Exam

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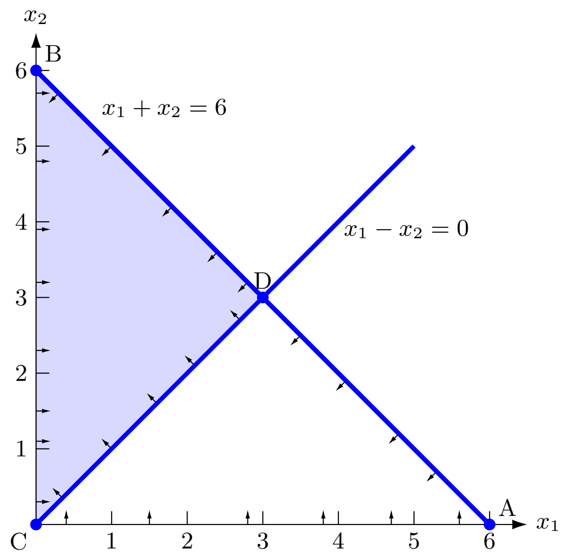

FIGURE 12
Feasible Region for the LP(16)

TABLE 32
Three Sets of Basic Variables Correspond to Corner point C

Basic Variables	Basic Feasible Solution	Corresponds to Extreme Point
x_1, x_2	$x_1 = x_2 = 3, s_1 = s_2 = 0$	D
x_1, s_1	$x_1 = 0, s_1 = 6, x_2 = s_2 = 0$	C
x_1, s_2	$x_1 = 6, s_2 = -6, x_2 = s_1 = 0$	Infeasible
x_2, s_1	$x_2 = 0, s_1 = 6, x_1 = s_2 = 0$	C
x_2, s_2	$x_2 = 6, s_2 = 6, s_1 = x_1 = 0$	B
s_1, s_2	$s_1 = 6, s_2 = 0, x_1 = x_2 = 0$	C

We can now discuss why the simplex algorithm often is an inefficient method for solving degenerate LPs. Suppose an LP is degenerate. Then there may be many sets(maybe hundreds) of basic variables that correspond to some nonoptimal extreme point.The simplex algorithm might encounter all these sets of basic variables before it finds that it was at a nonoptimal extreme point.This problem was illustrated(on a small scale) in solving(16): The simplex took two pivots before it found that point C was suboptimal. Fortunately,some degenerate LPs have a special structure that enables us to solve them by methods other than the simplex (see,foe example,the discussion of the assignment problem in Chapter 7).

P R O B L E MS

Group A

1 Even if an LP's initial tableau is nondegenerate, later tableaus may exhibit degeneracy. Degenerate tableaus often occur in the tableau following a tie in the ratio test. To illustrate this,solve the following LP:

$$\begin{aligned} max\ z &= 5x_1 + 3x_2 \\ s.t. \quad 4x_1 + 2x_2 &\le 12 \\ 4x_1 + x_2 &\le 10 \\ x_1 + x_2 &\le 4 \\ x_1, x_2 &\le 0 \end{aligned}$$

Also graph the feasible region and show which extreme points correspond to more than one set of basic variables.

2 Find the optimal solution of the following LP:

$$\begin{aligned} min\ z &= -x_1 - x_2 \\ s.t. \quad x_1 + x_2 &\le 1 \\ -x_1 + x_2 &\le 0 \\ x_1, x_2 &\ge 0 \end{aligned}$$

```
\documentclass[9pt]{extbook}
\usepackage[utf8]{inputenc}
\usepackage{tikz}
\usepackage[english]{babel}
\usepackage[paperheight=305mm,paperwidth=211mm,margin=20mm,heightrounded]{geometry}
\usepackage{pgfplots}
\usepackage{colortbl}
\usepackage{listings}
\usepackage{xcolor}
\usetikzlibrary{arrows.meta}
\usetikzlibrary{calc}
\usepackage{amsmath}
\usepackage{multicol}
\usepackage{array}
\usepackage{makecell}
\setlength{\parindent}{0pt}
\title{Exam}
\author{Monta Lokmane }
\date{May 2019}
 \begin{document}
\pagestyle{empty}
\maketitle

\begin{tabular}{  >{\raggedleft}m{4cm} b{12.5cm} }
 {\textcolor{blue}{FIGURE 12} \\ Feasible Region for the\\ LP(16)\\ }       &
 \begin{tikzpicture}


\draw [fill=blue!15] (0,0)  -- (0,6)  -- (3,3)  -- cycle;


\draw[ thin ,-{Latex[length=2mm,width=1.3mm]}](0,0) -- (6.5,0) node[anchor=west ]{$x_1$};
\draw[thin,-{Latex[length=2mm,width=1.3mm]}](0,0) -- (0,6.5) node[anchor=south]{$x_2$};

\foreach \x in{1,2,3,4,5,6}
    \draw (\x cm, 5pt) -- (\x cm, -0pt) node [anchor=north] {$\x$};
    \foreach \y in {1,2,3,4,5,6}
     \draw(5pt,\y cm) -- (-0pt,\y cm) node [anchor=east] {$\y$};

     \draw [ultra thick,blue](0,0) -- (5,5);
     \draw [ultra thick,blue](6,0) -- (0,6);
     \filldraw[blue] (3,3) circle (2pt) node[anchor=south] [black]{D};
     \filldraw[blue] (0,6) circle (2pt) node[anchor= south west ] [black]{B};
      \filldraw[blue] (6,0) circle (2pt) node[anchor=south west] [black]{A};
      \filldraw[blue] (0,0) circle (2pt) node[anchor=north east] [black]{C};


% bulti u zim  ana
                    % uz y ass
\draw[-{Latex[length=1.2mm,width=0.6mm]}] (0,0.3) -- (0.2,0.3);
\draw[-{Latex[length=1.2mm,width=0.6mm]}] (0,1.1) -- (0.2,1.1);
\draw[-{Latex[length=1.2mm,width=0.6mm]}] (0,1.5) -- (0.2,1.5);
\draw[-{Latex[length=1.2mm,width=0.6mm]}] (0,2.3) -- (0.2,2.3);
\draw[-{Latex[length=1.2mm,width=0.6mm]}] (0,3.2) -- (0.2,3.2);
\draw[-{Latex[length=1.2mm,width=0.6mm]}] (0,3.9) -- (0.2,3.9);
\draw[-{Latex[length=1.2mm,width=0.6mm]}] (0,4.8) -- (0.2,4.8);
\draw[-{Latex[length=1.2mm,width=0.6mm]}] (0,5.7) -- (0.2,5.7);
                    % uz x ass
\draw[-{Latex[length=1.2mm,width=0.6mm]}] (0.4,0) -- (0.4,0.2);
\draw[-{Latex[length=1.2mm,width=0.6mm]}] (1.5,0) -- (1.5,0.2);
\draw[-{Latex[length=1.2mm,width=0.6mm]}] (2.8,0) -- (2.8,0.2);
\draw[-{Latex[length=1.2mm,width=0.6mm]}] (3.8,0) -- (3.8,0.2);
\draw[-{Latex[length=1.2mm,width=0.6mm]}] (4.7,0) -- (4.7,0.2);
\draw[-{Latex[length=1.2mm,width=0.6mm]}] (5.6,0) -- (5.6,0.2);
                    % uz C-D ASS
\coordinate (A) at (5,5);
% PIRMAIS PUNKTS
\coordinate (M) at (0.35,0.35);
\draw [-{Latex[length=1.2mm,width=0.6mm]}] ($(M)!0cm!270:(A)$) -- ($(M)!0.2cm!90:(A)$);
```

```
% OTRAIS PUNKTS
\coordinate (M) at (1,1);
\draw [-{Latex[length=1.2mm,width=0.6mm]}] ($(M)!0cm!270:(A)$) -- ($(M)!0.2cm!90:(A)$);
% TRE AIS PUNKTS
\coordinate (M) at (1.6,1.6);
\draw [-{Latex[length=1.2mm,width=0.6mm]}] ($(M)!0cm!270:(A)$) -- ($(M)!0.2cm!90:(A)$);
% N PUNKTS
\coordinate (M) at (2.1,2.1);
\draw [-{Latex[length=1.2mm,width=0.6mm]}] ($(M)!0cm!270:(A)$) -- ($(M)!0.2cm!90:(A)$);
% N PUNKTS
\coordinate (M) at (2.7,2.7);
\draw [-{Latex[length=1.2mm,width=0.6mm]}] ($(M)!0cm!270:(A)$) -- ($(M)!0.2cm!90:(A)$);
                    % UZ A-B ASS
\coordinate (A) at (0,6);
\coordinate (B) at (6,0);
% PIRMAIS PUNKTS
\draw [ultra thick,blue](A) -- (B) coordinate (M) at (5.3,0.7);
\draw [-{Latex[length=1.2mm,width=0.6mm]}] ($(M)!0cm!90:(A)$) -- ($(M)!0.2cm!270:(B)$);
% OTRAIS PUNKTS
\draw [ultra thick,blue](A) -- (B) coordinate (M) at (4.8,1.2);
\draw [-{Latex[length=1.2mm,width=0.6mm]}] ($(M)!0cm!90:(A)$) -- ($(M)!0.2cm!270:(B)$);
% N PUNKTS
\draw [ultra thick,blue](A) -- (B) coordinate (M) at (4.1,1.9);
\draw [-{Latex[length=1.2mm,width=0.6mm]}] ($(M)!0cm!90:(A)$) -- ($(M)!0.2cm!270:(B)$);
% N PUNKTS
\draw [ultra thick,blue](A) -- (B) coordinate (M) at (3.5,2.5);
\draw [-{Latex[length=1.2mm,width=0.6mm]}] ($(M)!0cm!90:(A)$) -- ($(M)!0.2cm!270:(B)$);
% N PUNKTS
\draw [ultra thick,blue](A) -- (B) coordinate (M) at (2.8,3.2);
\draw [-{Latex[length=1.2mm,width=0.6mm]}] ($(M)!0cm!90:(A)$) -- ($(M)!0.2cm!270:(B)$);
% N PUNKTS
\draw [ultra thick,blue](A) -- (B) coordinate (M) at (2.4,3.6);
\draw [-{Latex[length=1.2mm,width=0.6mm]}] ($(M)!0cm!90:(A)$) -- ($(M)!0.2cm!270:(B)$);
% N PUNKTS
\draw [ultra thick,blue](A) -- (B) coordinate (M) at (1.8,4.2);
\draw [-{Latex[length=1.2mm,width=0.6mm]}] ($(M)!0cm!90:(A)$) -- ($(M)!0.2cm!270:(B)$);
% N PUNKTS
\draw [ultra thick,blue](A) -- (B) coordinate (M) at (1,5);
\draw [-{Latex[length=1.2mm,width=0.6mm]}] ($(M)!0cm!90:(A)$) -- ($(M)!0.2cm!270:(B)$);
% N PUNKTS
\draw [ultra thick,blue](A) -- (B) coordinate (M) at (0.3,5.7);
\draw [-{Latex[length=1.2mm,width=0.6mm]}] ($(M)!0cm!90:(A)$) -- ($(M)!0.2cm!270:(B)$);

 \node[] at (4.9,3.9) { $x_1 - x_2 = 0$};
 \node[] at (1.7,5.5) { $x_1 + x_2 = 6$};
     \draw [ultra thick,blue](0,0) -- (5,5);
     \draw [ultra thick,blue](6,0) -- (0,6);
\end{tikzpicture}  \\

 \bigbreak

  & \textcolor{blue}{TABLE 32}

Three Sets of Basic Variables Correspond to Corner point C

\begin{tabular}{ b{1.5cm} b{5cm} b{2.4cm}  }

 \noalign{\global\arrayrulewidth=0.5mm}
  \arrayrulecolor{blue}\hline

          \rowcolor{blue!5}\textcolor{blue}{Basic Variables} & \multicolumn{1}{c}{\textcolor{blue}
Extreme Point}  \\ [0.8ex]
   \noalign{\global\arrayrulewidth=0.5mm}
  \arrayrulecolor{blue}\hline
  \rowcolor{white}
 $x_1,x_2$ & $x_1=x_2=3,s_1=s_2=0$ &  \multicolumn{1}{c}{D}  \\
 $x_1,s_1$ & $x_1=0,s_1=6,x_2=s_2=0$ & \multicolumn{1}{c}{C} \\
 $x_1,s_2$ & $x_1=6,s_2=-6,x_2=s_1=0$ & \multicolumn{1}{c}{Infeasible} \\
 $x_2,s_1$ & $x_2=0,s_1=6,x_1=s_2=0$ & \multicolumn{1}{c}{C} \\
```

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 $x_2,s_2$ & $x_2=6,s_2=6,s_1=x_1=0$ & \multicolumn{1}{c}{B}    \\
 $s_1,s_2$ & $s_1=6,s_2=0,x_1=x_2=0$ & \multicolumn{1}{c}{C} \\


 \noalign{\global\arrayrulewidth=0.5mm}
  \arrayrulecolor{blue}\hline
\end{tabular}
 \bigbreak
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  \\
  {& We can now discuss why the simplex algorithm often is an inefficient method for solving degen
\end{tabular}

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\noindent
 \noalign {\global\arrayrulewidth=0.5mm}
  \arrayrulecolor{blue}\hline
%{\color{blue} \rule{\linewidth}{0.5mm} }
 \bigbreak
P R O B L E MS \\
\textcolor{blue}{Group A}
  \begin{multicols}{2}



\textcolor{blue}{1} Even if an LP's initial tableau is nondegenerate, later tableaus may exhibit d
\begin{align*}
\:\:\:\:max\:\: z=\deriv5x_1+3x_2\\
  s.t.\:\:\:\:\:\:\:\:\:\:\:\:\:\: \deriv    4x_1+2x_2 &\leq 12 \\
           \deriv  4x_1+x_2 &\leq 10 \\
           \deriv    x_1+x_2 &\leq 4 \\
            \deriv    x_1,x_2 &\leq 0 \\
\end{align*}

\columnbreak

Also graph the feasible region and show which extreme points correspond to more than one set of ba
\textcolor{blue}{2} Find the optimal solution of the following LP:

\begin{align*}
\:\:\:\:min\:\: z=\deriv-x_1-x_2\\
  s.t.\:\:\:\:\:\:\:\:\:\:\:\:\:\:\:\: \deriv    x_1+x_2 & \leq 1 \\
           \deriv   -x_1+x_2 & \leq 0 \\
           \deriv    x_1,x_2 & \geq 0 \\
           \deriv     \\
\end{align*}
 \end{multicols}

\begin{tabular}{ b{5cm} b{10.5cm} b{0.5cm}  }

$$$$

&{4.11 Degeneracy and the Convergence of the Simplex Algorithm} & \textcolor{blue}{171}
\\

\end{tabular}

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