Math 240: Discrete Structures I *Due 4:30pm Friday 29 September 2017.*
McGill University, Fall 2017 *Hand in to the mailbox at Burnside 1005.*

Homework assignment 1

Discussing the assignment with other students is allowed (and encouraged!) but you should hand in your own write-up. Only one name per assignment. Please cite the sources you used, including the books and websites you consulted, and the names of the people you collaborated with or who helped you. Late assignments are not accepted. Only your best four out of five homework assignments count. You can earn up to **5 bonus points** for submiting your homework assignment typed up in LaTeX.

Total: **60 points**

Problem 1 (Truth tables)**.** Use truth tables to decide whether each of the following sentences is a tautology, a contradiction, or a contigency. **[5 points]**

a) $(P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$

b) (DeMorgan's Law) $\neg(\neg P \wedge \neg Q) \Leftrightarrow P \vee Q$

c) $(P \Leftrightarrow Q) \Leftrightarrow ((P \wedge Q) \vee (\neg P \wedge \neg Q))$

d) $\neg(X \Rightarrow Y) \Rightarrow (Y \Rightarrow X)$

e) $(\neg P \wedge \neg Q) \Rightarrow (R \Rightarrow Q)$

Problem 2 (Logical identities)**.** Use the logical identities given in class to prove the equivalences listed below. The logical identities can be found on our Piazza site or at `http://www.math.mcgill.ca/~edecorte/math240/identities.pdf` **[4 points]**

a) $(X \Rightarrow Y) \vee (X \Rightarrow Z) \equiv (X \Rightarrow (Y \vee Z))$

b) $(P \Leftrightarrow Q) \equiv ((P \Rightarrow Q) \wedge (\neg P \Rightarrow \neg Q))$

Problem 3 (Order of precedence)**.** For each pair of sentences given below, show that they are not logically equivalent. **[4 points]**

a) $(P \Rightarrow Q) \Rightarrow R$; $\quad P \Rightarrow (Q \Rightarrow R)$

b) $(X \wedge Y) \vee Z$; $\quad X \wedge (Y \vee Z)$

Problem 4 (Symbolization)**.** Symbolize the following English sentences in logic, using the abbreviation scheme provided. **[18 points]**

a) I only leave my door open when I'm in my office. [D: I leave my door open; M: I'm in my office]

b) Neither Peter nor Jake has his driver's licence. [P: Peter has his driver's licence; J: Jake has his driver's licence]

c) If only kryptonite kills me, them I am superman. [k: kryptonite; $K(x)$: x kills me; S: I am superman]

d) Alice is not drinking beer underage. [B: Alice is drinking beer; Q: Alice is at least 18] *(Note: legal drinking age is 18.)*

e) $2 + 2 = 4$, even if my heart would break. [A: $2 + 2 = 4$; B: my heart would break]

f) You can't jump over buildings unless you're a superhero. [B: you can jump over buildings; H: you are a superhero.]

g) Only a knight can marry my daughter. [$K(x)$: x is a knight; $M(x, y)$: x can marry y; d: my daughter]

h) There are at least four distinct people. [$P(x)$: x is a person]

i) The square-root of 5 is irrational. [5: the number five; $x \cdot y$: the product of x and y; $I(x)$: x is a nonzero integer]

Problem 5 (Converse and contrapositive)**.** For each of the following implications, write both the converse and the contrapositive in English. **[6 points]**

a) If Susan is a sophomore, then she needs to take Math 240.

b) The only way to succeed is to keep trying.

c) You can only graduate if you have paid your library fines.

Problem 6 (Knights and knaves)**.** You encounter three inhabitants A, B, and C on the island of knights and knaves. **[6 points]**

A says, "B is a knave."

B says, "A and C are of the same type."

What is C? (I.e. a knight, or a knave?) Use truth tables, the logic identities, or a combination of both to justify your answer.

Problem 7 (Negation)**.** Match up each sentence on the lefthand side with its negation on the righthand side, and justify your answers with the logic identities given in class, together with the rules for negation with quantifiers. **[8 points]**

A) $\forall x(P(x) \wedge [(\exists y P(y)) \wedge (\exists y R(x,y)) \Rightarrow (\exists y P(y))])$

B) $\forall x \forall y(\neg R(x,y) \vee \neg P(x))$

C) $\forall x(P(x) \Leftrightarrow \exists y R(x,y))$

D) $\forall x \forall y(P(x) \wedge R(x,y) \Rightarrow R(y,x))$

1) $\exists x \exists y(P(x) \wedge R(x,y) \wedge \neg R(y,x))$

2) $\exists x([P(x) \vee \exists y R(x,y)] \wedge [\neg P(x) \vee \forall y \neg R(x,y)])$

3) $\exists x \exists y(P(x) \wedge R(x,y))$

4) $\exists x \neg P(x)$

Problem 8 (Thinking)**.** Let us introduce a new logical connective $*$, which we define with the following truth table. **[9 points]**

P	Q	$P * Q$
T	T	F
T	F	T
F	T	T
F	F	T

The sentence $P \vee \neg Q$ is logically equivalent to

$$(((P * P) * Q) * ((P * P) * Q)) * (((P * P) * Q) * ((P * P) * Q)).$$

Find a sentence written only with parentheses, $*$, and the proposition letters X and Y, which is logically equivalent to

$$\neg(X \Rightarrow (\neg X \vee \neg Y)).$$

Prove that your answer is correct. (For instance, you could provide a truth table.)