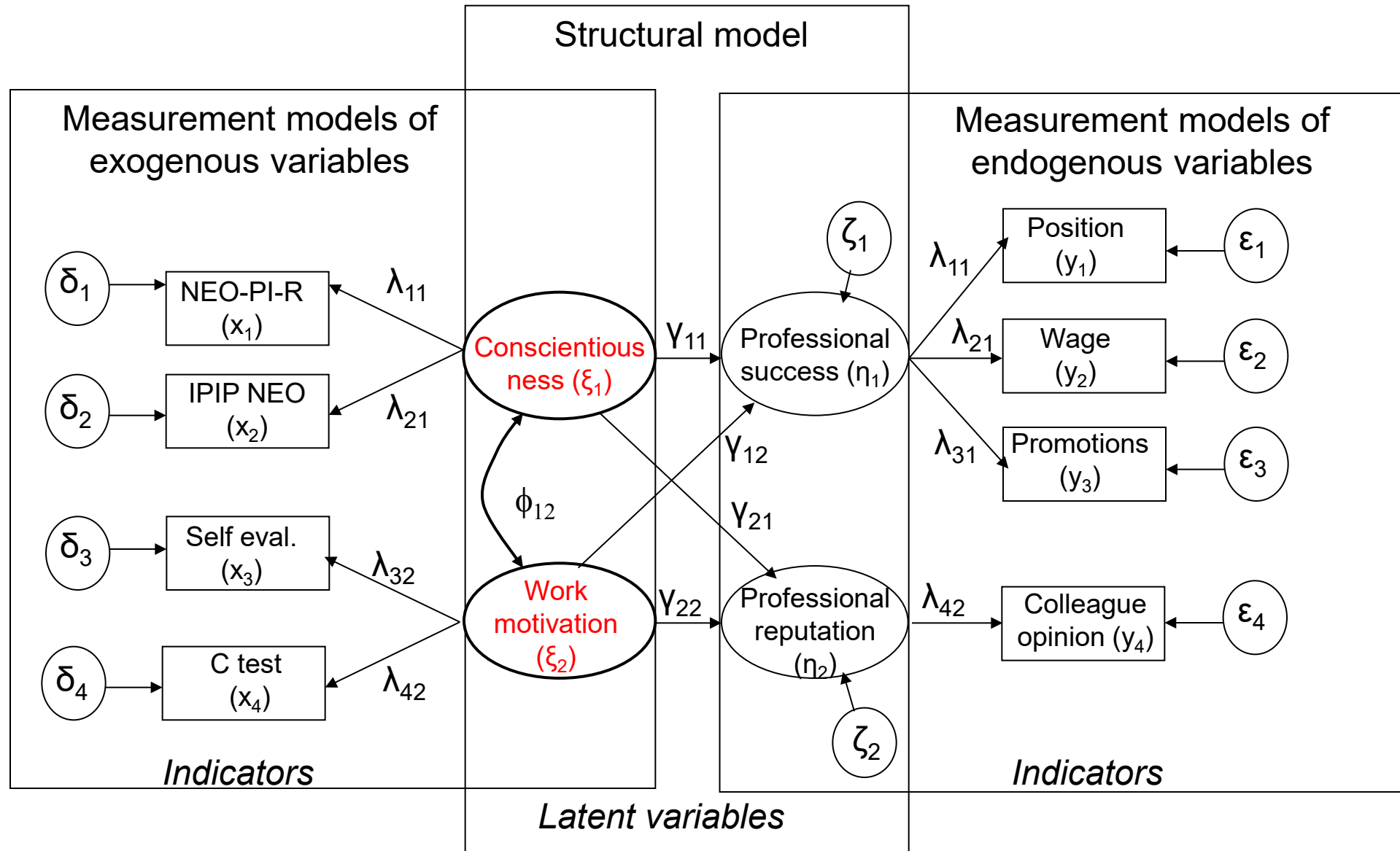
# > Hypotheses for a complex structural model: *Examples*

## **Additional structural hypotheses:**

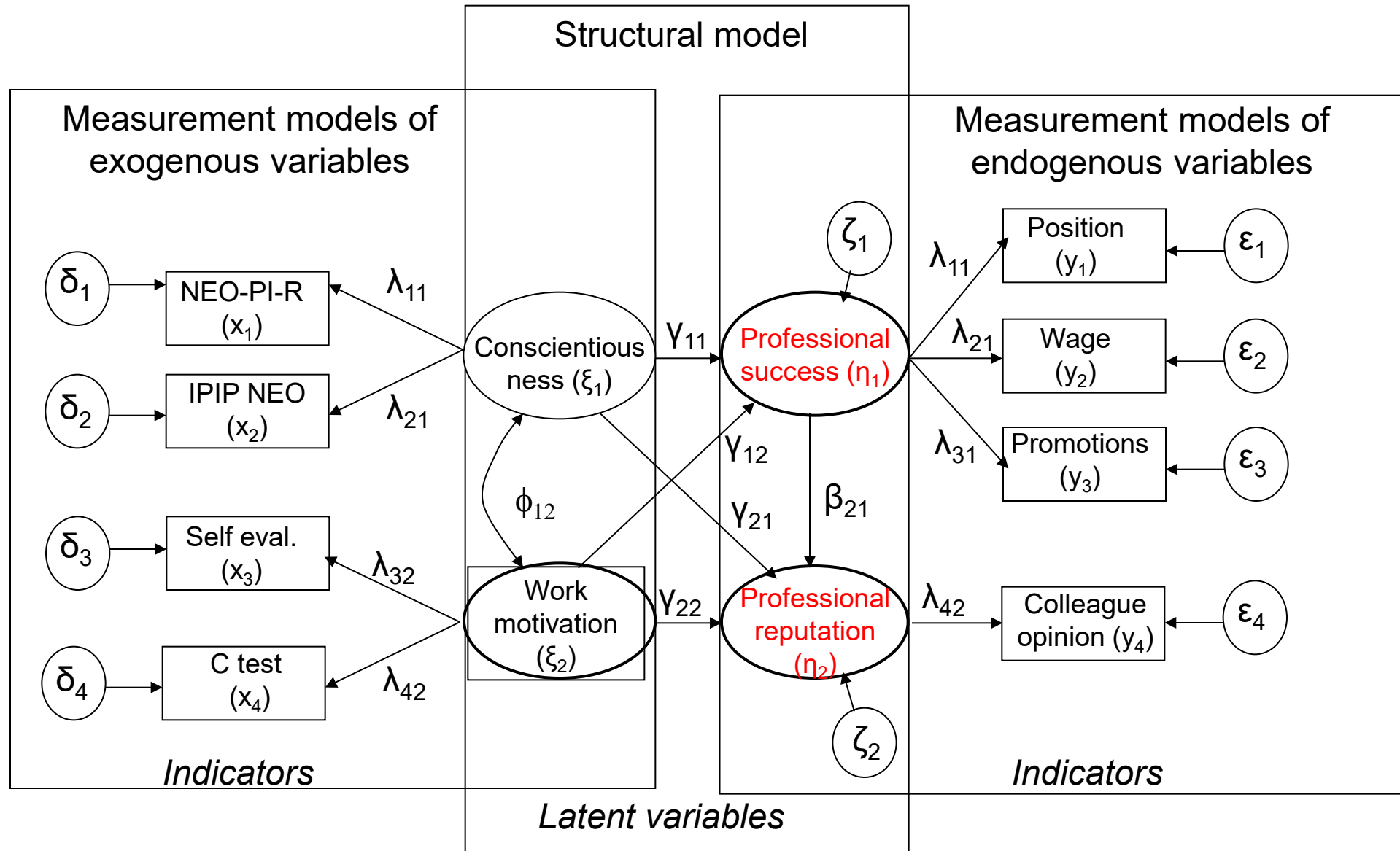
*(Remember: **the need to be complete**)*

- **H5:** Conscientiousness and work motivation are not independent of each other (= they are correlated).
- **H6:** Professional success exerts an influence on professional reputation.

# > Complex structural equation model



# > Complex structural equation model



## > the path diagram

- It includes ...
  - all manifest variables
  - all latent variables
  - all links representing structure
  - all parameters associated with the major links

# Steps in preparing data analysis

1. Formation of hypothesis
2. Path diagram and specification of model
- 3. Identification of the structure of the model**
4. Estimation of parameters
5. Evaluation of the results
6. (modification of the structure of the model)

### 3. Identification of the structure of models

Check whether enough empirical information is available for estimating the parameters of the model!

... without such information the results may not be valid!

*(this check is based on the assumption that either a covariance matrix or a correlation matrix provides the input to SEM)*

# 3. Identification of the structure of models

A note.

(... remember your high school education in mathematics)

A system of linear equations with unknowns can only be solved if there are *as many* equations *as* there are unknowns!



### 3. Identification of the structure of models

Check whether enough empirical information is available for estimating the parameters of the model!

*It is done by computing the degree of freedom (df):*

**The degree of freedom „df“ =  $s - t$**

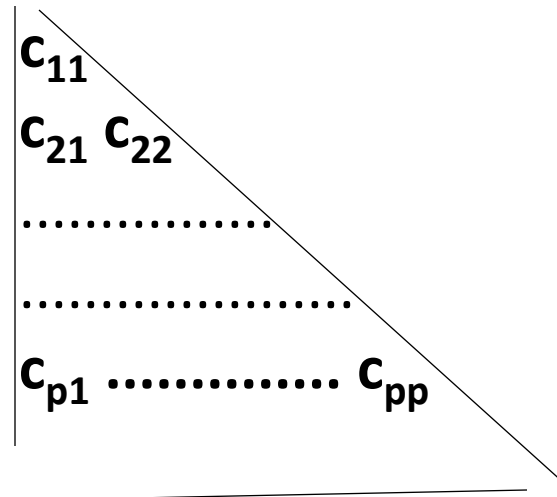
„... indication of whether parameter estimation is possible“

### 3. Identification of the structure of models

- ***The available information „s“ (items of information):***

there are  $n$  indicator variables (all manifest variables):

$$s = \frac{n(n + 1)}{2} = \text{number of unique variances and covariances of the covariance matrix}$$



### 3. Identification of the structure of models

- ***The available information „s“:***

there are  $n$  indicator variables (all manifest variables):

$$s = \frac{n(n + 1)}{2} = \text{number of unique variances and} \\ \text{covariances of the covariance matrix}$$

- ***The needed information „t“:***

$t$  = number of parameters to be estimated

### 3. Identification of the structure of models

- **The meanings** of the degree of freedom (df):

The need for *identification* means ...

$$df \geq 0$$

The need for *evaluating* the model means ...

$$df > 0$$

### 3. Identification of the structure of models

- **The meaning** of the degree of freedom  $(df) = s - t$

$df = 0$  ... is characterized as ... „just identified“

$df > 0$  ... is characterized as ... „identified“ (*desirable option*)

$df < 0$  ... is characterized as ... „not identified“

### 3. Identification of the structure of models

- **The meaning** of the degree of freedom (df) =  $s - t$

*... be aware if the degree of freedom is negative, the program is likely to run into problems and does not provide a result.*

*... otherwise, it is likely to be incorrect!*

### 3. Identification of the structure of models

- **Cautionary note:**

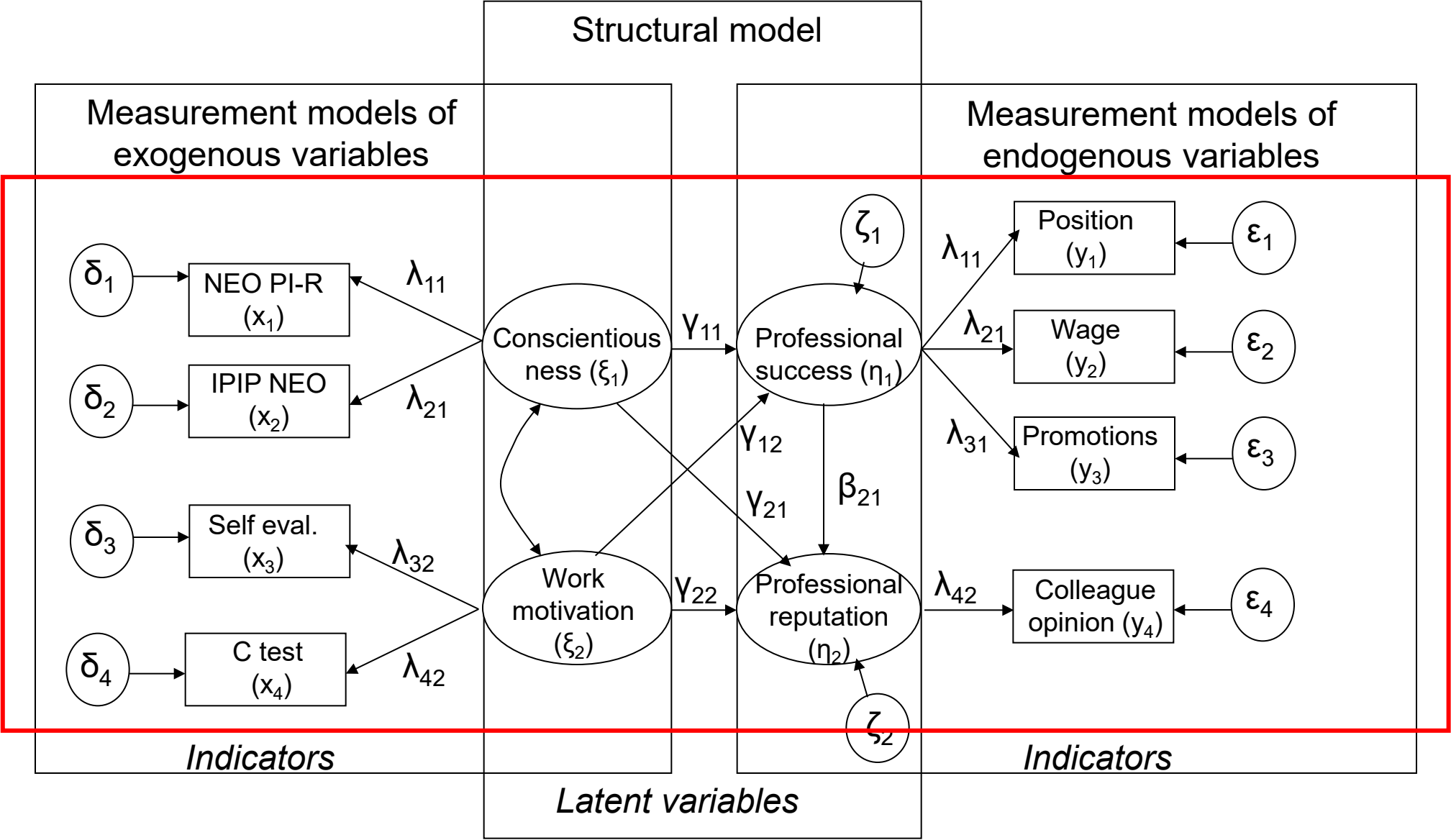
... if the model is complex, the degree of freedom of the .....

*complete model* and also of .....

*each measurement model*

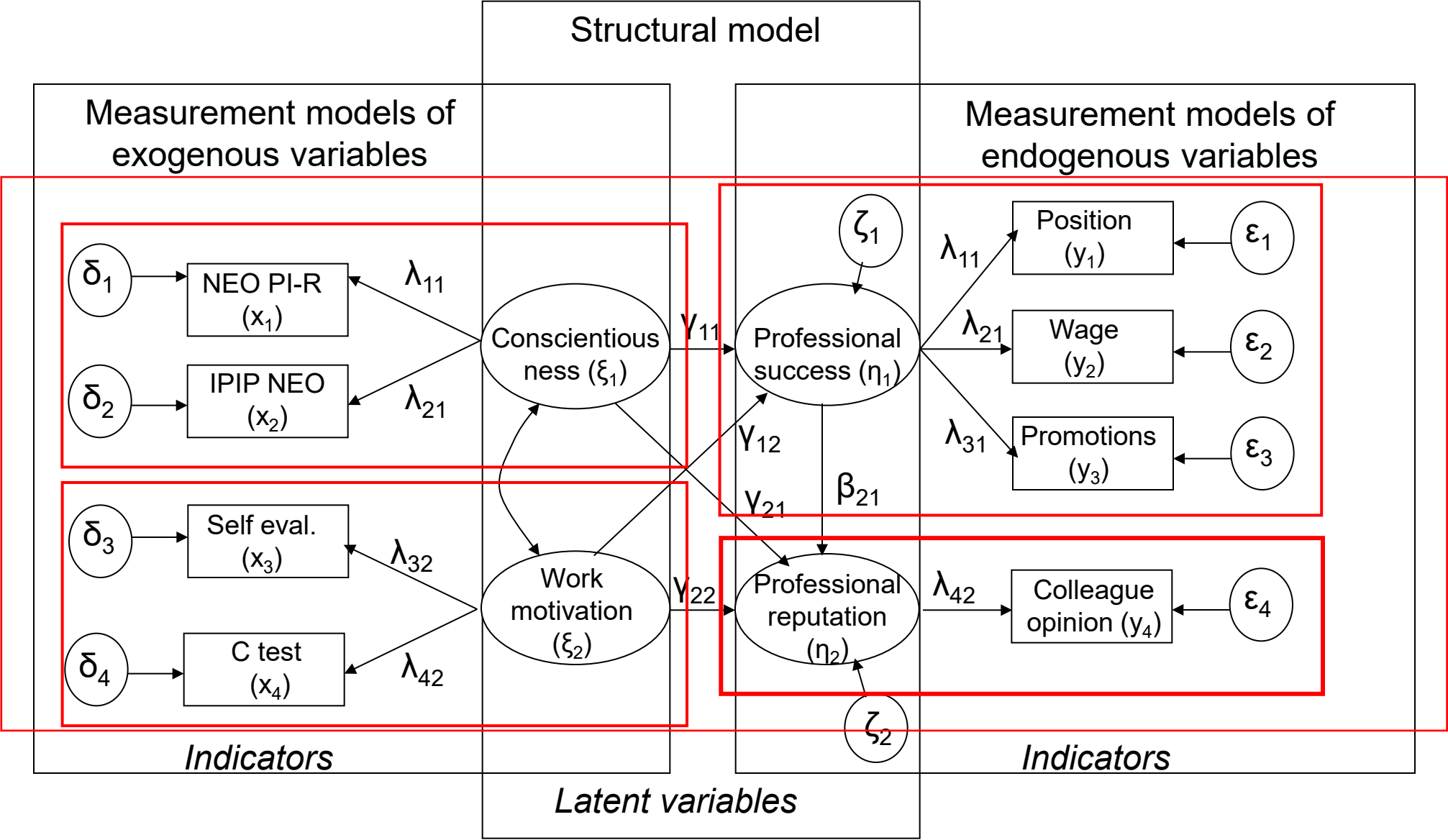
needs to be checked (sometimes measurement models are not good although the complete model is good).

Parts to be checked:

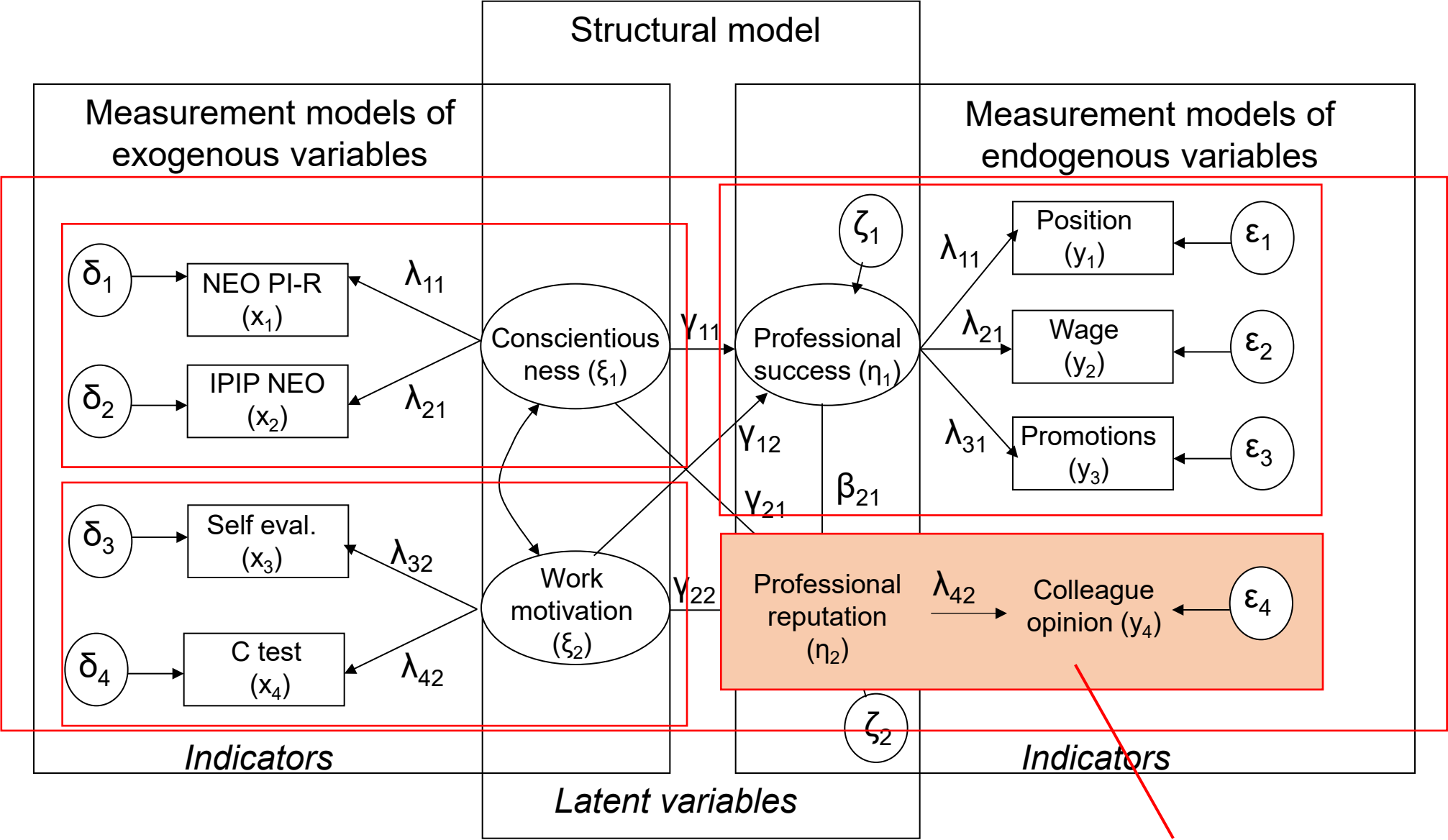




Parts to be checked:



Parts to be checked:



**Undesirable Case**

### 3. Identification of the structure of models

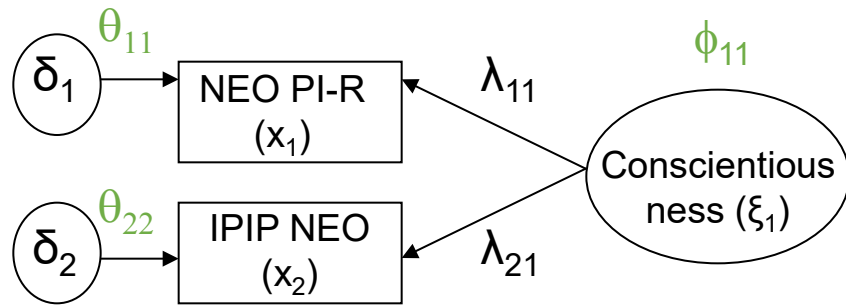
- There are combinations of manifest and latent variables that need *special treatment* for achieving correct results ...

Be aware that additional parameters need to be considered that are usually not included in the model of measurement:

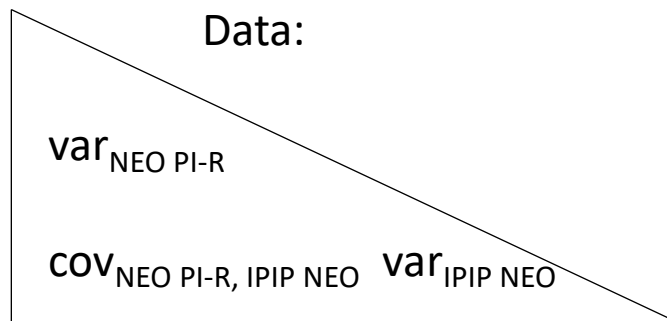
- the residual variance ( $\theta$ )
- the variance of an exogenous latent variable ( $\phi$ )
- the of  $\zeta$  ( $\psi$ )

# 3. Identification of the structure of models

A two-indicator example:



This model of measurement includes five parameters ( $\phi_{11}, \lambda_{11}, \lambda_{12}, \theta_{11}, \theta_{22}$ ) that need to be estimated,  $t = 5$ .



$N = 2$ :

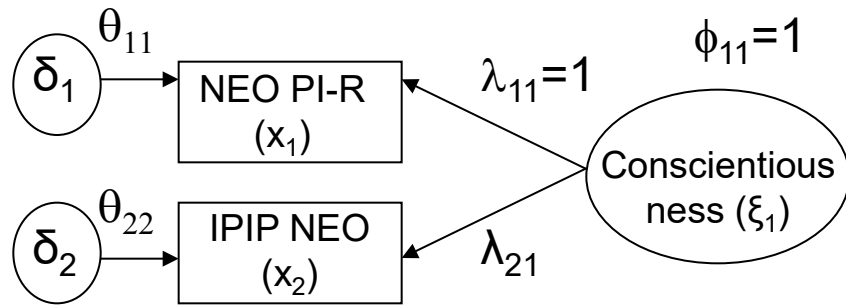
$$S = \frac{2(2+1)}{2} = 3$$

$\phi_{11}, \theta_{11}, \theta_{22}$  are part of the model of the covariance matrix (will be introduced in another section).

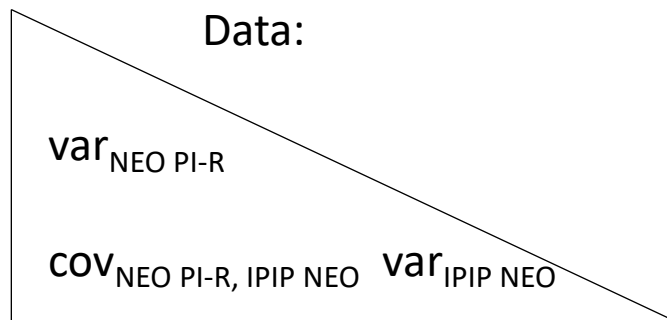
$df = s - t = 3 - 5 = -2 < 0$  (means ... is *not identified*)

### 3. Identification of the structure of models

A two-indicator example: What to do?



This model of measurement includes three parameters ( $\lambda_{21}$ ,  $\theta_{11}$ ,  $\theta_{22}$ ) that need to be estimated,  $t = 3$ .



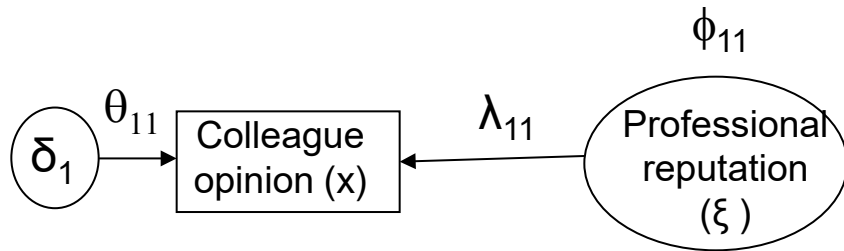
$N = 2$ :

$$S = \frac{2(2+1)}{2} = 3$$

$df = s - t = 3 - 3 = 0$  (means ... is *just identified*)

# 3. Identification of the structure of models

A one-indicator example:



This model of measurement includes three parameters ( $\lambda_{11}$ ,  $\theta_{11}$ ,  $\phi_{11}$ ) that need to be estimated,  $t = 3$ .

Data:

Variance (Colleague opinion)

$N = 1$ :

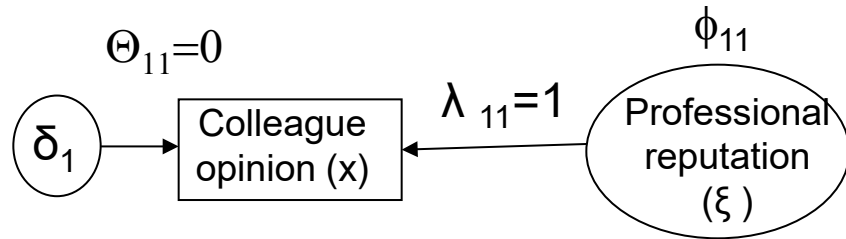
$$S = \frac{1(1+1)}{2} = 1$$

$\theta_{11}$  and  $\phi_{11}$  are part of the model of the covariance matrix (will be introduced in another section).

$df = s - t = 1 - 3 = -2 < 0$  (means ... is *not identified*)

### 3. Identification of the structure of models

A one-indicator example: What to do?



This model of measurement includes one parameter that need to be estimated,  $t = 1$ .

Data:

Variance (Colleague opinion)

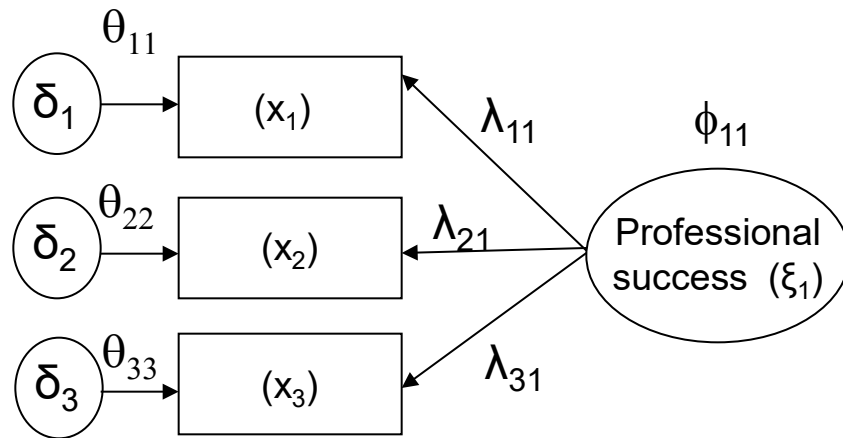
$N = 1$ :

$$S = \frac{1(1+1)}{2} = 1$$

$df = s - t = 1 - 1 = 0$  (means ... is *just identified*)

### 3. Identification of the structure of models

A three indicator example:



This model of measurement includes seven parameters ( $\phi_{11}$ ,  $\lambda_{11}$ ,  $\lambda_{21}$ ,  $\lambda_{31}$ ,  $\theta_{11}$ ,  $\theta_{22}$ ,  $\theta_{33}$ ) that need to be estimated,  $t = 7$ .

$N = 3$ :

$$S = \frac{3(3+1)}{2} = 6$$

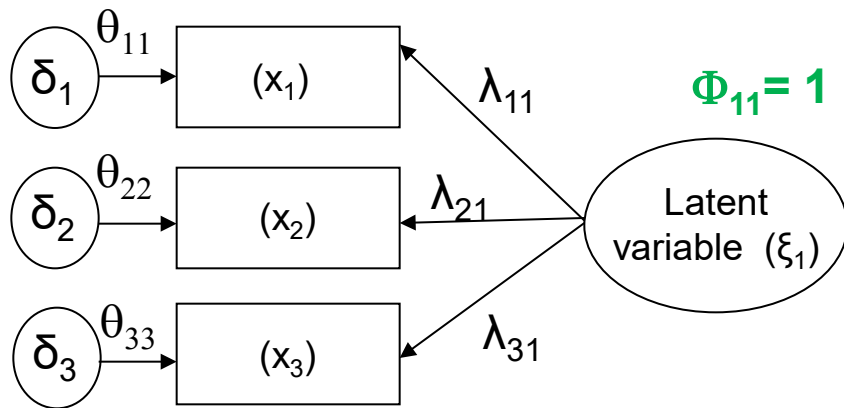
$\phi_{11}$ ,  $\theta_{11}$ ,  $\theta_{22}$  and  $\theta_{33}$  are part of the model of the covariance matrix (will be introduced in another section).

$df = s - t = 6 - 7 = -1 < 0$  (means ... is *not identified*)



# 3. Identification of the structure of models

A three indicator example: What to do?



This model of measurement includes six parameters ( $\lambda_{11}, \lambda_{21}, \lambda_{31}, \theta_{11}, \theta_{22}, \theta_{33}$ ) that need to be estimated,  $t = 6$ .

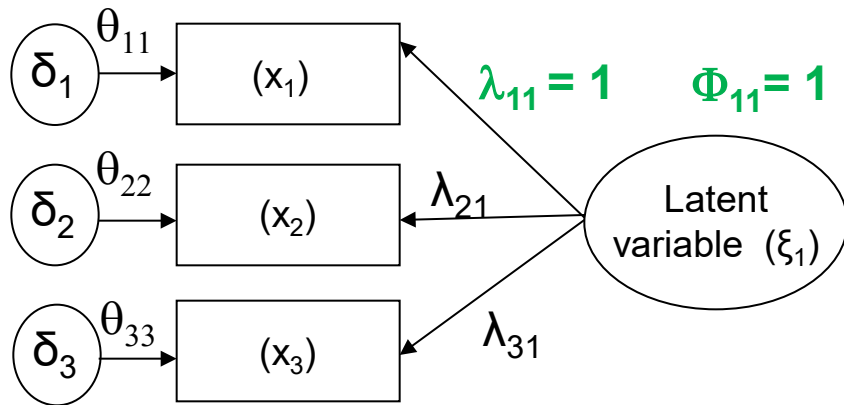
$N = 3$ :

$$S = \frac{3(3+1)}{2} = 6$$

$df = s - t = 6 - 6 = 0$  (means ... is *just identified*)

### 3. Identification of the structure of models

A three indicator example: *Better!*



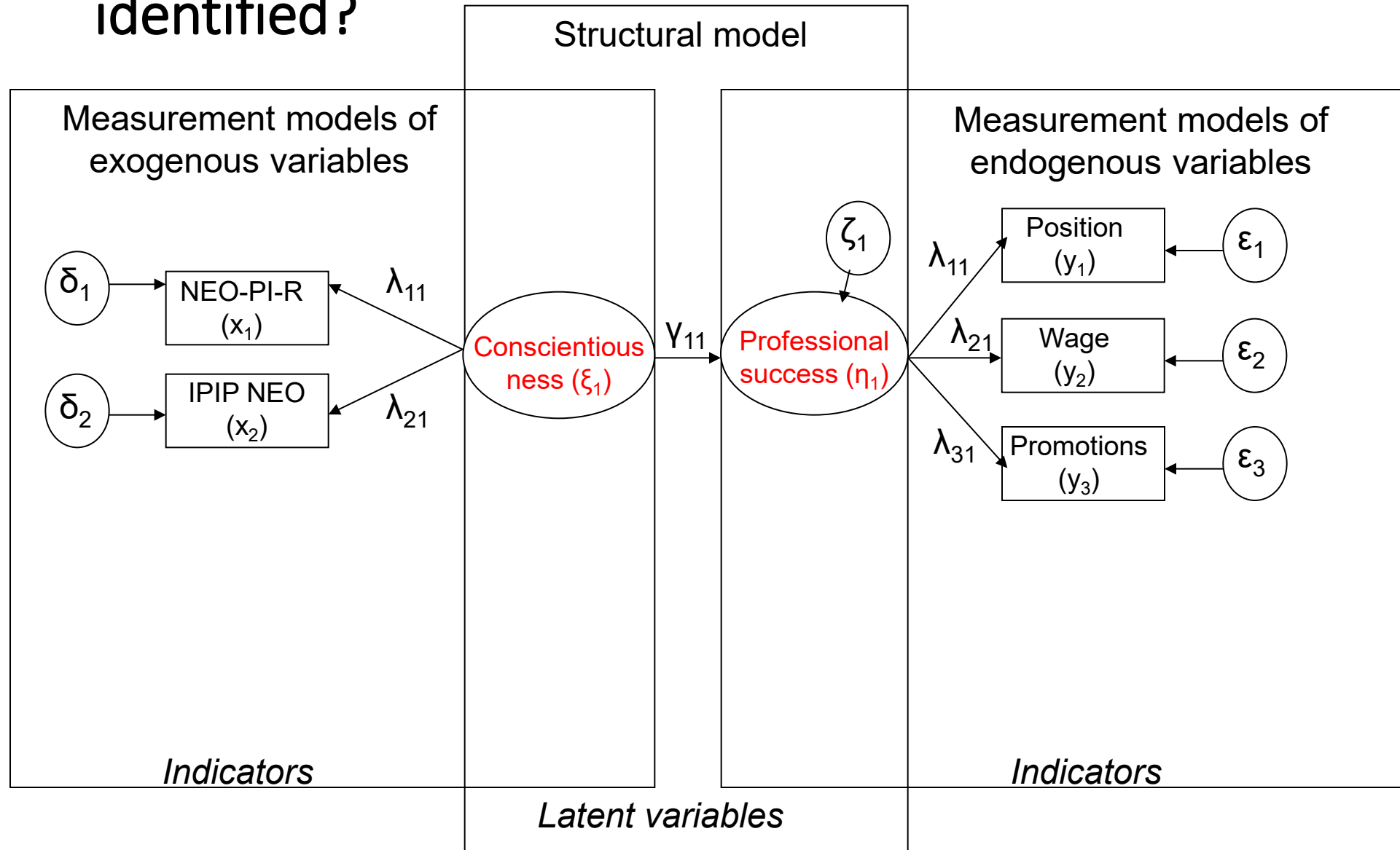
This model of measurement includes five parameters ( $\lambda_{21}$ ,  $\lambda_{31}$ ,  $\theta_{11}$ ,  $\theta_{22}$ ,  $\theta_{33}$ ) that need to be estimated,  $t = 5$ .

$N = 3$ :

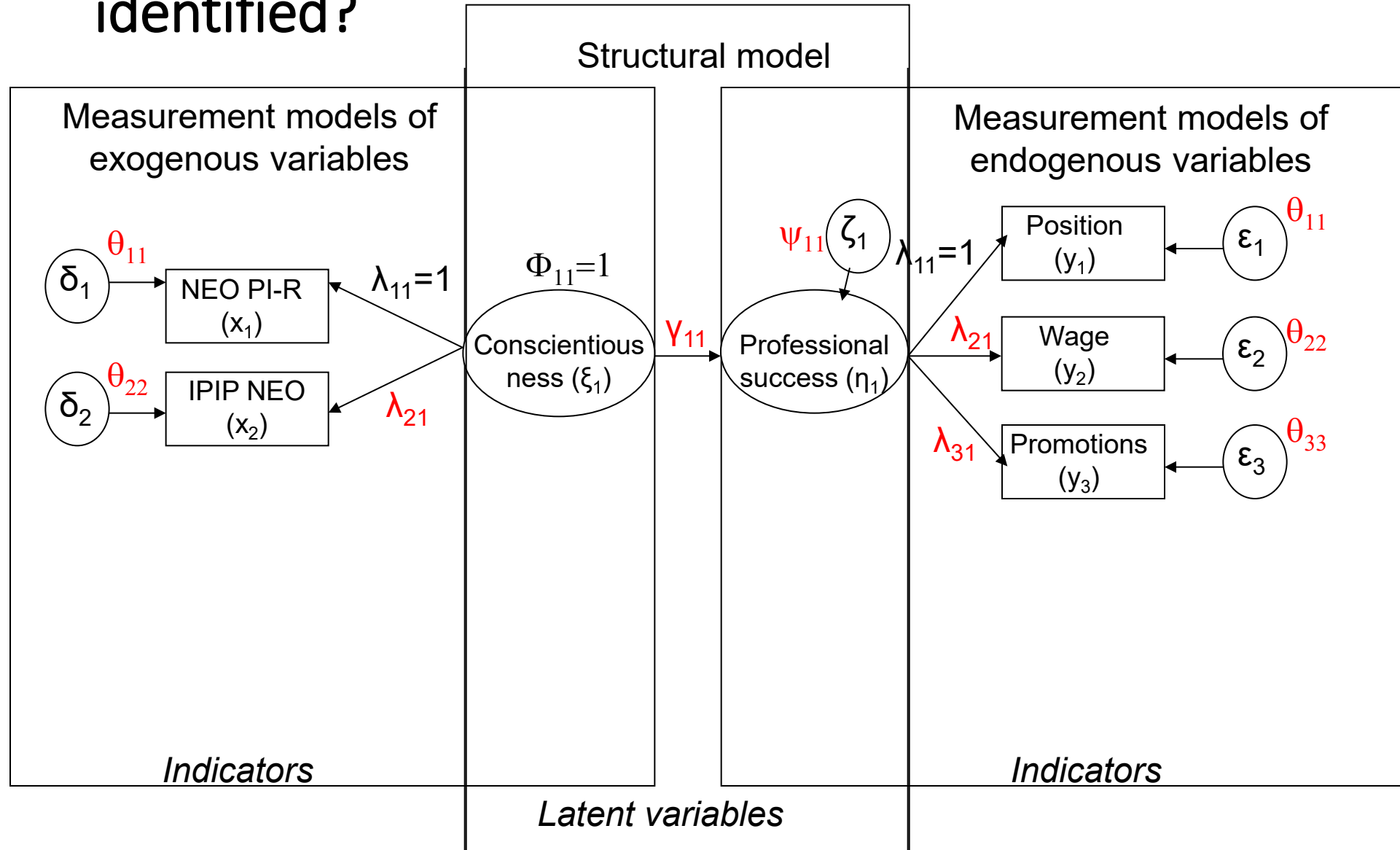
$$S = \frac{3(3+1)}{2} = 6$$

$$df = s - t = 6 - 5 = 1 \quad (\text{means ... is identified})$$

> Is this simple structural equation model identified?



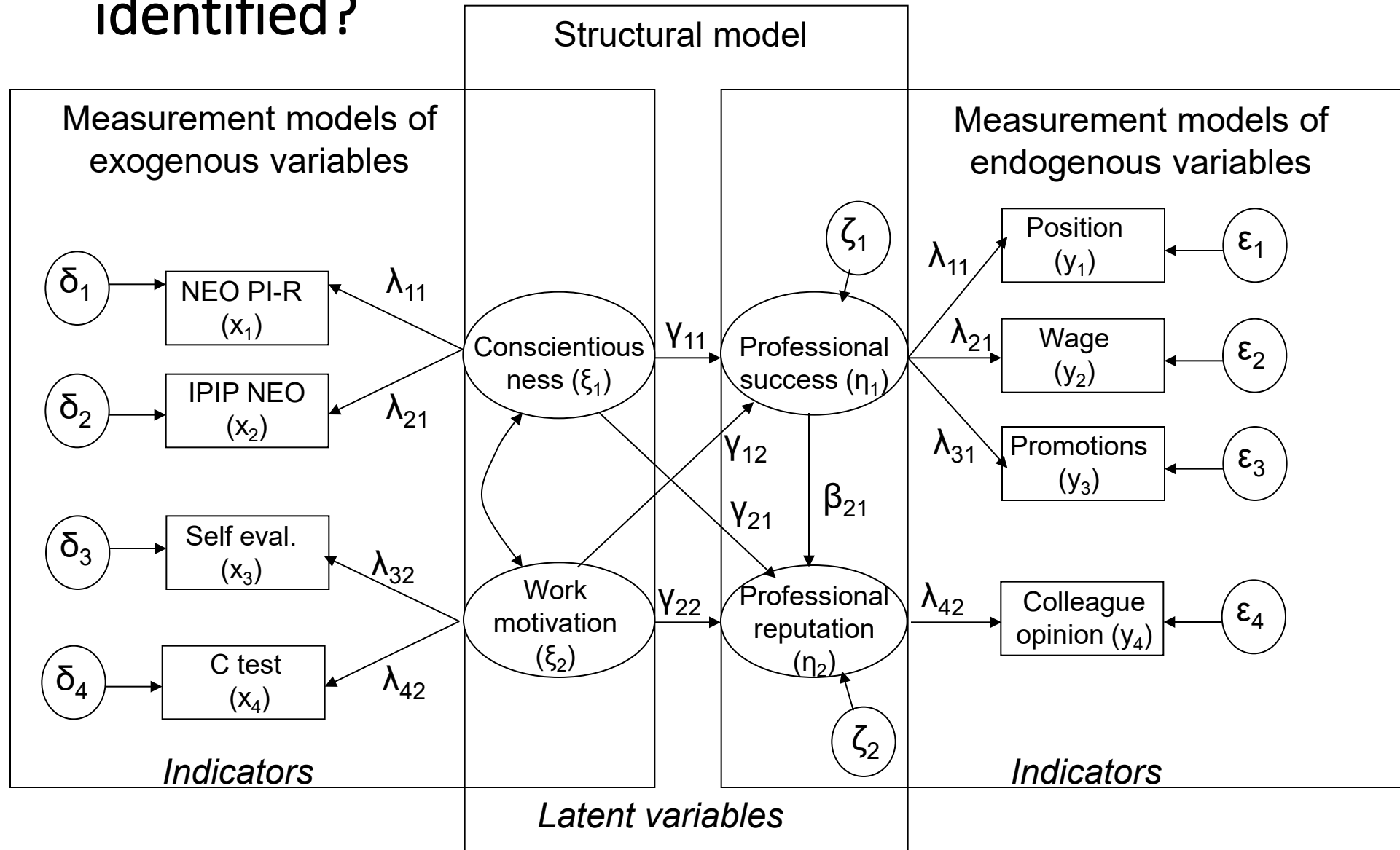
> Is this simple structural equation model identified?



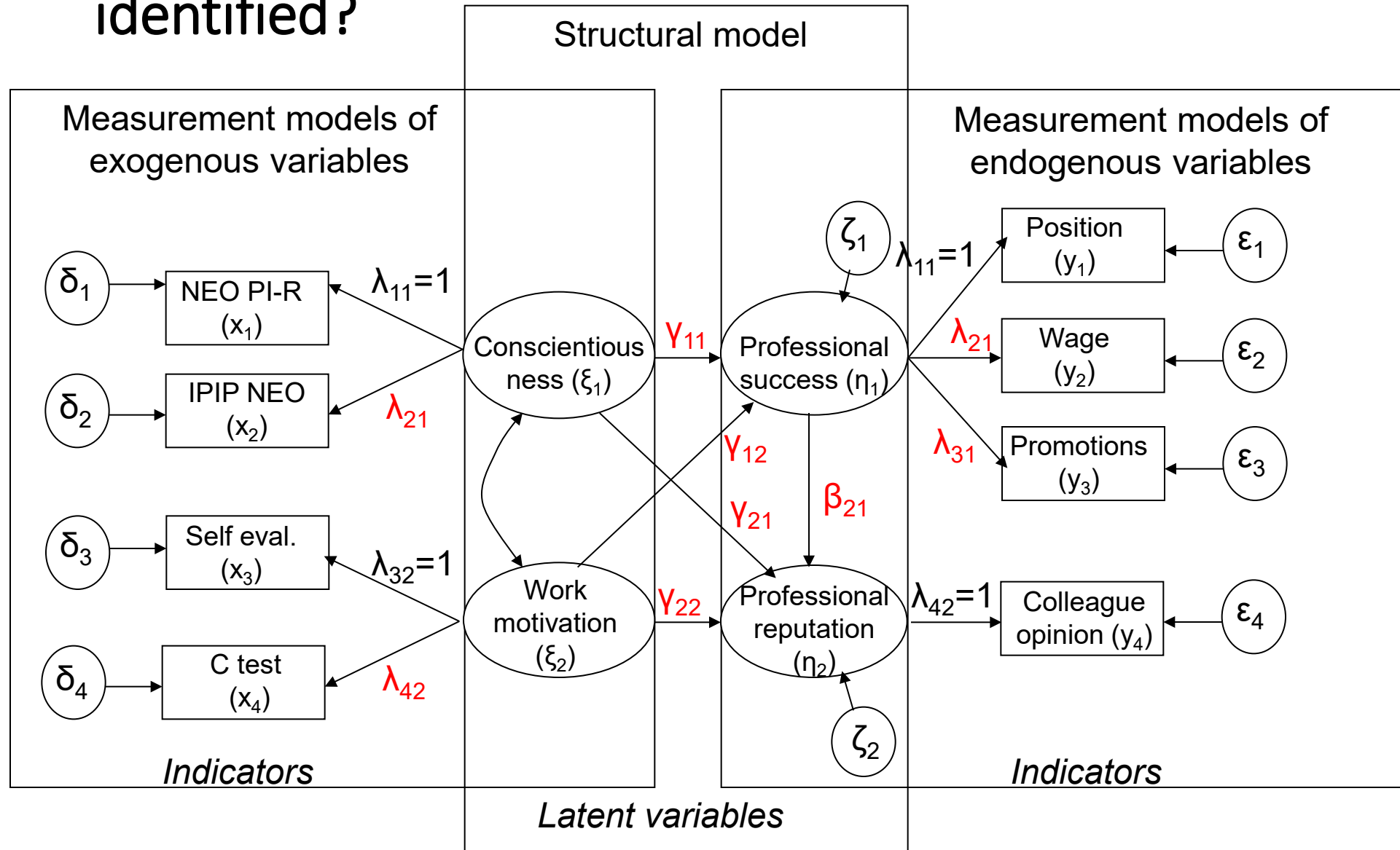
> Is this simple structural equation model identified?

- Number of parameter (according to counting): 10
- Number of items of information:  $5 \times (5+1) / 2 = 15$
- $(s - t) \quad \underline{df = 5}$

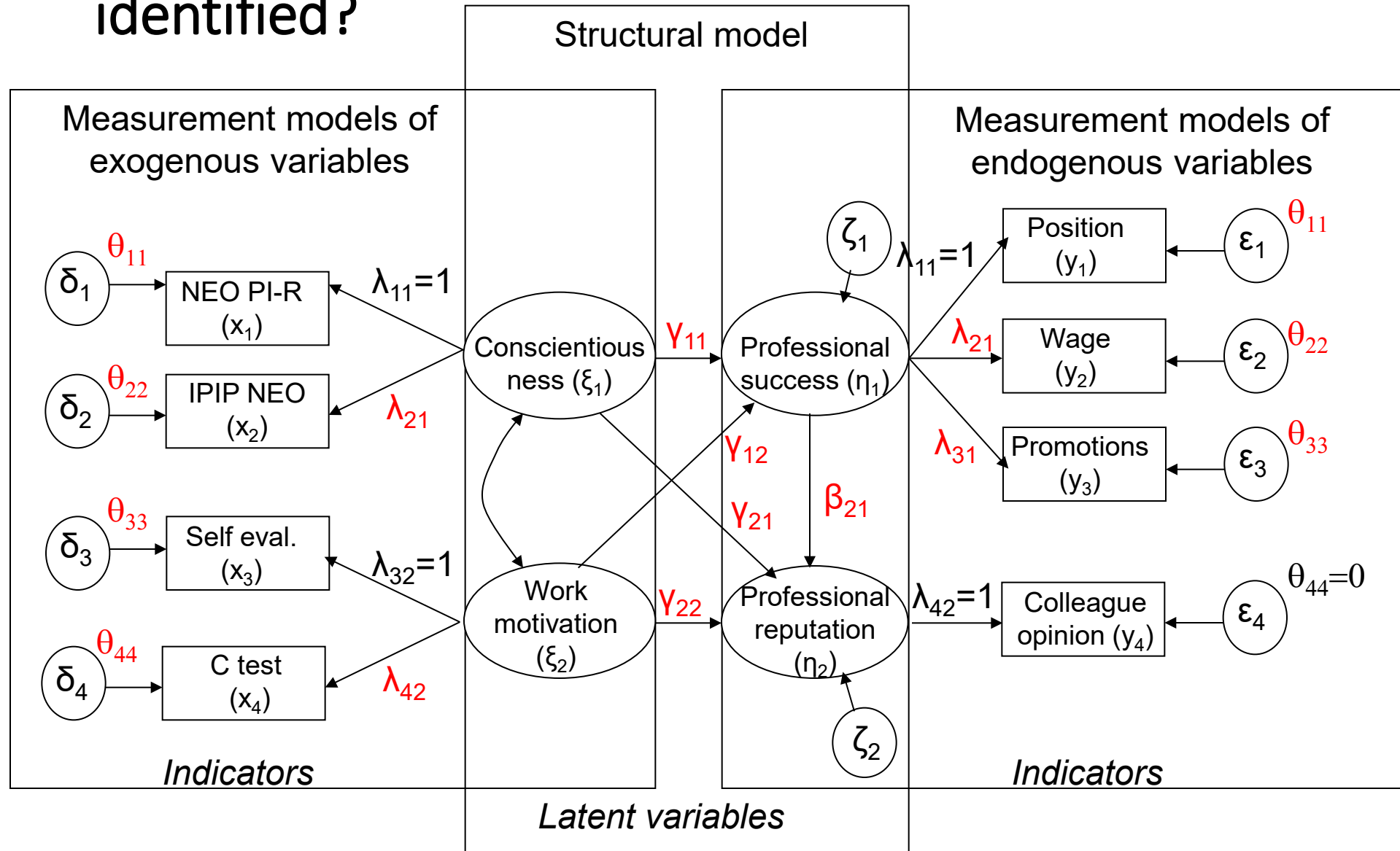
> Is this complex structural equation model identified?



> Is this complex structural equation model identified?

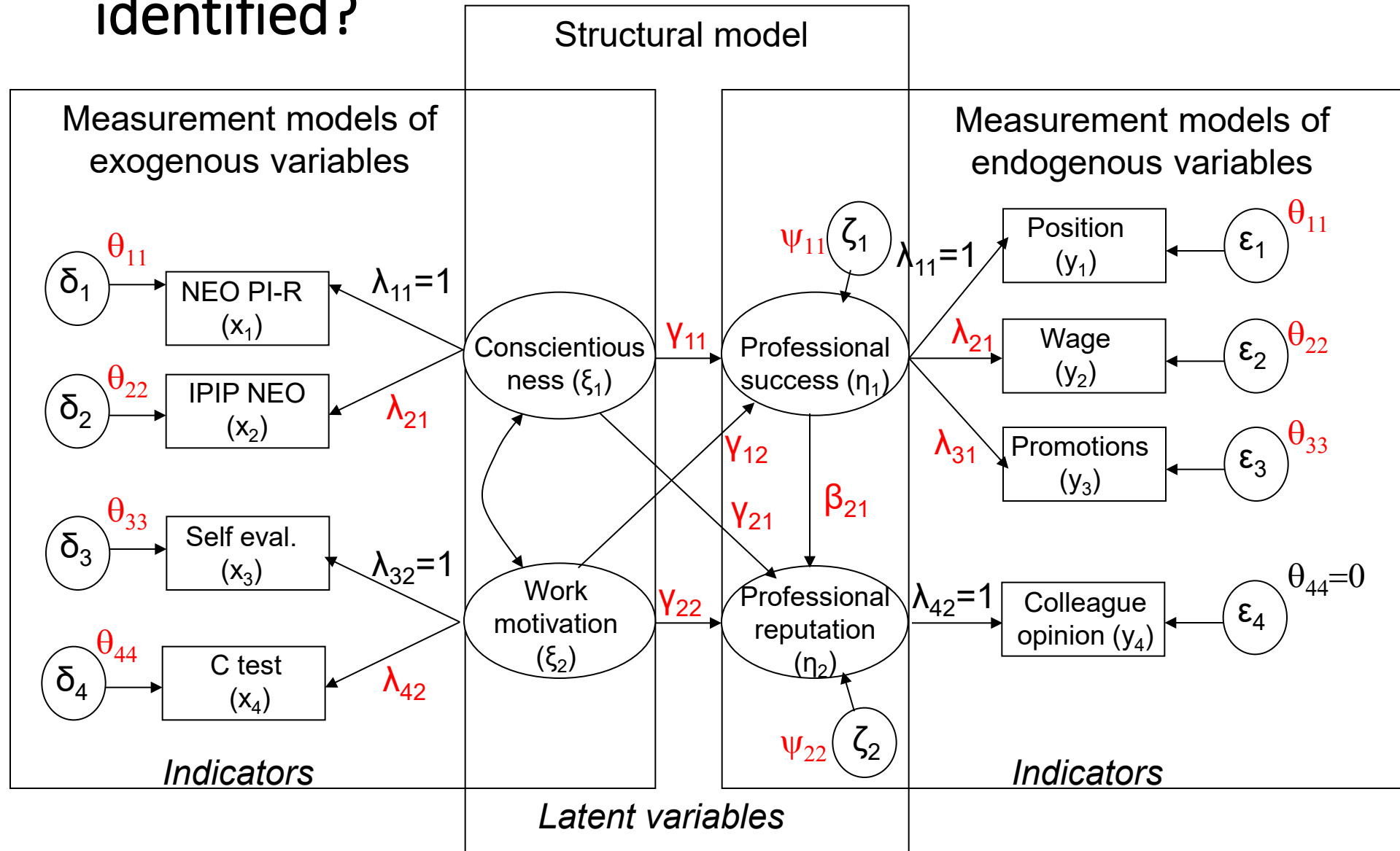


# > Is this complex structural equation model identified?





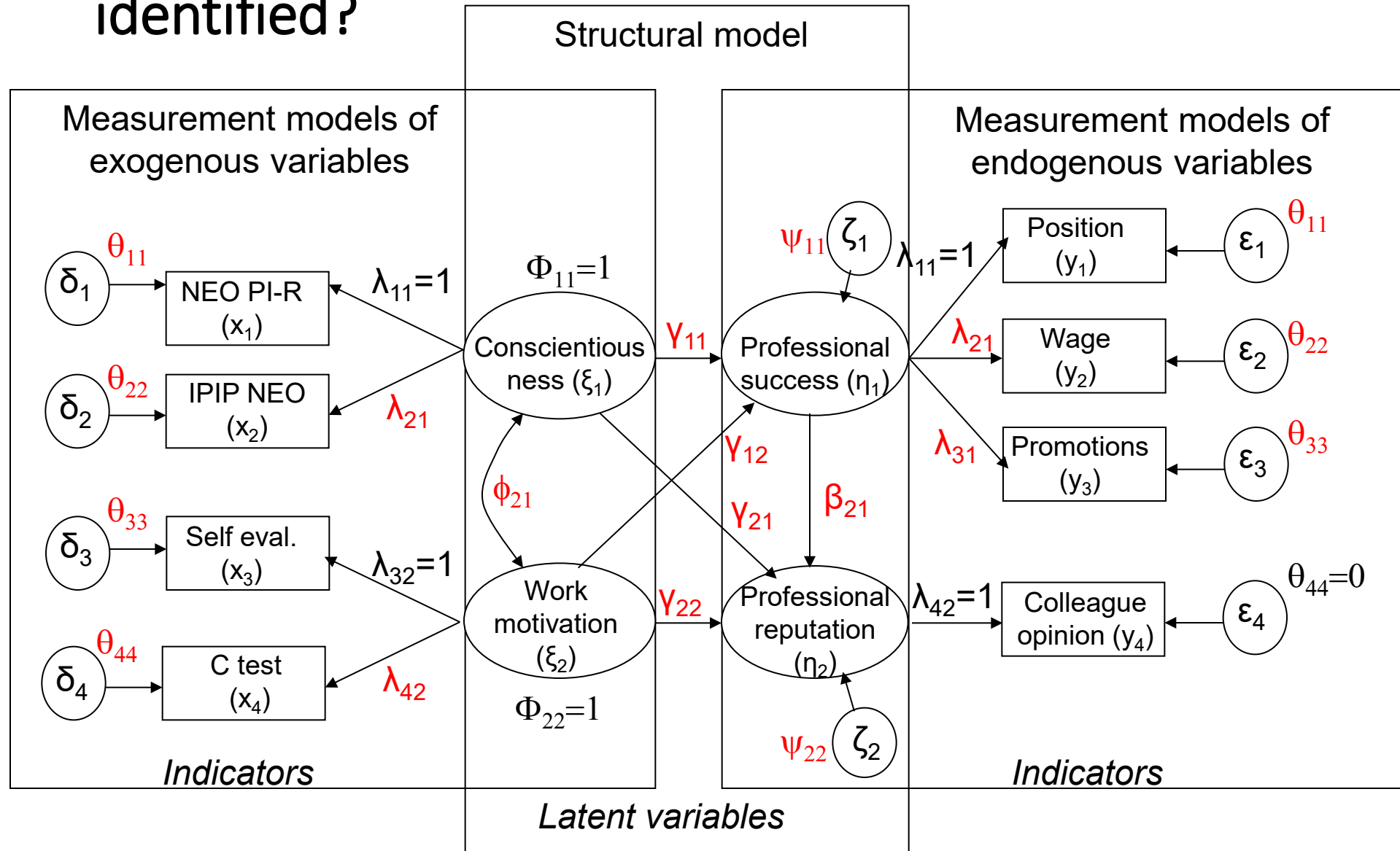
> Is this complex structural equation model identified?



# Practice C

- Determine: What is the number of to-be-estimated parameters?

> Is this complex structural equation model identified?



# Practice C

- What is the number of to-be-estimated parameters?
- 13
- 15
- 17
- 19

# Practice C

- What is the degree of freedom for the complete complex model?

$$n = 8$$

$$S = \frac{n(n+1)}{2}$$

$$df = s - t$$

# Practice C

- What are the degrees of freedom for the complete model?
- 13
- 14
- 15
- 16
- 17
- 19

### 3. Identification of the structure of models

- **Supplement**

- there are additional matrices including parameters
  - these matrices are of importance for the *estimation* of parameters and the evaluation of model fit

## > Vectors and matrices (4)

Further matrices of importance:

- $\Psi$       Psi                      Matrix of variances of the residuals of the *endogenous* latent variables ( $\zeta$ )
- $\Phi$       Phi                      Matrix of variances and covariances of the *exogenous* latent variables
- $\Theta_{\varepsilon}$       Theta-Epsilon                      Diagonal matrix of variances of the residual variables of the model of measurement of the *endogenous* part
- $\Theta_{\delta}$       Theta-Delta                      Diagonal matrix of variances of the residual variables of the model of measurement of the *exogenous* part



## > Vectors and matrices (4)

Further matrices .....

- what these matrices are useful for ?
  - ... they constitute the **model** of the covariance matrix  
(also referred to as *model-implied covariance matrix*)
  - ... in addition to the exogenous measurement model

$$\Sigma = \Lambda \Phi \Lambda' + \Theta_{\delta}$$

„sigma“

## > Vectors and matrices (4)

Further matrices .....

... in addition to the exogenous measurement model

$$\Sigma = \Lambda \Phi_{\xi} \Lambda' + \Theta_{\delta} \quad \left\{ = [\Lambda \xi + \delta]^2 \right\}$$

## A note.

- The **model of the covariance matrix** is another important model that plays a keyrole in CFA and SEM

$$\Sigma = \Lambda \Phi \Lambda' + \Theta$$



- **Sigma** ( $\Sigma$ ) is a  $p \times p$  matrix that is used for parameter estimation and the investigation of model fit.

## A note.

- The model of the covariance matrix is another important model that plays a keyrole in CFA and SEM

$$\Sigma = \Lambda\Phi\Lambda' + \Theta$$

- $\Sigma$  is designed in such a way that it potentially *corresponds to* the empirical  $p \times p$  covariance matrix  $S$  ( $S \sim \Sigma$ )
- Parameter estimation is expected to lead to the *best possible correspondence* with the empirical  $p \times p$  covariance matrix  $S$

# *Summary and brush up:*

0. Introductory remarks

What are exogeneous / endogeneous latent variables?

... independent / dependent latent variables

1. Formation of hypotheses

2. Path diagram and specification of model

3. Identification of the structure of models

# *Summary and brush up:*

0. Introductory remarks

What are exogeneous / endogeneous latent variables?

... independent / dependent latent variables

1. Formation of hypotheses

... being able to state research questions / hypotheses

.... being able to identify the constructs and find indicators

2. Path diagram and specification of model

... draw a path diagram and add parameters

... transform into the formal model

3. Identification of the structure of models

... being able to compute degrees of freedom for substructures

## *Questions regarding course unit 2*

- What are the components of a complete structural equation model?
- What is the basic assumption regarding the relationship of the true part (=systematic part) and the residual part of measurement?
- What means that „the model is identified“?
- What is the meaning of degree of freedom?

# Literature

- Kline, R. B. (2011). *Principles and practices of structural equation modeling* (3rd edition) (Chapter 1: Introduction). New York, NY: The Guilford Press.