

# An Exploration of the Specific Heat Capacities of Gases using the Clement and Desormes' Method and Ruchardt's Method

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## Abstract

The purpose of this experiment was to determine the ratio of the specific heat capacity of a gas under constant pressure and the specific heat capacity of the same gas under constant volume. Two methods were used based on air pressure; first Clement and Desormes', which utilizes adiabatic expansion, and secondly Ruchardt's method utilizing a piston. It was found that this ratio decreased as the gas' degrees of translational freedom increased, as expected, although the data of this experiment was not in strong agreement with expected values, likely do to systematic errors related to gas leaks. This experiment carries significance when comparing the molecular structure of various gases and can lead to insights in the field of acoustic physics.

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# 1 Introduction

The objective of this experiment was to determine the ratio of the specific heat of gases at constant pressure and at constant volume in order to determine the ratio of specific heat capacities ( $\gamma$ ). This relationship is characterized by

$$\gamma = \frac{C_p}{C_v} \quad (1)$$

where  $C_p$  is the specific heat capacity at constant pressure and  $C_v$  is the specific heat capacity at constant volume. If a gas expands while heating under constant pressure, additional work is performed on the gas to cause this change in volume. For this reason,  $C_p$  is greater than  $C_v$ . This lab calculated  $\gamma$  with two unique methods. The first method used was the Clement and Desormes method which uses a slightly pressurized flask which undergoes adiabatic expansion. The gas cools following this expansion and with that, the pressure decreases until the gas reaches thermal equilibrium with the surrounding environment. The second method used was the Rüchhardt method which uses a piston oscillating in a glass tube in response to pressure changes in a glass jar. The motion of the piston over time, specifically its period of oscillation ( $T$ ) is related to  $\gamma$  by

$$\gamma = \frac{4\pi^2 m V}{A^2 P T^2} \quad (2)$$

$m$  is the mass of the piston,  $V$  is the volume of the glass jar,  $A$  is the cross-sectional area of the piston and  $P$  is the pressure of the gas in the jar. The ratio of specific heat capacities ( $\gamma$ ) has implications in determining the molecular structure of gases. The ratio obeys the equation

$$\gamma = \frac{n + 2}{n} \quad (3)$$

$n$  is the number of degrees of translational freedom that the gas particles have. A monoatomic gas has only 3 degrees of freedom due to the 3 spatial dimensions of translational motion. More complex gas molecules have more degrees of freedom due to the motion of internal rotations between atoms. By comparing the experimental values of  $\gamma$  to hypothetical values based on degrees of freedom, useful insights may be made about the gases molecular structure.

## 2 Materials

Flask, air-tight stopper, gas source (air, argon, nitrogen gas, carbon dioxide), Pasco, gas pressure sensor, computer, barometer, large glass jar, glass tube fitted with graphite piston, motion sensor, hand pump.

## 3 Methodology

### 3.1 Clement and Desormes' Method

1. The flask was first verified to not leak when pressurized with gas.
2. Local ambient pressure ( $H$ ) was recorded.
3. Next, the flask was flushed with air such that only air remained inside.
4. The flask was sealed with stopper and pressure sensor.
5. Flask was filled with air to a pressure not exceeding 120 kPa.
6. Pressure ( $P_0$ ) was recorded once the pressure stopped decreasing.
7. Stopper was opened for a second, and then closed once again such that the flask's internal pressure could drop to atmospheric pressure.
8. Pressure ( $P_1$ ) was recorded once the pressure stopped decreasing again.
9. Steps 2-8 were repeated for 10 measurements of  $P_0$  and  $P_1$  of air.
10. Step 9 was repeated substituting air for other gases. First air was used, followed by  $N_2$ , then Ar, and lastly  $CO_2$ .

### 3.2 Ruchardt's Method

1. The motion sensor was aligned with the glass tube which is inside the jar.
2. The jar was gently pressurized such that the piston raised to an equilibrium point.
3. The jar was sharply pressurized with the hand pump such that the graphite piston began to oscillate about the equilibrium position.
4. The graphite piston's motion was recorded with the motion sensor for 10 seconds.
5. The jar was depressurized.
6. Steps 2-5 were repeated 10 times.

## 4 Results

This experiment sought to measure the ratio of specific heat ( $\gamma$ ) of four different gaseous substances: Air, Argon,  $\text{CO}_2$ , and  $\text{N}_2$ . Below is a portion of the relevant data from the experiment (Gingras, 2022). The full data is displayed in *Fourth Appendix*. Once experimental values for  $H$ ,  $P_0$ , and  $P_1$  were found as described in *Methodology*,  $C_p$  and  $C_v$  were calculated and plotted such that the slope of a linear fit through the origin represents  $\gamma$  (see equation 1). These graphs are shown below on the left and the corresponding plot of residuals is shown on the right. An additional fit that took the y-intercept ( $b$ ) as a parameter was produced for each data set which showed that  $b = 0$  was a reasonable assumption. The data from these additional fits can be found in *Fourth Appendix*. The last two images (figure 9 and table 1) represent a portion of the data from Ruchardt's method, and a summary of key values derived from the data, respectively.

Figure 1: The Specific Heat of Air at Constant Pressure ( $C_p$ ) and Constant Volume ( $C_v$ ) Collected Using the Clement And Desormes Method

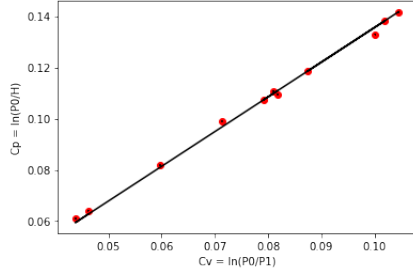


Figure 2: Plot of Residuals From The Graph of  $C_p$  over  $C_v$  of Air (Figure 1)

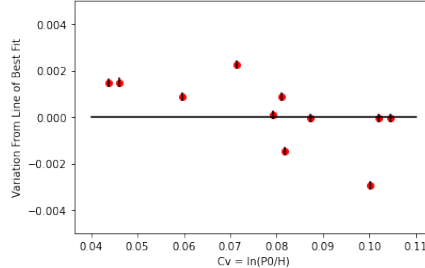


Figure 3: The Specific Heat of Argon at Constant Pressure ( $C_p$ ) and Constant Volume ( $C_v$ ) Collected Using the Clement And Desormes Method

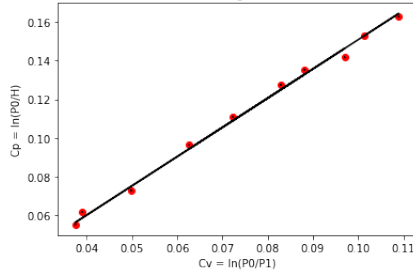


Figure 4: Plot of Residuals From The Graph of  $C_p$  over  $C_v$  of Argon (Figure 3)

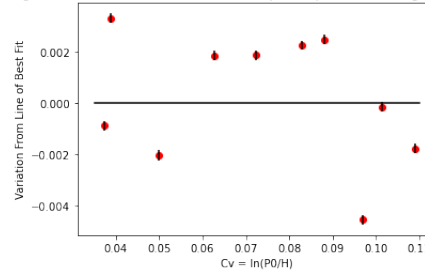


Figure 5: The Specific Heat of  $\text{CO}_2$  at Constant Pressure ( $C_p$ ) and Constant Volume ( $C_v$ ) Collected Using the Clement And Desormes Method

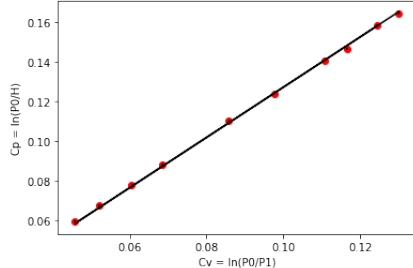


Figure 6: Plot of Residuals From The Graph of  $C_p$  over  $C_v$  of  $\text{CO}_2$  (Figure 5)

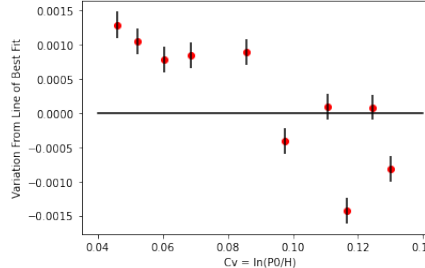


Figure 7: The Specific Heat of N2 at Constant Pressure ( $C_p$ ) and Constant Volume ( $C_v$ ) Collected Using the Clement And Desormes Method

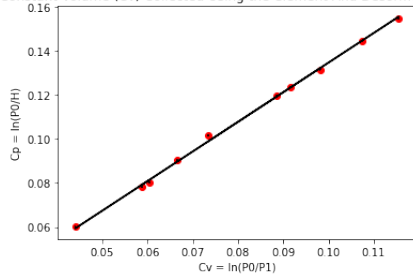


Figure 8: Plot of Residuals From The Graph of Cp Over Cv of N2 (Figure 7)

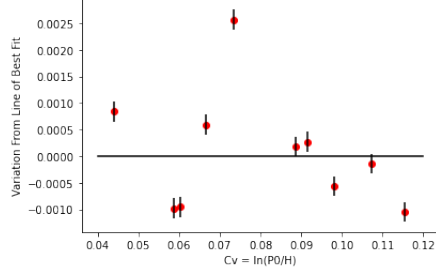


Figure 9: The Position of Graphite Piston After an Initial Displacement In Trials 1 and 2 (Ruchardt's Method)

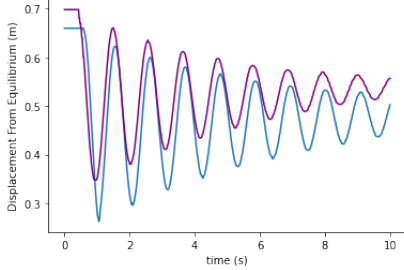


Table 1: Summary of key findings

Method 1:	Value	$\sigma$
$\gamma_{Air}$	1.356	0.006
$\gamma_{Arg}$	1.51	0.01
$\gamma_{CO_2}$	1.273	0.003
$\gamma_{N_2}$	1.348	0.004
Method 2:	Value	$\sigma$
$T$	1.0783	0.0006
$\gamma_{Air}$	1.344	0.004

## 5 Discussion

The results of this experiment found in table 1 demonstrates that argon gas has the greatest ratio of specific heat capacities ( $1.51 \pm 0.01$ ), followed by air ( $1.356 \pm 0.006$ ), nitrogen gas ( $1.348 \pm 0.004$ ), and lastly carbon dioxide ( $1.273 \pm 0.003$ ). The relative magnitudes of these values follows what is expected based on what is generally known about their molecular structure more complex molecules have lower values. However, in order to make proper assertions about the structure of the molecules, the experimental values must be compared to the theoretical  $\gamma$  values shown below in table 2. These assertions can be made because of the relationship shown in equation 3. For all data points, the experimental values of  $\gamma$  via the Clement and Desormes Method were found to be lower than was expected by the molecular structure of the gases. For example, Argon's value of  $1.51 \pm 0.01$  makes it in closer agreement to a diatomic molecular gas (1.40) than a monoatomic molecular gas (1.67), which we know it to be. Likewise,  $\gamma$  of nitrogen gas ( $1.348 \pm 0.004$ ) appears closer to the theoretical value of a polyatomic gas (1.33) than to the value for a diatomic gas (1.40) which was expected given that  $N_2$  is a diatomic molecule. The investigation of Air is a little more subtle. Because Air is made up of many different molecules it does not have a precise expectation value given by equation 3 (when  $n$  is an integer). However, given that more than 99% of molecules in Air are diatomic,  $N_2$  and  $O_2$  (Helmenstine, 2019), the assumption that Air will present as

relatively diatomic can be made. This assumption provides further evidence for a systematic bias because the experimental value ( $1.356 \pm 0.006$ ) is lower than the expected value of  $\gamma_{Air}$  (1.40).

A possible source of error that could decrease that could account for the observed systematic bias is the existence of gas leaks. An imperfect air-tight seal on the flask would let out gas causing  $P_1$  to be measured slightly lower. This would lower the experimental value for  $\gamma$  through the relationship described by equation 1. In fact, the seal of the flask was tested by increasing the pressure (from ambient) by about 19 kPa. From this data ("Leak Check Data" in Fourth Appendix) it appears that the pressure dropped due to leakage about 1 kPa every 3 seconds, however, this is not rigorously analyzed and further investigation is required to determine if this leakage could cause the systematic bias that is observed in the  $\gamma$  values.

The experimental value for  $\gamma_{Air}$  that was found using Richardt's Method ( $1.344 \pm 0.004$ ) is in reasonable agreement with the value that was found using the first method ( $1.356 \pm 0.006$ ). However, this value does furthers the trend of a systematic bias considering it is much lower than is the expected value for Air (1.40).

Table 2: Molecular Structure's effect on Specific Heat Capacity Ratio

Molecular Structure	$n$	Theoretical $\gamma$
Monoatomic	3	1.67
Diatomic	5	1.40
Polyatomic	6	1.33

## 6 Conclusions

The Clement and Desormes' method for measuring the ratio of specific heat capacities of gases was not in reasonable agreement with the expected values based on what is known about their molecular structure and their degrees of translational freedom. The method still proved useful for ranking the gases based on ratio of specific heat capacities, which does give some insight into the relative complexity of their molecular structure. Ruchardt's method, gave a  $\gamma$  value for air that is in reasonable agreement with Clement and Desormes' method. Both methods are in agreement with eachother, although in disagreement with expected values, likely due to a systematic error, namely gas leaks which decreased the measured value of  $\gamma$ .

## References

1. Gingras, C. (2022, January). Lab 1: Ratio of Specific Heats. Retrieved January 14, 2022, from MyCourses: <https://mycourses2.mcgill.ca/d2l/le/lessons/557407/topics/6221961>
2. Anne Marie Helmenstine, P. (2019, July 7). The Chemical Composition of Air. Retrieved January 19, 2022, from ThoughtCo.: <https://www.thoughtco.com/chemical-composition-of-air-604288>

## A First Appendix: Author Contribution Statements

Monte Mahlum

M.M contributed by taking raw data, analyzing this data via Data Analysis prompts, answering lab questions, writing the Lab Report, and completing the assignment.

Evan Henderson

E.H contributed by taking raw data, analyzing this data via Data Analysis prompts, answering lab questions, writing the Lab Report, and completing the assignment.

## B Second Appendix: Uncertainties

All measurements were taken using digital measurement tools. Because of this, the estimated error in a measurement was taken to be the smallest increment that was provided by the measurement device. To obtain the uncertainty in specific heat (C) this measurement error was then propagated through the function

$$C = \ln\left(\frac{x}{y}\right)$$

in the following manner:

$$\sigma_C^2 = \left( \left| \frac{\partial C}{\partial x} \right|_{[x_0, y_0]} \cdot \sigma_x \right)^2 + \left( \left| \frac{\partial C}{\partial y} \right|_{[x_0, y_0]} \cdot \sigma_y \right)^2$$

$$\sigma^2 \left[ \ln\left(\frac{x}{y}\right) \right]_{[x_0, y_0]} = \left( \frac{1}{x_0} \cdot \sigma_x \right)^2 + \left( \frac{-1}{y_0} \cdot \sigma_y \right)^2$$

It is important note that despite efforts to obtain accurate uncertainties, the plots of the residuals shown in *Results* seem to suggest that the uncertainties used for specific heat are too small. A plot of residuals with accurate uncertainties should show about  $\frac{2}{3}$  of data points being within  $1 \sigma$  of the line of best fit (x-axis). This is clearly not the case in any of the plots. However, the effect of this discrepancy on the derived values for  $\gamma$  is unknown.



## C Third Appendix: Lab Book

**Selectum** | 50 Sheets Feuilles | Made in Indonesia/Fabriqu  en Indon sie | www.linkproducts.ca | \$2.00

## Lab Book - Lab 1 "Finding $\gamma$ "

$\gamma$  - ratio of specific heat capacity of a gas

### Part 1 - Clement and Desormes method

- will investigate air, argon,  $N_2$ ,  $CO_2$
- adiabatic expansion  $\rightarrow$  isothermal pressure  $P_i$

$$\gamma = \frac{C_p}{C_v} = \frac{\ln\left(\frac{P_0}{P_i}\right)}{\ln\left(\frac{P_0}{P_i}\right)} = \frac{n+2}{n}$$

Monatomic $n=3$	if $\gamma = \frac{n+2}{2}$ , we should expect
diatomic $n=5$	argon to have the greatest $\gamma$
polyatomic $n=6$	$\therefore$ it is monatomic
	$CO_2$ should have the lowest $\gamma$ since it is polyatomic

- based on leak check data, the flask does appear to leak slightly. This will affect measurements of  $P_i$  and hence  $\gamma$ .

### Procedure - Part 1

- record ambient pressure
- fill flask w first gas
- increase internal pressure (to not exceed 120 kPa) this is  $P_0$  after the pressure levels out
- open stopper then close after 1 second, once pressure levels out record  $P_i$ . repeat 10 times.

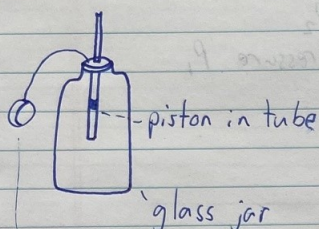


Repeat previous procedure for all gases. Beginning with the lightest gases and moving to heavier gases will facilitate properly flushing the flask. Order should be

air  $\rightarrow$   $N_2$   $\rightarrow$  Ar  $\rightarrow$   $CO_2$

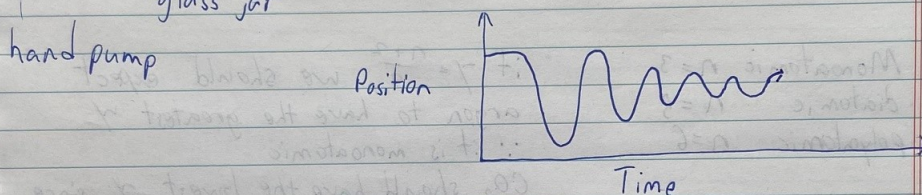
### Part 2 - Ruchardt's

□ - motion sensor



Goal - measure period ( $T$ ) of oscillations of piston in response to pressure change delivered by hand pump

Motion sensor + glass tube must be aligned



$$\text{Period} = \frac{\text{Last peak time} - \text{First Peak time}}{\# \text{ of peaks}}$$

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Lab1 Data Analysis

```
In [84]: import numpy as np
import matplotlib.pyplot as plt
import os
import pylab
from scipy.optimize import curve_fit
import csv
```

## 1. METHOD OF CLEMENT AND DESORMES

### 1.1 Air Data Analysis

```
In [186... #reading csv into an array titled "air_data"
air_data = []
with open("Air.csv") as air_raw:
    air_read = csv.reader(air_raw)
    for row in air_read:
        air_data.append(row)

#reading ln(P0/H) and ln(P0/P1) from air_data into lists titled air_y and air_x
air_y = []
air_x = []
for r in range(len(air_data)):
    if r > 1:
        air_y.append(float(air_data[r][6]))
        air_x.append(float(air_data[r][7]))

air_yerr = []
air_xerr = []
for r in range(len(air_data)):
    if r > 1:
        air_yerr.append(0.01*(1/float(air_data[r][2]))+0.01*(1/float(air_data[r]
        air_xerr.append((0.01*(1/float(air_data[r][2]))+0.01*(1/float(air_data[r]

#Generating figure
plt.figure()
plt.title("Figure 1: The Specific Heat of Air at Constant Pressure (Cp) and \n C
plt.xlabel("Cv = ln(P0/P1)")
plt.ylabel("Cp = ln(P0/H)")
plt.scatter(air_x,air_y, color="red")
plt.errorbar(air_x,air_y, xerr=air_xerr,yerr=air_yerr, fmt="none", color="black")

#Generating line of best fit with origin at 0
def func1(x,a):
    return a*x
a, covs1 = curve_fit(func1, air_x, air_y,)
print("a = ", a)
print("uncertainty on a = ", np.sqrt(covs1))
y1 = func1(air_x,a)
plt.plot(air_x,y1, color="black")

a = [1.35603593]
uncertainty on a = [[0.00554427]]
Out[186... [<matplotlib.lines.Line2D at 0x7faa3deb3040>]
```

localhost:8888/nbconvert/html/Lab1 Data Analysis.ipynb?download=false

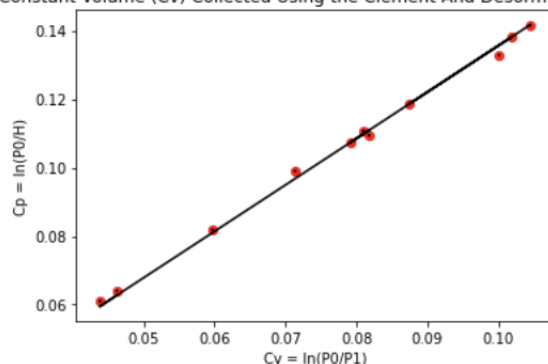
1/10

```

a = [1.35603593]
uncertainty on a = [[0.00554427]]
Out[186... [<matplotlib.lines.Line2D at 0x7faa3deb3040>]

```

Figure 1: The Specific Heat of Air at Constant Pressure ( $C_p$ ) and Constant Volume ( $C_v$ ) Collected Using the Clement And Desormes Method



```

In [189...
#Plotting residuals
plt.scatter(air_x,air_y-y1, color="red")
plt.errorbar(air_x, air_y-y1, yerr=air_yerr, fmt="none", color="black")
plt.ylim([-0.005, 0.005])
plt.title("Figure 2: Plot of Residuals From The Graph of Cp over Cv of Air (Figure 1)")
plt.xlabel("Cv = ln(P0/H)")
plt.ylabel("Variation From Line of Best Fit")
plt.plot([0.04, 0.11], [0,0], color="black")

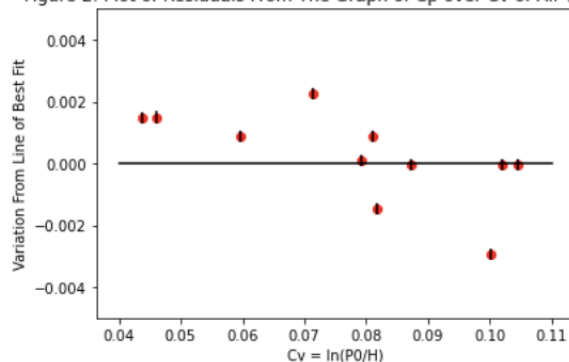
```

```

Out[189... [<matplotlib.lines.Line2D at 0x7faa3e127130>]

```

Figure 2: Plot of Residuals From The Graph of  $C_p$  over  $C_v$  of Air (Figure 1)



```

In [165...
#best fit where y-intercept = b
def func2(x,a,b):
    return a*x + b
para, covs2 = curve_fit(func2, air_x, air_y,)
para, covs2

print("a = ", para[0])
print("uncertainty on a = ", np.sqrt(covs2[0][0]))
print()
print("b = ", para[1])
print("uncertainty on b = ", np.sqrt(covs2[1][1]))

```

```

a = 1.3111357218438795
uncertainty on a = 0.017575508137920182

b = 0.003729959527432421
uncertainty on b = 0.001413788162775037

```

Table 1: Linear Fit With y-intercept As Parameter

Gas	$\gamma$	$\sigma$	y-intercept	$\sigma$
Air	1.31	0.02	0.004	0.001
Argon	1.49	0.03	0.002	0.003
CO <sub>2</sub>	1.250	0.006	0.002	0.0005
N <sub>2</sub>	1.34	0.01	0.001	0.001

## D Fourth Appendix: All Data

*folder/CD<sub>Air</sub>.csvAirDataCDMethod*

*folder/air.csvAirDataRMethod*

*folder/Ar.csvArgonDataCDMethod*

*folder/CO2.csvCO2DataCDMethod*

*folder/N2.csvN2DataCDMethod*

*folder/CD<sub>LeakCheck</sub>.csvLeakCheckData*