

Digital Signal Processing: Theory and Practice

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What is signal processing?

*"Signal processing is an enabling technology that encompasses the fundamental theory, applications, algorithms, and implementations of **processing or transferring information** contained in many different physical, symbolic, or abstract formats broadly designated as signals and uses **mathematical, statistical, computational, heuristic, and/or linguistic representations, formalisms, and techniques for representation, modeling, analysis, synthesis, discovery, recovery, sensing, acquisition, extraction, learning, security, or forensics.**"¹*

¹Moura, J.M.F. (2009). "What is signal processing?, Presidents Message". IEEE Signal Processing Magazine 26 (6). doi:10.1109/MSP.2009.934636

What is a signal?

Any physical quantity carrying information that varies with one or more independent variables.

$$s(t) = 1.23t^2 - 5.11t + 41.5$$

$$s(x, y) = e^{-(x^2+y^2+0.5xy)}$$

Mathematical representation will not be possible (e.g. *physiological signals, either because the exact function is not known or is too complicated.*)

What is a signal? (Contd ...)

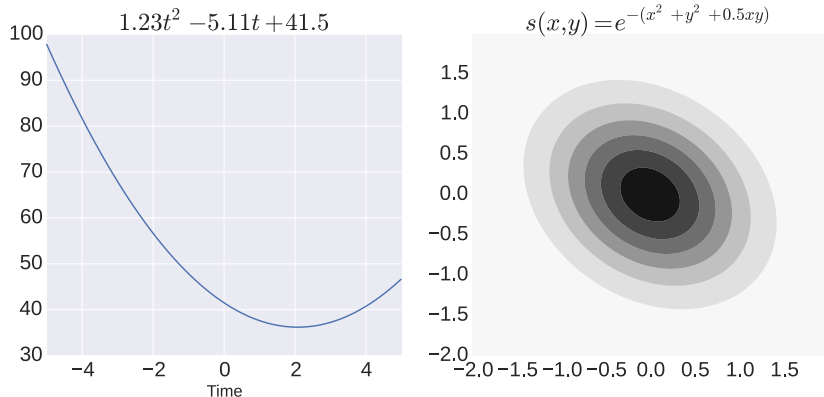


Figure : Example of 1D and 2D signals

Can you think of examples of 3D and 4D signals?

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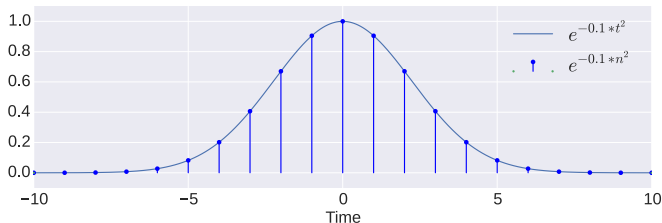
Classification of signals

- ▶ Based on the signal dimensions. *e.g.* 1D, 2D ...
- ▶ **Scalar** vs. **Vector** signals: *e.g.* gray scale versus RGB image

$$I_g(x, y) \in \mathbb{R} \text{ and } I_{color}(x, y) \in \mathbb{R}^3$$

- ▶ **Continuous-time** vs. **Discrete-time**: *based on the values assumed by the independent variable.*

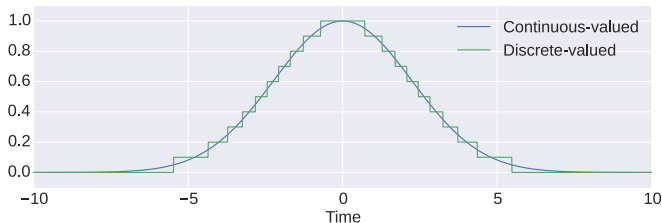
$$\begin{cases} x(t) = e^{-0.1t^2}, & t \in \mathbb{R} & \text{Continuous-time} \\ x[n] = e^{-0.1n^2}, & n \in \mathbb{Z} & \text{Discrete-time} \end{cases}$$



Classification of signals (Contd ...)

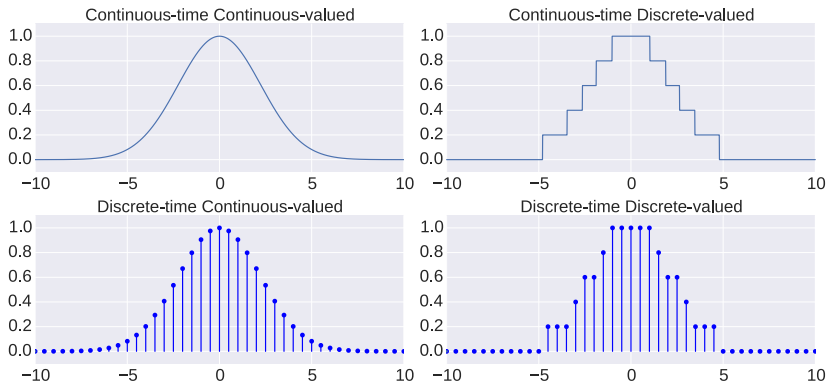
- **Continuous-valued vs. Discrete-valued:** *based on the values assumed by the dependent variable.*

$$\begin{cases} x(t) \in [a, b] & \text{Continuous-valued} \\ x(t) \in \{a_1, a_2, \dots\} & \text{Discrete-valued} \end{cases}$$



Classification of signals (Contd ...)

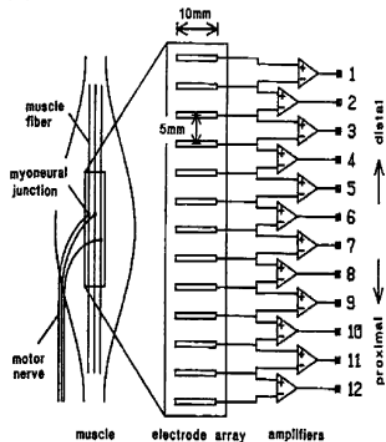
Four types of signals



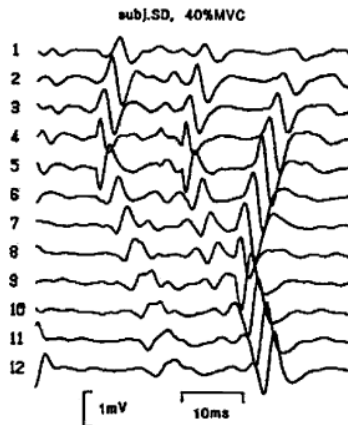
Classification of signals (Contd ...)

Example of a Continuous-valued discrete-time signal

(a)

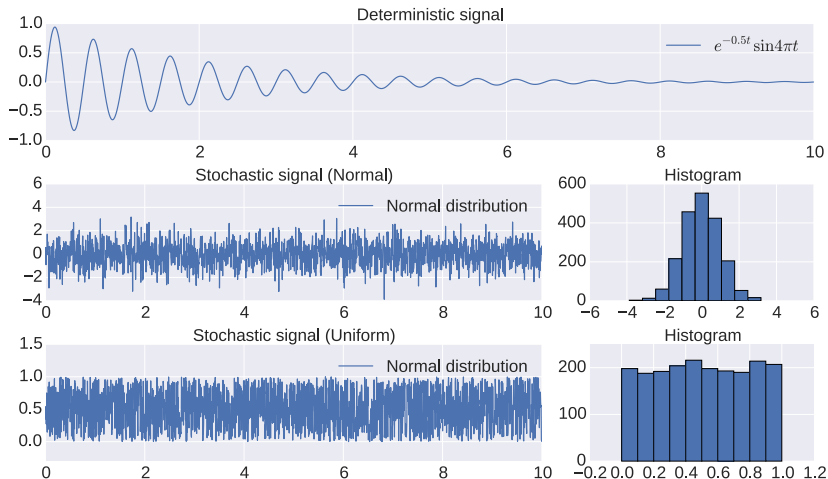


(b)



Classification of signals (Contd ...)

- **Deterministic vs. Stochastic:** *e.g. EMG is an example of a stochastic signal.*



Classification of signals (Contd ...)

- **Even vs. Odd:** *based on the symmetry about the $t = 0$ axis.*

$$\begin{cases} x(t) = x(-t), & \text{Even signal} \\ x(t) = -x(-t), & \text{Odd signal} \end{cases}$$

Can there be signals that are neither even nor odd?

Theorem

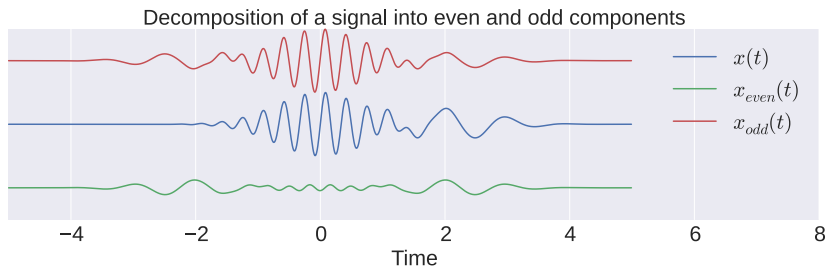
Any arbitrary function can be represented as a sum of an odd and even function.

$$x(t) = x_{\text{even}}(t) + x_{\text{odd}}(t)$$

$$\text{where, } x_{\text{even}}(t) = \frac{x(t) + x(-t)}{2} \quad \text{and} \quad x_{\text{odd}}(t) = \frac{x(t) - x(-t)}{2}.$$

Classification of signals (Contd ...)

Decomposition of an arbitrary signal into even and odd components



Classification of signals (Contd ...)

- **Periodic vs. Non-periodic:** *a signal is periodic, if and only if*

$$x(t) = x(t + T), \forall t$$

where, T is the fundamental period.

- **Energy vs. Power:** *indicates if a signal is short-lived.*

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \quad P = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$E = \sum_{-\infty}^{\infty} |x(t)|^2 \quad P = \frac{1}{2N+1} \sum_{-N}^N |x(t)|^2$$

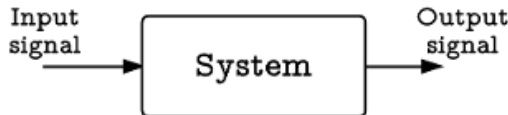
*A signal is an **energy** signal, if $0 < E < \infty$.*

*A signal is an **power** signal, if $0 < P < \infty$.*

What is a system?

A collection of objects united by some form of interaction or interdependence².

From the signal processing point of view, **a system is any physical device or algorithm that performs some operation on a signal to transform it into another signal.**



Can be thought of a mapping function. *e.g.*

$$y = f(x) = kx$$

²Zadeh, Lotfi A., and Charles A. Desoer. *Linear system theory*. New York: McGraw Hill, 1963.

Classification of systems

Based on the properties of the system.

- **Linearity:** *satisfies the properties of **scaling** and **superposition**.*

Let us assume,

$$f : x_i(t) \mapsto y_i(t)$$

The system f is linear, if and only if,

$$f : \sum_i a_i x_i(t) \mapsto \sum_i a_i y_i(t)$$

Which of these systems are linear?

1. $y(t) = k_1 x(t) + k_2 x(t - 2)$
2. $y(t) = \int_{t-T}^t x(\tau) d\tau$
3. $y(t) = 0.5x(t) + 1.5$

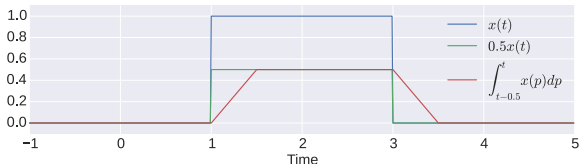
Classification of systems

Based on the properties of the system.

- **Memory:** *a system whose output depends on past or future values of its input is a system with memory, else the system is memoryless.*

Note: the system may or may not depend on its present.

$$\begin{cases} y(t) = 0.5x(t) & \text{Memoryless system} \\ y(t) = \int_{t-0.5}^t x(\tau) d\tau & \text{System with memory} \end{cases}$$



Classification of systems

- **Causality:** *a system whose output depends on the past and present only values of the input is a causal system.*

$$\begin{cases} y(t) = \int_{t-0.5}^t x(\tau) d\tau & \textbf{Causal} \\ y(t) = \int_{t-0.5}^{t+0.5} x(\tau) d\tau & \textbf{Non-causal} \end{cases}$$

- **Time invariance:** *system remains the same with time.*
If a system is time-invariant, then

$$f : x(t) \mapsto y(t) \iff f : x(t - t_0) \mapsto y(t - t_0)$$

- **Stability:** *bounded input produces bounded output.*

$$|x(t)| < M_x < \infty \mapsto |y(t)| < M_y < \infty$$

- **Invertibility:** *input can be recovered from output.*