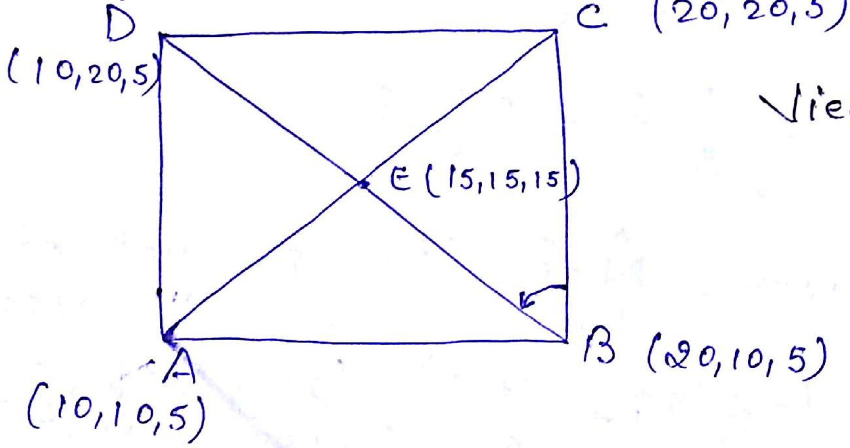
ALGORITHMS :-

(1) Flat Shading.

find intensity of one pixel of polygon and give same intensity to all pixels of surface.



Viewers Position $\equiv (100,100,100)$.

Total no. of surfaces = 4. (ABE, BCE, CDE, ADE).

for region ABE,

find intensity of any pixel of region ABE, and give all the pixels same intensity of surface ABE.

find normal of ABE,

$\vec{N} = \vec{AB} \times \vec{AE}$

$= 10\hat{i} \times (5\hat{i} + 5\hat{j} + 10\hat{k}) = 50\hat{k} - 100\hat{j}$.

Center point of ABE $= \left(\frac{20+15+10}{3}, \frac{10+15+10}{3}, \frac{5+15+5}{3}\right)$

$= (15, 12, 8)$ (take approx values)

[from formula of centroid $= \frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}$]

ray vector, $\vec{r} = (100-15)\hat{i} + (100-12)\hat{j} + (100-8)\hat{k}$.

$= 85\hat{i} + 88\hat{j} + 92\hat{k}$.

~~Norma~~ Unit vector of ray-vector,

$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{85\hat{i} + 88\hat{j} + 92\hat{k}}{\sqrt{(85)^2 + (88)^2 + (92)^2}}$

$$= \frac{85\hat{i} + 88\hat{j} + 92\hat{k}}{\sqrt{23433}} = \frac{85\hat{i} + 88\hat{j} + 92\hat{k}}{153.07}$$

Unit vector of normal,

$$\hat{N} = \frac{\vec{N}}{|\vec{N}|} = \frac{50\hat{k} - 100\hat{j}}{\sqrt{2500 + 10000}} \cancel{= \frac{50\hat{k} - 100\hat{j}}{111.8}}$$

$$= \frac{50\hat{k} - 100\hat{j}}{50\sqrt{5}}$$

$$= \frac{\sqrt{5}}{5}\hat{k} - \frac{2\sqrt{5}}{5}\hat{j}$$

Now,

Intensity $= \hat{N} \cdot \hat{r}$ (Dot product of both unit vectors).

$$= \left(-\frac{2\sqrt{5}}{5}\hat{j} + \frac{\sqrt{5}}{5}\hat{k}\right) \cdot \left(\frac{85}{153}\hat{i} + \frac{88}{153}\hat{j} + \frac{92}{153}\hat{k}\right)$$

$$= \frac{-2\sqrt{5}}{5} \times \frac{88}{153} + \frac{\sqrt{5}}{5} \times \frac{92}{153}$$

$$= (-0.23 + 0.12) \times \sqrt{5}$$

$$= -0.109 \times 2.23$$

$$= -0.25$$

-ve value can be there as we are not considering some constants.

Similarly calculate for other surfaces.

for surface ADE,

$\vec{N} = \vec{AE} \times \vec{AD}$ (in anti-clockwise dir^n).

$$= (5\hat{i} + 5\hat{j} + 10\hat{k}) \times (10\hat{j})$$

$$= 50\hat{k} - 100\hat{i}$$

$$\text{Center-point} = \left(\frac{35}{3}, \frac{45}{3}, \frac{25}{3}\right) = (12, 15, 8)$$

ray vector, $\vec{r} = (100-12)\hat{i} + (100-15)\hat{j} + (100-8)\hat{k}$

$$\vec{r} = 88\hat{i} + 85\hat{j} + 92\hat{k}.$$

$$\hat{r} = \frac{88\hat{i} + 85\hat{j} + 92\hat{k}}{\sqrt{23433}} = \frac{88\hat{i}}{153} + \frac{85}{153}\hat{j} + \frac{92\hat{k}}{153}.$$

Unit-vector of normal,

$$\hat{N} = \frac{\vec{N}}{|\vec{N}|} = \frac{50\hat{k} - 100\hat{i}}{50\sqrt{5}}$$

$$= \frac{\sqrt{5}}{5}\hat{k} - \frac{2\sqrt{5}}{5}\hat{i}.$$

$$\text{Intensity} = \hat{N}\cdot\hat{r}$$

$$= -\frac{2\sqrt{5}}{5} \times \frac{88}{153} + \frac{\sqrt{5}}{5} \times \frac{92}{153}.$$

$$= \frac{-84}{765} \times \cancel{2}\sqrt{5}$$

$$= \frac{-187.82}{765} = -0.245.$$

for surface BCE,

$$\vec{N} = \vec{BC} \times \vec{BE}$$

$$= (10\hat{j}) \times (-5\hat{i} + 5\hat{j} + 10\hat{k})$$

$$= -50 \times -\hat{k} + 100\hat{i}$$

$$= 50\hat{k} + 100\hat{i} = 100\hat{i} + 50\hat{k}.$$

$$\text{Center-point} = \left(\frac{55}{3}, \frac{45}{3}, \frac{25}{3}\right)$$

$$= (18, 15, 8)$$

ray vector, $\vec{r} = (100-18)\hat{i} + (100-15)\hat{j} + (100-8)\hat{k}.$

$$= 82\hat{i} + 85\hat{j} + 92\hat{k}.$$

Unit vector of ray vector,

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{82\hat{i} + 85\hat{j} + 92\hat{k}}{\sqrt{6784 + 7225 + 8464}} = \frac{82\hat{i} + 85\hat{j} + 92\hat{k}}{150}$$

Unit Normal vector,

$$\hat{N} = \frac{\vec{N}}{|\vec{N}|} = \frac{50\hat{k} + 100\hat{i}}{50\sqrt{5}}$$

$$= \frac{\sqrt{5}}{5}\hat{k} + \frac{2\sqrt{5}}{5}\hat{i}$$

Intensity, $I = \hat{N} \cdot \hat{r}$

$$= \left(\frac{\sqrt{5}}{5}\hat{k} + \frac{2\sqrt{5}}{5}\hat{i}\right) \cdot \left(\frac{82\hat{i}}{150} + \frac{85\hat{j}}{150} + \frac{92\hat{k}}{150}\right)$$

$$= \frac{\sqrt{5}}{5} \times \frac{92}{150} + \frac{2\sqrt{5}}{5} \times \frac{82}{150}$$

$$= \sqrt{5}\left(\frac{256}{750}\right) = 0.763$$

for surface DCE,

Normal Vector, $\vec{N} = \vec{CD} \times \vec{CE}$

$$= (-10\hat{i}) \times (-5\hat{i} - 5\hat{j} + 10\hat{k})$$

$$= 50\hat{k} + (-100) \times (-\hat{j})$$

$$= 50\hat{k} + 100\hat{j}$$

Center point $\equiv \left(\frac{45}{3}, \frac{55}{3}, \frac{25}{3}\right)$

$$\equiv (15, 18, 8)$$

ray vector, $\vec{r} = (100-15)\hat{i} + (100-18)\hat{j} + (100-8)\hat{k}$

$$= 85\hat{i} + 82\hat{j} + 92\hat{k}$$

Unit vector of $\vec{r}$,

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{85\hat{i} + 82\hat{j} + 92\hat{k}}{\sqrt{(85)^2 + (82)^2 + (92)^2}}$$

$$= \frac{85\hat{i} + 82\hat{j} + 92\hat{k}}{150}$$

$$= \frac{85}{150}\hat{i} + \frac{82}{150}\hat{j} + \frac{92}{150}\hat{k}$$

Unit Normal vector, $\hat{N} = \frac{\vec{N}}{|\vec{N}|} = \frac{50\hat{k} + 100\hat{j}}{\sqrt{(50)^2 + (100)^2}}$

Scanned by CamScanner

$$\hat{N} = \frac{50\hat{k} + 100\hat{j}}{50\sqrt{5}}.$$

$$= \frac{\sqrt{5}}{5}\hat{k} + \frac{2\sqrt{5}}{5}\hat{j}.$$

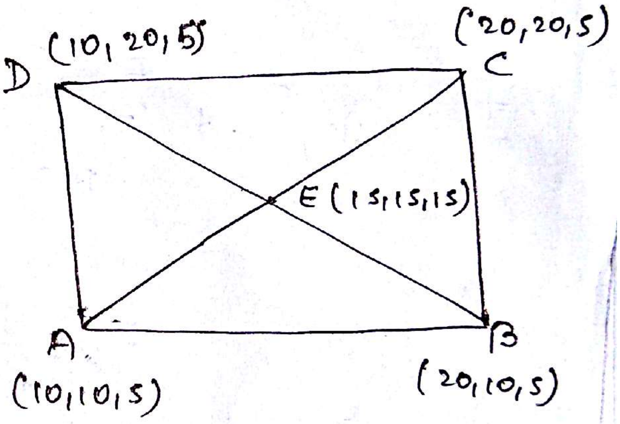
Intensity, $I = \hat{N}\cdot\hat{r}$.

$$= \left(\frac{\sqrt{5}}{5}\hat{k} + \frac{2\sqrt{5}}{5}\hat{j}\right)\cdot\left(\frac{85}{150}\hat{i} + \frac{82}{150}\hat{j} + \frac{92}{150}\hat{k}\right)$$

$$= \frac{\sqrt{5}}{5}\times\frac{92}{150} + \frac{2\sqrt{5}}{5}\times\frac{82}{150}.$$

$$= \sqrt{5}\cdot\left(\frac{256}{750}\right) = 0.763.$$

(2) Gaurand Shading.



Viewer's Position $\equiv (100,100,100)$.

Normal vectors, (to the surface)

$\vec{n}_{ABE} = 50\hat{k} - 100\hat{j}$.

$\vec{n}_{ADE} = -100\hat{i} + 50\hat{k}$.

$\vec{n}_{BCE} = 100\hat{i} + 50\hat{k}$

$\vec{n}_{CDE} = 100\hat{j} + 50\hat{k}$.

for Surface ABE,

$\vec{n}_A = -100\hat{i} - 100\hat{j} + 100\hat{k}$.

$\vec{n}_B = 100\hat{i} - 100\hat{j} + 100\hat{k}$.

$\vec{n}_E = 200\hat{k}$.

Unit vector for $\vec{n}_A$, $\hat{n}_A = -\frac{\sqrt{3}}{3}\hat{i} - \frac{\sqrt{3}}{3}\hat{j} + \frac{\sqrt{3}}{3}\hat{k}$.

Unit Vector for $\vec{n}_B$, $\hat{n}_B = \frac{\sqrt{3}}{3}\hat{i} - \frac{\sqrt{3}}{3}\hat{j} + \frac{\sqrt{3}}{3}\hat{k}$.

Ray vector, $\vec{r}_A = (00-10)\hat{i} + (100-10)\hat{j} + (100-5)\hat{k}$.

$= 90\hat{i} + 90\hat{j} + 95\hat{k}$.

$\vec{r}_B = (100-20)\hat{i} + (100-10)\hat{j} + (100-5)\hat{k}$.

$= 80\hat{i} + 90\hat{j} + 95\hat{k}$.

$\vec{r}_E = 85\hat{i} + 85\hat{j} + 85\hat{k}$.

Unit vectors,

$$\hat{r}_A = \frac{90}{159}\hat{i} + \frac{90}{159}\hat{j} + \frac{90}{159}\hat{k}.$$

$$\hat{r}_B = \frac{80}{153}\hat{i} + \frac{90}{153}\hat{j} + \frac{90}{153}\hat{k}$$

$$\hat{r}_E = \frac{\sqrt{3}}{3}\hat{i} + \frac{\sqrt{3}}{3}\hat{j} + \frac{\sqrt{3}}{3}\hat{k}.$$

Intensity at A,

$$I_A = \hat{n}_A \cdot \hat{r}_A = \left(-\frac{\sqrt{3}}{3}\hat{i} - \frac{\sqrt{3}}{3}\hat{j} + \frac{\sqrt{3}}{3}\hat{k}\right)\cdot\left(\frac{90}{159}\hat{i} + \frac{90}{159}\hat{j} + \frac{95}{159}\hat{k}\right)$$

$$= \frac{-30\sqrt{3} - 30\sqrt{3}}{159} + \frac{95\sqrt{3}}{159}$$

$$= -0.308.$$

Intensity at B,

$$I_B = \hat{n}_B \cdot \hat{r}_B = \left(\frac{\sqrt{3}}{3}\hat{i} - \frac{\sqrt{3}}{3}\hat{j} + \frac{\sqrt{3}}{3}\hat{k}\right)\cdot\left(\frac{80}{153}\hat{i} + \frac{90}{153}\hat{j} + \frac{95}{153}\hat{k}\right).$$

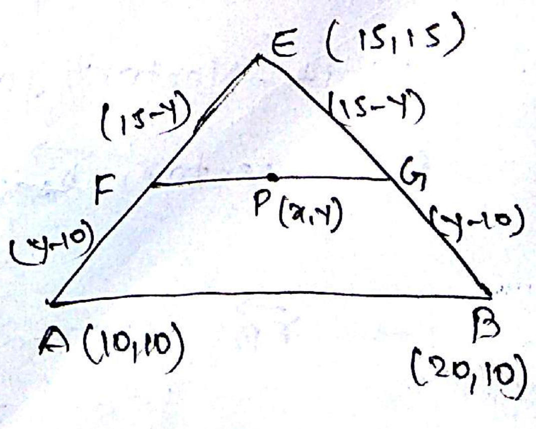
$$= \frac{80\sqrt{3}}{459} - \frac{30\sqrt{3}}{153} + \frac{95\sqrt{3}}{459} = 0.32.$$

Intensity at E,

$$I_E = \hat{n}_E \cdot \hat{r}_E = (\hat{k})\cdot\left(\frac{\sqrt{3}}{3}\hat{i} + \frac{\sqrt{3}}{3}\hat{j} + \frac{\sqrt{3}}{3}\hat{k}\right)$$

$$= \frac{\sqrt{3}}{3} = 0.58.$$

Now, find intensity of one pixel on that surface. z-coordinate is removed as only talking about projection.



let a pixel be P(x,y).

y-coord. at P = y = y-coord. at F.

y-coord. at A = 10.

distance AF = (y-10).

Similarly FE = (15-y).

∴ Ratio is obtained.

Intensity at F,

$$I_F = \frac{(y-10)I_E + (15-y)I_A}{(y-10)+(15-y)}$$ (by section formula for m:n ratio).

$$= \frac{(y-10)\,0.58 + (15-y)(-0.308)}{5} = \frac{0.89y - 10.5}{5}.$$

x-value at F,

$$X_F = \frac{(y-10)\,15 + (15-y)\cdot 10}{5} = \frac{5y}{5} = y$$ (section formula).

y-value at F = y-value at P = y.

Intensity at G,

$$I_G = \frac{(y-10)I_E + (15-y)I_B}{(y-10)+(15-y)} = \frac{(y-10)0.58 + (15-y)(0.32)}{5}$$

$$= \frac{0.26y - 1}{5}.$$

x-value at G,

$$X_G = \frac{(y-10)\cdot 15 + (15-y)\cdot 20}{5} = (30-y)$$

x-coordinate at F = y.

x-coordinate at P = x.

So, FP = x - y.

x-coordinate at G = 30 - y.

x- " at P = x.

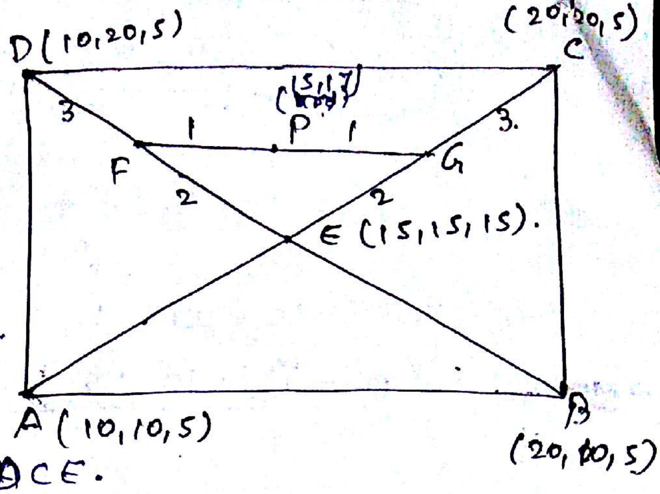
$\therefore$ PG = 30 - y - x.

Intensity at P,

$$I_P = \frac{(x-y)I_G + (30-y-x)I_F}{(x-y)+(30-y-x)}$$ (Section formula)

$$= \frac{(x-y)\left(\frac{0.26y-1}{5}\right) + (30-y-x)\times\left(\frac{0.89y-10.5}{5}\right)}{(30-2y)}$$

(3) Phong Shading.



Viewer's Position = (100,100,100).

for pixel P ≡ (15,17), Intensity = ?

Sol^n ⇒ find the surface on which the pixel (15,17) lies.

It will be on the surface DCE.

Normal for the surfaces.

$$\vec{n}_{ABE} = 50\hat{k} - 100\hat{j}.$$

$$\vec{n}_{BCE} = 100\hat{i} + 50\hat{k}$$

$$\vec{n}_{CDE} = 100\hat{j} + 50\hat{k}.$$

$$\vec{n}_{ADE} = -100\hat{i} + 50\hat{k}$$

normal,

$$\vec{n}_E = 200\hat{k}$$

$$\vec{n}_C = 100\hat{i} + 100\hat{j} + 100\hat{k}.$$

$$\vec{n}_D = -100\hat{i} + 100\hat{j} + 100\hat{k}.$$

for particular pixel P, ray vector and z-coordinate of P is also required.

~~Normal at F~~

y-coordinate at P = y-coordinate at F = 17.

EP = 17 − 15 = 2 and DF = 20 − 17 = 3.

∴ Ratio = 2 : 3.

Normal at F, $\vec{n}_F = \dfrac{3(\vec{n}_E) + 2(\vec{n}_D)}{5}$ (Section formula).

$$= \frac{3(200\hat{k}) + 2(-100\hat{i} + 100\hat{j} + 100\hat{k})}{5} = -40\hat{i} + 40\hat{j} + 160\hat{k}.$$

x-value at F, $x_F = \dfrac{3(15) + 2(10)}{5} = \dfrac{45+20}{5} = 13.$

Z-value at F, $z_F = \dfrac{3(15) + 2(5)}{5} = 11.$

Values at G,

normal at G, $\vec{n_G} = \dfrac{3(\vec{n_E}) + 2(\vec{n_C})}{5}$.

$= \dfrac{3(200\hat{k}) + 2(100\hat{i} + 100\hat{j} + 100\hat{k})}{5}$

$= 40\hat{i} + 40\hat{j} + 160\hat{k}$.

X-value at G, $X_G = \dfrac{3(15) + 2(20)}{5} = 17$.

Z-value at G, $Z_G = \dfrac{3(15) + 2(5)}{5} = 11$.

$PF = 15 - 13 = 2$.

$GP = 17 - 15 = 2$

ratio $= \dfrac{2}{2} = 1/1 = 1:1$.

normal at P, $n_p = \dfrac{1 \times \vec{n_G} + \vec{n_F} \times 1}{1+1}$.

$= \dfrac{(40\hat{i} + 40\hat{j} + 160\hat{k}) + (-40\hat{i} + 40\hat{j} + 160\hat{k})}{2}$

$= 40\hat{j} + 160\hat{k}$.

Z-value at P,

$Z_p = \dfrac{Z_F + Z_G}{2} = \dfrac{11+11}{2} = 11$.

unit vector of normal at P,

$\hat{n_p} = \dfrac{\sqrt{17}}{17}\hat{j} + \dfrac{4\sqrt{17}}{17}\hat{k}$.

ray vector at P,

$\vec{r_p} = (100-15)\hat{i} + (100-17)\hat{j} + (100-11)\hat{k}$.

$= 85\hat{i} + 83\hat{j} + 89\hat{k}$.

$\hat{r_p} = \dfrac{85\hat{i} + 83\hat{j} + 89\hat{k}}{\sqrt{(85)^2 + (83)^2 + (89)^2}} = \dfrac{85\hat{i}}{148} + \dfrac{83\hat{j}}{148} + \dfrac{89\hat{k}}{148}$.

Intensity,

$I_p = \hat{n_p} \cdot \hat{r_p} = \left(\dfrac{\sqrt{17}}{17}\hat{j} + \dfrac{\sqrt{17}}{17}\hat{k}\right) \cdot \left(\dfrac{85}{148}\hat{i} + \dfrac{83}{148}\hat{j} + \dfrac{89}{148}\hat{k}\right)$