

# COMP 350 Numerical Computing

## Assignment #5: Polynomial Interpolation, Spline Interpolation, and Least Squares Approximation

Date given: Wednesday, Nov 1. Date due: 5:00pm, Monday, Nov 15, 2017  
(To be marked by Yangchao Yi (yangchao.yi@mail.mcgill.ca))

1. (6 points) Use your hand to find the Vandermonde form, the Lagrange form, and the Newton form of the interpolating polynomial for these data

x	1	2	3	4
y	2	0	-10	-34

2. (14 points) (Programming by MATLAB) Let  $f(x) = 1/(1 + 25x^2)$ .

(a) Using 7 equally spaced nodes (or knots) on the interval  $[-1, 1]$ ,

- i. find the interpolating polynomial  $p(x)$  of degree 6 for  $f(x)$  by the Newton approach.
- ii. find the natural cubic spline function  $S(x)$  to interpolate  $f(x)$ .
- iii. find the function  $g(x) = a + bx^2 + cx^4$  to approximate  $f(x)$  by the least squares method.

- Print the coefficients of  $p(x)$ ,  $S(x)$  and  $g(x)$ .
- Print the four values  $f(x)$ ,  $f(x) - p(x)$ ,  $f(x) - S(x)$ , and  $f(x) - g(x)$  at 13 equally spaced points  $x_i = -1 + (1/6)i$  for  $i = 0, 1, \dots, 12$ .
- Plot  $y = f(x)$ ,  $y = p(x)$ ,  $y = S(x)$  and  $y = g(x)$  on the same plot.
- In addition to submitting your MATLAB programs, please put them in a PDF file so that it will be easy for the TA to read (note that the TA may or may not run your MATLAB programs).

(b) Using 7 Chebyshev nodes

$$x_i = \cos\left(\frac{2i+1}{2n+2}\pi\right), \quad i = 0, 1, \dots, n, \quad n = 6$$

to do the same things as in (a).

(c) Comparing the two plots obtained in (a) and (b), what do you observe?

**Note:** You are not allowed to use MATLAB built-in functions `polyfit`, `polyval`, and `spline`.