MATH 113: DISCRETE STRUCTURES
HOMEWORK DUE FRIDAY WEEK 12

Problem 1.

(a) Find the smallest positive integer n such that $7^n \equiv 1 \pmod{100}$.
(b) Use your solution to part (a) to find the last two digits of 7^{2020}. (You can use a computer to check your answer, but show how the solution can be derived easily by hand using part (a).
(c) (This part is optional and will not be graded.) What are the last two digits of

$$7^{7^{7^{\cdot^{\cdot^{\cdot^{7}}}}}}$$

in which the number of 7s appearing is 2020? Note $7^7 = 823543$ (or $43 \pmod{100}$), and $7^{7^7} = 7^{823543} \neq (7^7)^7 = 823543^7$.

Problem 2. Prove that if $a, b, c, m \in \mathbb{Z}$, $c \neq 0$, and $ac \equiv bc \pmod{mc}$, then $a \equiv b \pmod{m}$.

1

a

$7^1 \equiv 7 \pmod{100}$

$7^2 \equiv 49 \pmod{100}$

$7^3 \equiv 43 \pmod{100}$

$7^4 \equiv 43 \cdot 7 \pmod{100} = 1 \pmod{100}$

$7^5 \equiv 1 \cdot 7 \pmod{100} = 7 \pmod{100}$,

$\Rightarrow n = 4$

b

From the pattern, the last 2 digits should be 01. (07, 49, 43, 01, 07, ...)

2.

$ac \equiv bc \pmod{mc} \Leftrightarrow mc \mid ac - bc \Leftrightarrow q\,mc = ac - bc$

$\Updownarrow$

$a \equiv b \pmod{m} \Leftrightarrow m \mid a - b \Leftrightarrow q\,m = a - b$