MATH 240: Discrete Structures 1 Assignment #1

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Problem 1

a Truth Table

P	Q	$\neg P$	$\neg P \vee Q$	$P \implies Q$	$(P \implies Q) \iff (\neg P \vee Q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

This is a tautology

b Truth Table This is a tautology

P	Q	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$	$\neg(\neg P \wedge \neg Q)$	$P \vee Q$	$\neg(\neg P \wedge \neg Q) \iff P \vee Q$
T	T	F	F	F	T	T	T
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	F	T	T	T	F	F	T

c Truth Table This is a tautology

P	Q	$\neg P$	$\neg Q$	$P \iff Q$	$P \wedge Q$	$\neg P \wedge \neg Q$	$((P \wedge Q) \vee (\neg P \wedge \neg Q))$	$(P \iff Q) \iff ((P \wedge Q) \vee (\neg P \wedge \neg Q))$
T	T	F	F	T	T	F	T	T
T	F	F	T	F	F	F	F	T
F	T	T	F	F	F	F	F	T
F	F	T	T	T	F	T	T	T

d This is a tautology

X	Y	$X \implies Y$	$\neg(X \implies Y)$	$Y \implies X$	$(\neg(X \implies Y)) \implies (Y \implies X)$
T	T	T	F	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T

e This is a contingency

P	Q	R	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$	$R \implies Q$	$(\neg P \wedge \neg Q) \implies (R \implies Q)$
T	T	T	F	F	F	T	T
T	T	F	F	F	F	T	T
T	F	T	F	T	F	F	T
T	F	F	F	T	F	T	T
F	T	T	T	F	F	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	F	F
F	F	F	T	T	T	T	T

Problem 2

a The left side of the equation is: $(X \implies Y) \vee (X \implies Z)$
$\equiv (\neg X \vee Y) \vee (\neg X \vee Z)$ Condition Identity
$\equiv (\neg X \vee Y \vee \neg X) \vee (\neg X \vee Y \vee Z)$ Associative Property
$\equiv (\neg X \vee Y) \vee (\neg X \vee Y \vee Z)$ Idempotent Identity
$\equiv (\neg X \neg X \vee Y \vee Y) \vee (Z)$ Associative Identity
$\equiv (\neg X \vee Y) \vee (Z)$ Idempotent Identity
$\equiv \neg X \vee (Y \vee Z)$ Associative Identity
$\equiv X \implies (Y \vee Z)$ Conditional Identity

b Starting from the left side again: $(P \iff Q) \equiv (P \implies Q) \wedge (Q \implies P)$ Biconditional Identity
$\equiv (P \implies Q) \wedge (\neg Q \vee P)$ Conditional Identity
$\equiv (P \implies Q) \wedge (\neg(\neg P) \vee \neg Q)$ Commutative and Double Negative Properties
$\equiv (P \implies Q) \wedge (\neg P \implies \neg Q)$ Conditional Identity

Problem 3

a In this case, let us assume P = False. Therefore, as shown in class, $(P \Rightarrow Q)$ is always true. Similarly, $P \implies (Q \Rightarrow R)$ is always True. However, we can find R such that $(P \Rightarrow Q) \Rightarrow R$ is not true, that is, when R is False. So, for P = False, R = False, the two expressions are not the same and therefore the expressions are logically different.

b To show that the two expressions: $(X \wedge Y) \vee Z$ and $X \wedge (Y \vee Z)$ are not equivalent, let X = False. Now, if Z is picked as True, $(X \wedge Y)$ is False, and $(X \wedge Y) \vee Z$ is True. On the other hand, $X \wedge (Y \vee Z)$ is False since X is False. Therefore, the two sides are not equivalent here.

Problem 4

a D $\Rightarrow$ H

b $\neg P \wedge \neg J$

c $\forall x (K(x) \wedge (x = k)) \implies S$

d B $\Rightarrow$ Q

e A

f (If you jump over buildings, then you must be a superhero) $B \Rightarrow H$

g (Rephrasing: if x can marry my daughter, then x must be a knight) $\forall x M(x, d) \Rightarrow K(x)$

h (There exists at least four distinct x_i's, such that P(x_i) holds and the x_i's are all distinct). This is written as: $(\exists x_1 P(x_1)) \wedge (\exists x_2 P(x_2)) \wedge (\exists x_3 P(x_3)) \wedge (\exists x_4 P(x_4)) \wedge (x_1 \neq x_2 \neq x_3 \neq x_4)$

i For this, let x be the squart root of 5. For a number to be irrational, it cannot be written as a fraction of two integers, ie, there are not both numbers m, n such that $x \neq \frac{m}{n}$
This can be written as: $\forall x (x \cdot x = 5) \implies \neg[\exists m I(m) \wedge \exists n I(n) \wedge (m = n \cdot x)]$

Problem 5

a Converse: If Susan needs to take Math 240, then she is a sophomore.
Contrapositive: If Susan is not a sophomore, then she does not need to take Math 240

b Converse: If one keeps trying, one will succeed.
Contrapositive: If one does not keep trying, then one will not succeed

c Converse: If you've paid your library fines, then you can graduate
Contrapositive: If you haven't paid your library fines, then you can't graduate.

Problem 6

Knights only tell the truth, and knaves only lie.
Let, P : A is a knight
Q : B is a knight
R: C is a knight
A says: B is a knave (1)
B says: A and C are the same (2)
Formulating (1) into a logical expression: $P \iff \neg Q$ (3)
(If and only if A is a knight, then A is telling the truth and B is a knave)
$Q \iff ((P \wedge R) \vee (\neg P \wedge \neg R))$ (4)
If B is a knight, then either A and C are both knights, or A and C are both knaves
From (3): $P \iff \neg Q \equiv (P \wedge \neg Q) \vee (\neg P \wedge Q)$ (From the biconditional identity)
This means either P or Q is true, but not both.
From (4):
$Q \iff ((P \wedge R) \vee (\neg P \wedge \neg R))$
$\equiv Q \wedge ((P \wedge R) \vee (\neg P \wedge \neg R)) \vee \neg Q \wedge \neg((P \wedge R) \vee (\neg P \wedge \neg R))$ (Using the Biconditional identity)
$\equiv Q \wedge ((P \wedge R) \vee (\neg P \wedge \neg R)) \vee \neg Q \wedge ((P \vee R) \wedge (\neg P \vee \neg R))$ (By DeMorgan's laws)
$\equiv Q \wedge ((P \wedge R) \vee (\neg P \wedge \neg R)) \vee \neg Q \wedge ((\neg P \wedge R) \vee (P \wedge \neg R))$ (By distributive and complement and identity laws)
$\equiv P \wedge ((Q \wedge R) \vee (\neg Q \wedge \neg R)) \vee \neg P \wedge ((Q \wedge \neg R) \vee (\neg Q \wedge R))$ (By the distributive and associative laws)
This means, that if P is true, then either Q and R are both true or Q and R are both false $\Rightarrow$ B and C are the same type if A is a knight $\Rightarrow$ From (1) B must be a knave and so C must be a knave
Alternatively, if P is false, then either Q is true or R is true but not both $\Rightarrow$ B and C are different types if A is a knave $\Rightarrow$ Q is true (from (3)) i.e., B is a knight and therefore, C must be a knave.
So, in either case, C is a knave

Problem 7

A $\forall x(P(x) \wedge ((\exists y P(y)) \wedge (\exists y R(x,y)) \implies (\exists y P(y))))$
$\equiv \forall x(P(x) \wedge ((\exists y \neg P(y)) \vee \forall y \neg R(x,y) \vee \exists y P(y)))$ (By the deMorgan's law)
$\equiv \forall x(P(x) \wedge T)$ (By commutative and complement and dominant property)
$\equiv \forall x(P(x))$ (By the dominant property again)
Negating the statement gives: $\neg(\forall x P(x)) \equiv \exists x \neg P(x)$ Match (4)

B $\forall x \forall y(\neg R(x,y) \vee \neg P(x))$
The negation is $\exists x \exists y \neg(\neg R(x,y) \vee \neg P(x)) \equiv \exists x \exists y(R(x,y) \wedge P(x))$, using first deMorgan's law, and then double negative rule.
Match: (3)

C $\forall x(P(x) \iff \exists y R(x,y)) \equiv \forall x((P(x) \wedge \exists y R(x,y)) \vee (\neg P(x) \wedge \forall y \neg R(x,y)))$ (From the biconditional rule).
The negation is $\exists x \neg((P(x) \wedge \exists y R(x,y)) \vee (\neg P(x) \wedge \forall y \neg R(x,y)))$
$\equiv \exists x(\neg(P(x) \wedge \exists y R(x,y)) \wedge \neg((\neg P(x) \wedge \forall y \neg R(x,y))))$ (By deMorgan's Thm.)
$\exists x((\neg P(x) \vee \forall y \neg R(x,y)) \wedge (P(x) \vee \exists y R(x,y)))$ (By deMorgan's Thm.)
$(P(x) \vee \exists y R(x,y)) \wedge (\neg P(x) \vee \forall y \neg R(x,y))$ (By commutative property)
Match (2)

D $\forall x \forall y(P(x) \wedge R(x,y) \Rightarrow R(y,x))$
$\equiv \forall x \forall y((P(x) \wedge R(x,y)) \Rightarrow R(y,x))$ (By order of precedence of operations)

The negation is: $\exists x \exists y \neg(\neg(P(x) \wedge R(x,y)) \vee R(y,x))$ (By Conditional property)
$\equiv \exists x \exists y (P(x) \wedge R(x,y) \wedge \neg R(y,x))$ (By deMorgan's thm)
Match (1)

Problem 8

From the truth table, the equation $P * Q$ is the same as $\neg(P \wedge Q)$ First, I will try to reduce the given expression $\neg(X \Rightarrow (\neg X \vee \neg Y))$ into a simpler form.
$\neg(X \Rightarrow (\neg X \vee \neg Y))$
$\equiv \neg(\neg X \vee (\neg X \vee \neg Y))$ By the conditional identity
$\equiv \neg(\neg X \vee \neg Y)$ By the idempotent identity
$\equiv (X \wedge Y)$ By DeMorgan's law
Therefore, we need to write $X \wedge Y$ in terms of $P * Q \equiv \neg(P \wedge Q)$
Proof:
$X \wedge Y \equiv \neg(\neg(X \wedge Y))$ (Property of negation)
$\equiv \neg((\neg(X \wedge Y)) \wedge (\neg(X \wedge Y)))$ (Idempotent property)
$\equiv (\neg(X \wedge Y)) * (\neg(X \wedge Y))$ (From the definition of $*$)
$\equiv (X * Y) * (X * Y)$ (From the definition of $*$)