# MATH 202: VECTOR CALCULUS
## CAES 9.3 AND 9.4
## HOMEWORK DUE FRIDAY WEEK 10

*Problem* 1. For $n = 4$, $k = 1$,

$$(1), (2), (3), (4)$$

for $n = 3$, $k = 2$,

$$(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)$$

We can see that there is $n^k$ ordered $k$-tuples from $\{1, .., n\}$.
We see that for $k = 1$,

$$(1), (2), (3), (4)$$

$k = 2$,

$$(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)$$

$k = 3$,

$$(1,2,3), (1,2,4), (1,3,4), (2,3,4)$$

$k = 4$,

$$(1,2,3,4)$$

which we can see the number of increasing $k-$tuples is $\binom{n}{k}$ as there are the number of ways to choose $k$ distinct indices.

*Problem* 2. For $k = 0$,

$$\omega = f$$

$k = 1$,

$$\omega = f_1 dx + f_2 dy + f_3 dz + f_4 dw$$

$k = 2$,

$$\omega = f_{(1,2)} dx \wedge dy + f_{(1,3)} dx \wedge dz + f_{(1,4)} dx \wedge dw + f_{(2,3)} dy \wedge dz + f_{(2,4)} dy \wedge dw + f_{(3,4)} dz \wedge dw$$

$k = 3$,

$$\omega = f_{(1,2,3)} dx \wedge dy \wedge dz + f_{(1,2,4)} dx \wedge dy \wedge dw + f_{(1,3,4)} dx \wedge dz \wedge dw + f_{(2,3,4)} dy \wedge dz \wedge dw$$

$k = 4$,

$$\omega = f_{(1,2,3,4)} dx \wedge dy \wedge dz \wedge dw$$

*Problem* 3. (1) Firstly, we can see,

$$\gamma'(t) = \begin{pmatrix} 2t \\ 3t^2 - 1 \end{pmatrix}$$

then,

$$\int_\gamma \omega = \int_{t=-1}^{1} (t^2 - 1)(3t^2 - 1) - (t^3 - t)(2t) = \int_{t=-1}^{1} t^4 - 2t^2 + 1 = \left( \frac{t^5}{5} - t^2 + t \right) \Bigg|_{t=-1}^{1} = \frac{16}{15}$$

(2) Firstly, we can see,

$$\gamma'(t) = \begin{pmatrix} 1 \\ 2t \end{pmatrix}$$

then,

$$\int_\gamma \omega = \int_{t=0}^{2} (t)(2t) - (t^2)(1) = \int_{t=0}^{2} 2t^2 - t^2 = \left(\frac{t^3}{3}\right)\Big|_{t=0}^{2} = \frac{8}{3}$$

*Problem* 4. (1) Firstly, we can see,

$$\gamma'(t) = \begin{pmatrix} 1 \\ 2at \\ 3bt^2 \end{pmatrix}$$

then,

$$\int_\gamma \omega = \int_{t=-1}^{1} (bt^3)(1)+(t)^2(2at)+(at^2)(3bt^2) = \int_{t=-1}^{1} bt^3+2at^3+3abt^4 = \left(\frac{bt^4}{4}+\frac{at^4}{2}+\frac{3abt^5}{5}\right)\Big|_{t=-1}^{1} = \frac{6ab}{5}$$

(2) Firstly, we can see,

$$\gamma'(t) = \begin{pmatrix} -a\sin t \\ a\cos t \\ b \end{pmatrix}$$

then,

$$\int_\gamma \omega = \int_{t=0}^{2\pi} (bt)(-a\sin t)+(a\cos t)^2(a\cos t)+(a\sin t)(b) = \int_{t=0}^{2\pi} (ab\sin t)(1-t)+a^3\cos^3 t = -ab\int_{t=0}^{2\pi} t\sin t$$

which we evaluate with integration by parts, so let $u = t$, $v = -\cos t$, so,

$$-ab\left(\int_{t=0}^{2\pi} t\sin t\right) = -ab\left((-t\cos t)\Big|_{t=0}^{2\pi} - \int_{t=0}^{2\pi} -\cos t\right) = 2\pi ab$$

*Problem* 5. (1)

$$\int_\gamma \omega = \int_a^b (f\circ\gamma)dy = \int_a^b (\varphi\circ\gamma_2)\gamma_2' = \int_{\gamma_2(a)}^{\gamma_2(b)} \varphi$$

(2) Let $\gamma = (\gamma_1, \gamma_2)$, $f(x, y) = \varphi(x)$, $g(x, y) = \phi(y)$ and noting from part (1)

$$\int_\gamma \omega = \int_{\gamma_1(a)}^{\gamma_1(b)} \varphi + \int_{\gamma_2(a)}^{\gamma_2(b)} \phi = 0$$

since $\gamma(a) = \gamma(b)$.