Homework: 6.3, 6.5

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Math 202: Vector Calculus

Due October 20th, 2020

6.3.5

Optional

6.3.6

We can imagine that x and $\tilde{x}$ are vectors in $\mathbb{R}^n$. $|\tilde{x} - x|$ is a vector in the box J. We can show this symbolically,

$$x = (x_1, x_2, ..., x_n) \qquad \tilde{x} = (\tilde{x}_1, \tilde{x}_2, .., \tilde{x}_n)$$

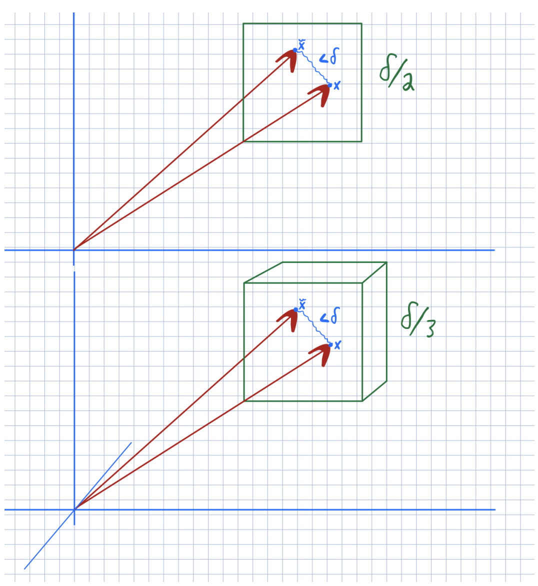
thus,

$$|\tilde{x}_i - x_i| < \delta/n$$

Then we see by size bounds,

$$|\tilde{x} - x| \leq \sum_{i=0}^{n} |\tilde{x}_i - x_i| < \sum_{i=0}^{n} \delta/n = \delta$$

In $\mathbb{R}^2$ and $\mathbb{R}^3$ it would look like,



6.3.7

6.2.5 is a prior example when we could integrate a discontinuous function.

Since f is monotonic increasing, we know that its largest value taken on f is the largest value on the domain, so we can bound f by having a ball $B(0, \max\{|f(0)|, f(1)\})$ thus f is bounded. Note that the interval $f(x) \in [f(0), f(1)], x \in B$ is compact as it is closed and bounded.

Firstly, if f is continuous then it is clearly integrable. If f is not continuous, then we see a stair-step function. Suppose

$$(f(1) - f(0))/n < \varepsilon$$

Then we can partition the interval into n subintervals of length $1/n$. Then we see,

$$L(f, P) = 1/n(f(0) + f(1/n) + ... + f((n-1)/n)$$

$$U(f, P) = 1/n(f(1/n) + f(2/n) + ... + f(1)$$

Then

$$U(f, P) - L(f, P) = (f(1) - f(0))/n < \varepsilon$$

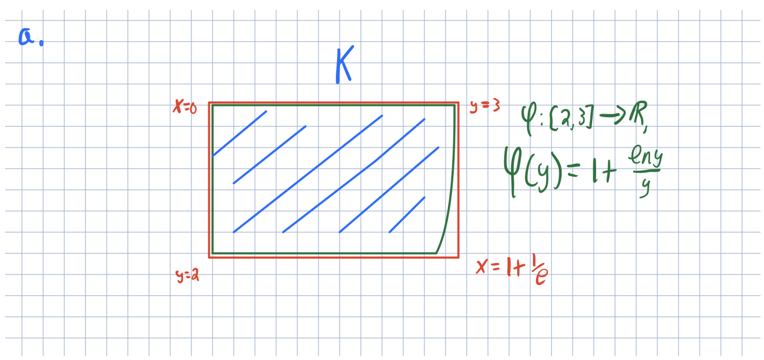
Then we see that $\int_B f$ exists.

6.5.9

Note that all K are compact and that all f is continuous on the K.

a.

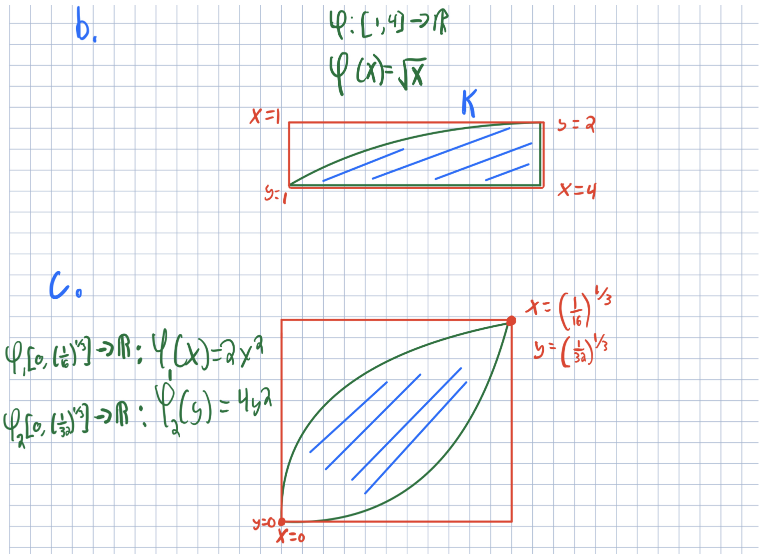
We can see, that we put the blue set K into the red box $[0, 1 + 1/e] \times [2, 3]$



We extend f to B by setting $f = 0$ on $B - K$, then f is only discontinuous on the curve between K and $B - K$, which is boundary curve which is a subset of the graph φ defined above. We know that the boundary curve has volume zero by Proposition 6.5.3 and thus the integral $\int_B f$ exists by Theorem 6.5.4, which is exactly $\int_K f$ by Definition 6.5.6.

b.

We can see, that we put the blue set K into the red box $[1, 4] \times [1, 2]$



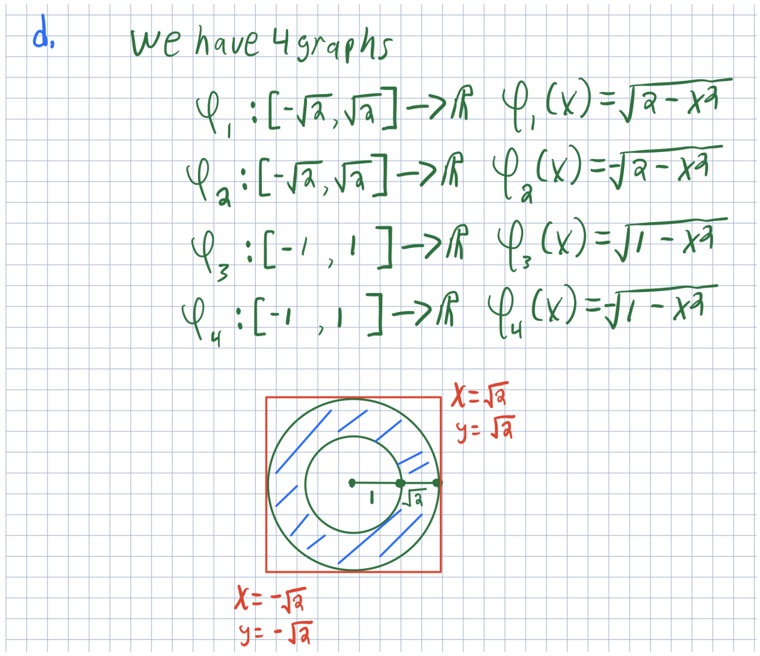
We extend f to B by setting $f = 0$ on $B - K$, then f is only discontinuous on the curve between K and $B - K$, which is boundary curve which is a subset of the graph φ defined above. We know that this boundary curve has volume zero by Proposition 6.5.3 and thus the integral $\int_B f$ exists by Theorem 6.5.4, which is exactly $\int_K f$ by Definition 6.5.6.

c.

We can see, that we put the blue set K into the red box $\left[0, \left(\frac{1}{16}\right)^{1/3}\right] \times \left[0, \left(\frac{1}{32}\right)^{1/3}\right]$. We extend f to B by setting $f = 0$ on $B - K$, then f is only discontinuous on the curves between K and $B - K$, which are the boundary curves which are subsets of the graphs φ_1, φ_2 defined above. We know that these boundary curves has volume zero by Proposition 6.5.3 and thus the integral $\int_B f$ exists by Theorem 6.5.4, which is exactly $\int_K f$ by Definition 6.5.6.

d.

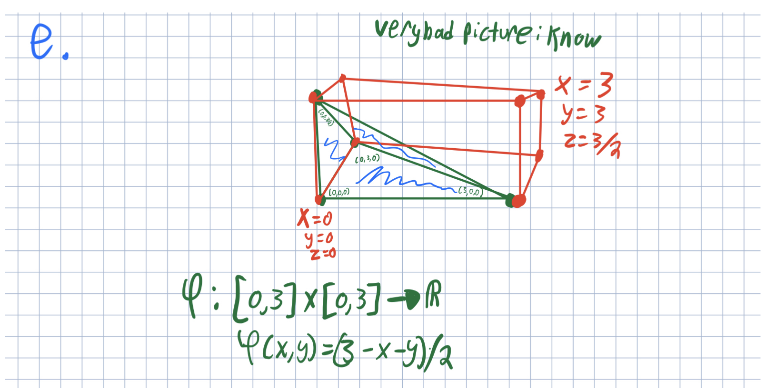
We can see, that we put the blue set K into the red box $[-\sqrt{2}, \sqrt{2}] \times [-\sqrt{2}, \sqrt{2}]$.



We extend f to B by setting $f = 0$ on $B - K$, then f is only discontinuous on the curves between K and $B - K$, which are the boundary curves which are the graphs φ_1, φ_2, φ_3, and φ_4 defined above. We know that these boundary curves has volume zero by Proposition 6.5.3 and thus the integral $\int_B f$ exists by Theorem 6.5.4, which is exactly $\int_K f$ by Definition 6.5.6.

e.

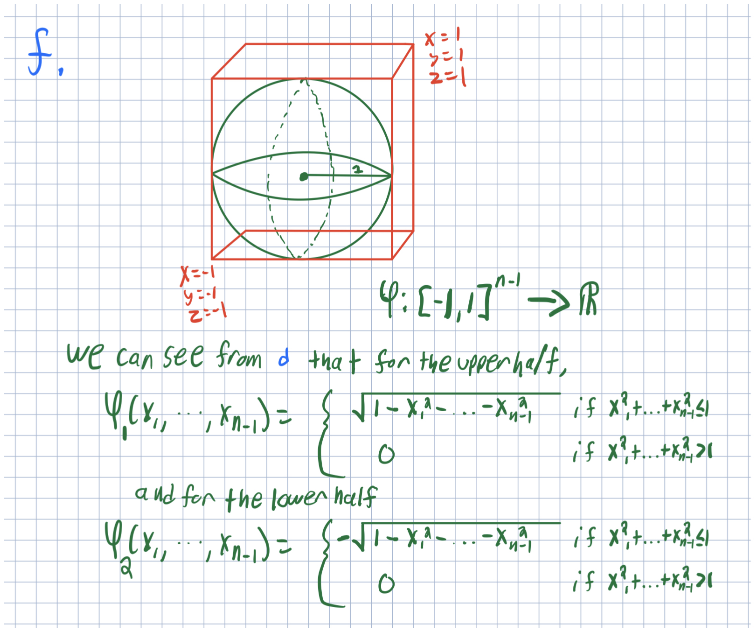
We can see, that we put the blue set K into the red box $[0, 3] \times [0, 3] \times [0, 3/2]$.



We extend f to B by setting $f = 0$ on $B - K$, then f is only discontinuous on the plane between K and $B - K$, which is the boundary plane which is a subset of the graph φ defined above. We know that these boundary plane has volume zero by Proposition 6.5.3 and thus the integral $\int_B f$ exists by Theorem 6.5.4, which is exactly $\int_K f$ by Definition 6.5.6.

f.

We can see, that we put the blue set K into the red box $[-1, 1]^n$.



We extend f to B by setting $f = 0$ on $B - K$, then f is only discontinuous on the curves between K and $B - K$, which are the boundary curves which are subsets of the graphs φ_1, φ_2 defined above. We know that these boundary curves has volume zero by Proposition 6.5.3 and thus the integral $\int_B f$ exists by Theorem 6.5.4, which is exactly $\int_K f$ by Definition 6.5.6.