Math 240: Discrete Structures I
McGill University, Fall 2017
Prof: Evan DeCorte

Due 4:30pm Thursday 12 October 2017.
Hand in to the mailbox at Burnside 1005.

Homework assignment 2

Discussing the assignment with other students is allowed (and encouraged!) but you should hand in your own write-up. Only one name per assignment. Please cite the sources you used, including the books and websites you consulted, and the names of the people you collaborated with or who helped you. Late assignments are not accepted. Only your best four out of five homework assignments count. You can earn up to **5 bonus points** for submiting your homework assignment typed up in LaTeX.

Total: **60 points**

Problem 1. Prove by contradiction that $\sqrt{91}$ is irrational. [**5 points**]

Problem 2. [**6 points**]

a) Calculate $\gcd(2016, 208)$.

b) Can you find integers a and b such that $2016a + 208b = 1000$? If so, what are they? If not, why not?

c) Can you find integers a and b such that $2016a + 208b = 1024$? If so, what are they? If not, why not?

Problem 3. Find an integer x which satisfies the congruence. Justify each answer. [**9 points**]

a) $32x \equiv 8 \mod 13$

b) $39x \equiv 65 \mod 169$

c) $x^2 - 7x \equiv 10 \mod 11$

[Hint for c): complete the square.]

Problem 4. What is the last digit of the number 323^{4097}? Explain. [**6 points**]

Problem 5. Prove by induction that [**6 points**]

$$1 + 3 + 3^2 + 3^3 + \cdots + 3^n = \frac{3^{n+1} - 1}{2}$$

for all $n \geq 0$.

Problem 6. **[8 points]**

a) Prove that every positive integer is congruent to the sum of its (base-10) digits modulo 9.

b) Let $n_1 > 0$ be a multiple of 9. Suppose that we add up all the (base-10 digits) of n_1; denote this sum by n_2. Then add up all the digits of n_2 to get n_3, and all the digits of n_3 to get n_4, and so on. This produces a sequence of numbers $n_1, n_2, n_3, \ldots, n_k, \ldots$. Use induction and part a) to prove that $n_k = 9$ for all large enough k. When you write up your solution, clearly state your *induction hypothesis*.

[`https://youtu.be/Q53GmMCqmAM`]

Problem 7. A crowd of at least two people stands in a room and each one holds a cake. At the sound of a whistle, each person throws their cake at the person closest to them. (Before you ask: no one throws cake at himself.) If the number of people in the crowd is odd, then there is someone who does not get a cake thrown at them. Prove this. Assume that all the distances between pairs of people are distinct. **[10 points]**

Problem 8. How many solutions does the following congruence have? **[10 points]**

$$51x \equiv 34 \mod 646$$

Consider two solutions the same if they are congruent modulo 646. It wouldn't hurt to check your answer with a computer (and you're encouraged to do that), but we're looking for a mathematical solution. [Hint: First find one solution. Then, if x and y are both solutions, what can you say about $x - y$?]