Homework: 3.10

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Math 202: Vector Calculus

Due September 18th, 2020

3.10.4

$$\begin{aligned}(u+v)\times(u-v) &= u\times(u-v)+v\times(u-v) && \text{by bilinearity of the cross product}\\ &= u\times u - u\times v + v\times u - v\times v && \text{by bilinearity of the cross product}\\ &= -2(u\times v) && \text{by skew-symmetry of the cross product}\end{aligned}$$

Thus as we see $(u+v)\times(u-v)$ is a scalar multiple of $u\times v$.

3.10.6

$$u\times v = u\times w \implies u\times v - u\times w = 0$$

and by bilinearity,

$$u\times(w-v) = 0$$

Thus for cancellation to work, u and $w-v$ must be colinear, or scalar multiples of each other.

3.10.10

Since this relation is satisfied, we know that (x,y,z) lie in both the $l(p,d)$ line and the $l(p,D)$ line. For the first condition,

$$\begin{aligned}x_d x_D + y_d y_D + z_d z_D &= 0\\ \langle d, D\rangle &= 0\end{aligned}$$

Which implies the lines are orthogonal.

For the second condition, it can be rethought of as

$$cx_D = x_d$$

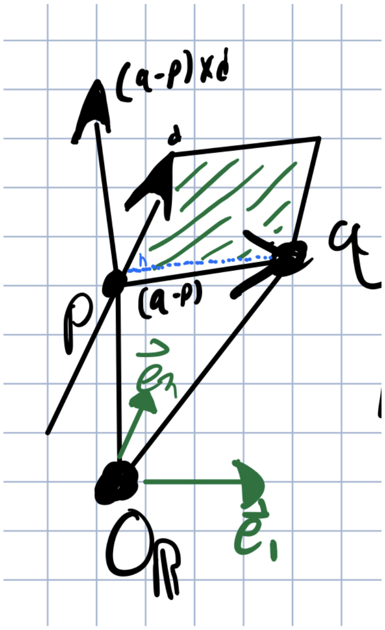
$$cy_D = y_d$$

$$cz_D = z_d$$

$$\implies cD = d$$

Thus implying the lines are the same line since d and D are scalar multiples of each other.

3.10.12



As we can see, and calculate, the absolute value of the determinate is equal to the area of the parallelogram formed by d and $q-p$. Let us call h the shortest distance from the vector d to the point q, or the height of the parallelogram. $h = |(q-p)e_1|$ as we only are considering the component in the e_1 direction for the height. Using $bh =$ Area of parallelogram, we find

$$|d||(q-p)e_1| = |(q-p) \times d| \implies h = \frac{|(q-p) \times d|}{|d|}$$

3.10.15

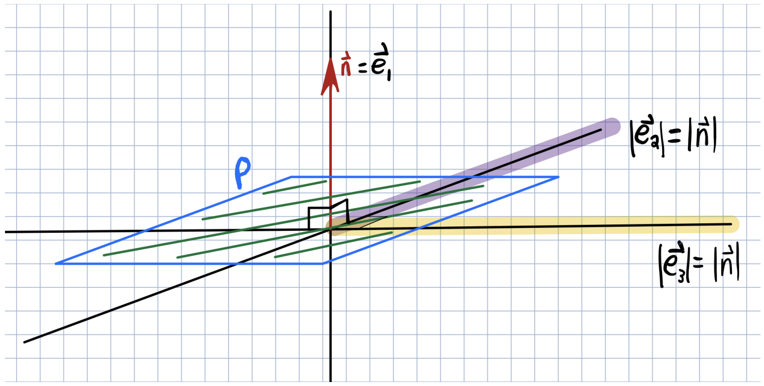
Let us choose one component to be nonzero, take x without loss of generality, then

$$\frac{x}{a} = 1$$

which is true when $x = a$. From here we can see that the plane intersects each axis at $(a, 0, 0)$,$(0, b, 0)$, and $(0, 0, c)$

3.10.17

Let us consider the inner product with the basis such that $|ne_1| = |n|$ and $|ne_2| = 0$. Or in other words if we imagine three dimensional space, then our n vector is one axis, and the two orthogonal vectors embedded in the plane P are the other two axes.



Then

$$\begin{aligned}
&|\langle (q-p), n\rangle| \\
&\langle (|(q-p)_{\perp n}|, |(q-p)_{\| n}|), (|n|, 0)\rangle \\
\implies &|(q-p)_{\| n}||n| = |\langle (q-p), n\rangle| \\
\implies &(q-p)_{\| n} = \frac{|\langle (q-p), n\rangle|}{|n|}.
\end{aligned}$$

And $(q-p)_{\| n}$ = the distance to the plane, as seen below.