$\mathcal{T}$ Interpretation of the A-Series: A Response to McTaggart's Unreality of Time

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1 Introduction

McTaggart's presentation of the unreality of time is controversial and tangled. We will first define his terms used: the A-Series is tensed terms, for example, past, present, and future, which take on different truth values at different points in time. While the B-Series is untensed time, for example earlier, and later, they take on the same truth value given 2 different events.

2 Outline of McTaggart's Argument for the Unreality of Time

His bare-bones structure relies on the following: The A-Series of time is the necessary condition for the reality of time, but the A-Series of time does not exist, therefore time does not exist. Symbolically, where T is the property of time, and A is the property of the A-Series,

$$(\forall t)(At \iff Tt) \tag{1}$$

$$(\forall t)(\neg At) \tag{2}$$

$$\overline{\hspace{6cm}} \tag{3}$$

$$\therefore \neg \exists T \tag{4}$$

We can see that the overarching argument is valid and irrefutable, however the sub-arguments his premises rely on are not as clear in their validity.

His first premise (**1**) is explicit that there is no other theory of time that will be sufficient for showing the existence of time, because the other explanations (B-Series and the "C-Series", which will not be addressed here) lack either the direction of time (strictly increasing) or lack change. Once again, we will symbolically notate this,

$$(\exists X)(\forall t)(Xt \iff Tt) \implies (Xt = At) \tag{5}$$

Note that there exists a unique time series that can possibly imply time's existence, which McTaggart claims is the A-Series. McTaggart argues against the B-Series for its seeming lack of change, by using the example of a hot poker that is hot but changes its temperature over time. For example, "this poker is hot in the present," and later, "this poker was hot in the past." Describing this in the B-Series seems to not allow for change, as we can only say, "this poker was hot earlier" and "this poker will be hot." Change does not occur here since the moment when the poker is hot will always be earlier than the moment later when the poker is not hot. McTaggart sees this lack of temporal change to be an issue, because he believes that change is essential to time's existence and without it reality would be atemporal.

Finally, McTaggart explicitly denies the existence of the A-Series, citing an "infinite vicious regress" (1). First, he presents the idea that any event must be present at a given time, past at another, and future at another in order to imply time's existence. However, these properties of past, present, and future are mutually exclusive, as, for example, nothing can be present and past at the same moment. Therefore, time does not exist. Symbolically, where S is the property of the "pastness," P is the property of the "presentness," and F is the property of the "futureness."

$$(\forall t)(Pt \wedge St \wedge Ft) \implies Tt \tag{6}$$

$$(\forall t)(Pt \wedge St) \vDash \bot \tag{7}$$

$$(\forall t)(Ft \wedge St) \vDash \bot \tag{8}$$

$$(\forall t)(Ft \wedge Pt) \vDash \bot \tag{9}$$

$$\overline{\hphantom{(\forall t)(Pt \wedge St \wedge Ft) \implies Tt}} \tag{10}$$

$$\therefore \neg\exists T \tag{11}$$

Seemingly though, we can reply to (**6**) that there are specific times which have the properties of past, present, and future relative to an event. McTaggart states that this is where the "infinite vicious regress" arises. We can describe a time by saying phrases such as "present in the present," "past in the present," and "future in the present," but terms like this still yield a contradiction similar to above. There are 9 of these second level phrases, and in order to describe the second level A-Series properties we must go to the third level, in which there are 27, and so on and so forth implying his "infinite vicious regress." McTaggart claims the only way to avoid this contradiction is by rejecting the A-Series and accepting the unreality of time.

We could simply reject this contradiction by invoking the B-Series and rejecting the A-Series, but we would have to argue for an atemporal reality, or that time does not change, both of which have issues. On the other hand, the contradiction that arises with the A-Series could be the lack of semantical care that seemingly causes this contradiction; I will argue for this.

3 Logic and Topology of $\mathcal{T}$

I will construct a explanation of time that avoids McTaggart's contradiction, which we will call the $\mathcal{T}$ interpretation of the A-Series. Let $\mathcal{T}$ be the property of existence at a given time, thus we can translate our prior formulae to,

$$[(\forall t)(\exists \mathcal{T}_1)(St \implies \mathcal{T}_1 t)] \tag{12}$$

$$[(\forall t)(\exists \mathcal{T}_2)(Pt \implies \mathcal{T}_2 t)] \tag{13}$$

$$[(\forall t)(\exists \mathcal{T}_3)(Ft \implies \mathcal{T}_3 t)] \tag{14}$$

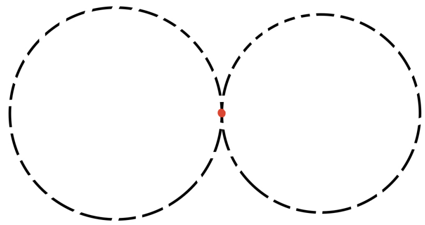
$$\mathcal{T}_1 t \wedge \mathcal{T}_2 t \wedge \mathcal{T}_3 t \implies Tt \tag{15}$$

$$\overline{} \tag{16}$$

$$\therefore \exists T \tag{17}$$

Within these formulae we see that time does in fact exist. We will break these down our premises to show that our premises are in fact valid.

Take time as a topological space, now these properties $\mathcal{T}$ can be imagined as a set of points. So given a set of points, those points are either present, future, or past to the event t. In our case, $\mathcal{T}_1$ and $\mathcal{T}_2$ are $\epsilon-$balls, open sets of radius ϵ centered at a point in time, where $\epsilon > 0$. $\mathcal{T}_2$ on the other hand is a single point in time. Arguments could be made to have $\mathcal{T}_2$ to be a ball, but there are implications that arise that are distasteful, such as the presentness and pastness both "being" simultaneously. When taking the conjunction of the three as seen in (**15**), we can imagine it visually as such,



where the left-most ball is $\mathcal{T}_1$, the red point is $\mathcal{T}_2$, and the right-most ball is $\mathcal{T}_3$. We can see that there exists a point in which there is the future, past, and present at once, but it is not contradictory since the balls are open and do not contain the present or each other. Topologically speaking, the limit approaching the present from the past and limit approaching from the future converges to the present, thus the present is a limit point of the 2 balls. Metaphysically, at any given moment we see that we are in the present, but always the past and future are always almost the moment that is the present. This transcends two objections, first the McTaggart's infinite vicious regress and second the deferral to B-Series.

First, note that we can aptly describe the moments in time by giving the property of $\mathcal{T}$, so the property $\mathcal{T}$ is one that transcends the semantical fickleness of the tensed words of "being." $\mathcal{T}$ is not the property like the untensed terms, but instead one implied by these properties. The conjunction of the $\mathcal{T}$ implies the existence of a single point that is the present, but is also the past and future's limit points, thus satisfying an event's property of having pastness, presentness, and futureness and therefore implying the existence of time without McTaggart's contradiction. It is not entirely clear what semantical form $\mathcal{T}$ takes, which we will discuss in the next section, however it is clear that it avoids the infinite regress because describing $\mathcal{T}$'s positions does not rely on the tenses of "being".

Secondly, perhaps more controversially, the $\mathcal{T}$ interpretation does not invoke the B-Series. Suppose we had relations $\triangleright$ and $\triangleleft$ which correspond to precedes and follows respectively. These binary operations are asymmetric, irreflexive, and transitive. Let C denote the onset of COVID-19, and G denote the Gettysburg Address,

$$G \triangleright C \iff C \triangleleft G$$

where the other properties follow by some symbolic computation. We then see that for our $\mathcal{T}$ that,

$$\mathcal{T}_1 t \triangleright \mathcal{T}_2 t \triangleright \mathcal{T}_3 t$$

But if we imagine taking the temporal limit over $\mathcal{T}_1$ and $\mathcal{T}_3$ then we also can see,

$$\lim_{\text{t}\to\text{present}} \mathcal{T}_1 t = \mathcal{T}_2 t$$

$$\lim_{\text{t}\to\text{present}} \mathcal{T}_3 t = \mathcal{T}_2 t$$

where the only logical conclusion is that $\mathcal{T}_1$ and $\mathcal{T}_3$ share a temporal limit point of the present. The B-Series alone cannot sufficiently to describe this phenomenon, as the limit point over the B-operators would imply that $St = Pt = Ft$, which the contradiction we wish to avoid. The operators from the B-Series are not sufficient to describe the phenomenon of $\mathcal{T}$ interpretation alone; the A-Series is needed. Once again, we see that $\mathcal{T}$ interpretation of the A-Series transcends the semantical pitfalls that McTaggart presents.

4 Semantics of $\mathcal{T}$

The logic of $\mathcal{T}$ does describe time well, but it does not fit nicely into our everyday temporal vocabulary like those of the A-Series and B-Series. $\mathcal{T}$ does use, implicitly, the untensed terms of the A-Series though. Constructing $\mathcal{T}_1$, we can imagine stating that "the event given is the past as far as 2ϵ away and as close as can be to the present." Similarly, for $\mathcal{T}_3$ and for $\mathcal{T}_2$, we can say "the event given is present." While mathematically and logically the limit of $\mathcal{T}_1$ and $\mathcal{T}_3$ is easily described, semantically it is not. Semantically stating that an event is almost present implies that it is strictly past or strictly future. However, this is not the case with the $\mathcal{T}$ interpretation as it is not strictly the past nor strictly the future. To describe $\mathcal{T}$ semantically, we will take an example. A train approaches a station and stops one centimeter behind/ahead of where it is expected, but this does not stop passengers from entering from one side of the train and exiting from the other side of the train. Let us deconstruct this example. When we say behind/ahead by one centimeter we are referring to the past/future an epsilon away from the present. The people entering and existing the train denote the properties of change and the direction of strictly increasing. We see that within a certain range the trains position is indistinguishable from some other positions, similar to the limit points of $\mathcal{T}_1$ and $\mathcal{T}_3$. This example does not aptly describe the $\mathcal{T}$ interpretation and has some questions left to the interpreter. Its intention is not to be exhaustive explanation of $\mathcal{T}$, but only an analogy. For the time being, $\mathcal{T}$ is restricted to logical language and intuition without a correct semantic interpretation on the horizon.

5 Conclusion

McTaggart's interpretation of the unreality of time relies on two premises that are not definite in their validity. The first premise relies on the the rejection of the B-Series, which is a debatable position taken by McTaggart, but his second premise, that the A-Series is contradictory, is the premise focused on in this essay. The construction of the $\mathcal{T}$ interpretation of the A-Series in this essay avoids McTaggart's contradiction by invoking a topological interpretation of time. While the logical interpretation of $\mathcal{T}$ is clear, the semantic interpretation is not yet as clear, because the semantic statement of the limit has unintended implications.

References

[1] J. M. E. McTaggart *The Nature of Existence (Volume 2): The Unreality of Time*. Cambridge: Cambridge University Press (1927)