Exams

FAI2 - 16/06/2020

$\exists X.\, stays\_home(X)$

$\exists X.\, (works\_from\_home(X))$

$\exists X.\, limits\_contact(X)$

$\exists X.\, (\forall Y contact(X, Y) \rightarrow min\_distance(X, Y, 2))$

$\exists X.\, (\forall Y.\, (live\_together(X, Y) \wedge has\_symptoms(Y)) \rightarrow not\_leave(X))$

$\exists X.\, washes\_hands(X)$

$\forall X.\, (stays\_home(X) \wedge works\_from\_home(X) \wedge limits\_contact(X) \wedge$
$(\forall Y contact(X, Y) \rightarrow min\_distance(X, Y, 2)) \wedge$
$(\forall Y.\, (live\_together(X, Y) \wedge has\_symptoms(Y)) \rightarrow not\_leave(X))$
$\wedge\, washes\_hands(X)) \rightarrow safe(X)$

$\forall (X).\, (\neg safe(X) \rightarrow not\_safe(X))$

$\forall (X).\, (not\_safe(X) \rightarrow might\_die(X))$

$\forall X, Y (not\_safe(X) \wedge not\_safe(Y) \wedge infected(X) \rightarrow might\_transmit\_virus(X, Y))$

$\exists X.\, (not\_safe(X) \rightarrow died(X))$

$stays\_home(simmy).$

Query

$? - \exists X.\, (might\_transmit\_virus(simmy, X))$

ALC

FAI2 - 07/07/2020

First Order Logic

$\forall X.\, bus(x) \rightarrow (\exists Y.\, wheels(X, Y) \wedge Y > 3) \wedge powered(X, engine)$

$\forall X.\, car(X) \rightarrow wheels(X, 4) \wedge powered(X, engine)$

$\forall X.\, bike(X) \rightarrow wheels(X, 2) \wedge powered(X, human)$

$\forall X.\, ebike(X) \leftrightarrow bike(X) \wedge powered(X, engine)$

$\forall (X, Y).\, Drives(X, Y) \wedge car(Y) \rightarrow HasLicense(X, B) \wedge adult(X)$

$\forall (X, Y).\, Drives(X, Y) \wedge bus(Y) \rightarrow HasLicense(X, C) \wedge adult(X)$

$\forall X.\, HasLicense(X, C) \rightarrow HasLicense(X, B)$

$\forall X.\, drives(X, Y) \wedge bike(Y) \rightarrow person(X)$

$\exists X, Y, T, Z.\, (drives(X, T) \wedge drives(Y, Z) \wedge X \neq Y \wedge T \neq Z)$

$? - \exists X.\, (drives(driver1, X) \wedge \neg drives(driver2, X))$

ALC

$Bus \sqsubseteq (\geq 4 hasComponent. wheel \sqcap poweredBy. engine)$

$Car \sqsubseteq (\geq 4 hasComponent. wheel \sqcap \leq 4 hasComponent. wheel \sqcap \exists poweredBy. engine)$

$Bike \sqsubseteq (\geq 2 hasComponent. wheel \sqcap \leq 2 hasComponent. wheel \sqcap \exists poweredBy. human)$

$eBike \sqsubseteq (\geq 2 hasComponent. wheel \sqcap \leq 2 hasComponent. wheel \sqcap \exists poweredBy. human \sqcap \exists poweredBy. engine)$

$CarDriver \sqsubseteq (Adult \sqcap \exists hasLicense. BLicense)$

$CarDriver \equiv Adult \sqcap \exists Drives. Car$

$BusDriver \equiv Adult \sqcap \exists Drives. Bus$

$BikeDriver \equiv Adult \sqcap \exists Drives. Bus$

$BusDriver \sqsubseteq (Adult \sqcap \exists hasLicense. CLicense)$

$CLicense \sqsubseteq BLicense$

$BikeDriver \sqsubseteq Person$

$Vehicle \equiv Bus \sqcup Car \sqcup Bike \sqcup eBike$

The *There are different type of drivers is already encoded in the fact that* $CarDriver$ and $BusDriver$ are two different entities.

$hasLicense(driver1, X) \land hasLicense(driver2, Y) \land (X \sqsubseteq Y)$

FAI2 - 10/6/2021

We can define a $contains(molecule, element, quantity, ion)$ predicate that states that some element in contained in a molecule with given quantity and ionization (for for neutral). For example, H_2O would be encoded as

$contains(h2o, h, 2, 0) \land contains(h2o, o, 1, 0)$.

Then, we can encode possible reactions in our becher as implications:

$becher\_contains(h2) \land becher\_contains(o) \to becher\_contains(h2o)$

Query

$\exists X, Y, I. contains(h2o, X, Y, I)$ and continue iterating to find them all, which is something we could do in Prolog with a `findall([X,Y,I], contains(h2o, X, Y, I), R)`.

ALC

$Molecule \sqsubseteq \exists contains. Element$

$Molecule \sqsubseteq \exists hasFormula. Formula$

$Formula \sqsubseteq String$

$Ion \equiv (Atom \sqcap ((\exists hasIonization. PositiveIonization) \sqcup (\exists hasIonization. NegativeIonization)))$

$Atom \sqsubseteq isComposedOf. Element$

$Ion \sqsubseteq Atom$

$Element \sqsubseteq Molecule$ An element can be considered as a single molecule?

$ChemicalReaction \sqsubseteq \geq 2 hasReactants.\ Molecule$

$ChemicalReaction \sqsubseteq \exists hasProducts.\ Molecule$

Query

$q(x) \leftarrow isComposedOf(querymolecule, x)$

Exercise sheet

Exercise 1

Build a knowledge base in which the following knowledge is represented: Father, Mother, GrandMother, GrandFather, Aunt, Uncle, Niece, Nephew, Mother of at least 3 sons, Father of at most 2 Daugthers.

First Order Logic

$$
\begin{gathered}
parent(Fabrizio, Simone) \\
parent(Rossella, Simone) \\
woman(Rossella) \\
man(Fabrizio) \\
parent(X,Y) \land man(X) \rightarrow father(X,Y) \\
parent(X,Y) \land woman(X) \rightarrow mother(X,Y) \\
parent(X,Y) \land parent(Y,Z) \land woman(X) \rightarrow grandmother(X,Z) \\
parent(X,Y) \land sister(Z,Y) \rightarrow aunt(Z,X) \\
parent(X,Y) \land brother(Z,Y) \rightarrow uncle(Z,X) \\
sibling(X,Y) \land man(X) \rightarrow brother(X,Y) \\
sibling(X,Y) \land woman(X) \rightarrow sister(X,Y) \\
(uncle(X,Z) \lor aunt(X,Z)) \land woman(Z) \rightarrow niece(Z,X) \\
(uncle(X,Z) \lor aunt(X,Z)) \land man(Z) \rightarrow nephew(Z,X) \\
mother(X,A) \land mother(X,B) \land mother(X,C) \land (A \neq B \neq C) \land \\
man(A) \land man(B) \land man(C) \rightarrow mother_3(X) \\
father(X,A) \land father(X,B) \land (A \neq B) \land woman(A) \land woman(B) \rightarrow father_2(X)
\end{gathered}
$$

ALC

$$Father \equiv Parent \sqcap Man$$
$$Son \equiv (Man \sqcap \exists hasParent.\ Person)$$
$$Daughter \equiv (Woman \sqcap \exists hasParent.\ Person)$$
$$Mother \equiv Parent \sqcap Woman$$
$$GrandMother \equiv (Mother \sqcap \exists hasChild.\ Parent)$$
$$GrandFather \equiv (Father \sqcap \exists hasChild.\ Parent)$$
$$Aunt \equiv (Woman \sqcap \exists hasSibling.\ Parent)$$
$$Uncle \equiv (Man \sqcap \exists hasSibling.\ Parent)$$
$$Sibling \equiv (Brother \sqcup Sister)$$
$$Niece \equiv (Woman \sqcap \exists Parent.\ Sibling)$$
$$Nephew \equiv (Man \sqcap \exists Parent.\ Sibling)$$
$$Mother_3 \equiv (Woman \sqcap \geq 3 Son)$$
$$Father_2 \equiv (Man \sqcap \leq 2 Daughter)$$

Exercise 2

Build a knowledge base in which the following knowledge is represented: All humans are mammals; all mammals are warm blooded. All dogs are mammals. Humans own animals. There are animals that are not warm blooded. All mammals are animals. A human cannot own another human.

First Order Logic

$$human(X) \rightarrow mammal(X)$$
$$mammal(X) \rightarrow warmblooded(X)$$
$$dog(X) \rightarrow mammal(X)$$
$$own(X,Y) \rightarrow human(X), animal(Y)$$
$$\exists X.\ (animal(X) \wedge \neg warmblooded(X))$$
$$mammal(X) \rightarrow animal(X)$$
$$own(X,Y) \rightarrow human(X) \wedge \neg human(Y)$$

ALC

$$Human \sqsubseteq Mammal$$
$$Mammal \sqsubseteq WarmBlooded$$
$$Dog \sqsubseteq Mammal$$
$$AnimalOwner \equiv Human \sqcap \exists owns.\ Animal$$
$$ColdBloodedAnimal \equiv Animal \sqcap \neg WarmBlooded$$
$$Mammal \sqsubseteq Animal$$
$$(Human \sqcap \exists owns.\ Human).\ \bot$$

Exercise 3

First Order Logic

$student(X) \rightarrow smart(X)$

$\exists X.\ student(X)$

$\exists X.\ (student(X) \wedge smart(X))$

$\forall X \exists Y.\ (student(X) \wedge student(Y) \wedge loves(X,Y))$

$\forall X \exists Y.\, (student(X) \land student(Y) \land loves(X, Y) \land X \neq Y)$

$\exists X.\, (\forall Y.\, student(X) \land student(Y) \land loves(Y, X))$

$student(mark)$

$student(paul)$

$takes(mark, analysis) \leftrightarrow \neg takes(mark, geometry)$

$\neg takes(mark, analysis) \leftrightarrow takes(mark, geometry)$

$takes(paul, analysis) \land takes(paul, geometry)$

$\neg takes(mark, analysis)$

$\forall Y.\, (student(Y) \to \neg loves(Y, paul))$

ALC

$Student \sqsubseteq Smart$

$SmartStudent \equiv Student \sqcap Smart$

$(Student \sqcap \exists loves.\, Student) \equiv Student$

Sentence 5 is not encodable in ALC (I think)

$Student \sqcap \forall loves.\, Student$

$Mark \sqsubseteq Student$

$Paul \sqsubseteq Student$

$Mark \sqsubseteq ((\exists takes.\, GeometryExam \sqcap \neg \exists takes.\, AnalysisExam) \sqcup$

$(\exists takes.\, AnalysisExam \sqcap \neg \exists takes.\, GeometryExam))$

$Paul \sqsubseteq (\exists takes.\, GeometryExam \sqcap \exists takes.\, AnalysisExam)$

$Mark \sqsubseteq \neg \exists takes.\, AnalysisExam$

$(Student \sqcap \exists loves.\, Paul).\, \bot$

Other exercises

Not all students attend both the History and the Biology courses. The best mark in History is higher than the best mark in Biology. One only student did not pass the exam in History. One only student did not pass both the exam in History and the exam in Biology.

$\exists X.\, (attends(X, history) \land \neg attends(X, biology))$

$\forall X, Y.\, (best\_mark(history, X) \land best\_mark(biology, Y) \to (X > Y)$

$\forall X, Y.\, (\neg pass(X, history) \land \neg pass(Y, history) \to X = Y)$

$\forall X, Y.\, (\neg pass(X, history) \land \neg pass(Y, history) \land \neg pass(Y, biology) \land \neg pass(X, biology) \to X = Y)$

Politicians always fool someone, and sometimes fool everyone, but they do not always fool everyone.

$\forall X.\, policitian(X) \exists Y.\, fools(X, Y)$

$\exists X.\, policitian(X) \land (\forall Y.\, fools(X, Y))$

$\neg(\forall X.\, politician(X) \to (\forall Y.\, fools(X, Y)))$

Every person who does not like any vegetarian is intelligent. Nobody likes a vegetarian who is intelligent. There is a woman who likes every man who is not a vegetarian.

$\forall X, Y.\, (vegetarian(Y) \to not\_like(X, Y)) \to intelligent(X)$

$\neg\exists X, Y.\, (likes(X, Y) \land vegetarian(Y) \land intelligent(Y))$

$\exists X.\, woman(X)(\forall Y(\neg vegetarian(Y) \land man(Y)) \to likes(X, Y))$

1. *(a) Tony, Mike and John are members of the Alpine Club*
2. *(b) Every member of the Alpine Club is a skier or a climber*
3. *(c) No climber likes rain*
4. *(d) Every skier likes snow*
5. *(e) Mike does not like everything that Tony likes*
6. *(f) Mike likes everything Tony does not like*
7. *(g) Tony likes both rain and snow*

$member(alpine, tony)$

$member(alpine, mike)$

$\forall x.\, member(alpine, x) \to (\neg skier(x) \land \neg climber(x))$

$\neg\exists x.\, (climber(x) \land likes(rain, x))$

$\forall x.\, skier(x) \to likes(snow, x)$

$\exists x.\, (likes(x, tony) \land likes(y, mike))$

$likes(rain, tony) \land likes(snow, tony)$

bachelor $\doteq \neg\, \exists$married.$\top \sqcap$ man	*„bachelors are unmarried men“*
married $\doteq$ married^{-1}	(being married to so. is reflexive)
$\exists$married.$\top \sqsubseteq$ happy	*„all married people are happy“*
$\exists_{\geqq 2}$ love $\sqsubseteq \bot$	*„you can love at most one person“*
$\exists$married.woman $\sqsubseteq \exists$love.woman	*„someone married to a woman also loves a woman“*