Digital Signal Processing: Theory and Practice

Some useful and important signals

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Useful discrete-time signals

We will look at some important signals, that we will often come across and are useful in the analysis of signals and systems.

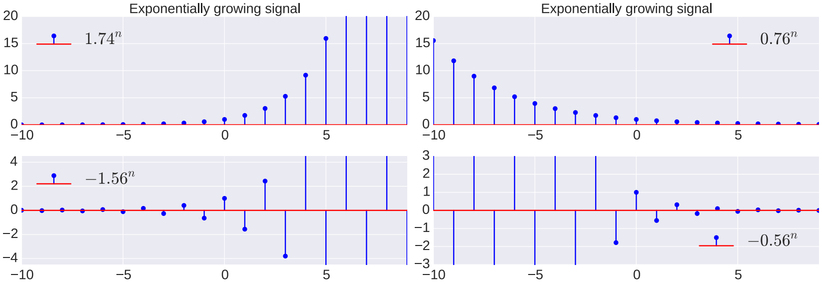
- Exponential signals
- Sinusoids
- Exponential sinusoids
- Impulse function
- Step function

Real Exponentials

Discrete-time version

$$x[n] = b\,(a)^n$$

where, $a, b \in \mathbb{R}$ and $n \in \mathbb{Z}$. b is the amplitude and a is the exponential growth or decay rate.



Real Exponentials

These are encountered as solution to first order difference equations.

$$x[n] = kx[n-1] \implies x(t) = C(k)^n$$

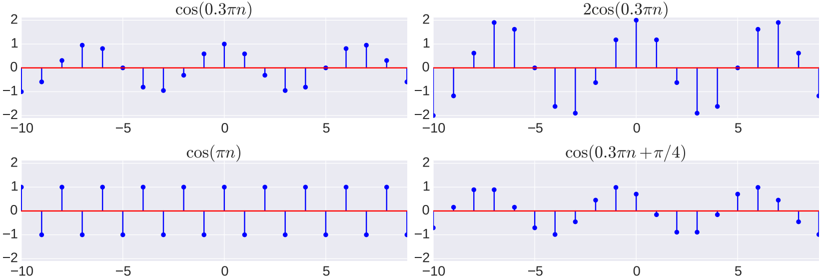
Can you think of practical examples of systems that result in such signals?

Sinusoidal signals

Discrete-time version

$$x[n] = A\sin\left(\Omega n + \phi\right)$$

where, A is the amplitude, Ω is the digital frequency $\left(\text{rad.sample}^{-1}\right)$, and ϕ is the phase angle.



What is the fundamental period?

Sinusoidal signals (Contd ...)

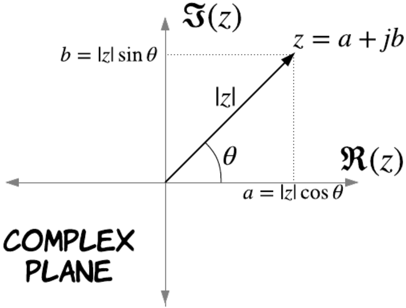
There are some peculiarities to the discrete sinusoid:

- Not all sinusoids are periodic! e.g. $\sin(n)$
- There is a maximum frequency for discrete sinusoids. What is it?
- Two sinusoids that differ by a discrete frequency of 2π are the same sinusoids.

Sinusoidal signals (Contd ...)

Complex exponential representation of sinusoids

$$z = a + jb = |z|\, e^{j\theta} = |z| \cos\theta + j\, |z| \sin\theta$$

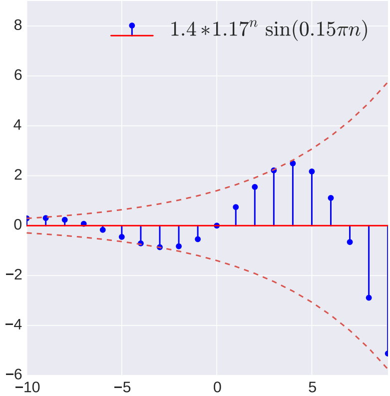


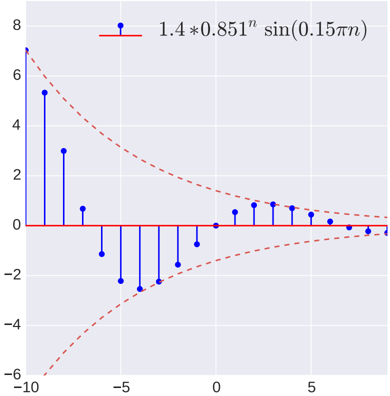
$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \quad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Exponential sinusoids

Continuous-time version
Amplitude modulated sinusoids

$$x[n] = ab^n \sin(\Omega n + \phi), \quad a, b, \Omega, \phi \in \mathbb{R}$$



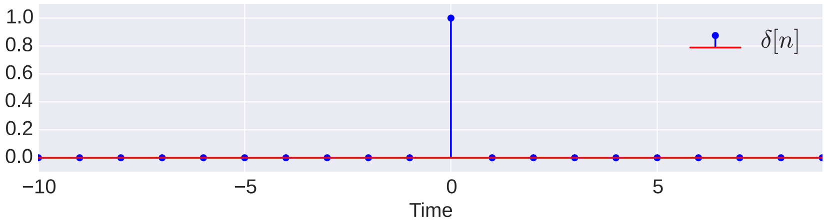


Impulse function $\delta[n]$

Kronecker delta function or sequence $\delta[n]$

- Very easy to understand unlike the continuous-time version.

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{Otherwise} \end{cases}$$



Step squence $u[n]$

Definition of **discrete-time** unit step sequence,

$$u[t] = \sum_{k=-\infty}^{n} \delta[k]$$