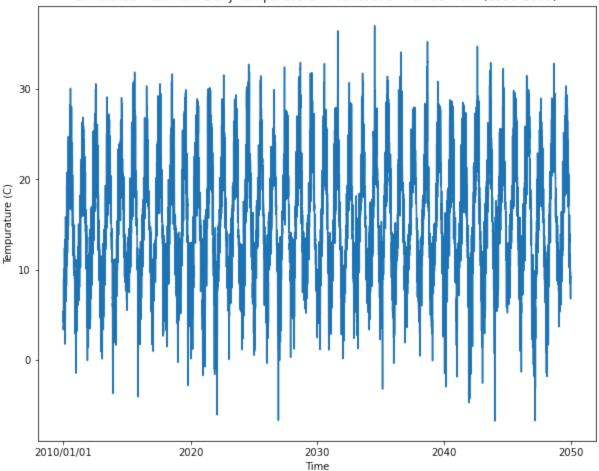
```
In [1]:
         import os
         import numpy as np
         import pandas as pd
         import matplotlib.pyplot as plt
         from scipy import stats
         from sklearn.metrics import pairwise_distances
         import math
         from scipy.stats import pearsonr, spearmanr
In [2]:
         # Read in data
         T_file = os.path.join('desktop', 'daily_T.csv')
         P_file = os.path.join('desktop', 'daily_P.csv')
         T_data = np.genfromtxt(T_file, delimiter=',', dtype=str)
         P_data = np.genfromtxt(P_file, delimiter=',', dtype=str)
         # 3 different scenarios
         temp data s1 = [float(T data[i][1])  for i in range(21917,36500)]
         prec_data_s1 = [float(P_data[i][1])  for i  in range(21917, 36500)]
         temp_data_s2 = [float(T_data[i][4])  for i in range(21917,36500)]
         prec_data_s2 = [float(P_data[i][4]) for i in range(21917,36500)]
         temp data s3 = [float(T data[i][5]) for i in range(21917,36500)]
         prec_data_s3 = [float(P_data[i][5])  for i  in range(21917, 36500)]
         t_index = [i for i in range(len(temp_data_s1))]
In [3]:
         # Plot time vs temp just to check
         plt.figure(figsize=(10, 8))
         plt.plot(t_index, temp_data_s1)
         plt.xticks([0,3658,7296,10944,14582], ['2010/01/01', '2020', '2030', '2040', '20
         plt.xlabel('Time')
         plt.ylabel('Tempurature (C)')
         plt.title('Simulated Maximum Daily Tempurature In Vancouver Trained From (1950-2
```

plt.show()

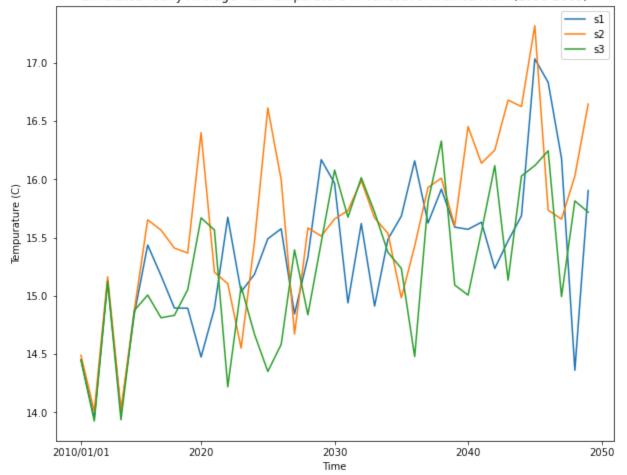
### Simulated Maximum Daily Tempurature In Vancouver Trained From (1950-2009)



```
In [4]:
         # Yearly Average
         plt.figure(figsize=(10, 8))
         # Scenario 1
         ya_T = []
         ystd_T = []
         i = 0
         while i < len(temp_data_s1)-365:</pre>
              ya_T.append(np.average([temp_data_s1[j] for j in range(i,i+365)]))
             ystd_T.append(np.std([temp_data_s1[j] for j in range(i,i+365)]))
              i += 365
         T_avg = np.average(ya_T)
         ystd_avg = np.average(ystd_T)
         ya_t_index = [k for k in range(len(ya_T))]
         plt.plot(ya_t_index, ya_T, label='s1')
         # Scenario 2
         ya T = []
         ystd_T = []
         i = 0
         while i < len(temp_data_s1)-365:</pre>
              ya_T.append(np.average([temp_data_s2[j] for j in range(i,i+365)]))
              ystd_T.append(np.std([temp_data_s2[j] for j in range(i,i+365)]))
              i += 365
         T \text{ avg} = \text{np.average(ya T)}
         ystd_avg = np.average(ystd_T)
```

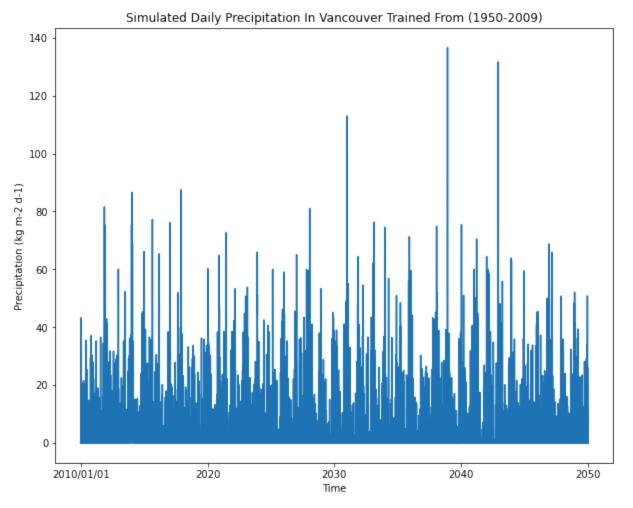
```
ya_t_index = [k for k in range(len(ya_T))]
plt.plot(ya_t_index, ya_T, label='s2')
# Scenario 3
ya_T = []
ystd_T = []
i = 0
while i < len(temp_data_s1)-365:</pre>
    ya_T.append(np.average([temp_data_s3[j] for j in range(i,i+365)]))
    ystd_T.append(np.std([temp_data_s3[j] for j in range(i,i+365)]))
    i += 365
T_avg = np.average(ya_T)
ystd_avg = np.average(ystd_T)
ya_t_index = [k for k in range(len(ya_T))]
plt.plot(ya_t_index, ya_T, label='s3')
plt.xticks([0,9,19,29,39], ['2010/01/01', '2020', '2030', '2040', '2050'])
plt.xlabel('Time')
plt.ylabel('Tempurature (C)')
plt.title('Simulated Yearly Average Max Tempurature In Vancouver Trained From (1
plt.legend()
plt.show()
```

#### Simulated Yearly Average Max Tempurature In Vancouver Trained From (1950-2009)



```
In [5]: # Plot time vs precipitation just to check
plt.figure(figsize=(10, 8))
```

```
plt.plot(t_index, prec_data_s1)
plt.xticks([0,3658,7296,10944,14582], ['2010/01/01', '2020', '2030', '2040', '20
plt.xlabel('Time')
plt.ylabel('Precipitation (kg m-2 d-1)')
plt.title('Simulated Daily Precipitation In Vancouver Trained From (1950-2009)')
plt.show()
```



```
In [6]:
         # Probability density of set X at x
         def P(X,x):
             X = np.histogram(X)
             for i in range(len(X[1])-1):
                 if X[1][i] < x and x < X[1][i+1]:
                     return X[0][i]/np.sum(X[0])
         # Calculate entropy of prob. distribution generated from sample X with histogram
         def entropy(X_hist):
             H = 0
             N = np.sum(X_hist)
             if N != 0:
                 for nx in X_hist:
                     if nx != 0:
                         H = nx*math.log(nx/N)/N
             return H
```

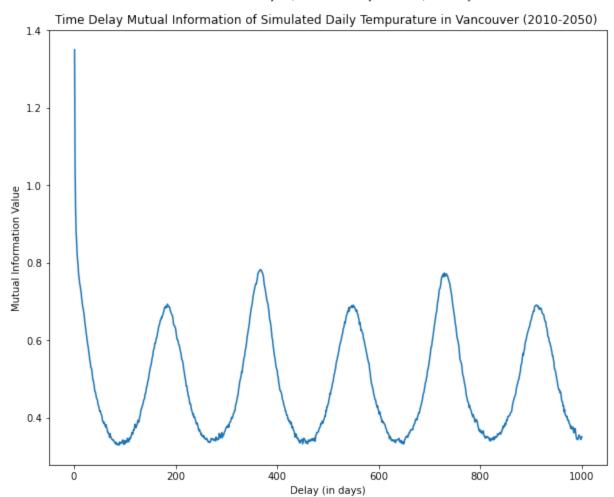
```
# Calculate entropy of joint PDF generated from samples X,Y with 2D histogram li
def joint entropy(XY hist):
   H = 0
   N = np.sum(XY_hist)
   if N != 0:
        for row_i in XY_hist:
            for n_xy in row_i:
                if n xy != 0:
                    H = n_xy*math.log(n_xy/N)/N
    return H
\# Mutual information from two samples X, Y
def mi(X, Y, bins):
   X_hist, X_edges = np.histogram(X, bins)
   Y hist, Y edges = np.histogram(Y, bins)
   XY_hist, XYx_edges, XYy_edges = np.histogram2d(X, Y, bins)
   MI = entropy(X_hist) + entropy(Y_hist) - joint_entropy(XY_hist)
    return MI
# Time delay mutual information:
def tdmi(X, delay, bins):
    return mi(X[:len(X)-delay], X[delay:], bins)
```

```
In [7]: # Temp data analysis:
# Set bin number
bins = 100

# Delay
max_delay = 1000

D = [i + 1 for i in range(max_delay)]
TDMI = [tdmi(temp_data_s1, delay, bins) for delay in D]

plt.figure(figsize=(10,8))
plt.plot(D, TDMI)
plt.xlabel('Delay (in days)')
plt.ylabel('Mutual Information Value')
plt.title('Time Delay Mutual Information of Simulated Daily Tempurature in Vancount plt.show()
```



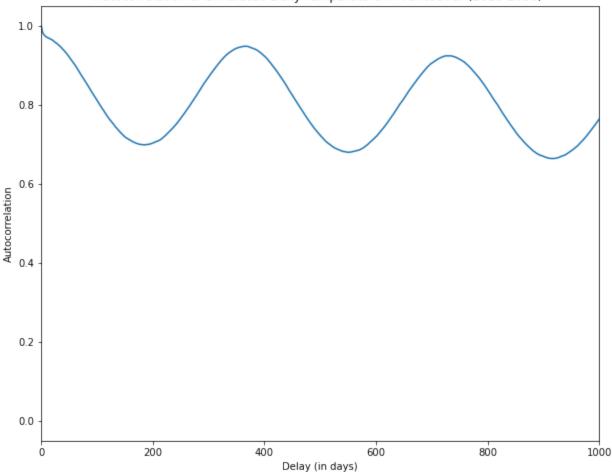
```
In [8]:
# Calculate the autocorrelation
Autocorr = np.correlate(temp_data_s1, temp_data_s1, mode='full')

# Normalization
Autocorr /= np.max(Autocorr)

# Generate the lag values for the x-axis
Lags = np.arange(-len(temp_data_s1) + 1, len(temp_data_s1))

# Plotting
plt.figure(figsize=(10,8))
plt.plot(Lags, Autocorr)
plt.xlim(0,1000)
plt.xlabel('Delay (in days)')
plt.ylabel('Autocorrelation')
plt.title('Autocorrelation of Simulated Daily Tempurature in Vancouver (2010-205 plt.show())
```

#### Autocorrelation of Simulated Daily Tempurature in Vancouver (2010-2050)



```
print('Mutual information Between Temperature And Precipitation')
print('Scenario 1: ', mi(temp_data_s1, prec_data_s1, bins))
print('Scenario 2: ', mi(temp_data_s2, prec_data_s2, bins))
print('Scenario 3: ', mi(temp_data_s3, prec_data_s3, bins))
```

Mutual information Between Temperature And Precipitation

Scenario 1: 0.1325726441609003 Scenario 2: 0.16378914722520133 Scenario 3: 0.14494353392700177

```
In [16]:
```

```
from scipy.spatial import distance
from scipy.stats import pearsonr, spearmanr
import dcor

dc1 = dcor.distance_correlation(np.array(temp_data_s1), np.array(prec_data_s1))
dc2 = dcor.distance_correlation(np.array(temp_data_s2), np.array(prec_data_s2))
dc3 = dcor.distance_correlation(np.array(temp_data_s3), np.array(prec_data_s3))

pearson_corr1, _ = pearsonr(np.array(temp_data_s1), np.array(prec_data_s1))
pearson_corr2, _ = pearsonr(np.array(temp_data_s2), np.array(prec_data_s2))
pearson_corr3, _ = pearsonr(np.array(temp_data_s3), np.array(prec_data_s3))

spearman_corr1, _ = spearmanr(np.array(temp_data_s2), np.array(prec_data_s2))
spearman_corr2, _ = spearmanr(np.array(temp_data_s2), np.array(prec_data_s2))
spearman_corr3, _ = spearmanr(np.array(temp_data_s3), np.array(prec_data_s3))
```

```
print("Distance Correlation")
print('Scenario 1: ', dc1)
print('Scenario 2: ', dc2)
print('Scenario 3: ', dc3)
print()
print("Pearson's Correlation")
print('Scenario 1: ', pearson_corr1)
print('Scenario 2: ', pearson_corr2)
print('Scenario 3: ', pearson_corr3)
print()
print("Spearman's Correlation ")
print('Scenario 1: ', spearman_corr1)
print('Scenario 2: ', spearman_corr2)
print('Scenario 3: ', spearman_corr3)
```

```
Distance Correlation
Scenario 1: 0.27261905773096046
Scenario 2: 0.30372356990866695
Scenario 3: 0.2785289916336257

Pearson's Correlation
Scenario 1: -0.22333959358269667
Scenario 2: -0.22746386197080795
Scenario 3: -0.24474907044833538

Spearman's Correlation
Scenario 1: -0.29278829783629445
Scenario 2: -0.3115243714891102
Scenario 3: -0.2984585486390431
```

Phase Space Reconstruction of Rossler System:

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from scipy.integrate import solve_ivp
```

```
In [22]:
          def rossler_system(x, y, z, a, b, c):
              dx = -y - z
              dy = x + a * y
              dz = b + z * (x - c)
              return dx, dy, dz
          a = 0.2
          b = 0.2
          c = 5.0
          dt = 0.01  # Time step
          num_steps = 10000 # Number of steps
          x = np.zeros(num_steps)
          y = np.zeros(num_steps)
          z = np.zeros(num steps)
          x[0], y[0], z[0] = [1, 2, 3]
          print('xyz_0 = ', [x[0], y[0], z[0]])
          for i in range(1, num_steps):
              dx, dy, dz = rossler_system(x[i-1], y[i-1], z[i-1], a, b, c)
```

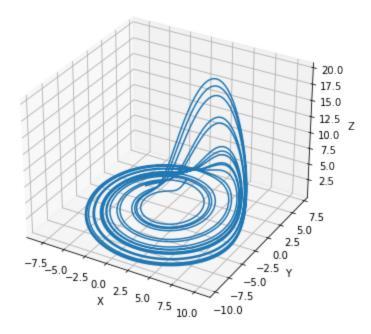
```
x[i] = x[i-1] + dx * dt
y[i] = y[i-1] + dy * dt
z[i] = z[i-1] + dz * dt

embedding_dim = 3  # Embedding dimension
time_delay = 10  # Time delay

fig = plt.figure(figsize=(8,6))
ax = fig.add_subplot(111, projection='3d')
#fig figure(figsize=(6,6))
ax.plot3D(x, y, z)
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title('Rossler System Trajectory Starting From xyz_0')
plt.show()
```

 $xyz_0 = [1.0, 2.0, 3.0]$ 

Rossler System Trajectory Starting From xyz\_0



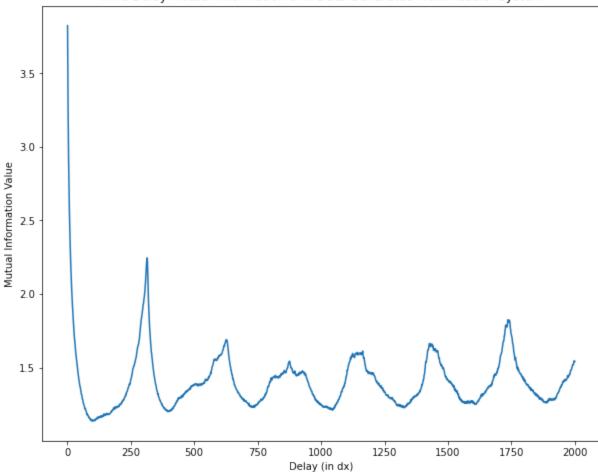
```
In [26]: # Reconstruction from x data

bins = 100
# Delay
max_delay = round(num_steps/5)

D = [i + 1 for i in range(max_delay)]
TDMI = [tdmi(x, delay, bins) for delay in D]

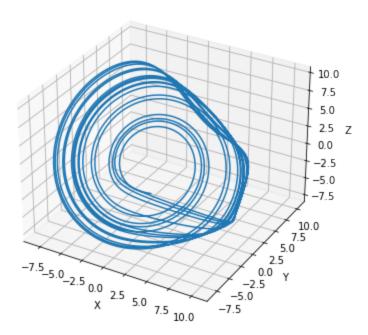
plt.figure(figsize=(10,8))
plt.plot(D, TDMI)
plt.xlabel('Delay (in dx)')
plt.ylabel('Mutual Information Value')
plt.title('Time Delay Mutual Information of X Data Generated From Rossler System plt.show()
```

#### Time Delay Mutual Information of X Data Generated From Rossler System



```
In [28]:
          # Get optimal delay with min(TDMI)
          min_delay_mi = [0,TDMI[0]]
          for i in range(1, 250):
              if TDMI[i] < min_delay_mi[1]:</pre>
                  min_delay_mi = [i,TDMI[i]]
          optimal_delay = min_delay_mi[0]
          #now set new y to x(t + optimal_delay) and z = x(t + 2*optimal_delay)
          x_{recon} = x[:len(x)-2*optimal_delay]
          y_recon = x[optimal_delay:len(x)-optimal_delay]
          z_recon = x[2*optimal_delay:]
          fig = plt.figure(figsize=(8,6))
          ax = fig.add_subplot(111, projection='3d')
          #fig figure(figsize=(6,6))
          ax.plot3D(x_recon, y_recon, z_recon)
          ax.set_xlabel('X')
          ax.set_ylabel('Y')
          ax.set_zlabel('Z')
          ax.set_title('Rossler System Trajectory Starting From xyz_0')
          plt.show()
```

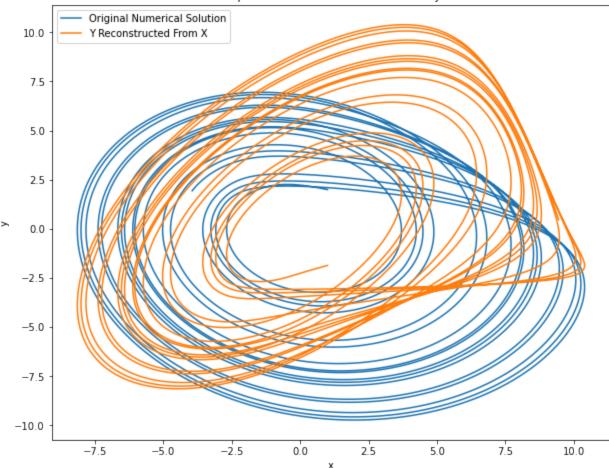
# Rossler System Trajectory Starting From xyz\_0



```
In [32]: # More plotting

plt.figure(figsize=(10,8))
plt.plot(x,y, label='Original Numerical Solution')
plt.plot(x_recon,y_recon, label='Y Reconstructed From X')
plt.xlabel('x')
plt.ylabel('y')
plt.title('Phase Space Reconstruction of Rossler System')
plt.legend()
plt.show()
```

## Phase Space Reconstruction of Rossler System



```
In [33]: # More plotting

plt.figure(figsize=(10,8))
 plt.plot(x,z, label='Original Numerical Solution')
 plt.plot(x_recon,z_recon, label='Z Reconstructed From X')
 plt.xlabel('x')
 plt.ylabel('z')
 plt.title('Phase Space Reconstruction of Rossler System')
 plt.legend()
 plt.show()
```

# Phase Space Reconstruction of Rossler System

