Session 3: Time Series using INLA

Learning Objectives

After this lecture you should be able to

- Introduce the concept of time series data
- Identify key features of time series
- Specify the two main types of temporal models (RW and AR)
- Run a time model in R-INLA (Practical 2)

The content in this lecture is based on the data and code available in Chapter 8 of Gomez-Rubio, V. (2020). Bayesian inference with INLA (1st ed.) Link: `https://becarioprecario.bitbucket.io/inla-gitbook/`

Motivation

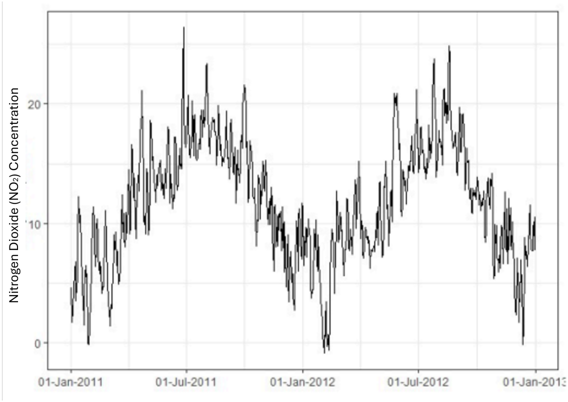
Taking a step back from spatial models ...

What is interesting about time models?

- We live in a complex world and it is often not sufficient to consider just snapshots of a spatial process at a given time.
- The behaviour from one time point to the next is important and this temporal dependence can be modelled.
- Unlike space, the temporal data hold a natural order.

Time Series

- A time series is a set of observations taken sequentially in time
- We define a discrete time series as Z_t, with data taken at specific time intervals $t = \{1, 2, ...\}$



Time series: mean and autocorrelation

A time series Z_t has:

- Mean function: $\mu_t = E(Z_t)$
- Autocovariance function: $C(t, r) = Cov(Z_t, Z_r)$
- Autocorrelation function: $\rho(t, r) = \frac{C(t,r)}{\sqrt{C(t,t), C(r,r)}}$ where $\rho(t, r) \in [-1, 1]$
- Variance $C(t, t) = var(Z_t) = \sigma_t^2$

(a special case of autocovariance):

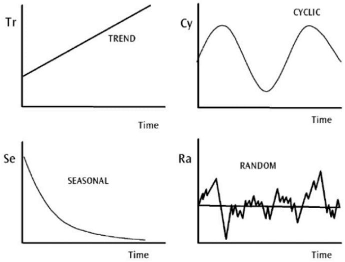
Autocorrelation is the correlation of a variable with itself:

i.e. between Z_t and Z_r for times t and r

Components of time series

Time series can exhibit different patterns or irregular fluctuations:

- Trend refers to long-term change in the mean level
- Seasonal variation refers to periodic fluctuations which occur periodically within a year
- Cyclic changes refers to recurrent rise and falls not over a fixed period
- Irregular fluctuations refers to variations that do not follow any regularity or seasonality



Stationarity

In studying time series, a very important concept is given by stationarity, the stability of the statistical properties of the process through time.

- A time series is said to be strongly stationary if: for any subset of size n and an integer h, $(Z_{t_1}, Z_{t_2}, ..., Z_{t_n})$ has the same distribution as $(Z_{t_1+h}, Z_{t_2+h}, ..., Z_{t_n+h})$
- A time series is said to be weak (or second order) stationary if:
  - $E(Z_t) = \mu$: the mean is constant for all t
  - $(Z_t) = \sigma^2$: the variance does not depend on t
  - $Cov(Z_t, Z_r) = C(t-r)$: the autocovariance only depends on the elapsed time between t and r

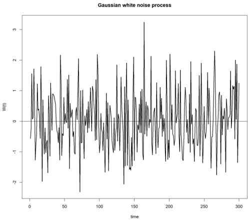
The white noise process

The Gaussian white noise process is a sequence of independent normally identically distributed random variables:

$$W_t \overset{\text{iid}}{\sim} N(0, \sigma^2_W)$$

This process is stationary:

- $E(W_t) = 0$
- $var(W_t) = \sigma^2_W$
- $cov(W_t, W_r) = 0$ for $t \neq r$



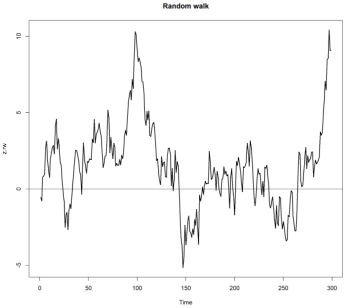
In `R-INLA`, this model is specified using `iid`, e.g. f(time, model=`"iid"`)

Random walk (RW)

The random walk (RW) describes how an observation directly depends upon one or more previous measurements plus a white noise process. The random walk of order 1 (RW1), is defined as:

$$Z_t = Z_{t-1} + W_t$$

where W_t is a white noise process.



Random walk (RW)

For RW1, we have:

$$
\begin{aligned}
Z_1 &= Z_0 + W_1 \\
Z_2 &= Z_1 + W_2 = Z_0 + W_1 + W_2 \\
&\dots \\
Z_t &= Z_0 + W_1 + \cdots + W_t \\
&= Z_0 + \sum_{j=1}^{t} W_j
\end{aligned}
$$

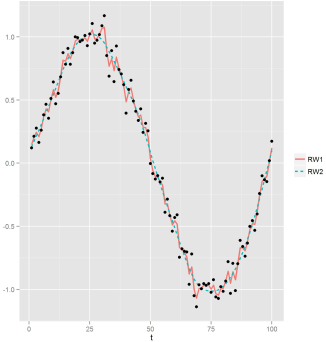
- $E(Z_t) = Z_0 + \sum_{j=1}^{t} E(W_j) = Z_0$, which is independent of t.
- $Var(Z_t) = var(\sum_{j=1}^{t} W_j) = \sum_{j=1}^{t} \sigma_W^2 = t\sigma_W^2$, which depends on t.

Hence the random walk process is not stationary.

Random walk (RW)

For the random walk of order 2, RW2, we have:

$$Z_t = 2Z_{t-1} - Z_{t-2} + W_t$$



In R-INLA, the RW1 and RW2 models are implemented through the model specification `rw1` and `rw2`, e.g. f(time, model="`rw1`")

Autoregressive (AR) process

The autoregressive process of order (p), AR(p) is time series model where the original data is expressed as a function of its previous values in time.
It is defined as:

$$Z_t + \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \cdots + \phi_p Z_{t-p} + W_t$$

where W_t is a white noise process and ϕ_i is a sequence of unknown autoregressive parameters.

The AR(1) process

The simplest model is for $p = 1$, the AR(1) model:

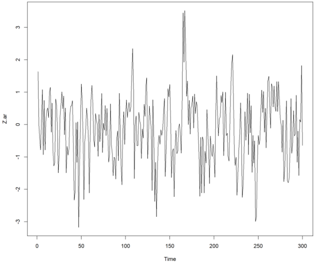
$$Z_t = \rho Z_{t-1} + W_t$$

where we set $\phi_i = \rho$ for $|\rho| < 1$, an unknown temporal correlation term.

The AR1 process can be written as an infinite series of white noise random variables, and since $E(W_t) = 0$ and $var(W_t) = \sigma^2_W$:

- $E(Z_t) = 0$
- $var(Z_t) = \frac{\sigma^2_w}{1-\rho^2}$

which does not depend on time, thus the process is stationary.



In `R-INLA`, the AR1 model is implemented through the model specification `ar1`, e.g. f(time, model=`"ar1"`)