MATH 113: DISCRETE STRUCTURES
HOMEWORK DUE WEDNESDAY WEEK 12

Problem 1. Use the Euclidean algorithm to compute the following (showing your work):

(a) $\gcd(20, 45)$ (b) $\gcd(247, 299)$ (c) $\gcd(51, 897)$.

Problem 2. Use the Euclidean algorithm to compute the $\gcd$ of 198 and 168 and find integers m and n such that

$$\gcd(198, 168) = 198m + 168n.$$

Show your work.

Problem 3.

(a) Show that if n is positive integer of the form $4k + 3$ for some integer k, then n is not a perfect square. (Hint: Suppose $n = m^2$. We can then write $m = 4q + r$ for some $r \in \{0, 1, 2, 3\}$. Consider the remainders of the quantities $(4q)^2$, $(4q+1)^2$, $(4q+2)^2$, and $(4q+3)^2$ upon division by 4.)

(b) Show that no integer in the sequence

$$11, \quad 111, \quad 1111, \quad 11111, \quad \ldots$$

is a perfect square. [Hint: Use the fact that $111\ldots1111 = 111\ldots1108 + 3$.]

1

a. $45 = 2 \cdot 20 + 5$

$20 = 5 \cdot 4 + 0$

$\therefore \gcd(45, 20) = 5$

b. $299 = 247 \cdot 1 + 52$

$247 = 52 \cdot 4 + 39$

$52 = 39 \cdot 1 + 13$

$39 = 13 \cdot 3 + 0$

$\therefore \gcd(247, 299) = 13$

c. $897 = 51 \cdot 17 + 30$

$51 = 30 \cdot 1 + 21$

$30 = 21 \cdot 1 + 9$

$21 = 9 \cdot 2 + 3$

$9 = 3 \cdot 3 + 0$

$\therefore \gcd(51, 897) = 3$

2. $198 = 168 \cdot 1 + 30$

$168 = 30 \cdot 5 + 18$

$30 = 18 \cdot 1 + 12$

$18 = 12 \cdot 1 + 6$

$12 = 6 \cdot 2 + 0$

$\therefore \gcd(198, 168) = 6$

$6 = (33 \cdot 6)m + (28 \cdot 6)n$

$1 = 33m + 28n$

let $n = 13$. Then we see that

$28(n) = 364$

&

let $m = -11$,

$33(-11) = 363$

$\therefore m = -11, n = 13$

3.

a. $4K+3 = m^2$

$= (4q+r)^2$

$= 16q^2 + 8qr + r^2$

thus we find the remainders from division by 4, as the LHS is 3,

if $r=1$, $r^2=1$, thus remainder is 1

$r=2$, $r^2=4$, thus remainder is 0

$r=3$, $r^2=9$, thus remainder is 1

$r=4$, $r^2=16$ thus remainder is 0

Since the remainders on the RHS never match LHS they never equal. Thus n is not a perfect square.

b. we can see the sequence is also,

$$\underset{8+3,}{\overset{11}{\parallel}} \quad \underset{100+8+3,}{\overset{111}{\parallel}} \quad \underset{1000+100+8+3,}{\overset{1111}{\parallel}} \quad \ldots$$

we can see that all the numbers of the form 10^i, $i \in \mathbb{Z}_{\geq 2}$, are divisible by 4, and so is 8. thus $111\ldots111$ is of the form $4k+3$. But we know numbers in this form are not perfect squares. Thus numbers of the form $111\ldots111$ are not perfect squares.