Homework 2

Physics 201

Due September 11th, 2020

Exercise 1. Let $p = 0$,

$$\frac{d}{dx}\left(\sum_{j=0}^{\infty} a_j(j)x^{j+1}\right) + \sum_{j=0}^{\infty} a_j x^{j+2} - \sum_{j=0}^{\infty} 2a_j x^j = 0$$

$$\sum_{j=0}^{\infty} a_j(j)(j+1)x^j + \sum_{j=0}^{\infty} a_j x^{j+2} - \sum_{j=0}^{\infty} 2a_j x^j = 0$$

$$a_0(0)(1)(x^0) + a_1(1)(2)(x) - 2a_0(x^0) - 2a_1 x + \sum_{j=2}^{\infty} a_j(j)(j+1)x^j + \sum_{j=2}^{\infty} a_{j-2}x^j - \sum_{j=2}^{\infty} 2a_j x^j = 0$$

Cancelling and combining terms, and let $a_0 = 0$,

$$\sum_{j=2}^{\infty} x^j(a_j((j)(j+1) - 2) + a_{j-2})$$

$$\sum_{j=0}^{\infty} x^{j+2}(a_{j+2}((j+2)(j+3) - 2) + a_j)$$

Thus the recursion relation is,

$$a_{j+2} = -\frac{a_j}{(j+2)(j+3) - 2}$$

And the first few terms are, for a given a_1,

$$a_3 = -\frac{a_1}{10}$$
$$a_5 = -\frac{a_3}{28} = \frac{a_1}{280}$$
$$a_7 = -\frac{a_5}{54} = -\frac{a_1}{1512}$$

Exercise 2. For ease we will set $x = i\omega t$. Using the different forms given we see,

$$\frac{1}{2}Ae^x + \frac{1}{2}Ae^{-x} + \frac{1}{2i}Be^x + \frac{1}{2i}Be^{-x} = \bar{A}e^x + \bar{B}e^{-x}$$

Thus we can derive $\bar{A}$ from A,

$$\bar{A} = \frac{1}{2}A(\frac{e^x + e^{-x}}{e^x}) = \frac{1}{2}A(1 + \frac{1}{e^{2x}})$$

and

$$\bar{B} = \frac{1}{2i}B(\frac{e^x - e^{-x}}{e^x}) = \frac{1}{2i}B(1 - \frac{1}{e^{2x}})$$

Exercise 3.

$$i^2 = \sqrt{-1}\sqrt{-1} = -1$$
$$i^3 = i^2 \cdot i = -i$$
$$i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$$
$$i^5 = i^3 \cdot i^2 = (-i)(-1) = i$$

Exercise 4. $z = s_z^{i\phi_z}$, $p = s_p e^{i\phi_p}$, $s_z^2 = a^2 + b^2$, $s_p^2 = u^2 + v^2$.

$$\frac{z}{p} = \frac{s_z}{s_p} e^{i\phi_z} e^{i\phi_p} = \frac{z(p^*)}{s_p^2} = \frac{(a+bi)(u-vi)}{u^2+v^2}$$

$$\implies \Re(\frac{z}{p}) = \frac{au+bv}{u^2+v^2} = \Im(\frac{z}{p}) = \frac{ub-av}{u^2+v^2}$$

Exercise 5. a.

$$(z+p)^* = ((a+u)+(b+v)i)^* = ((a+u)-(b+v)i) = (a-bi)+(u-vi) = z^* + p^*$$

b.

$$(zp)^* = ((au-bv)+i(av+bu))^* = ((au-bv)-i(av+bu)) = (a-bi)(u-vi) = z^* p^*$$

c.

Following results form 1.5.3,

$$\left(\frac{z}{p}\right)^* = \left(\frac{(a+bi)(u-vi)}{u^2+v^2}\right)^* = \frac{au+bv-i(ub-av)}{u^2+v^2} = \frac{(-a+bi)(-u-vi)}{u^2+v^2} = \frac{-(z^*)(-p)}{s_p^2} = (z^*)\left(\frac{e^{-i\phi_p}}{s_p}\right) = \frac{z^*}{p^*}$$

Exercise 6.

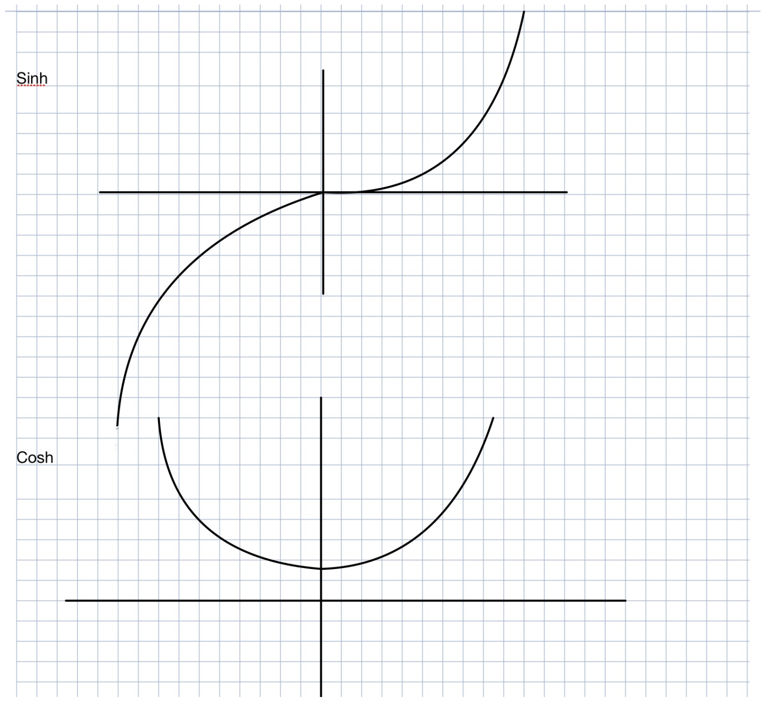
$$\begin{aligned}
\cos(\phi+\theta) &= \frac{1}{2}(e^{i(\phi+\theta)} + e^{-i(\phi+\theta)}) \\
&= \frac{1}{2}(e^{i\phi}e^{i\theta} + e^{-i\phi}e^{-i\theta}) \\
&= \frac{1}{2}((\cos(\phi)+i\sin(\phi))(\cos(\theta)+i\sin(\theta)) + (\cos(\phi)-i\sin(\phi))(\cos(\theta)-i\sin(\theta))) \\
&\text{after cancellation of } i\cos(\phi)\sin(\theta) \text{ and } i\cos(\theta)\sin(\phi) \text{ terms} \\
&= \frac{1}{2}(2\cos(\phi)\cos(\theta) - 2\sin(\phi)\sin(\theta)) \\
&= \cos(\phi)\cos(\theta) - \sin(\phi)\sin(\theta)
\end{aligned}$$

Exercise 7. Since all sinh covers all of the even terms and cosh covers all the odd, the exponential expression becomes,

$$\sum_{i=0}^{\infty} \frac{x^i}{i!} = \sum_{i=0}^{\infty} \frac{x^{2i}}{(2i)!} + \sum_{i=0}^{\infty} \frac{x^{2i+1}}{(2i+1)!} \implies e^x = \cosh(x) + \sinh(x)$$

Doing similar to before,

$$
\begin{aligned}
e^x &= \cosh(x) + \sinh(x) \\
\pm e^{-x} &= \cosh(x) - \sinh(x) \\
\implies \cosh(x) &= \frac{e^x + e^{-x}}{2} \\
\cosh(x) &= \frac{e^x - e^{-x}}{2}
\end{aligned}
$$



Exercise 8.

$$
\begin{aligned}
\sin(i\theta) &= \frac{1}{2i}(e^{i(i)\theta} - e^{-i(i)(\theta)}) \\
&= \frac{1}{2i}(e^{-\theta} - e^{\theta}) \\
&= -\frac{1}{i}\frac{1}{2}(e^{\theta} - e^{-\theta}) \\
&= -\frac{\sinh(\theta)}{i} \\
&= i\sinh(\theta)
\end{aligned}
$$

$$
\begin{aligned}
\sinh(i\eta) &= \frac{1}{2}(e^{i\eta} - e^{-i\eta}) \\
&= i\frac{1}{2i}(e^{i\eta} - e^{-i\eta}) \\
&= i\sinh(\eta)
\end{aligned}
$$