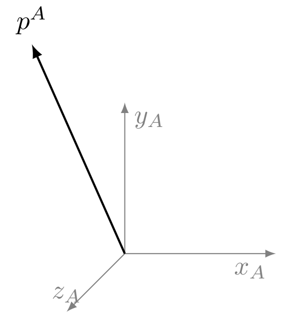
Introduction to Robotics

Spatial Transformation and Kinematics

1. Consider the following figure with the (orthonormal) coordinate frame $\{A\}$. The representation of a point P in the coordinate frame is given by $p^A = \begin{bmatrix}-1 & 1 & -1\end{bmatrix}^T$.



Consider a new stange coordinate frame with three vectors $\{x_B, y_B, z_B\}$ that is not orthogonal, i.e. the vectors are not mutually perpendicular to each other, but have length 1. The direction angles (all in degrees) of the three vectors $\{x_B, y_B, z_B\}$ are given

$$\text{Direction angles of } x_B = (45, 45, 90)$$

$$\text{Direction angles of } y_B = (90, 45, 45)$$

$$\text{Direction angles of } z_B = (45, 90, 45)$$

Let p^B be the repsentation of the point P in the new frame $\{B\}$.

(a) What are the transformation matrices T_B^A and T_A^B that lets us go from frames $\{A\}$ to $\{B\}$ and frame $\{B\}$ to $\{A\}$, respectively?

(b) How many parameters do you need to represent this transformation?

(c) What is the value of p^B?

(d) What happens to T_B^A and T_A^B when the vectors in frame $\{B\}$ are orthonormal?

2. Consider a series of spatial transformations from frames $\{A_i\}_{i=1}^n$, represented by a homogenous transformation matrix representing frame $\{A_i\}$ in $\{A_{i-1}\}$,

$$T_i^{i-1} = \begin{bmatrix} R_i^{i-1} & p_i^{i-1} \\ 0_{1\times 3} & 1 \end{bmatrix}$$

Find the expression for T_1^n.

3. Consider a point P represented by the position vector $x_P = \begin{bmatrix}-2 & 3 & 10\end{bmatrix}^T$. The following transformations are applied to the point in a sequential order. What is the resulting position of the of the transformed point.

(a) Rotate x_P by 25 deg about the fixed x-axis.

(b) Rotate x_P by 90 deg about the fixed z-axis.

(c) Rotate x_P by 45 deg about the new x-axis resulting from the previous two rotations.

Provide detailed derivation of the calculations, along with the final operator R which lets you carry out the aforementioned operations on the point P.

4. Consider a point P represented by the position vector $x_P = \begin{bmatrix}1 & 1 & 1\end{bmatrix}^T$. The following transformations are applied to the point in a sequential order. What is the resulting position of the of the transformed point.

(a) Rotate x_P by 45 deg about the fixed z-axis.

(b) Rotate x_P by 45 deg about the new x-axis resulting from the previous rotation.

(c) Translate the point transformed point P by $\begin{bmatrix}-1 & 0 & -1\end{bmatrix}$ with respect to the new frame resulting from the previous two rotations.

(d) Rotate x_P by 90 deg about the fixed y-axis.

(e) Translate the point transformed point P by $\begin{bmatrix}0 & 2 & 2\end{bmatrix}$ with respect to fixed frame.

Provide detailed derivation of the calculations, along with the final operator R which lets you carry out the aforementioned operations on the point P.

5. All three angle representation run into singularity. For each of the following representations, for what values of α, β or γ a sigularity would occur? Explain why this happens.

(a) Euler angle Z-Y-Z

(b) Euler angle Y-X-Z

(c) Fixed angle X-Y-Z

(d) Euler angle Z-X-Y

What about the angle-axis representation? Does a singluarity occur in this case?

6. What are the Euler angle Z-Y-Z and fixed angle Z-Y-Z angles required to take a point $x = \begin{bmatrix}1 & 1 & 0\end{bmatrix}^T$ to $y = \begin{bmatrix}-1 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}^T$?

7. Consider a planetary system with a star at the origin of the global reference frame $\{S\}$. There is a single planet, at a distance of r_P, orbiting the star in the global XY plane. At any give time the angular position of the planet is given by $\theta_P(t)$ measured with respect to the global X axis. The local reference frame of the planet $\{P\}$ of the planet has its origin at the center of the planet and has the same orientation with respect to $\{S\}$ at time $t = 0$, i.e. $R_P^S(0) = I$. The planet has a moon, located at a distance of r_M, from the origin of $\{P\}$, and is orbiting the planet in the local XY plane of frame $\{P\}$. The angular position of the moon $\theta_M(t)$ is measured with respect the local X axis of frame $\{P\}$. The moon has its own local reference frame

$\{M\}$ located at its center, and has the same orientation as $\{P\}$ at time $t = 0$, i.e. $R_M^P(0) = I$. Find an expression for the homogenous transformation representing the star's reference frame $\{S\}$ in terms of that of the moon $\{M\}$.

8. How many parameters does one require to fully specify a homogenous transformation between two coordinate frames? How many do we require when using the DH conventions? Why is this number different? Explain your answer with the necessary mathematical argument.

9. Obtain the DH parameters for the following robots:

   (a) Planar 2D cartesian robot

   (b) 3D cartesian robot with a spherical wrist.

   (c) 3D cartesian robot with a spherical wrist.

   (d) 3DOF robots in Fig. 1

   (e) Robots in Fig. 2

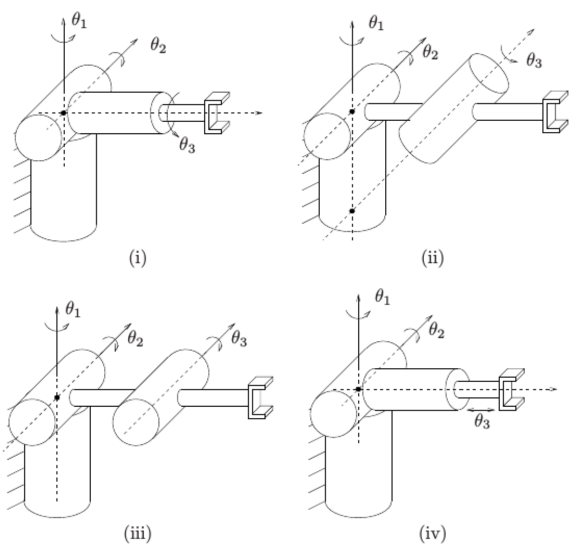


Figure 1: Image taken from [1]

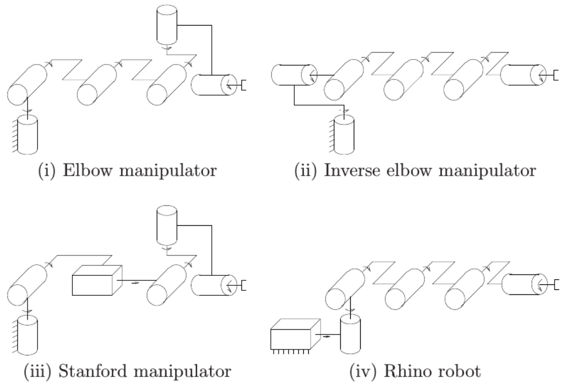


Figure 2: Image taken from [1]

References

[1] Murray, Richard M., Zexiang Li, S. Shankar Sastry, and S. Shankara Sastry. A mathematical introduction to robotic manipulation. CRC press, 1994