Homework 1

Physics 201

Due September 4th, 2020

Exercise 1. Using the general solution

$$x(t) = A\cos(\omega t) + B\sin(\omega t) + a$$

then using the boundary values,

$$x(0) = x_0 = A + a$$

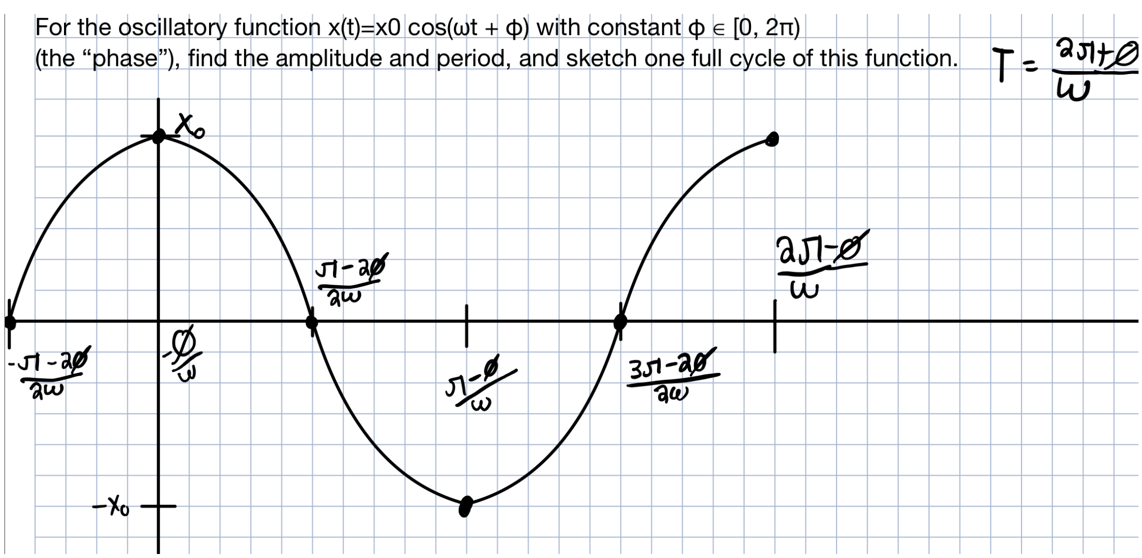
$$x(t^*) = x_* = (x_0 - a)\cos(\omega t^*) + B\sin(\omega t^*) + a \implies B = -\frac{(x_0 - a)\cos(\omega t^*) - x_* + a}{\sin(\omega t^*)}$$

Thus the final solution is

$$x(t) = (x_0 - a)\cos(\omega t) - \frac{(x_0 - a)\cos(\omega t^*) - x_* + a}{\sin(\omega t^*)}\sin(\omega t) + a$$

Exercise 2. If $\omega t^* = n\pi$, then there arises division by zero because the B term has $\sin(n\pi) = 0$ in the denominator. Thus the solution fails.

Exercise 3.



Exercise 4.

$$m\ddot{x}(t) = F_0\cos(\omega t) \implies \ddot{x}(t) = \frac{F_0\cos(\omega t)}{m}$$

Integrate once

$$\int \ddot{x}(t)dt = \int \frac{F_0\cos(\omega t)}{m}dt \implies \dot{x}(t) = -\frac{F_0}{m\omega}\sin(\omega t) + A$$

Integrating again

$$\int \dot{x}(t)dt = \int -\frac{F_0}{m\omega}\sin(\omega t) + Adt \implies x(t) = -\frac{F_0}{m\omega^2}\cos(\omega t) + At + B$$

Using the boundary conditions,

$$x(0) = x_0 = -\frac{F_0}{m\omega^2} + B \implies B = x_0 + \frac{F_0}{m\omega^2}$$
$$\dot{x}(0) = v_0 = A$$

Thus the final solution is,

$$x(t) = -\frac{F_0}{m\omega^2}\cos(\omega t) + v_0 t + x_0 + \frac{F_0}{m\omega^2}$$

Exercise 5. Since we are using Taylor expansion we need to determine an x_0. For our case we will use 0. Thus,

$$\sin(\Delta x) = f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x + \frac{1}{2}f''(x_0)\Delta x^2 \implies \sin(\Delta x) \approx \Delta x$$

$$\cos(\Delta x) = f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x + \frac{1}{2}f''(x_0)\Delta x^2 \implies \cos(\Delta x) \approx 1 - \frac{1}{2}\Delta x^2$$

Thus we get the table

Δx	$\sin(\Delta x)$	$\cos(\Delta x)$	$\sin(\Delta x)$ approx	$\cos(\Delta x)$ approx
0.1	0.0998	0.9950	0.1	0.995
0.2	0.1987	0.9801	0.2	0.98
0.4	0.3894	0.9211	0.4	0.92
0.8	0.7174	0.6967	0.8	0.68

Exercise 6. For the approximation with $x_0 = 0$ and $a = 1$,

$$f(x_0 + \Delta x) = (1 + x_0 + \Delta x)^n \approx f(x_0) + f'(x_0)\Delta x = 1 + n\Delta x$$

Thus for the examples,

$$\sqrt{1+x} \approx 1 + \frac{1}{2}\Delta x$$

$$\frac{1}{\sqrt{1+x}} \approx 1 - \frac{1}{2}\Delta x$$

$$\frac{1}{(1+x)^2} \approx 1 - 2\Delta x$$

Exercise 7. We will use the function $f(x) = x^{-2}$ with $x_0 = 10$ and $\Delta x = 1$. So our Taylor expansion is

$$f(x_0 + \Delta x) \approx f(x_0) + \Delta f'(x_0) = 0.01 - 2(0.001)\Delta x$$

Thus we can quickly calculate 1/121,

$$\frac{1}{121} = f(x_0 + \Delta x) \approx 0.01 - 2(0.001)(1) = 0.008$$

Which is close to the true value of 0.00826.