

Linear Systems

Dept. of Bioengineering, CMC Vellore

Instructor: Sivakumar Balasubramanian

TA: Ann David

Office hours: SB: Thur 2:00-3:00PM, AD: Sat 10:00-11:00AM

Course Webpage: Please refer to this page for the up-to-date lecture notes and assignments.

<https://siva82kb.github.io/teaching/lce/lce.html>

What is the course about?

- Introduction to applied linear control and estimation.
- Focuses on linear algebra, state space representation and analysis, state feedback control and state estimation.
- First half of the course focuses on applied linear algebra, and the second half focuses on linear dynamical systems, control and estimation.

What to expect from the course?

- Important concepts in applied linear algebra
- State space representation and analysis of physical systems
- Design and analysis of linear state feedback controllers
- Design and analysis of linear state observers

Course Scoring and Grading

Course Activities

- **Homework assignment** 25%
Assignments will be provided to the student by the instructor and will be due one week after the assignments are provided. Late submissions will not be evaluated. Assignments will include both regular paper-and-pencil and programming problems. The student is free to use any programming language to solve the problems. You are encouraged to work in groups to solve these problems, and learn from each other. But write down your own solutions and do not copy.
- **Surprise Quiz** 15%
These will be given throughout the duration of the course. They will be short 20-30 min open book, in-class quizzes.
- **Mid-term** 15%
Take home exam, due the next day. This can include both paper-and-pencil and programming problems. Students are not allowed to discuss among themselves in solving these problems.
- **Final** 45%
Take home exam, due the two days after it is given. This can include both paper-and-pencil and programming problems. Students are not allowed to discuss among themselves in solving these problems.

Grading policy: No relative grading

A+ : $90 \leq \text{Score}$; **A** : $80 \leq \text{Score} < 90$; **B** : $70 \leq \text{Score} < 80$;
C : $60 \leq \text{Score} < 70$; **D** : $50 \leq \text{Score} < 60$; **E** : $40 \leq \text{Score} < 50$; **F** : $\text{Score} < 40$;

Policy for academic dishonesty

There will be zero tolerance towards academic dishonesty, and anyone found carrying such activities will receive an 'F' grade in the course.

References

1. G Strang, *Introduction to linear algebra*. Wellesley, MA: Wellesley-Cambridge Press, 1993.
2. CD Meyer, *Matrix analysis and applied linear algebra*. Siam; 2000 Jun 1.
3. S Boyd and L Vandenberghe, *Introduction to Applied Linear Algebra – Vectors, Matrices, and Least Squares*. [Online book](#)

Other Resources

1. Online course on *Linear Algebra* by G Strang at MIT OCW. [Course Link](#)
2. Online course on *Linear Dynamical Systems* by S Boyd. [Course Link](#)

Course Content

1. Vectors
2. Matrices
3. Orthogonality
4. Matrix Inverses
5. Least squares methods
6. Eigenvectors and eigenvalues
7. Positive definiteness and matrix norm
8. Singular Value Decomposition
9. Linear dynamical systems (LDS)
10. Modelling physical systems
11. Solution to LDS
12. Stability
13. Controllability
14. Observability
15. State feedback control
16. Linear observers
17. *Linear quadratic regulators*
18. *Kalman Filter*

Course content details

Vectors

1. Vectors; 2. Vector spaces; 3. Subspaces; 4. Linear independence; 5. Span and spanning sets; 6. Inner product; 7. Norm; 8. Angle between vectors; 9. Basis; 10. Dimension of a vector space; 11. Linear functions

Matrices

1. Matrices; 2. Matrix Operations; 3. Matrix Multiplication; 4. Properties of Matrix Multiplication; 5. Geometry of Linear Equations; 6. Gaussian Elimination; 7. Gauss-Jordan Method; 8. Row Echelon Forms; 9. Homogeneous systems; 10. Non-homogeneous systems; 11. **LU** factorization; 12. Linear transformation; 13. Four fundamental subspaces; 14. Matrix inverse

Orthogonality

1. Orthogonality; 2. Orthogonal subspaces; 3. Relationship between the four fundamental subspaces; 4. Gram-Schmidt orthogonalization; 5. **QR** factorization; 6. Orthogonal projection

Matrix Inverses

1. Representation of vectors in a basis; 2. Matrix Inverse; 3. Left Inverse; 4. Right Inverse; 5. Pseudo-inverse; 6. Inverses and **QR** factorization

Least squares methods

1. Overdetermined System of linear equations 2. Least Squares Problem 3. Multi-Objective Least Squares 4. Constrained Least Squares

Eigenvectors and eigenvalues

1. Linear transformation; 2. Representation of linear transformations in different basis; 3. Similarity transformation; 4. Complex vectors and matrices; 5. Eigenvectors and Eigenvalues; 6. Diagonalization of matrix $\mathbf{X}\mathbf{\Lambda}\mathbf{X}^{-1}$; 7. Jordan form

Matrix norm, Positive definiteness

Singular Value Decomposition

Linear dynamical systems (LDS)

Modelling physical systems

Solution to LDS

Stability

Controllability

Observability

State feedback control

Linear observers

Linear quadratic regulators

Kalman Filter