

Statistical Information Theory

$$H(W) = -\log_2(1/25) = \log_2(25)$$

where W is the complete process of weighing.

However, in contrast to the previous problem, each separate weighing may have three outcomes: the right pan goes down, the left pan goes down, and both pans come to the equilibrium. Thus each weighing gives $\log_2 3$ bits of information. When the set W consists of m weighing, we have

$$m \log_2 3 \geq \log_2 25$$

Consequently,

$$m \geq (\log_2 25 / \log_2 3) \approx 2.93$$

i.e., $m \geq 3$ as m is a whole number. This shows that we need to look for an algorithm with $m = 3$.

Algorithm of a minimal solution for finding the lighter coin is given is given below:

Step 1: Divide 25 coins into three groups: 9, 8 and 8 coins, and go to the next step.

Step 2: Compare two groups of 8 coins, using the balance. This action (experiment) may give two results. In one case, the weights of both groups are equal. It means that the lighter coin is in the group of 9 coins because there is only one lighter coin. Then go to Step 3. In the other case, the balance shows that the weight of one group of 8 coins is less than the weight of the other group of 8 coins. It means that the lighter coin is in the lighter group of 8 coins because there is only one lighter coin. Then go to Step 8.

Step 3: Divide the group of 9 coins into three groups: 3, 3 and 3 coins, and go to the next step.

Step 4: Compare two selected groups of 3 coins, using the balance. This action (experiment) may give two results. In one case, the weights of both groups are equal. It means that the lighter coin is in the third group of 3 coins because there is only one lighter coin. Then go to Step 5. In the other case, the balance shows that the weight of one group of 3 coins is less than the weight of the other group of 3 coins. It means that the lighter coin is in the lighter group of 3 coins because there is only one lighter coin. Then go to Step 7.