Homework: 2F

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Math 113: Discrete Structures

Due September 11th, 2020

Problem 1

a.

There are 2 cases. First, if the 2 sets are disjoint, then the cardinalities of each set adds up to cardinality of the union because there are no overlapping elements. For 2 sets that are not disjoint, we are counting shared elements twice, thus we must subtract the elements A and B have in common. Thus the equality follows.

Problem 2

a.

The maximal amount of elements is $m+n$ which occurs when $A \cap B = \emptyset$. The minimum is m which occurs when $A \subseteq B$.

b.

The maximal is n which occurs when $A = B$. The minimal is 0, when $A \cap B = \emptyset$

Problem 3

a.

Any number whose prime factorization is 2^x, where $x \in \mathbb{Z}_{\geq 0}$ since this indicates that a single digit in the bit string is 1. Also including 1, since $2^0 = 1$.

b.

The number would be $2^{|A|} - 1$ since all b_i would be 1 implying

$$b_{n-1}2^{n-1} + b_{n-2}2^{n-2} + ... + b_1 2^1 + b_0 2^0 = 1 \cdot 2^{n-1} + 1 \cdot 2^{n-2} + ... + 1 \cdot 2^1 + 1 \cdot 2^0 = 2^n - 1$$

c.

Take $A = \{x_0, x_1, ..., x_{n-1}\}$ with $B \in 2^A$, that does not have x_{n-1} with the subset strictly ordered, then the number is even because the bit string is in the form

$$[...0]_2 = b_{n-1}2^{n-1} + b_{n-2}2^{n-2} + ... + b_1 2^1 + 0 \cdot 2^0 = 2(b_{n-1}2^{n-2} + b_{n-2}2^{n-3} + ... + b_1 2^0),$$

which is even. To put it more formally, all sets B need to be in the form $B \in \{A : A \in 2^A, A \cap \{x_{n-1}\} = \emptyset\}$