Digital Signal Processing: Theory and Practice

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What is signal processing?

"Signal processing is an enabling technology that encompasses the fundamental theory, applications, algorithms, and implementations of ***processing or transferring information*** *contained in many different physical, symbolic, or abstract formats broadly designated as signals and uses* ***mathematical, statistical, computational, heuristic, and/or linguistic representations****, formalisms, and techniques for* ***representation, modeling, analysis, synthesis, discovery, recovery, sensing, acquisition, extraction, learning, security, or forensics.****"*[1]

[1] Moura, J.M.F. (2009). "What is signal processing?, Presidents Message". IEEE Signal Processing Magazine 26 (6). doi:10.1109/MSP.2009.934636

What is a signal?

Any physical quantity carrying information that varies with one or more independent variables.

$$s\left(t\right) = 1.23t^2 - 5.11t + 41.5$$

$$s\left(x, y\right) = e^{-(x^2+y^2+0.5xy)}$$

Mathematical representation will not be possible (e.g. *physiological signals, either because the exact function is not known or is too complicated.*)

What is a signal? (Contd ...)

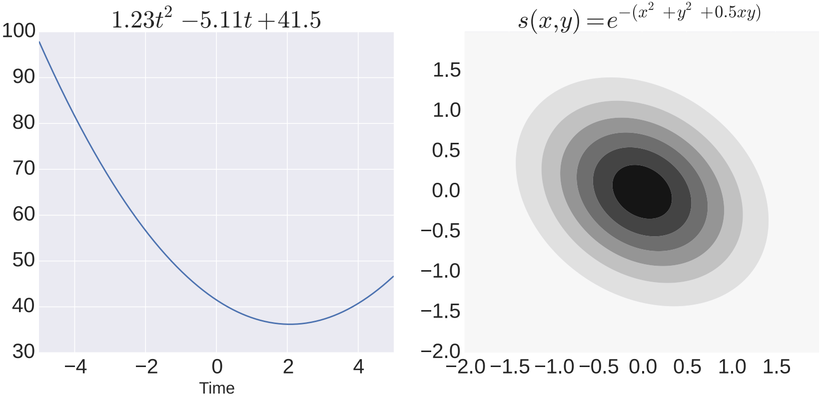
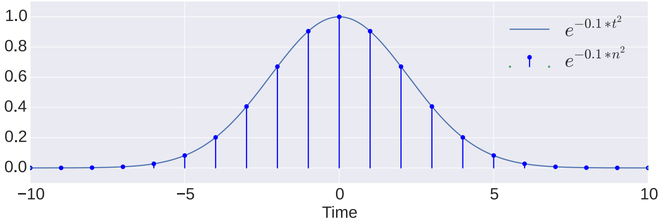


Figure : Example of 1D and 2D signals

Can you think of examples of 3D and 4D signals?

Classification of signals

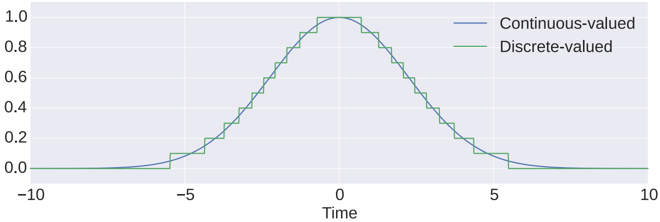
- Based on the signal dimensions. *e.g.* 1D, 2D ...
- **Scalar** vs. **Vector** signals: *e.g. gray scale versus RGB image*
$$I_g\left(x, y\right) \in \mathbb{R} \text{ and } I_{color}\left(x, y\right) \in \mathbb{R}^3$$
- **Continuous-time** vs. **Discrete-time**: *based on the values assumed by the independent variable.*
$$\begin{cases} x(t) = e^{-0.1t^2},\ t \in \mathbb{R} & \text{Continuous-time} \\ x[n] = e^{-0.1n^2},\ n \in \mathbb{Z} & \text{Discrete-time} \end{cases}$$



Classification of signals (Contd ...)

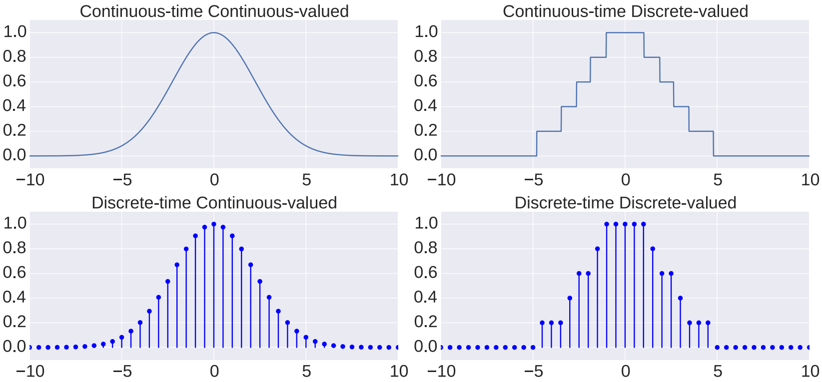
- **Continuous-valued** vs. **Discrete-valued**: *based on the values assumed by the dependent variable.*

$$\begin{cases} x(t) \in [a, b] & \text{Continuous-valued} \\ x(t) \in \{a_1, a_2, \cdots\} & \text{Discrete-valued} \end{cases}$$



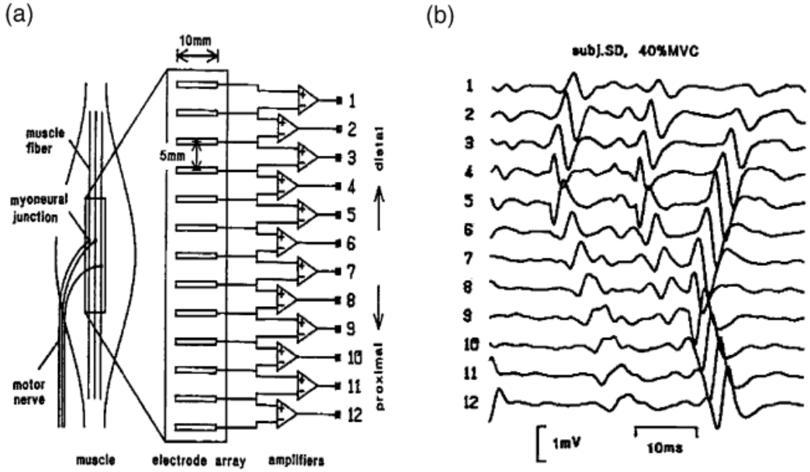
Classification of signals (Contd ...)

Four types of signals



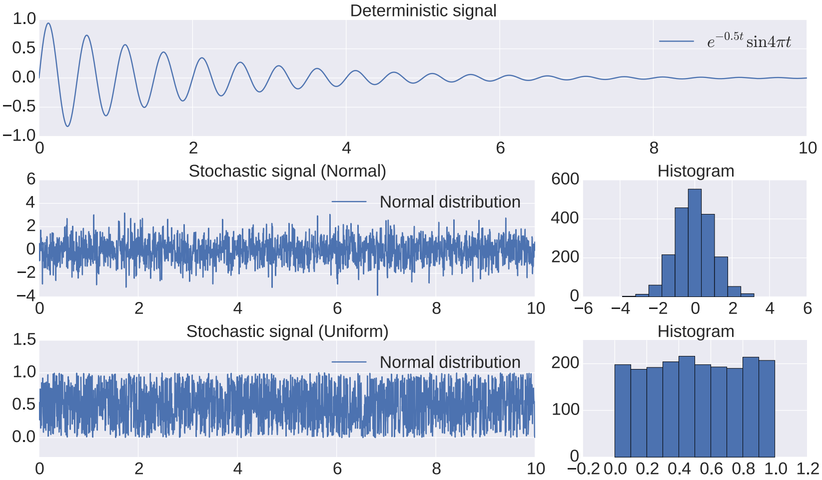
Classification of signals (Contd ...)

Example of a Continuous-valued discrete-time signal



Classification of signals (Contd ...)

- **Deterministic** vs. **Stochastic**: *e.g. EMG is an example of a stochastic signal.*



Classification of signals (Contd ...)

- **Even** vs. **Odd**: *based on the symmetry about the* $t = 0$ *axis.*

$$\begin{cases} x(t) = x(-t), & \text{Even signal} \\ x(t) = -x(-t), & \text{Odd signal} \end{cases}$$

Can there be signals that are neither even nor odd?

Theorem

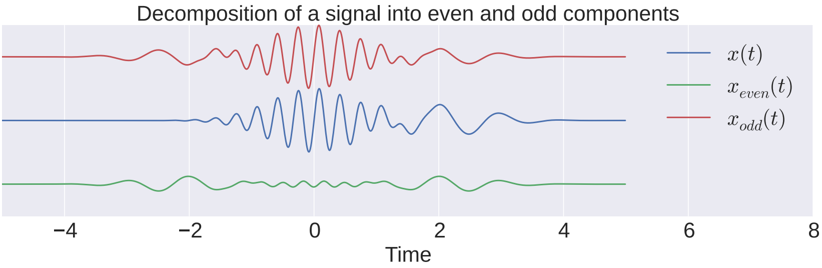
Any arbitrary function can be represented as a sum of an odd and even function.

$$x(t) = x_{even}(t) + x_{odd}(t)$$

where, $x_{even}(t) = \frac{x(t)+x(-t)}{2}$ and $x_{odd}(t) = \frac{x(t)-x(-t)}{2}$.

Classification of signals (Contd ...)

Decomposition of an arbitrary signal into even and odd components



Classification of signals (Contd ...)

- **Periodic** vs. **Non-periodic**: *a signal is periodic, if and only if*

  $$x\left(t\right)=x\left(t+T\right),\forall t$$

  where, T is the fundamental period.

- **Energy** vs. **Power**: *indicates if a signal is short-lived.*

  $$E=\int_{-\infty}^{\infty}\left|x(t)\right|^{2}dt \qquad P=\frac{1}{T}\int_{-T/2}^{T/2}\left|x(t)\right|^{2}dt$$

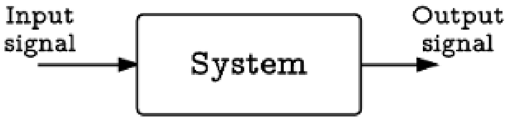
  $$E=\sum_{-\infty}^{\infty}\left|x(t)\right|^{2} \qquad P=\frac{1}{2N+1}\sum_{-N}^{N}\left|x(t)\right|^{2}$$

  *A signal is an **energy** signal, if* $0<E<\infty$.
  *A signal is an **power** signal, if* $0<P<\infty$.

What is a system?

A collection of objects united by some form of interaction or interdependence[2].

From the signal processing point of view, **a system is any physical device or algorithm that performs some operation on a signal to transform it into another signal.**



Can be thought of a mapping function. *e.g.*

$$y = f(x) = kx$$

[2]Zadeh, Lotfi A., and Charles A. Deoser. *Linear system theory*. New York: McGraw Hill, 1963.

Classification of systems

Based on the properties of the system.

- **Linearity**: *satisfies the properties of* ***scaling*** *and* ***superposition.***

  Let us assume,

  $$f : x_i(t) \mapsto y_i(t)$$

  The system f is linear, if and only if,

  $$f : \sum_i a_i x_i(t) \mapsto \sum_i a_i y_i(t)$$

  Which of these systems are linear?

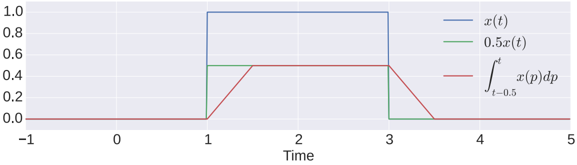
  1. $y(t) = k_1 x(t) + k_2 x(t-2)$
  2. $y(t) = \int_{t-T}^{t} x(\tau) d\tau$
  3. $y(t) = 0.5x(t) + 1.5$

Classification of systems

Based on the properties of the system.

- **Memory**: *a system whose output depends on past or future values of its input is a system with memory, else the system is memoryless.*
  Note: the system may or may not depends on its present.

$$\begin{cases} y(t) = 0.5x(t) & \textbf{Memoryless system} \\ y(t) = \int_{t-0.5}^{t} x(\tau)d\tau & \textbf{System with memory} \end{cases}$$



Classification of systems

- **Causality**: *a system whose output depends on the past and present only values of the input is a causal system.*

$$\begin{cases} y(t) = \int_{t-0.5}^{t} x(\tau)d\tau & \textbf{Causal} \\ y(t) = \int_{t-0.5}^{t+0.5} x(\tau)d\tau & \textbf{Non-causal} \end{cases}$$

- **Time invariance**: *system remains the same with time.* If s system is time-invariant, then

$$f : x\left(t\right) \mapsto y\left(t\right) \iff f : x\left(t - t_0\right) \mapsto y\left(t - t_0\right)$$

- **Stability**: *bounded input produces bounded output.*

$$\left|x\left(t\right)\right| < M_x < \infty \mapsto \left|y\left(t\right)\right| < M_y < \infty$$

- **Invertibility**: *input can be recovered from output.*