First Exam

Monta Lokmane (REBCO2)

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with

$$\begin{aligned}[L^2, L_z] &= [L_x^2 + L_y^2 + L_z^2, L_z] \\ &= L_x[L_x, L_z] + [L_x, L_z]L_x \\ &\qquad + L_y[L_y, L_z] + [L_y, L_z]L_y + (L_z^2, L_z) \\ &= L_x(-i\hbar)L_y + (-i\hbar)L_yL_x + L_y(i\hbar)L_x + (i\hbar)L_xL_y + 0 \\ &= 0\end{aligned}$$

Thus operators L^2 and L_z commute and, continuing in the same way, it can be shown that

$$[L^2, L_x] = [L^2, L_y] = [L^2, L_x] = 0 \tag{19.29}$$

Eigenvalues of the angular momentum operators

We will now use the commutation relations for L^2 and its components to find the eigenvalues of L^2 and L_z, without reference to any specific wavefunction. In other words, the eigenvalues of the operators follow from the structure of their commutators. There is nothing particular about L_z,and L_x or L_y could equally well have been chosen, though, in general, it is not possible to find states that are simultaneously eigenstates of two or more of L_x, L_y and L_z.

To help with the calculation, it is convenient to define the two operators

$$U{\equiv}L_x + iL_y \quad\text{and}\quad D{\equiv}L_xiL_y.$$

These operators are not Hermitian; they are in fact Hermitian conjugates, in that U†=D and D†=U, but they do not represent measurable physical quantities. We first note their multiplication and commutation properties:

$$\begin{aligned}UD &= (L_x + iL_y)(L_x - iL_y) = L_x^2 + L_y^2 + i[L_y, L_x] \\ &= L^2L_z^2 + hL_z, \end{aligned} \tag{19.30}$$

$$\begin{aligned}DU &= (L_x - iL_y)(L_x + iL_y) = L_x^2 + L_y^2 - i[L_y, L_x] \\ &= L^2 - L_z^2 - \hbar L_z, \end{aligned} \tag{19.31}$$

$$[L_z, U] = [L_z, L_x] + i[L_z, L_y] = i\hbar L_y + \hbar L_x = \hbar U, \tag{19.32}$$

$$[L_z, D] = [L_z, L_x]i[L_z, L_y] = i\hbar L_y - \hbar L_x = -\hbar D \tag{19.33}$$

In the same way as was shown for matrices, it can be demonstrated that if two operators commute they have a common set of eigenstates. Since $L^2$and L_z commute they possess such a set; let one of the set be $|\psi\rangle$with

$$L^2|\psi\rangle = a|\psi\rangle \quad\text{and}\quad L_z|\psi\rangle = b|\psi\rangle.$$

Now consider the state $|\psi'\rangle$= U$|\psi\rangle$and the actions of L^2 andL_z upon it.

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\documentclass[10pt]{book}
\usepackage[english]{babel}
\usepackage[utf8]{inputenc}
\normalsize
\usepackage{listings}
\usepackage{fancyhdr}
\usepackage{amsmath}
\renewcommand{\baselinestretch}{1.14}
%\usepackage[b5paper, total={6.8in, 9.8in}]{geometry}
\usepackage[paperheight=9.8in,paperwidth=6.8in,margin=25mm]{geometry}
\title{First Exam}
\author{Monta Lokmane (REBCO2)}
\date{ 2019}
 \begin{document}

\maketitle

\pagestyle{fancy}
\fancyhf{}
\rhead{}
\chead{QUANTUM OPERATORS}
\fancyfoot[C]{~660~}

%\section{First Section}
\setlength{\parindent}{0in}
with
\begin{align*}
[L^2,  L_z]&=[L^2_x  +L^2_y + L^2_z, L_z]\\
&=L_x[L_x,L_z]+[L_x,L_z]L_x\\
 &\:\:\:\:\:\:\:\:\:\:\:\:+L_y[L_y,L_z]+[L_y,L_z]L_y+(L^2_z,L_z)\\
&=L_x(-i\hbar)L_y+(-i\hbar)L_yL_x+L_y(i\hbar)L_x+(i\hbar)L_xL_y+0\\
 &= 0
\end{align*}
Thus operators $L^2$ and $L_z$ commute and, continuing in the same way
\bigskip
\begin{equation}
[L^2,L_x]=[L^2,L_y]=[L^2,L_x]=0 \tag{$19.29$}
\end{equation}

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{\centering\textit {Eigenvalues of the angular momentum operators}\par
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To help with the calculation, it is convenient to define the two opera
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\end{align*}
These operators are not Hermitian; they are in fact Hermitian conjugate
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\begin{align*}
UD&=(L_x+iL_y)(L_x-iL_y)=L^2_x+L^2_y+i[L_y,L_x]\\\tag{$19.30$}
&=L^2  L  ^2_z+hL_z,\\
DU&=(L_x - iL_y)(L_x+iL_y)=L^2_x+L^2_y -i[L_y,L_x]\\\tag{$19.31$}
&=L^2-L^2_z-\hbar L_z,\\\tag{$19.32$}
[L_z,U]&=[L_z,L_x]+i[L_z,L_y]=i\hbar L_y+\hbar L_x=\hbar U,\\\tag{$19.
[L_z,D]&=[L_z,L_x]  i  [L_z,L_y]=i\hbar L_y-\hbar L_x=-\hbar D
\end{align*}
\medskip
In the same way as was shown for matrices, it can be demonstrated that
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\begin{align*}
L^2 |\psi\rangle =a |\psi\rangle \:\:\: \text{and}\: \:\:
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Now consider the state \textbar$\psi'$ \rangle $= U$ |$\psi$\rang
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