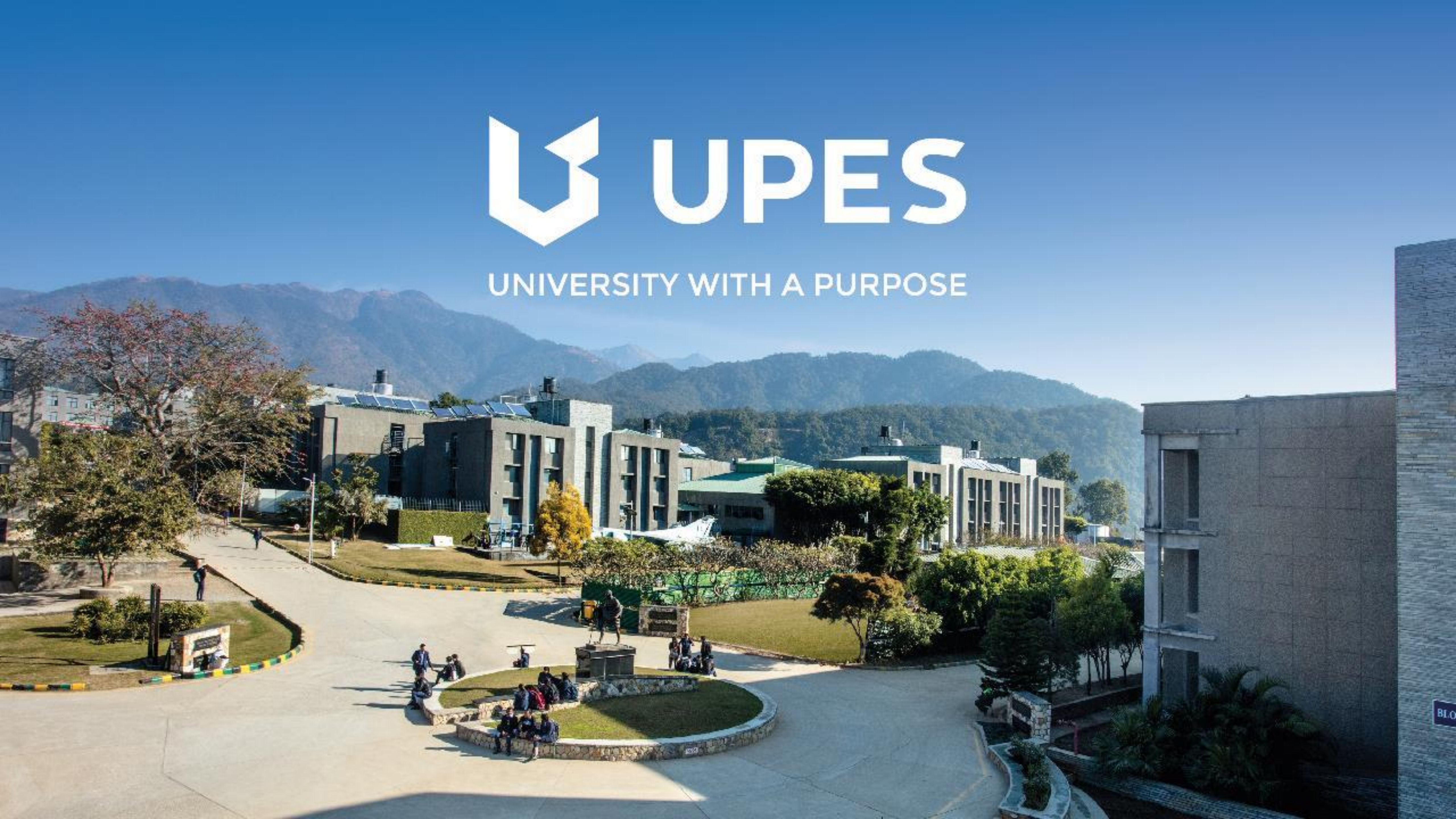
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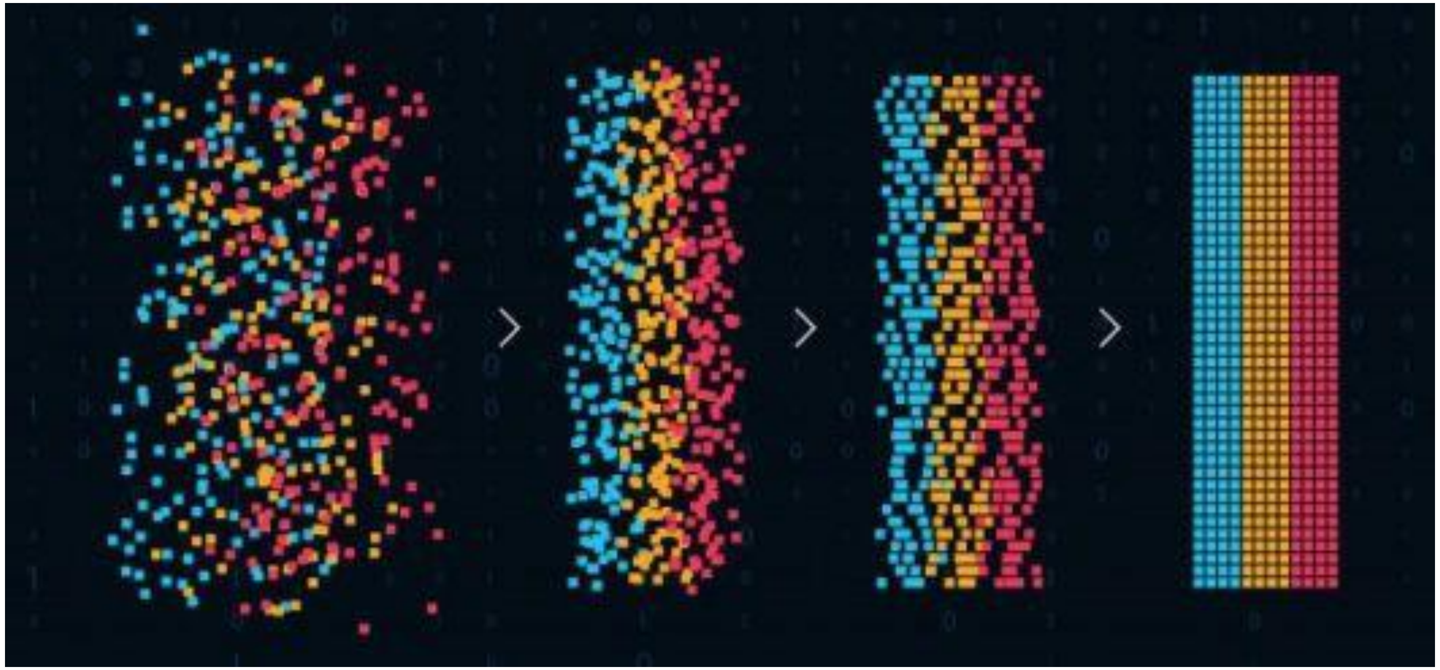
Pattern and Anomaly Detection



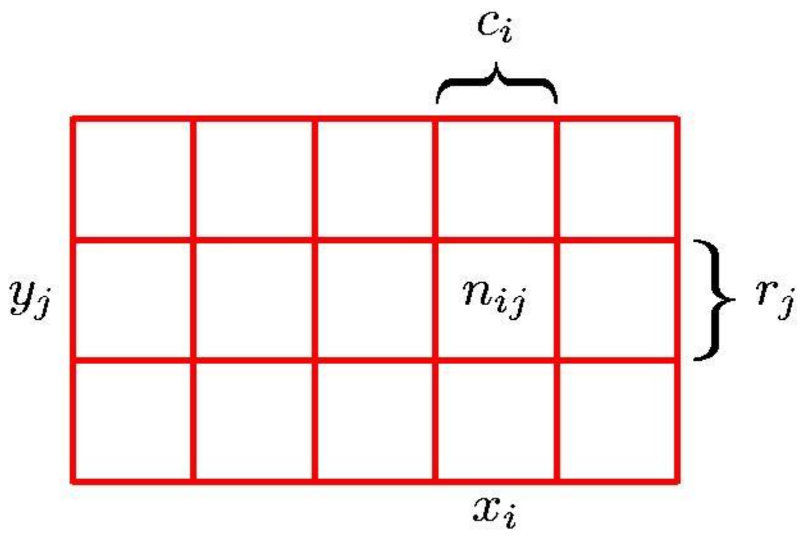
Source: Edureka

B. Tech., CSE + AI/ML

Dr Gopal Singh Phartiyal

16/08/2021

Probability Theory



Sum Rule

$$p(X = x_i) = \frac{c_i}{N} = \frac{1}{N}\sum_{j=1}^{L} n_{ij}$$

$$= \sum_{j=1}^{L} p(X = x_i, Y = y_j)$$

Product Rule

$$p(X = x_i, Y = y_j) = \frac{n_{ij}}{N} = \frac{n_{ij}}{c_i} \cdot \frac{c_i}{N}$$

$$= p(Y = y_j | X = x_i) p(X = x_i)$$

Probability Theory

Baye’s Theorem

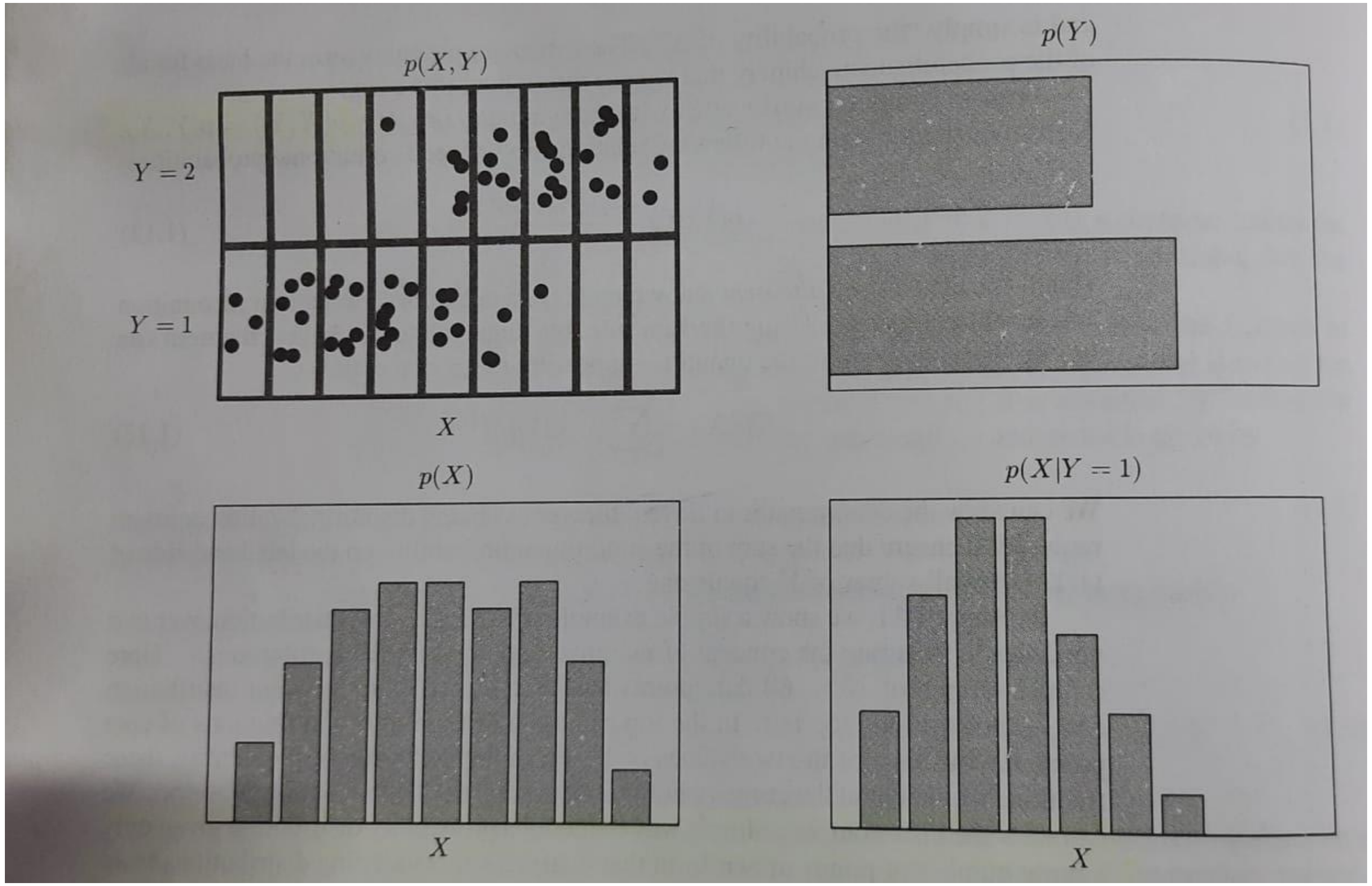
$$p(Y|X) = \frac{p(X|Y)p(Y)}{p(X)}$$

$$p(X) = \sum_Y p(X|Y)p(Y)$$

posterior $\propto$ likelihood × prior

- Plays critical role in pattern recognition and machine learning

Example: Y =2, X = 9



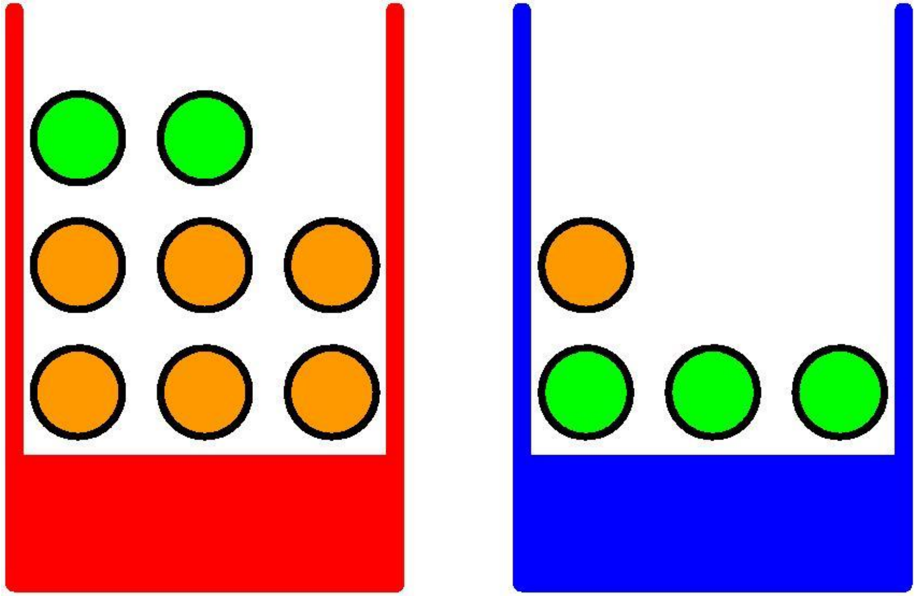
- The histogram depicts the distribution
- It can be interpreted as probability if N tends to infinity

Example: Apples (a) and Oranges (o)

- Given

$$p(B=r) = 4/10$$
$$p(B=b) = 6/10$$
$$p(B=r)+p(B=b)=1.$$

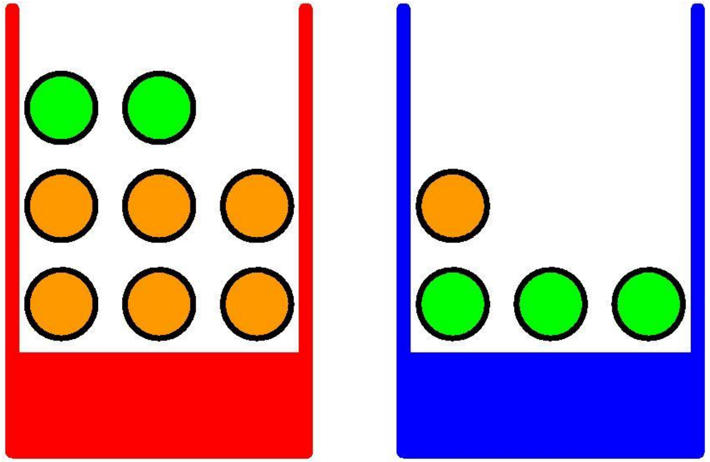
- What is the probability of picking an apple given box is red?
- Overall probability of choosing an apple?
- What is the probability of picking a a red box given fruit is apple?



Red (r)
2 apples
6 oranges

Blue (b)
1 orange
3 apples

Example: Apples (a) and Oranges (o)



- Given

$$p(B = r) = 4/10$$
$$p(B = b) = 6/10$$
$$p(B = r) + p(B = b) = 1.$$

- What is the probability of picking an apple given box is red?

$$p(F = o|B = r) = 3/4$$

- Overall probability of choosing an apple?

$$p(F = a) = p(F = a|B = r)p(B = r) + p(F = a|B = b)p(B = b)$$

- What is the probability of picking a a red box given fruit is apple?

$$p(B = r|F = o) = \frac{p(F = o|B = r)p(B = r)}{p(F = o)} = \frac{3}{4} \times \frac{4}{10} \times \frac{20}{9} = \frac{2}{3}.$$

Probability Densities

- Probability Density

It is the probability of a variable x, i.e. p(x) over an interval (x, x+$\dot{\Delta}$x) when $\dot{\Delta}$x tends to zero.

- Based on the above assumption, the probability p(x) over interval (a, b) is given as

$$p(x \in (a,b)) = \int_a^b p(x)\,\mathrm{d}x.$$

- With the consideration

$$\begin{aligned} p(x) &\geqslant 0 \\ \int_{-\infty}^{\infty} p(x)\,\mathrm{d}x &= 1. \end{aligned}$$

Expectations

- The average value of some function f(x) under a probability distribution p(x) is called the expectation of f(x) and denoted as $\mathbb{E}$(f)

- For discrete distribution

$$\mathbb{E}[f] = \sum_x p(x) f(x)$$

For continuous

$$\mathbb{E}[f] = \int p(x) f(x) \, \mathrm{d}x$$

Approximate Expectation (discrete and continuous)

$$\mathbb{E}[f] \simeq \frac{1}{N} \sum_{n=1}^{N} f(x_n)$$

Conditional Expectation (discrete)

$$\mathbb{E}_x[f|y] = \sum_x p(x|y) f(x)$$

Variance and Covariance

- Variance

$$\operatorname{var}[f] = \mathbb{E}\left[(f(x) - \mathbb{E}[f(x)])^2\right] = \mathbb{E}[f(x)^2] - \mathbb{E}[f(x)]^2$$

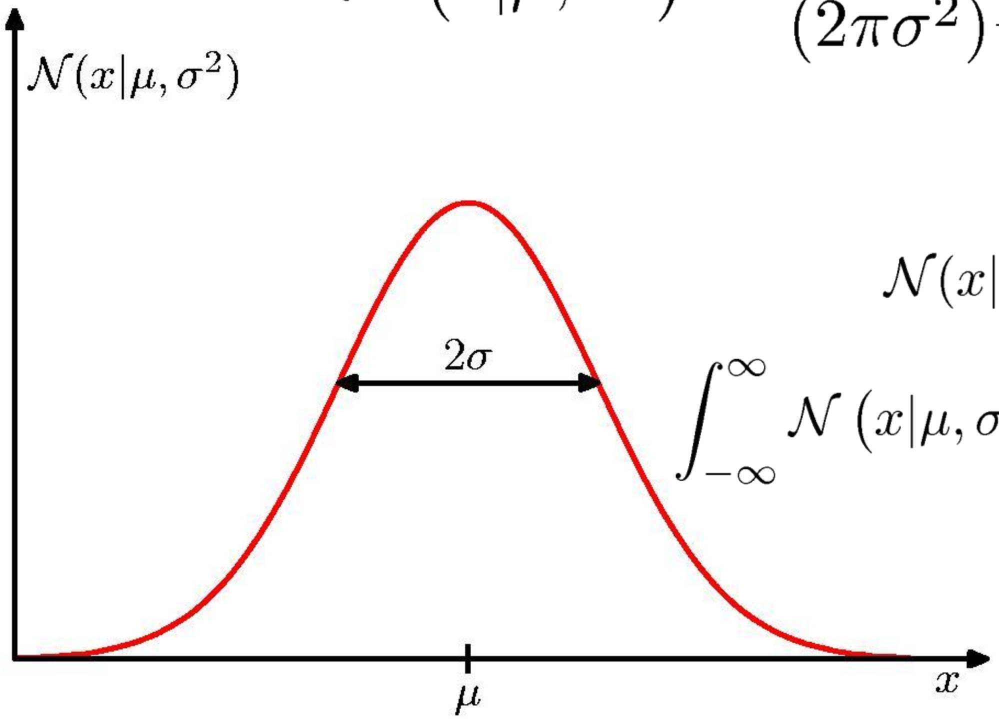
- Covariance

$$\begin{aligned} \operatorname{cov}[x, y] &= \mathbb{E}_{x,y}\left[\{x - \mathbb{E}[x]\}\{y - \mathbb{E}[y]\}\right] \\ &= \mathbb{E}_{x,y}[xy] - \mathbb{E}[x]\mathbb{E}[y] \end{aligned}$$

$$\begin{aligned} \operatorname{cov}[\mathbf{x}, \mathbf{y}] &= \mathbb{E}_{\mathbf{x},\mathbf{y}}\left[\{\mathbf{x} - \mathbb{E}[\mathbf{x}]\}\{\mathbf{y}^{\mathrm{T}} - \mathbb{E}[\mathbf{y}^{\mathrm{T}}]\}\right] \\ &= \mathbb{E}_{\mathbf{x},\mathbf{y}}[\mathbf{x}\mathbf{y}^{\mathrm{T}}] - \mathbb{E}[\mathbf{x}]\mathbb{E}[\mathbf{y}^{\mathrm{T}}] \end{aligned}$$

The Gaussian Distribution

$$\mathcal{N}\left(x|\mu,\sigma^2\right) = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\}$$



$$\mathcal{N}(x|\mu,\sigma^2) > 0$$

$$\int_{-\infty}^{\infty} \mathcal{N}\left(x|\mu,\sigma^2\right) \mathrm{d}x = 1$$

Thank You

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