Shading → 3 types

1) flat shading or constant intensity or per polygon shading.
2) Gouraud shading or per vertex shading.
3) Phong shading or per pixel shading.

Flat shading

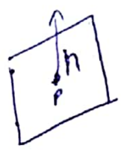
→ Fast & simple method for rendering an object with polygon surfaces.

→ In this method, a single intensity is calculated for each polygon (i.e at the center of the polygon)

→ All points over the surface of the polygon are then displayed with the same intensity value.

This method provides an accurate rendering for an object if all of the foll. assumptions are valid.

1) The object is a polyhedron and is not an approximation of an object with a curved surface.
2) All light sources illuminating the object are sufficiently far from the surface so that N·L and the attenuation function are constant over the surface.
3) The viewing position is sufficiently far from the surface so that V·R is constant over the surface.



→ Calculate intensity at P and color the complete polygon with the intensity at P.

drawback → ~~[illegible]~~ Intensity discontinuities are present. Mach band effect.

Gouraud shading

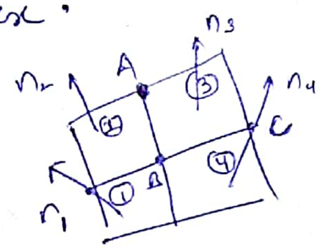
Also called intensity interpolation scheme, developed by Gouraud.

→ This method renders a polygon surface by linearly interpolating intensity values across the surface.

→ Intensity values for each polygon are matched with the values of adjacent polygons along the common edges, thus eliminating the intensity discontinuities that can occur in flat shading.

Steps to perform Gouraud shading

1). Determine the average unit normal vector at each polygon vertex.



- At each polygon vertex, we obtain a normal vector by averaging the surface normals of all polygons sharing that vertex.

$$N_B = \frac{\sum_{K=1}^{n} N_K}{\left| \sum_{K=1}^{N} N_K \right|}$$

$\hat{N}_B = \hat{n}_1 + \hat{n}_2 + \hat{n}_3 + \hat{n}_4$

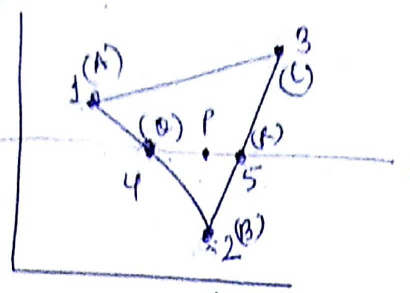
$\hat{N}_C = \hat{n}_3 + \hat{n}_4$

$N_A = \hat{n}_2 + \hat{n}_3$

2) Apply an illumination model to each vertex to calculate the vertex insity.

(i.e calculate I_A, I_B, I_C)

3) linearly interpolate the vertex intensities over the surface of the polygon.



For Gouraud shading, the intensity at point 4 is linearly interpolated from the intensities at vertices 1 and 2. The intensity at point 5 is linearly interpolated from intensities at vertices 2 and 3. An interior point p is then assigned an intensity value that is linearly interpolated from intensities at positions 4 & 5.

$$I_4 = \frac{y_4 - y_2}{y_1 - y_2} I_1 + \frac{y_1 - y_4}{y_1 - y_2} I_2 \quad \text{or} \quad I_4 = u I_1 + (1-u) I_2$$

where $u = AQ/AB$

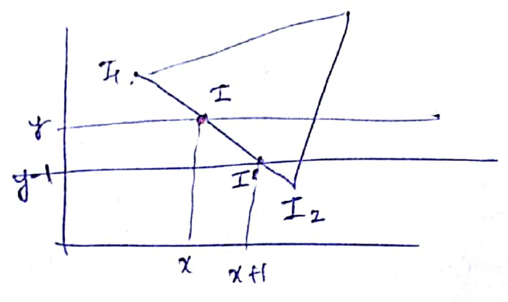
similarly, $$I_5 = \frac{y_3 - y_5}{y_3 - y_2} I_2 + \frac{y_5 - y_2}{y_3 - y_2} I_3 \quad \text{or} \quad I_5 = w I_2 + (1-w) I_3$$

where $w = \frac{BR}{BC}$

Similarly $$I_p = \frac{x_5 - x_p}{x_5 - x_4} I_4 + \frac{x_p - x_4}{x_5 - x_4} I_5 \quad \text{or} \quad I_p = t I_4 + (1-t) I_5$$

$t = \frac{QP}{QR}$

→ Incremented calculations are used to obtain successive edge intensity values b/w scan lines -



$$I = \frac{y - y_2}{y_1 - y_2} I_1 + \frac{y_1 - y}{y_1 - y_2} I_2 \quad \text{——} \quad (1)$$

for I', substitute $y = y-1$ in eq (1)

$$\therefore I' = \frac{y - 1 - y_2}{y_1 - y_2} I_1 + \frac{y_1 - y + 1}{y_1 - y_2} I_2$$

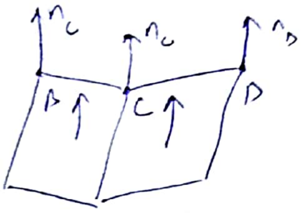
$$I' = \frac{y-y_2}{y_1-y_2} I_1 - \frac{I_1}{y_1-y_2} + \frac{y_1-y}{y_1-y_2} I_2 + \frac{I_2}{y_1-y_2}$$

$$= \left(\frac{y-y_2}{y_1-y_2} I_1 + \frac{y_1-y}{y_1-y_2} I_2\right) + \frac{(I_2 - I_1)}{y_1-y_2}$$

$$\Downarrow$$

$$\boxed{I' = I + \frac{I_2 - I_1}{y_1-y_2}}$$

Drawback →



If the normals at the vertices B, C, D are computed using polygon averaging, then they all have the same direction and hence the same intensity, then linear interpolation yields a constant intensity value from B to D, which makes the surface appear flat in that area.

→ still some amount of mach band effect present.

Phong shading

→ A more accurate method for rendering a polygon surface is to illuminate normal vectors.

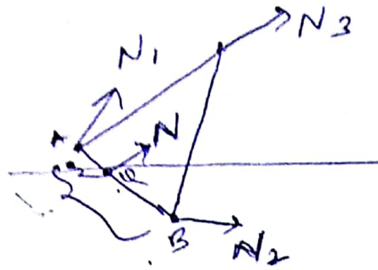
→ Developed by Phong Bui Tuong, is called phong shading or normal vector interpolation shading.

→ greatly reduces the mach band effect.

Steps to perform phong shading

1) Determine the average unit normal vector at each polygon vertex.

inearly Interpolate the vertex normals over the surface of the polygon.



$$N = \frac{y - y_2}{y_1 - y_2} N_1 + \frac{y_1 - y}{y_1 - y_2} N_2$$

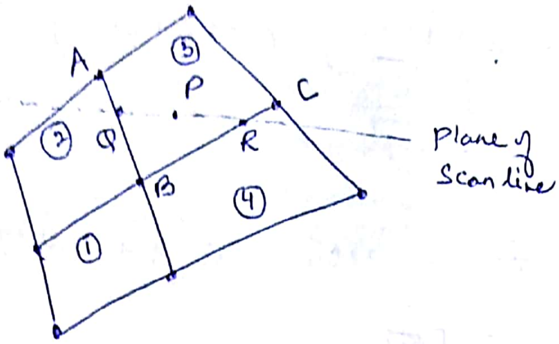
or

$$N = u N_1 + (1-u) N_2 \qquad \boxed{u = \frac{AQ}{AB}}$$

3) Apply an illumination model along each scanline to calculate projected intensities for the surface points.

Drawback :→ requires considerably more calculations.

consider the segment of a surface shown in fig. The equations of the four planes are -



1: $2z - 4 = 0$

2: $-x + 1.732y + 7.5z - 17 = 0$

3: $-2.25x + 3.897y + 10z - 24.5 = 0$

4: $5.5z - 11 = 0$

→ The vector to the eye is $V[1\ 1\ 1]$, and a single point light source is located at positive infinity on the z-axis. The light vector is thus $L[0\ 0\ 1]$. Given info is:

$d = 0,\ k = 1,\ I_a = 1,\ I_l = 10,\ n = 2,\ k_s = 0.8,\ K_d = K_a = 0.15.$

Apply all the shading techniques to calculate the Intensity at Point P. Given ⇒ $\frac{AQ}{AB} = 0.4$, $\frac{BR}{BC} = 0.7$, $\frac{QP}{QR} = 0.5$

Sol:→ Flat shading

Since point P is in polygon 3

$$\therefore\ \hat{n}_3 = \frac{n_3}{|n_3|} = \frac{-2.25\hat{i} + 3.897\hat{j} + 10\hat{k}}{\sqrt{(-2.25)^2 + (3.897)^2 + (10)^2}}$$

$$= \frac{-2.25\hat{i} + 3.897\hat{j} + 10\hat{k}}{\sqrt{120.25}} = -0.21\hat{i} + 0.36\hat{j} + .91\hat{k}$$

The angle b/w the normal & light vector.

$$\hat{n}_3 \cdot \hat{L} = (-0.21\hat{i} + 0.36\hat{j} + 0.91\hat{k}) \cdot (\hat{k})$$

$$\hat{n}_3 \cdot \hat{L} = 0.91$$

$$\hat{R} = 2(\hat{n}_3 \cdot \hat{L})\hat{n}_3 - \hat{L} = 2 * 0.91(-0.21\hat{i} + 0.36\hat{j} + 0.91\hat{k}) - \hat{k}$$

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$\hat{R} = -0.38\hat{i} + 0.66\hat{j} + 0.66\hat{k}$

$\hat{V} = \frac{i+j+k}{\sqrt{3}} = 0.58\hat{i} + 0.58\hat{j} + 0.58\hat{k}$

```
printf("3.Display all elements of queue \n");
printf("4.Quit \n");
printf("Enter your choice : ");
scanf("%d", &choice);
switch (choice)
{
case 1:
insert();
break;
case 2:
delete();
break;
case 3:
display();
break;
case 4:
exit(1);
default:
printf("Wrong choice \n");
End of switch*/
of while*/
```

$\hat{R}\cdot\hat{V} = -0.2204 + .3828 + 0.3828$

$\hat{R}\cdot\hat{V} = 0.55$

Acc. to Illumination model :

$$\boxed{I_p = I_a k_a + \frac{I_l}{d^2+k}\left[k_d(\hat{n}\cdot\hat{L}) + k_s(\hat{R}\cdot\hat{V})^m\right]} \quad (1)$$

$$= 1 * 0.15 + \frac{10}{0+1}\left[0.15(.94) + 0.8(0.55)^2\right]$$

$$= 0.15 + 10\left[0.136 + .242\right]$$

$\boxed{I_p = 3.93}$ → using flat shading.

The whole polygon will be colored with the intensity I_p in flat shading.

2 Gauraud Shading

$n_A = n_2 + n_3 = -3.25\hat{i} + 5.63\hat{j} + 17.5\hat{k}$

$n_B = n_1 + n_2 + n_3 + n_4 = -3.25\hat{i} + 5.63\hat{j} + 25\hat{k}$

$n_C = n_3 + n_4 = -2.25\hat{i} + 3.897\hat{j} + 15.5\hat{k}$

Scanned by CamScanner

unit vectors of n_A, n_B, n_C are

$$\hat{n}_A = \frac{n_A}{|n_A|} = \frac{-3.25\hat{i} + 5.63\hat{j} + 17.5\hat{k}}{\sqrt{(-3.25)^2 + (5.63)^2 + (17.5)^2}} = -0.17\hat{i} + 0.3\hat{j} + 0.94\hat{k}$$

$$\hat{n}_B = \frac{n_B}{|n_B|} = \frac{-3.25\hat{i} + 5.63\hat{j} + 25\hat{k}}{\sqrt{(-3.25)^2 + (5.63)^2 + (17.5)^2}} = -0.12\hat{i} + 0.22\hat{j} + 0.97\hat{k}$$

$$\hat{n}_C = \frac{n_C}{|n_C|} = \frac{-2.25\hat{i} + 3.897\hat{j} + 15.5\hat{k}}{\sqrt{(-2.25)^2 + (3.897)^2 + (15.5)^2}} = -0.14\hat{i} + 0.24\hat{j} + 0.96\hat{k}$$

Calculate Intensity of each vertex using eq (1)

$$I_A = 1 * 0.15 + \frac{10}{0+1}\left[0.15(\hat{n}_A \cdot \hat{L}) + 0.8(\hat{R}_A \cdot \hat{V})^m\right]$$

∴ The unit reflected vectors are :-

$$\hat{R}_A = 2(\hat{n}_A \cdot \hat{L})\hat{n}_A - \hat{L}$$

$$= 2\left((-0.17\hat{i} + 0.3\hat{j} + 0.94\hat{k}) \cdot (\hat{k})\right)(-0.17\hat{i} + 0.3\hat{j} + 0.94\hat{k}) - \hat{k}$$

$$= 2 * 0.94(-0.17\hat{i} + 0.3\hat{j} + 0.94\hat{k}) - \hat{k}$$

$$\hat{R}_A = -0.33\hat{i} + 0.57\hat{j} + 0.76\hat{k}$$

$$\hat{R}_B = -0.24\hat{i} + 0.42\hat{j} + 0.87\hat{k}$$

$$\hat{R}_C = -0.27\hat{i} + 0.46\hat{j} + 0.84\hat{k}$$

$I_A = 0.15 + 10 \left[0.15(0.94) + 0.8(.58)^2\right]$

$= 0.15 + 10\left[0.14 + 0.27\right]$

$\boxed{I_A = 4.25}$

$I_B = 0.15 + 10\left[(0.15) + (0.30)\right] = 4.65$

$I_C = 0.15 + 10\left[(0.14) + (0.29)\right] = 4.45$

Now,

linearly Interpolate the intensities at the vertex.

$I_Q = u I_A + (1-u) I_B$

$I_R = w I_B + (1-w) I_C$

$I_P = t I_Q + (1-t) I_R$.

Given

$u = AQ/AB = 0.4$

$w = BR/BC = 0.7$

$t = \frac{QP}{QR} = 0.5$

$I_Q = 0.4 * 4.25 + (1-0.4) * 4.65 = 4.49$

$I_R = 0.7 * 4.65 + (1-0.7) * 4.45 = 4.59$

$I_P = 0.5 * 4.49 + (1-0.5) * 4.59 = 4.54$

$\therefore \boxed{I_P = 4.54}$ $\longrightarrow$ using Gouraud shading

Phong shading

Interpolates the normal

$$\hat{n}_Q = u\hat{n}_A + (1-u)\hat{n}_B$$

$$\hat{n}_R = \cancel{w}\,\hat{n}_B + (1-w)\hat{n}_C$$

$$\hat{n}_P = t\hat{n}_Q + (1-t)\hat{n}_R$$

$$\hat{n}_Q = 0.4\left[-0.17\hat{i} + 0.3\hat{j} + 0.94\hat{k}\right] + (1-0.4)\left[-0.12\hat{i} + 0.22\hat{j} + 0.97\hat{k}\right]$$

$$= -0.14\hat{i} + 0.25\hat{j} + 0.96\hat{k}$$

$$\hat{n}_R = 0.7\left[-0.12\hat{i} + 0.22\hat{j} + 0.97\hat{k}\right] + (1-0.7)\left[-0.14\hat{i} + 0.24\hat{j} + 0.96\hat{k}\right]$$

$$= -0.13\hat{i} + 0.23\hat{j} + 0.97\hat{k}$$

$$\hat{n}_P = 0.5\left[-0.14\hat{i} + 0.25\hat{j} + 0.96\hat{k}\right] + (1-0.5)\left[-0.13\hat{i} + 0.23\hat{j} + 0.97\hat{k}\right]$$

$$= -0.14\hat{i} + 0.24\hat{j} + 0.97\hat{k}$$

$$\hat{R}_P = 2(\hat{n}_P \cdot \hat{L})\hat{n}_P - \hat{L}$$

$$= 2\left(\left(-0.14\hat{i} + 0.24\hat{j} + 0.97\hat{k}\right) \cdot \hat{k}\right)\left(-0.14\hat{i} + 0.24\hat{j} + 0.97\hat{k}\right) - \hat{k}$$

$$= 2 \times 0.97\left(-0.14\hat{i} + 0.24\hat{j} + 0.97\hat{k}\right) - \hat{k}$$

$$\hat{R}_P = -0.27\hat{i} + 0.46\hat{j} + 0.87\hat{k}$$

(11)

$\therefore$ The intensity at P is

$$I_p = 1 \times 0.15 + \frac{10}{6+1}\left[0.15(\hat{n}_p \cdot \hat{L}) + 0.8(\hat{R}_p \cdot \hat{V})^n\right]$$

$$= 0.15 + 10\left[0.15(.97) + 0.8(.615)^2\right]$$

$$= 0.15 + 10\left[0.15 + .30\right]$$

$$= 4.65$$

$\boxed{I_p = 4.65}$ using phong shading

Flat $\rightarrow$ $I_p = 3.93$

Gouraud $\rightarrow$ $I_p = 4.54$

phong $\rightarrow$ $I_p = 4.65$