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Assignment 4

Assumption:

- There are no obstacles on any paths that may prevent the group from taking that path.
- The group is able to backtrack or repeat any path or return to sites that are already visited.
- The group can pass by the exit without having to exit the zoo.

Justification of assumptions:

- If there were obstacles on the paths, some paths would no longer be accessible. This could potentially make the model unsolvable without additional information.
- The map has more than 2 vertices of odd degrees which means that a Eulerian path is not possible. Therefore, the group will need to backtrack some paths otherwise the problem would not be solvable.
- It is assumed that we can pass by point F without having to immediately exit since if the group has to exit immediately after reaching point F, the model created would not be valid. Furthermore, this would drastically increase the value of the minimum distance the group will walk to cover all sites.

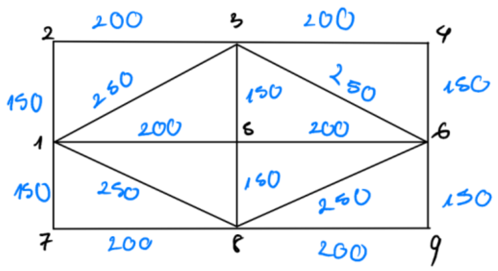
Model:

Variables:

From the question, we can create the following variables to represent each of the different kinds of animals sites.

Where:

1. **1** is D.
2. **2** is A.
3. **3** is B.
4. **4** is C.
5. **5** is E.
6. **6** is F.
7. **7** is G.
8. **8** is H.
9. **9** is I.



Therefore, the distances between the sites can be represented by the following matrix:

	1	2	3	4	5	6	7	8	9

1		150	250		200		150	250	
2	150		200						
3	250	200		200	150	250			
4			200			150			
5	200		150			200		150	
6			250	150	200			250	150
7	150							200	
8	250				150	250	200		200
9						150		200	

Where blank spaces indicate that there is no direct path from one site to another or no way to reach the targeted sites without also visiting another site. We can make these path weights be equal to infinity.

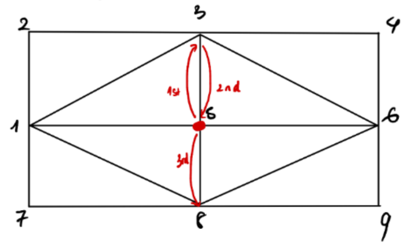
	1	2	3	4	5	6	7	8	9
1	∞	150	250	∞	200	∞	150	250	∞
2	150	∞	200	∞	∞	∞	∞	∞	∞
3	250	200	∞	200	150	250	∞	∞	∞
4	∞	∞	200	∞	∞	150	∞	∞	∞
5	200	∞	150	∞	∞	200	∞	150	∞
6	∞	∞	250	150	200	∞	∞	250	150
7	150	∞	∞	∞	∞	∞	∞	200	∞
8	250	∞	∞	∞	150	250	200	∞	200
9	∞	∞	∞	∞	∞	150	∞	200	∞

It is possible to brute force all combinations of paths to find the shortest distance however I will introduce some constraints and rules to make the process more efficient.

Rules:

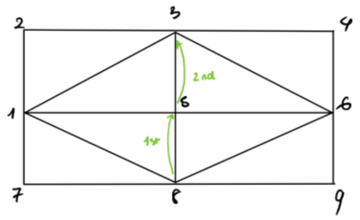
1) The group will have to repeat some paths; therefore, they should always repeat the shortest path (150) to minimise travel cost.
    - By observation, the map has more than 2 vertices of odd degrees and therefore a Eulerian path is not possible, forcing the group to revisit some sites in order to cover all nodes.
    - Thus, the shortest paths should be chosen as the paths that would be repeated. In other words, the group should never repeat a path that costs 200 or 250 and only repeat a path that costs 150.
2) The group should never start covering the middle column at node 5.
    - If the group starts at node 5 this means that a route will need to be repeated (either from 3 to 5 or from 8 to 5) so that all 3 sites are covered.
    - This means that we have added an extra 150 to the cost unnecessarily as shown by the illustration:

If the group starts at node 5

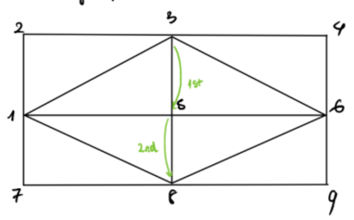


- On the other hand, if the group starts at either node 3 or 8, no path repetition is required to cover all 3 sites as shown below:

If the group starts at node 3



If the group starts at node 3



3) The group always choose to take the next path with the lowest weight while minimising repeated routes.
    - The group will maximise the use of 150 paths.
    - The group will minimise the use of 200 and 250 paths.

Constraints:

1) The last site visited must be 6

2) Start at 1

Goal:

We start at 1 and proceed through the map in an attempt cover all sites and end up at 6. Our goal is to cover each column one by one. This means that we will attempt to cover site 1, 2 and 7 first. Then, we will cover site 3, 5 and 8 and finally, we will cover site 4, 6 and 9. We do this as it will maximise the use of 150 paths as outlined in rule 3. It is note worthy that we will also need to repeat 1 path in column 1 and column 3 as both our start point, and end point are in the middle of the column. This adheres to rule 1. Furthermore, we seek to start covering the middle column by first traveling to site 3 or site 8 in order to avoid having to repeat more times than we needed to.

The solution and working outs.

We first start at 1:

	1	2	3	4	5	6	7	8	9
1	∞	150	250	∞	200	∞	150	250	∞
2	150	∞	200	∞	∞	∞	∞	∞	∞
3	250	200	∞	200	150	250	∞	∞	∞
4	∞	∞	200	∞	∞	150	∞	∞	∞
5	200	∞	150	∞	∞	200	∞	150	∞
6	∞	∞	250	150	200	∞	∞	250	150
7	150	∞	∞	∞	∞	∞	∞	200	∞
8	250	∞	∞	∞	150	250	200	∞	200
9	∞	∞	∞	∞	∞	150	∞	200	∞

By examining the weight table, we can see that the next possible nodes are to 2,3,5, 7 or 8. Here, the 2 lowest possible paths are to site 2 and site 7. We select site 2.

	1	2	3	4	5	6	7	8	9
1	∞	150	250	∞	200	∞	150	250	∞

Then, from site 2, we can either travel to site 1 or site 3. We want to maximise the number of 150 paths used and we want to finish the first column before moving onto the second column so the group will revisit site 1.

	1	2	3	4	5	6	7	8	9
2	150	∞	200	∞	∞	∞	∞	∞	∞

Then, from site 1, we can see that the next possible nodes are to 2,3,5, 7 or 8. Here, the 2 lowest possible paths are to site 2 and site 7. However, since we have already travelled to site 2, we can go to site 7.

	1	2	3	4	5	6	7	8	9	
1	∞	150	250	∞	200	∞	150	250	∞	

Then, from site 7, we can either travel to site 1 or site 8. Though we want to maximise the number of 150 paths, site 1 has already been visited twice and all 3 sites of the first column is covered. Therefore, the group will then visit site 8.

	1	2	3	4	5	6	7	8	9
7	150	∞	∞	∞	∞	∞	∞	200	∞

Then, from site 8, we can either travel to site 1, 5, 6, 7, or 9. We want to maximise the number of 150 paths, and site 1 has already been visited twice. Therefore, the group will then visit site 5.

	1	2	3	4	5	6	7	8	9
8	250	∞	∞	∞	150	250	200	∞	200

Then, from site 5, we can either travel to site 1, 3, 6, or 8. We want to maximise the number of 150 paths, and the sites 1 and 8 has already been visited. Therefore, the group will then visit site 3.

	1	2	3	4	5	6	7	8	9
5	200	∞	150	∞	∞	200	∞	150	∞

Then, from site 3, we can either travel to site 1, 2, 4, 5 or 6. We want to maximise the number of 150 paths; however, we have just come from site 5. Therefore, to minimise the distance travelled, the group will then take the next shortest path available which is 200 and visit site 4 since site 2 has already been visited also.

	1	2	3	4	5	6	7	8	9
3	250	200	∞	200	150	250	∞	∞	∞

Then, from site 4, we can either travel to site 3 or 6. We want to maximise the number of 150 paths; and site 3 has also been visited. Therefore, to minimise the distance travelled, the group will visit site 6.

	1	2	3	4	5	6	7	8	9
4	∞	∞	200	∞	∞	150	∞	∞	∞

Even though this is the exit, we have not visited the site 9, we must visit site 9 and then return to the exit at site 6.

	1	2	3	4	5	6	7	8	9
6	∞	∞	250	150	200	∞	∞	250	150

	1	2	3	4	5	6	7	8	9
9	∞	∞	∞	∞	∞	150	∞	200	∞

Final solution:

Therefore, a possible path that the group can take to have the lowest distance is:

1 to 2 to 1 to 7 to 8 to 5 to 3 to 4 to 6 to 9 and then to 6 where the group can exit. Or this can be interpreted as D to A to D to G to H to E to B to C to F to I and then back to F. It is noteworthy that this is not the only possible route for the group to minimise the distance and cover all sites.

Adding up all the distances will give us the minimum distance to cover all sites:

150 + 150 + 150 + 200 + 150 + 150 + 200 + 150 + 150 + 150 = 1600 metres

The minimum distance for the group tour walk to cover all sites is 1600 metres.

Discussion:

- The solution was obtained by initially setting up the variables 1 to 9 to represent each of the sites A to I. This question was attempted with a brute force approach. Yet, several rules are introduced to assist us in solving the problem. Ultimately, by following these rules, a valid solution was found. It is worth noting that the solution is only possible due to the assumptions made above.
- Reflection of model and general discussions:
    - There are several advantages of using this model. Firstly, it is easy to be visualised by using a table to demonstrate the concept. Furthermore, by assigning numbers to represent each of the site instead of letters, the model could be run more efficiently

and effectively using code. Also, modelling it this way allows for the problem to be possibly solved by a TSP algorithm. Therefore, it could be scaled to the problem size as needed.

- Generally, there are more than 1 path that can minimise the cost of travel for the group however only 1 path was found as the question did not require the knowledge of these paths, only the minimal travel distance.

- Special considerations and cases:
  - Nodes are different to edges and thus this model only attempt to cover all sites not all paths.
  - If we have to exit as soon as we visit site F, the minimum distance would be much greater than 1600 as we would have to back track multiple times to the second column.