Homework: 6.3, 6.5

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Math 202: Vector Calculus

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6.6.1

First, note that double integral does not take into account the orientation and thus the sign, while the iterated integral does because the one-dimensional integrals do. Thus, we see that this follows as the top and bottom areas equal each other, and so the value of the respective integrals follows.

6.6.2

We can see by illustration that our other iterated integral is

$$\int_{y=a}^{b} \int_{x=y}^{b}$$

6.6.3

We can see that through illustration that our iterated integral with the inner order changed is

$$\int_{x=0}^{1} \left(\int_{z=0}^{x} \int_{y=0}^{1-x} + \int_{z=x}^{1} \int_{y=z-x}^{1-x} \right) f$$

6.6.4

We can see that through illustration that our iterated integral with the inner order changed is

$$\int_{x=0}^{1} \left(\int_{z=0}^{x^2} \int_{y=0}^{1} + \int_{z=x^2}^{1+x^2} \int_{y=\sqrt{z-x^2}}^{1} \right) f$$

6.6.5

a.

$$\int_{y=2}^{3} \int_{x=0}^{1+\ln y/y} e^{xy} = \int_{y=2}^{3} ((1/y)e^{xy}) \Big|_{0}^{1+\ln y/y} = \int_{y=2}^{3} e^{y} - 1/y = e^{y} - \ln y \Big|_{y=2}^{3} = e^{3} - e^{2} - \ln\left(\frac{3}{2}\right)$$

b.

$$\int_{x=1}^{4}\int_{y=1}^{\sqrt{x}} e^{x/y^2}/y^5 = \int_{y=1}^{2}\int_{x=y^2}^{4} e^{x/y^2}/y^5 = \int_{y=1}^{2} e^{x/y^2}/y^3\Big|_{x=y^2}^{4} = \int_{y=1}^{2} e^{4/y^2}/y^3 - e/y^3$$

$$= -(1/8)e^{4/y^2} + (1/2)e/y^2\Big|_{y=1}^{2} = e^4/8 - e/2$$

c.

$$\int_{x=0}^{\left(\frac{1}{16}\right)^{1/3}}\int_{2x^2}^{\sqrt{x}/2} 1 = \int_{x=0}^{\left(\frac{1}{16}\right)^{1/3}} \sqrt{x}/2 - 2x^2 = x^{3/2}/3 - \frac{2}{3}x^3\Big|_{x=0}^{\left(\frac{1}{16}\right)^{1/3}} = \left(\frac{1}{3}\cdot\sqrt{\frac{1}{16}} - \frac{2}{3}\cdot\frac{1}{16}\right) - 0 = \frac{1}{24}$$

f.

$$\int_{x_1=0}^{1}\int_{x_2=0}^{1}\cdots\int_{x_n=0}^{1} x_1x_2\ldots x_n = \int_{x_1=0}^{1} x_1\int_{x_2=0}^{1} x_2\ldots\int_{x_n=0}^{1} x_n = (1/2)^n$$