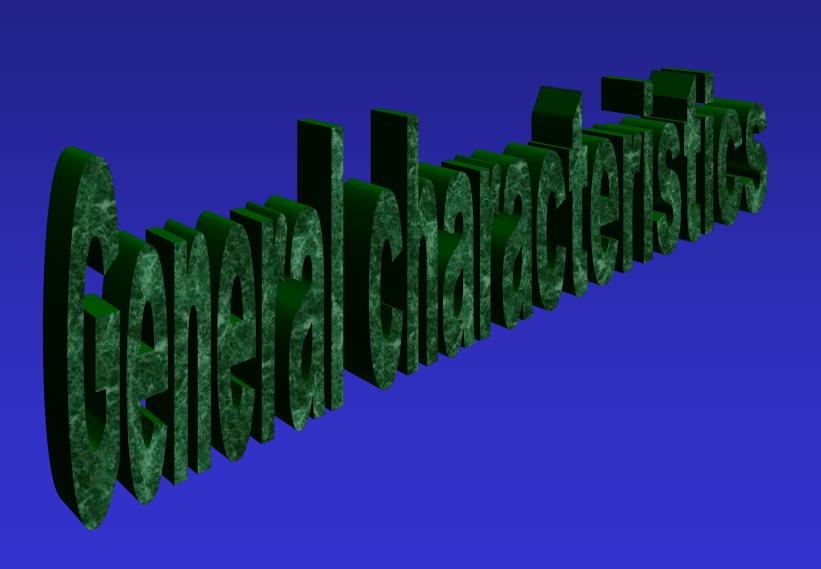
General Introduction to Structural Equation Modeling and Its Application

Outline

- General characteristics
- A short history
- The latent manifest contrast
- The priority of theory
- The specificities of the model
- The need to be complete
- The step from the manifest to latent levels
- The graphical potential



A. Methodological background

- An important characteristic of this approach is the focus on **non-observable random variables** that represent theoretical **constructs**
- It incorporates factor-analytic techniques and methods of regression analysis

B. The denotation

- Modeling latent variables (also applies to the IRT and ML approaches)
- Structural equation modeling
- SEM (Structural equation modeling)
- Confirmatory factor analysis
- CFA (Confirmatory factor analysis)

Outdated denotation:

- LISREL: *Li*near *s*tructural *rel*ationships
- LISREL als program package

- a variable:

..... a *place holder* that can take different values or levels or states

.... is something unknown

.... must be defined (for being psychologically useful)

... in psychological applications variables are usually restricted to real numbers or integers or categories

- a variable:

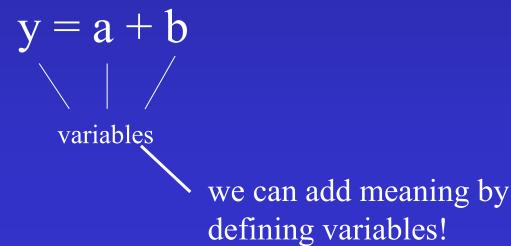
```
.... a place holder that can take different values or levels or states
.... it is something unknown
.... it must be defined (for being useful)
it is often used for representing something
```

- a construct:

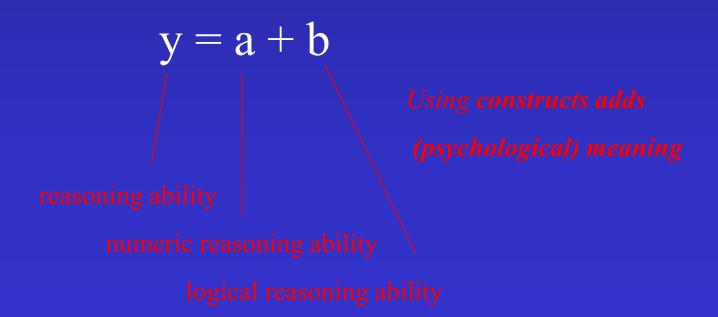
- a theoretical concept / an idea
- is a concept of science
- is a concept that has been shaped as the result of conducting research e.g. competency, extraversion, mathematical reasoning

- relationships according to mathematical structures
- an example:

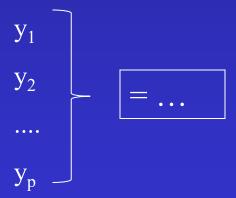
... one of many possible mathematical structures of y, a, b ...



- relationships according to mathematical structures
- an example:



- a multivariate method for data analysis (in contrast to multiple methods)



- a multivariate method for data analysis
- a confirmatory method (... explained in one of the following sections)

- a multivariate method for data analysis
- a confirmatory method
- consideration of multiple indicators

e.g. *construct*: reasoning ability reasoning ability

indicator: ability scale score ability scale score 1 ability scale score 2

- a multivariate method for data analysis
- a confirmatory method
- consideration of multiple indicators
- consideration of errors of measurement

Observed score

Area where the **true** score is likely to be found!

A short history



Spearman is credited with the idea of a latent variable



In 1904 Spearman included the idea of a latent variable in his early concept of factor analysis (FA)



... represents the latent source as latent variable

In about 1935 Thurstone introduced common factor analysis

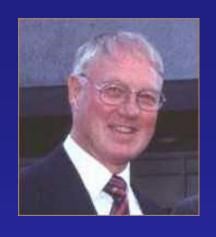


... allows no comparison with alternative structures!

In about 1935 Thurstone introduced common factor analysis

... allows no comparison with alternative structures:
no model check possible!

i.e. it cannot be excluded that the response is driven by another source (—— capitalization on chance)!



 Confirmatory factor analysis becomes available (Jöreskog, 1970, 1971)

In CFA a latent variable is presented as part of a model ... allows check of correctness of model (i.e. it is possible to decide whether there is good model fit or model misfit)



 Confirmatory factor analysis becomes available (Jöreskog, 1970, 1971)

- good model fit ______ confirmation of hypothesis (assumptions) incl. in model

- model misfit rejection of hypothesis (assumptions) incl. in model

The further development consists in adapting the approach to ...

```
... specific research problems
```

... data types

... method effects

٠.,

that makes it really work.



Manifest Contrast

- related to: theory of observational and theoretical languages, etc.

- related to: theory of observational and theoretical languages, etc.
 - i.e. discrimination of two types of language language of what is observable / language of ideas
 - ... was developed by the so-called Vienna group
 - (z. B. Carnap)

- What do you see now (give a name)?



- What do you see now (give a name)?



- What kind of experience do you have with it?
- Is it *manifest* or *latent*?

- related to: discrimination of two types of language ...
- What is the manifest level? What does it mean?



A real burgler!

- related to: discrimination of two types of language ...
- What is the manifest level? What does it mean?
- What is the latent level? What does it mean?

A unreal burgler!



... there is a specific person (,,Max")



When we talk about it, we use the concept of what he is doing

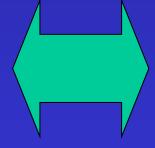
(a burgler)



... because the concept is more general.

- related to: discrimination of two types of language ...
- What is the manifest level? ...
- What is the latent level? ...
- How are the levels related to each other?







How is the contrast reflected in psychological statistics?

(related statistical concepts)

- Things of the manifest level:
- Things of the latent level:

- **⇒** manifest variable
- ⇒ latent variable

How is the contrast reflected in psychological statistics?

(related statistical concepts)

- Things of the manifest level:
- Things of the latent level:

→ manifest variable

latent variable

How are these levels related to each other? ...by the model of measurement

The model of measurement:

it explains a manifest variable (e.g., X) by means of several latent variables (e.g., τ , .. and ε)

$$X = \tau + \dots + \varepsilon$$

(mostly by assuming linear relationships among the latent variables *but other relationships are also possible*)

The model of measurement:

it explains a manifest variable (e.g., X) by means of several latent variables (e.g., τ , .. and ε)

$$X = \tau + \dots + \varepsilon$$



(mostly by assuming linear relationships among the latent variables)

The priority of theory

- research starts with a theory or detailed construct
- data collection is designed according to theory
- data analysis is conducted according to theory

- research starts with a theory or detailed construct
- data collection is designed according to theory
- data analysis is conducted according to theory



Confirmatory approach of research

- Confirmatory approach of research

means the checking of hypotheses (...) regarding ...

- Confirmatory approach of research

means the checking of hypotheses (...) regarding ...



... e.g. food of parrots

- Confirmatory approach of research

means the checking of hypotheses (...) regarding ...



The beak suggests that parrots eat nuts!

- Confirmatory approach of research

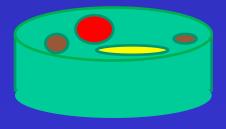
means the checking of hypotheses (...) regarding ...

Hypothesis: parrots

eat nuts!

Search for supporting information





- Confirmatory approach of research

means the checking of hypotheses regarding ...

- Exploratory approach of research

means searching of something



- Confirmatory approach of research

means the checking of hypotheses regarding ...

- Exploratory approach of research

means searching of something

- Origin: Empiricism

Research without assumptions (e.g. John Lock)

- Confirmatory approach of research

means the checking of hypotheses regarding ...

- Exploratory approach of research

means searching of something

- Origin: Empiricism

Research without assumptions (e.g. John Lock)

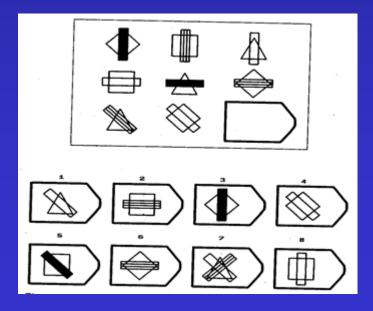
Problems (perceptual errors, random influences, etc.)

An important advantage of the confirmatory approach is that it is instrumental in dealing *with* the *complexity* of the empirical reality.

An example is provided to demonstrate this advantage!

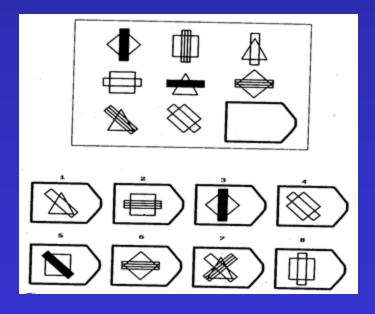
Example: investigation of the validity of a scale - APM

Figure: APM item



Example: investigation of the validity of a scale, APM

- ... is id valid as scale of fluid reasoning?
- ... is id valid as scale of spatial ability?



Example: investigation of the validity of a scale, APM

A popular way of *investigating validity* is to <u>relate</u> the scale to be validated (APM) to other scales that are expected to measure the same construct.

This means that APM is to be related to

- a scale measuring *fluid reasoning*
- a scale measuring spatial ability

Example: investigation of the validity of a scale, APM

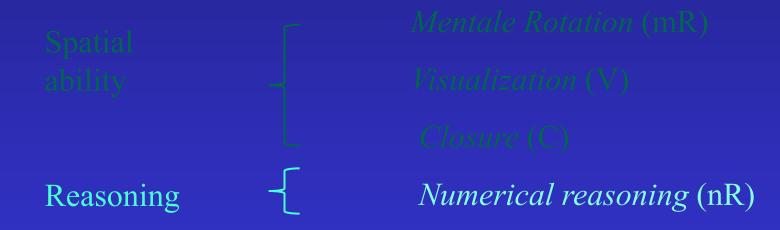
Spatial ability

Mentale Rotation (mR)

Visualization (V)

Closure (C)

Example: investigation of the validity of a scale, APM



Example: investigation of the validity of a scale, APM

Scales used to collect the available data

APM

Mentale Rotation (mR)

Visualization (V)

Closure (C)

Numerical reasoning (nR)

Example: investigation of the validity of a scale, APM

Next step: statistical investigation using

- *exploratory approach*: estimation of the weights w
- *confirmatory approach*: use of already available knowledge for theory-guided estimation the weights *w*

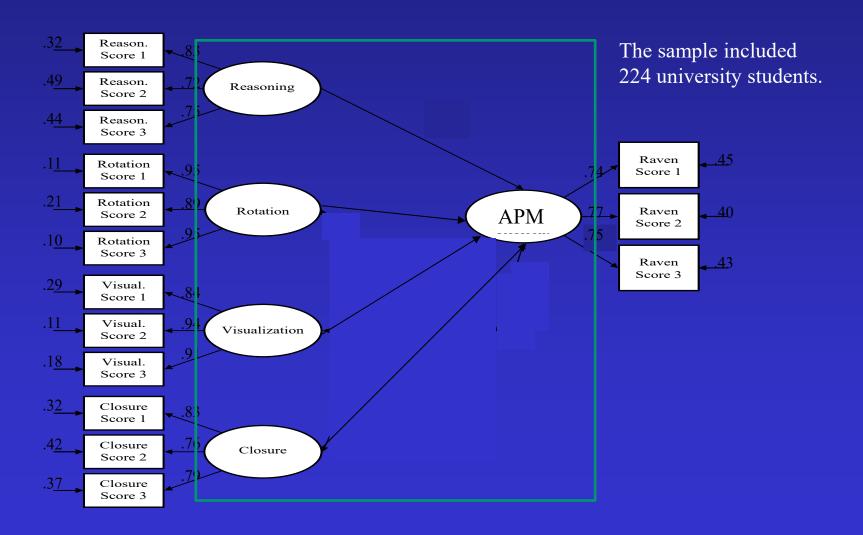
Example: investigation of the validity of a scale, APM

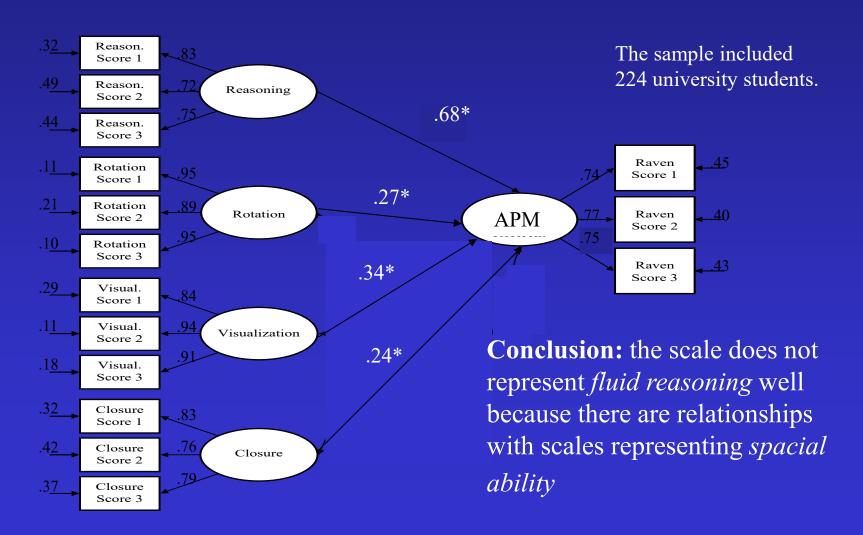
Next step using the

• exploratory approach: estimation of the weights w

Exploratory:
$$Y_{APM} = k + wX_{mR} + wX_{V} + wX_{C} + wX_{nR} + e$$

The estimates of the weights have to show which scale contributes to the scale that is to be validated!





Example: investigation of the validity of a scale, APM

Next step using the ...

• *confirmatory approach*: use of already available knowledge for *theory-guided* estimation the weights w

What we know is that mental rotation (mR), visualization (V) and closure (C) are integrated into spacial ability!

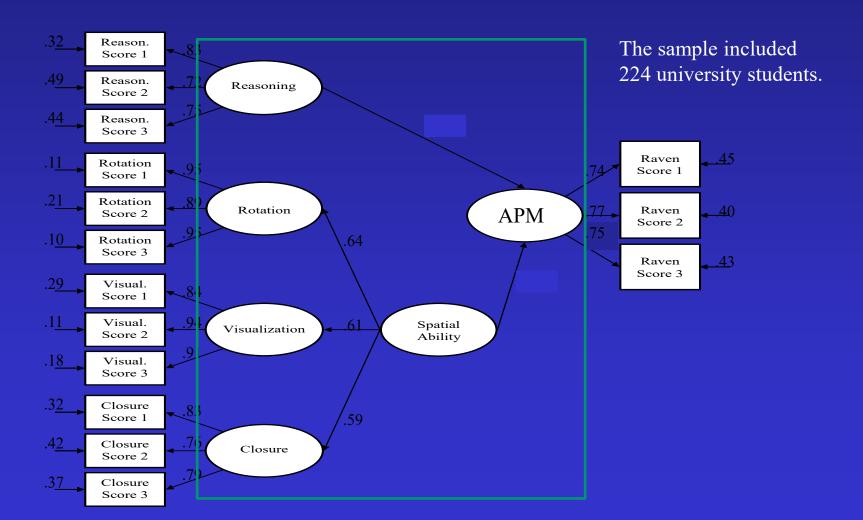
Example: investigation of the validity of a scale, APM

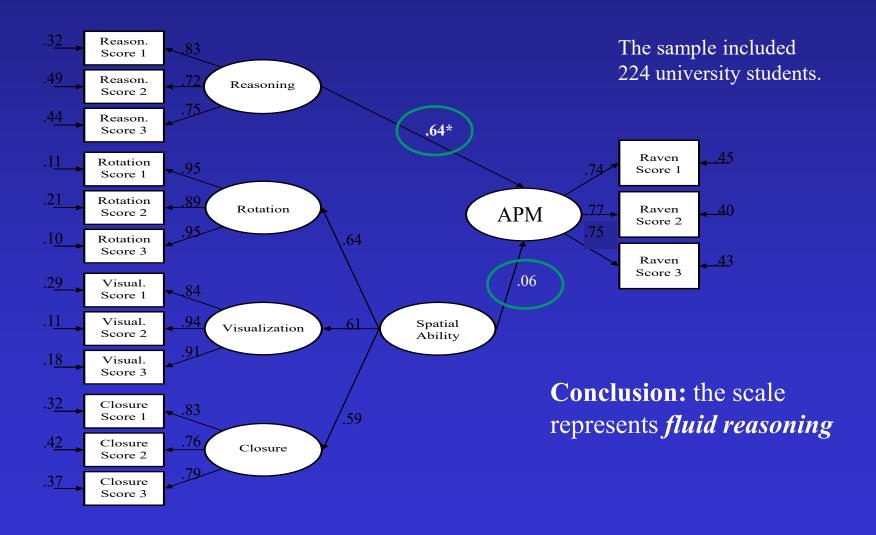
Next step using the

• *confirmatory approach*: use of already available knowledge for *theory-guided* estimation the weights w

Therefore, $\mathbf{w}\mathbf{X}_{mR} + \mathbf{w}\mathbf{X}_{V} + \mathbf{w}\mathbf{X}_{C}$ needs to be integrated into one component: $\mathbf{w}\mathbf{X}_{V mR C}$

so that
$$\mathbf{Y}_{APM} = \mathbf{k} + \mathbf{w} \mathbf{X}_{V_{\underline{m}R}\underline{C}} + \mathbf{w} \mathbf{X}_{\mathbf{n}R} + \mathbf{e}$$

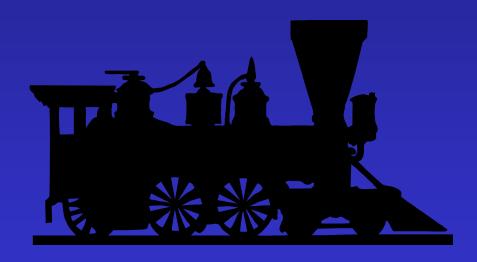




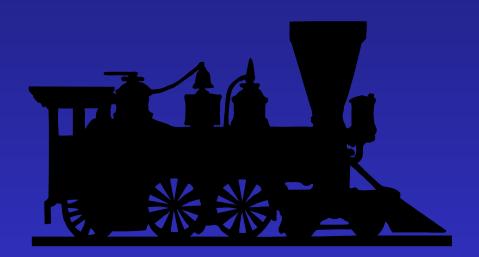
The Special States

The Specific States

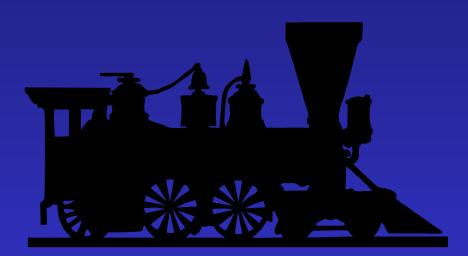
... concerning the question: Why is the focus on the model and not on the theory?







The train-engine theory



The train-engine theory

The train-engine model



The model

.... is a <u>construction</u> that serves the picturing of reality.

.... is to a small degree an abstraction of reality.

The theory

.... is also a construction.

.... is focused on what is essential

A major differerence: the details

• a model is usually rather specific (many details)

e.g. it may say "the train engine has **eight** wheels for rails"



• a theory is usually rather general

e.g. it may say "the train engine has wheels"



The consequences of specificity

- there is one theory (e.g. of the train engine)
- there are various models (e.g. of specific train engines)
- but both of them refer to the same reality

The consequences of specificity

Advantages of models:

- ♦ they are so specific that they **turn out as wrong** when related to observations
- ♦ different models can be compared with each other with respect to observations
- ♦ the best model can be identified

All this means that models are especially useful for investigating properties of ...!

The consequences of specificity

Disadvantage of theory:

- ♦ they can hardly fail (some researchers even argue that it is impossible to disprove a theory)
- ♦ progress in science is hardly possible

An example

An example from cognitive load theory

♦	Theory of cognitive load says a too large <i>load</i> in cognitive process	ing
	leads to errors!	

... it implies that increasing the <u>difficulty</u> (=<u>load</u>) of arithmetic tasks leads to more errors.

..... the difficulty (=load) by integrating <u>more</u> rules to be <u>processed</u> into the arithmetic tasks leads to more errors.

An example

Models for investigating properties: an example from cognitive load theory

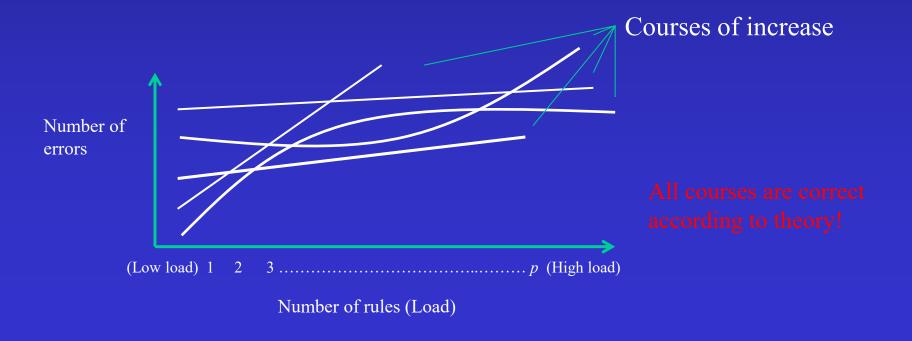
Theory of cognitive load says ... a too large *load* in cognitive processing leads to errors!

- But, cognitive <u>theory</u> is <u>imprecise</u> regarding the relationship of load and errors
 - it only suggests: the more <u>rules</u> (that means more difficult, a higher load), the more errors.

An example

Models for investigating properties: an example from cognitive load theory

Ways of modeling ,,the more rules (...), the more errors ,, can mean



An example

Models for investigating properties: an example from cognitive load theory

.

Models must be specific!

```
e.g. model 1: assumes a linear increase
```

e.g. model 2: assumes a quadratic increase

e.g. model 3: assumes an increase within a specific range

. . . .



This way models are likely to improve our knowledge

On specificities – final comments

- ♦ Models are expected to account for observed data either well or not at all.
- ◆ One aim of research is to identify the best model out of the set of all reasonable models
- ◆ Further research aims at designing models that provide a *good* account of reality.
- ♦ There is agreement that all models are *wrong* but *useful*.

to be complete

- The statistical evaluation of models is conducted by means of *Goodness-of-fit tests*

... this means a "high" error probability is desirable

... compare inferential statistics: a "small" error probability is desirable

- The statistical evaluation of models is conducted by means of

Goodness-of-fit tests

i.e. it is checked whether the data in general are explained by the model

- The statistical evaluation of models is conducted by means of

Goodness-of-fit tests

The general problem with goodness-of-fit testing is ...

- ... that all systematic sources of responding must be represented in the model
- ... otherwise "good model fit" is not reachable

An example:

The investigation of the structural/factorial validity of a reasoning scale

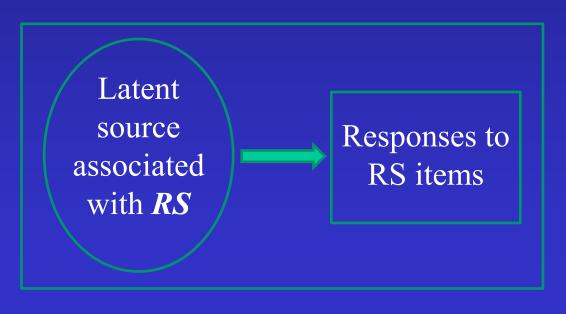
i.e. it is to be investigated whether the items of the

- scale are homogeneous
 - are unidimensional

A specific example:

.... data collection by a reasoning scale (RS)

.... investigation of the data by the following rational:

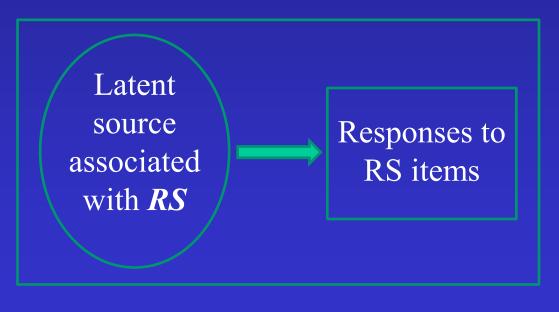


If there is no other source of responding, the corresponding model is likely to yield *good model fit*

A specific example:

.... data collection by a reasoning scale (RS)

.... investigation of the data by the following rational:



But, if there is another source of responding ...

A specific example:

... for demonstrating the "need to be complete", we assumed that something went <u>wrong</u> in data collection:

... the participants did <u>not get enough time</u> to complete all items.

.

The consequence: the statistical investigation indicates a failure; i.e. model misfit.

A specific example:

• • • • •

The consequence: the statistical investigation indicates model misfit.

What does this mean?

- ... the outcome seems to suggest that either
 - model does not fit to data

or

the hypothesis is incorrect

... the true reason for failure is <u>incompleteness</u> (... *not* an incorrect hypothesis)

A specific example:

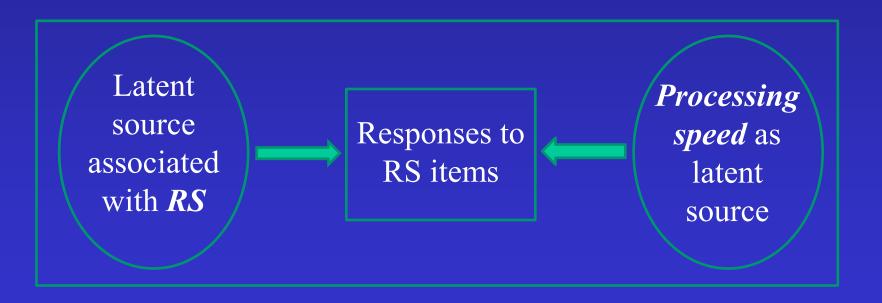
... ,the participants did <u>not</u> get enough time to complete all items" ... means that the participants' processing speed influenced results.

There were two sources of responding instead of one.

Therefore, the model must consider *two* sources in order to account for the data - or - sufficient time must be made available.

A specific example:

... ,the participants did <u>not</u> get enough time to complete all items" ... means that the participants' processing speed influenced results.



In sum

The *need to be complete* creates the following situation:

all important sources of performance must be integrated into the model that is investigated to achieve good model fit.

..... although the researcher may be interested in only one of them.

In sum

••••

If the model is incomplete, it is likely to fail! (... although the investigated hypothesis may be correct)



.... This section is on the consequences of latent variable modeling for the results of investigating the relationship between two constructs!

It demonstrates an advantage of modeling latent variables because the statistical investigation is conducted *at the latent level* (not the manifest level)

customary investigations are conducted at the manifest level

investigations using modeling latent variables are conducted at the latent level.

An example: the relationship between *intelligence* and *natural-science competency* (n-competency)

The data:

- Intelligence data: APM, WAIS
- N-competency data: chemestry and physics grades

An example: the relationship between *intelligence* and *n-competency*

Empirical results for estimating the relationship from student sample:

```
- correlation APM – chemistry grade: .3
```

- correlation *APM physics grade*: .4
- correlation WAIS chemistry grade: .3
- correlation *WAIS physics grade*: .2

What is an estimate of the true relationship?

An example: the relationship between *intelligence* and *n-competency*

Result of averaging: correlation intelligence \emptyset – grade \emptyset : 3

An example: the relationship between *intelligence* and *n-competency* with the transfer to the latent level

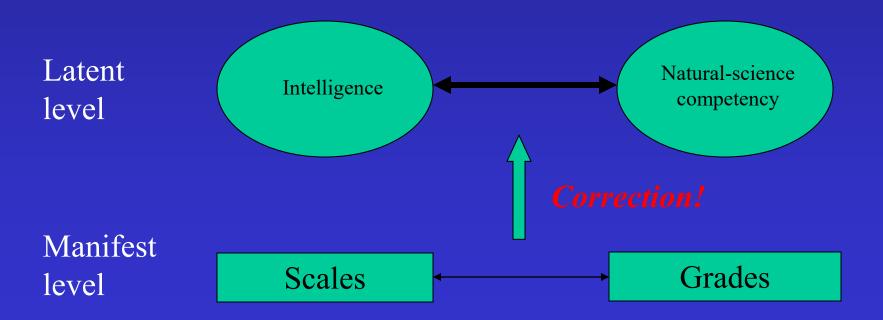
The idea is:

the correlation at the manifest level is impaired by the influence of error.

Therefore, the elimination of error means the transfer to the latent level

The transfer is accomplished by a correction procedure that eliminates the influence of error

An example: the relationship between *intelligence* and *n-competency* with the transfer to the latent level



An example: the relationship between *intelligence* and *n-competency* with the transfer to the latent level

The outset:

covarianz on manifest level

correlation =

SDs

Note. SD means standard deviation

An example: the relationship between *intelligence* and *n-competency* with the transfer to the latent level

A correction procedure eliminates the influence of error!

Note. SD means standard deviation

An example: the relationship between *intelligence* and *n-competency* with the transfer to the latent level

Auxiliary estimates of the relative amount of true variation included in the standard deviation are provided by correlations of the indicators of the same construct!

```
- Correlation WAIS – APM: .77 = \mathbf{c}_{\text{true part}}

- Correlation chemistry grade – physics grade: .77 (Reliability)
```

(+ assumption of equal standard deviation)

An example: the relationship between *intelligence* and *n-competency* with the transfer to the latent level

A correction procedure eliminates the influence of error!

correlation intelligence - n-c... = covarianz on manifest level

An example: the relationship between *intelligence* and *n-competency* with the transfer to the latent level

A correction procedure eliminates the influence of error!

covarianz on manifest level

correlation intelligence-nc... =

 $c_{true_intelligence}SD_{intelligence} \times c_{true_n\text{-competency}}SD_{n\text{-competency}}$

An example: the relationship between *intelligence* and *n-competency* with the transfer to the latent level

 $c_{\text{true}}^{2} SD_{\text{intelligence}} \times SD_{\text{n-competency}}$

An example: the relationship between *intelligence* and *n-competency* with the transfer to the latent level

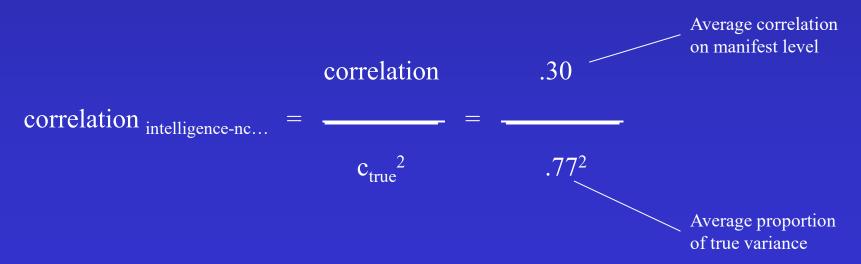
$$correlation \times SD_{intelligence} \times SD_{n-competency}$$

$$c_{true}^{2} SD_{intelligence} \times SD_{n-competency}$$

An example: the relationship between *intelligence* and *n-competency* with the transfer to the latent level

$$correlation \\ correlation \\ = \\ \frac{c_{true}^2}{}$$

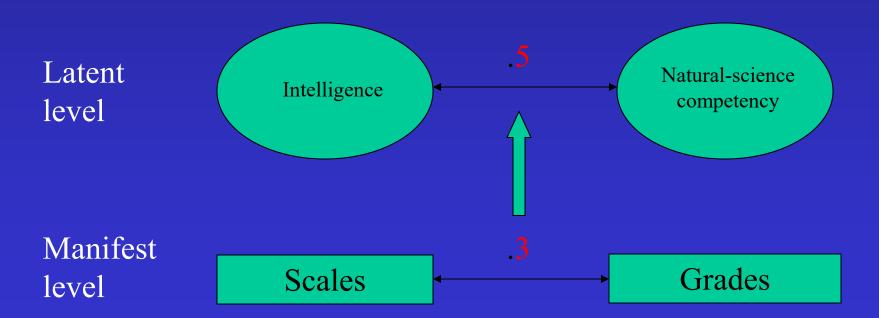
An example: the relationship between *intelligence* and *n-competency* with the transfer to the latent level



An example: the relationship between *intelligence* and *n-competency* with the transfer to the latent level

correlation .30
$$c_{\text{true}}^{2} = \frac{.30}{c_{\text{true}}^{2}} = \frac{.30}{.77^{2}}$$

An example: the relationship between *intelligence* and *n-competency* with the transfer to the latent level





Investigate the relationship between <u>optimismus</u> and <u>extraversion</u> on the latent level when you have only data from the manifest level!

The data:

- -Optimism data: RS (Rosenberg scale), PO (POSO scale)
- -Extraversion data: E-EPI, E-BF (Big FIVE questionnaire)

The correlations:

-RS – E-EPI: .25
$$RS - PO (c_{true})$$
: .8

$$-RS - E-BF$$
: .4

-PO – E-EPI:
$$.2$$
 E-EPI – E-BF (c_{true}) : $.6$

The correction formula:

$$\overline{\text{correlation}} \times \text{SD} \times \text{SD}$$

$$c_{true} \times c_{true} \times SD \times SD$$

The data:

- -Optimism data: RS (Rosenberg scale), PO (POSO scale)
- -Extraversion data: E-EPI, E-BF (Big FIVE questionnaire)

The correlations:

$$-RS - E-BF$$
: .4

$$RS - PO (c_{true})$$
: .8

$$E-EPI-E-BF$$
 (c_{true}): .6

The correction formula:

$$\overline{\text{correlation}} \times \text{SD} \times \text{SD}$$

$$c_{true} \times c_{true} \times SD \times SD$$

The data:

- -Optimism data: RS (Rosenberg scale), PO (POSO scale)
- -Extraversion data: E-EPI, E-BF (Big FIVE questionnaire)

The correlations:

The Formula:

$$correlation_{optimism-extraversion} = \frac{.30 \times SD \times SD}{c_{true} \times c_{true} \times SD \times SD}$$

What is the result?

- .450
- .506
- .625
- .705

Supplement: abuse in IQ research!

How can there be abuse?

* using unreliable tests

Supplement: abuse in IQ research!

Previous example: Reliability Improvement in correlation
.77 .30 .50

Supplement: abuse in IQ research!

Previous example: Reliability Improvement in correlation
.77 .30 .50

Other

example: .60 .30 .83

Consequently:

The worse the reliability of the scale is, the larger the improvement due to correction.

Not good – leads to results that are not replicable

Demonstration of abuse due to unreliability by Gignac using artificial data

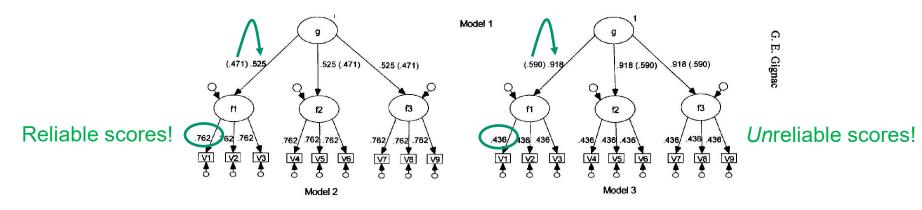


Figure 1:

Model 1 = higher-order model with phantom variables and corresponding constraints applied for the purposes of identification/scaling; Models 2 and 3 = high reliability/low reliability higher-order models with completely standardized maximum likelihood estimation parameter estimates (attenuated parameter estimates within parentheses).

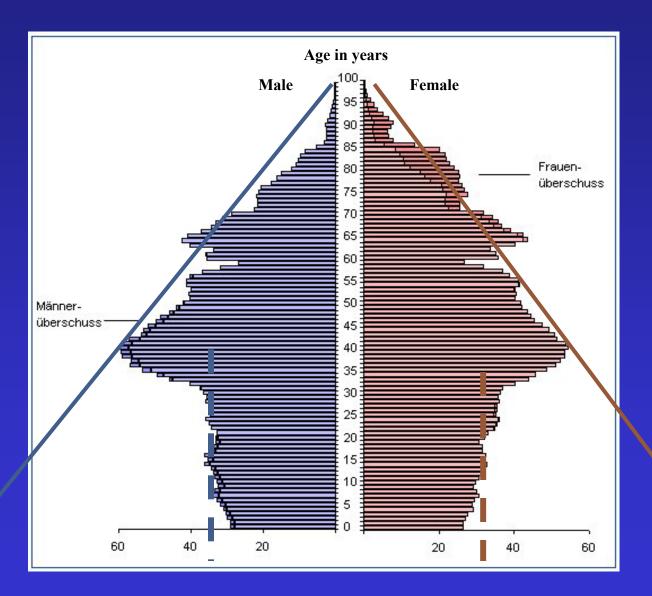


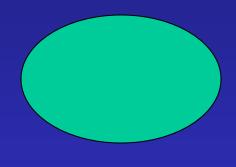
- Structural equation modeling heavily uses graphical illustration
 - for communicating structural features of models
 - for communicating important results

- latent variable modeling within the factor-analytic approach heavily uses graphical illustration
 - for communicating structural features of models
 - for communicating important results

Advantage:

• it uses the high capacity for processing visual informantion





latent variable



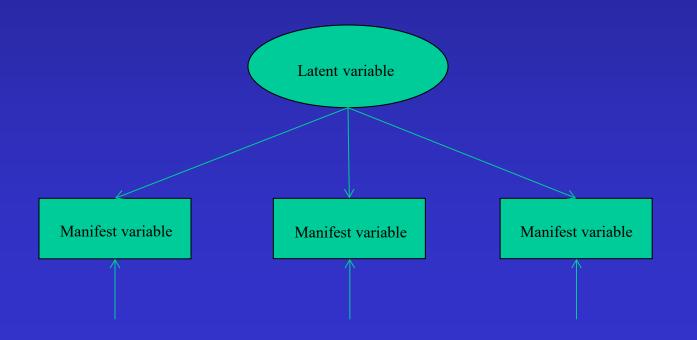
manifest variable

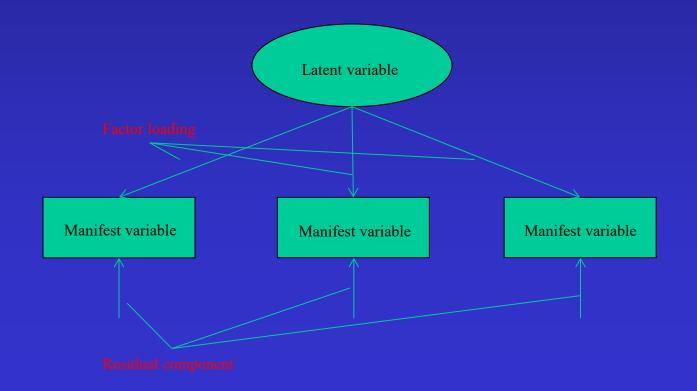


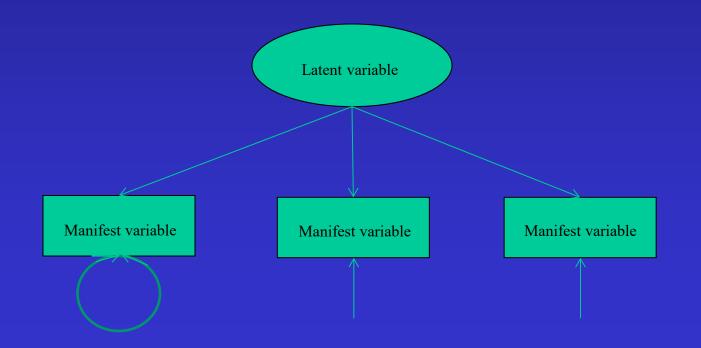
directed relationship

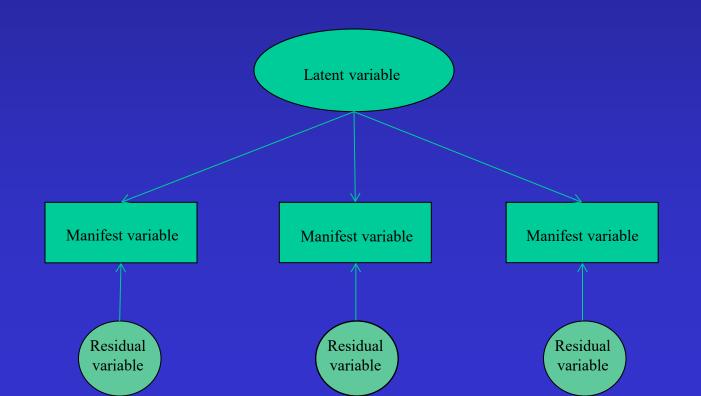


non-directed relationship / correlation









Summary and brush up:

- Some basics
 A short history
 The latent manifest contrast
 The priority of theory
 The specificities of the model
 The need to be complete

 What is a variable / a construct?
 ... a placeholder / a scientifically elaborated idea
 ... observable / not observable
 ... it is a confirmatory approach
 ... models are more detailed than theories
 ... testing extends to the complete model
- The step from the manifest to latent levels
- The graphical potential

Summary and brush up:

- Some basics
- A short history
- The latent manifest contrast
- The priority of theory
- The specificities of the model
- The need to be complete
- The step from the manifest to latent levels
- The graphical potential

The gain due to LVM?

.. estimates free of error

... use of figures and diagrams

Questions regarding the first unit:

- What is a construct?
- What distinguishes manifest from latent?
- What is the meaning of confirmatory?

