Homework: 3F

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Math 113: Discrete Structures

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Problem 1

a.

$\binom{n}{k} - \binom{n-3}{k}$
For the left hand side we see the $\binom{n}{k}$ represents the number of k-subsets of the set of size n including the elements a, b, c. $\binom{n-3}{k}$ represents the k-subsets of the set of size n excluding the elements a, b, c, thus is size $n - 3$. So this represents the amount of subsets of size k of a subset n with one, two or three of the a, b, c elements included.

b.

For the right hand side, $\binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}$. Without loss of generality, we can choose a single element, say a. $\binom{n-1}{k-1}$ represents the number of subsets of size $k - 1$ from the set of size, $n - 1$, from here we can imagine inserting a. So this covers all of the subsets of size k from a set of size n, that contain a. For the latter two terms, we can imagine similar, except we are removing a and b from the sets and a, b, c from the sets. Then we take the subsets of size $k - 1$ from the set of size $n - 2$ and add elements a, b back into the subsets. However, this seems problematic, because then we are taking subsets of size $k + 1$. We do this because otherwise we would be over counting, since there exists subsets from the left most term that have the element b or c in them. Thus by allowing subsets of size $k + 1$ we are counting them correctly. This logic matches to the subsets of size $k - 1$ of the set of size $n - 3$. Thus we have now counted all of the subsets that contain either one, two, or three of a,b,c. By doing so we see that the left hand side is exactly the right hand side, as we are counting the amount of subsets of size k of a subset n with one, two or three of the a, b, c elements included.

Problem 2

Let us take a fun example, in spirit of the Hungarian saying, if you can't fit a piece of bread in your pocket, break it in half. Suppose you have 17 pieces of bread, and you want to give some to 5 of your friends. So you then have 17 choose 5 options. But you aren't sure about how to fit 5 pieces of bread in one pocket. So you take the stack of bread and split it into 10 pieces and 7 pieces. But you wonder how there are still 17 choose 5 options. You sit and ponder. Then you realize, that you can go through the sets, choosing n from the stack of 10, and $5-n$ from the stack of 7. Multiplying the 10 choose n and the 7 choose $5 - n$ gives you the number of combinations of 5 pieces of bread from the 17 by MCP, given that you can only get n from the 10 stack and $5 - n$ from the 7 stack.

Thus, adding all of these possible cases, by ACP, or adding each $n = 0, 1, 2, 3, 4, 5$, gives you the total combinations for choosing 5 pieces out of 17 pieces. Thus the equality holds.