McGill Physical Journal

A Steamy Exploration of the Latent Heat of Vaporization of Water And Its Triple Point Pressure Through Various Data Fitting Models

Evan Henderson, Monte Mahlum

McGill University Department of Physics

January 26$^{\text{th}}$, 2022

Abstract

The purpose of this experiment was to determine the latent heat of vaporization of water along with its triple point pressure. A liquid boils when the saturated vapor pressure above the liquid is greater than the external pressure. As external pressure increases, it takes more energy for the saturated vapor pressure to overcome this external pressure, thus, it boils at a greater temperature. From this relation, a value for latent heat can be determined. Working under the assumption that latent heat is independent of temperature, this experiment has measured the Latent Heat of Vaporization of Water to be 40780 ± 50 J/mol using a linear least-squares fit and 40900 ± 50 J/mol using a non-linear least-squares fit. Additionally these models were extrapolated to find the triple point pressure of water, calculated to be 0.65 ± 0.02 using the linear fit, and 0.6 ± 0.6 cm Hg using the non-linear fit. Data analysis shows an underestimation of uncertainties, which can explain discrepancies between derived triple point pressure and the accepted value.

1 Introduction

When water is heated, energy is required not only to increase the temperature but also to facilitate the phase change from liquid to gaseous vapour. The purpose of this experiment was to determine the energy required to initiate that phase change of water from liquid to gas at constant temperature, this is also called the latent heat of vaporization of water (L). This quantity can be determined by measuring the boiling point of water under variable pressure.

Liquid water can only exist in equilibrium with its vapour phase at specific pressures and temperatures as described by the phase diagram shown below. All lines in this diagram correspond to pressure and temperature combinations that produce a state in which two phases can exist in equilibrium. The point where these lines intersect is known as a triple point in which three phases can coexist. As seen in the diagram, lower pressures correspond to lower boiling points and vice-versa.

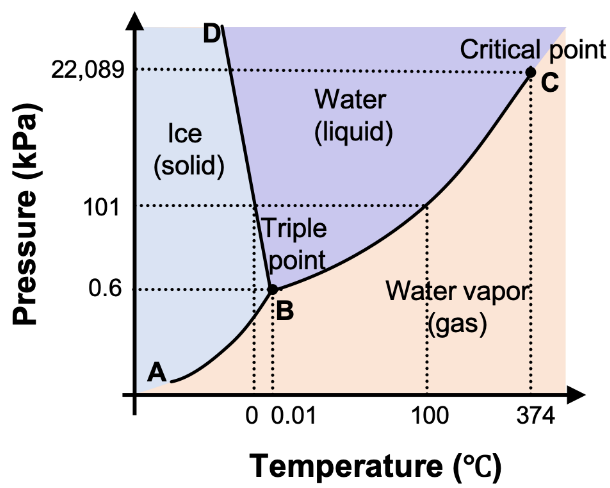


Figure 1: Phase diagram for water. Taken with permission. (Univesity of Wisconsin. 2016)

Part of the line connecting points B and C can be described more quantitatively through the following equation:

$$P = P_0 \cdot e^{\frac{-L}{RT}}, \tag{1}$$

where P is the boiling pressure at temperature T, P_0 is the pressure at which water can no longer boil, L is the latent heat of vaporization, and R is the universal gas constant (8.314

$J * mol^{-1} * K^{-1}$). This equation can also be presented in the following linear form:

$$\ln(P) = -\left(\frac{L}{R}\right)\left(\frac{1}{T}\right) + \ln(P_0). \tag{2}$$

Once the latent heat of vaporization of water (L) is experimentally determined, the triple point pressure can be estimated using its known temperature value of 273.16 K (Universiti Tunku Abdul Rahman).

2 Materials

Water-flask equipped with a direct-reading thermometer, water cooled condenser, ballast tube, mercury manometer, air valve, thermocouple, temperature to voltage converter, Pasco, computer, barometer, water, heater, vacuum,

3 Methodology

1. First the ambient pressure was recorded using the barometer.
2. Lab set up was configured as shown in Figure 1.
3. Water-cooled condenser was turned on.
4. The ballast tube was maximally depressurized using a vacuum.
5. Heater was turned on, and temperature was monitored with Pasco temperature sensor.
6. After a few minutes, the water in flask began to boil as a temperature equilibrium was reached. When the temperature leveled out, the temperature and the column heights on the manometer were recorded.
7. Next, the air valve was opened for a moment and the ballast tube was allowed to increase in pressure minutely. The height of the left manometer was lowered 1-2cm.
8. Steps 6-7 were repeated 14 times, until the internal pressure reached atmospheric pressure and the the two columns of the manometer were even.

4 Results

In this experiment, two distinct values for the latent heat of vaporization of water (L) was found with two different least squares fits. The first method uses equation 2 to obtain a slope corresponding to $\frac{-L}{R}$ (*Figure 2*). The second uses equation 1 to find a similar value for $\frac{-L}{R}$ through an exponential fit (*Figure 3*). These two values were found to be 40780 ± 50 J/mol and 40900 ± 50 J/mol respectively. The triple point pressure was then calculated by setting T in both equations to 273.15 K, the known triple point temperature (Universiti Tunku Abdul Rahman). These values were found to be 0.65 $\pm$ 0.02 using the linear fit, and 0.6 $\pm$ 0.6 cm Hg using the non-linear fit. Lastly, note that error bars are present in all figures only they are too small to be visualized. Further investigation into errors can be found in *Discussion* and the error propagation process that was used is outlined in *Second Appendix*.

Figure 2: Linear Regression of Data Collected by Measuring Temperature Equilibrium of Boiling Water In Flask At Variable Pressures

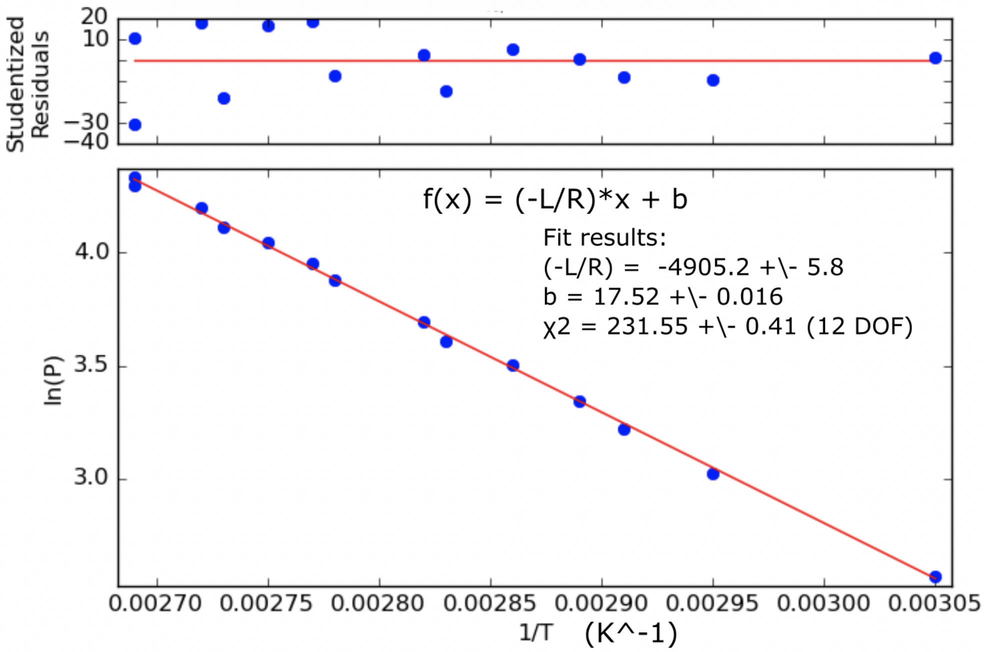


Figure 3: Non-linear Fit of Data Collected by Measuring Temperature Equilibrium of Boiling Water In Flask At Variable Pressures

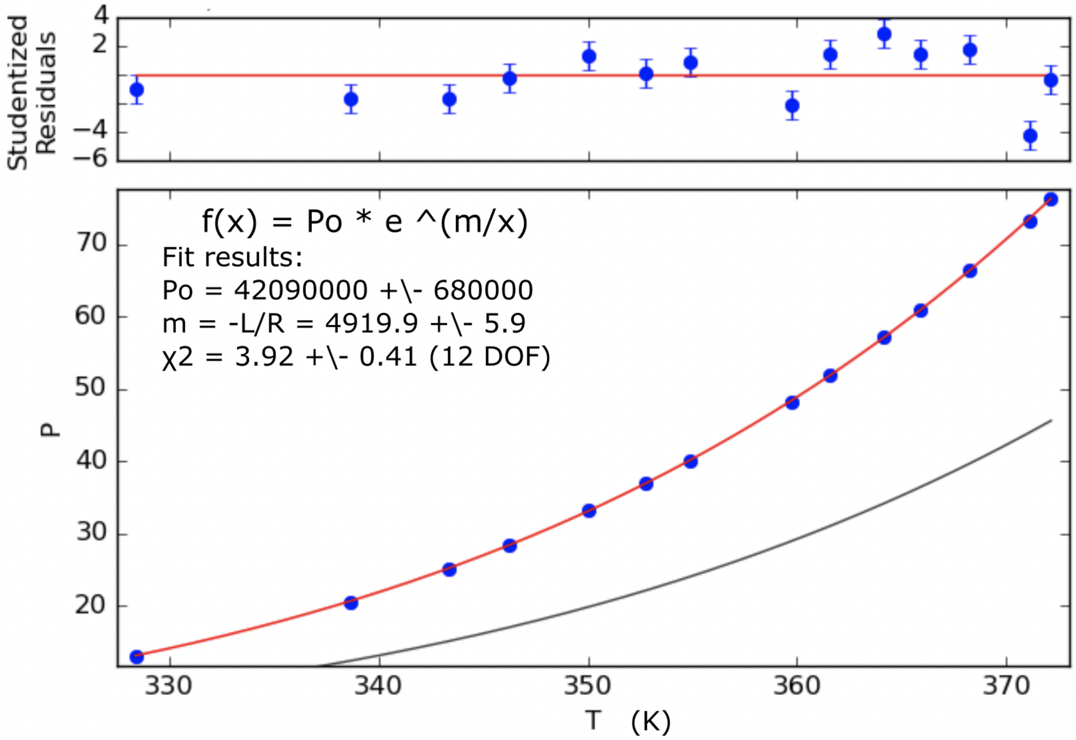


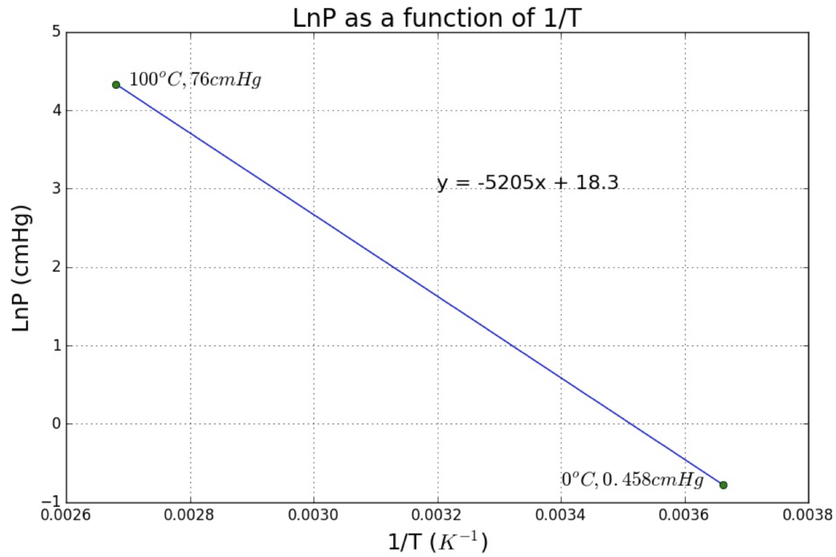
Table 1: Latent Heat of Vaporization of Water (L) Obtained Through Various Methods

Method	L (J/mol)	σ_L ($\pm$J/mol)
Linear Fit	40780	50
Non-linear Fit	40900	50
Accepted Value (Datt P. 2011)	40800	

Table 2: Triple Point Pressure P_{tp} Obtained Through Various Methods

Method	P_{tp} (cm Hg)	σ_{Ptp} ($\pm$ cm Hg)
Linear Fit	0.65	0.02
Non-linear Fit	0.6	0.6
Accepted Value (Conner N. 2019)	0.45878	

Figure 4: Linear Interpolation of Data Fond in Appendix of Lab Manual (Mcgill University. Winter 2022).



5 Discussion

This experiment has measured the latent heat of vaporization of water (L) to be 40780 ± 50 J/mol using a linear least-squares fit and 40900 ± 50 J/mol using a non-linear least-squares fit. Both values are in reasonable agreement with the accepted value of 40800 J/mol as seen in table 1. The triple point pressure of water was then calculated to be 0.65 $\pm$ 0.02 using the linear fit, and 0.6 $\pm$ 0.6 cm Hg using the non-linear fit. The value derived from the non-linear fit is in strong agreement with the accepted value (0.45878 cm Hg) while the value from the linear fit is not in agreement likely due to underestimated uncertainties.

The linear fit obtained in this experiment differs slightly from the linear interpolation found in the lab manual (Mcgill University. Winter 2022). The interpolation is defined by the line of best fit $y = -5205x + 18.3$ while the the line of best fit found during this experiment is $y = (4900 \pm 50)x + (17.52 \pm 0.02)$. The slope and y-intercept of the linear regression fit performed in this experiment is not in agreement with the interpolated data.

This experiment successfully has measured the latent heat of vaporization of water to within a maximum of 2 σ of its known value(40800 J/mol). Additionally, the non-linear model was useful in calculating the triple point pressure of water. The linear model also approached the known triple point pressure but with a much smaller uncertainty, leading to disagreement with that known value.

One explanation for why the linear model fails to accurately predict is the triple point pressure is that the latent heat of vaporization may be temperature-dependant. The latent heat of vaporization in this experiment was only calculated from data collected at a elevated range of temperatures (325 K to 375 K). An extrapolation from this temperature range to cooler temperatures may cause deviations from true values. Data should be collected at a wider range of temperatures in future experiments to describe these deviations.

Another reason for experimental disagreement could be an underestimation of uncertainties which were determined using common practices. The manometer is an analog measuring device with smallest divisions of 0.1 cm. Therefore the uncertainty on measurements made with the manometer is ± 0.05 cm. This results in an uncertainty on pressure of ± 0.07 cm of Hg. The repeatability of the Pasco temperature sensor is ± 0.1 K which was taken as the uncertainty on temperature. While not much can be said about the error in temperature, the error in pressure was clearly taken to be too small considering that far fewer than $\frac{2}{3}$ of the data points are in agreement with the least-squares fit (see residuals in Figures 1, and 2).

6 Conclusions

It has been seen that the boiling point of water can occur at a range of temperatures depending on the pressure. By observing this process, the value latent heat of vaporization of water was found using two different analysis methods in this experiment. 40780 ± 50 J/mol by the linear fit and 40900 ± 50 J/mol by the non-linear fit. The triple point pressure was then calculated to be 0.6 ± 0.06 cm of Hg (by the linear fit), and 0.65 ± 0.02 cm of Hg (by the non-linear fit). The values found for latent heat of vaporization align well with theoretical values, but the calculated triple point pressure from the linear fit shows disagreement with the accepted value (0.45878 cm Hg). This is possibly due to the assumption that latent heat is independent of temperature. More data taken at a wider range of temperatures is required to justify this assumption.

References

1. Univesity of Wisconsin. (2016). UW-MADISON CHEMISTRY 103/104 RESOURCE BOOK. Retrieved January 25, 2022, from University of Wisconsin Pressbooks: https://wisc.pb.unizin.org/minimisgenchem/chapter/features-of-phase-diagrams-m11q1/

2. Universiti Tunku Abdul Rahman. Chapter 07 Thermodynamics. Retrieved January 25, 2022, from Physics: http://staff.utar.edu.my/limsk/Physics/07%20Thermodynamics.pdf

3. Datt P. (2011) Latent Heat of Vaporization/Condensation. In: Singh V.P., Singh P., Haritashya U.K. (eds) Encyclopedia of Snow, Ice and Glaciers. Encyclopedia of Earth Sciences Series. Springer, Dordrecht. https://doi.org/10.1007/978-90-481-2642-2_327

4. Conner, N. (2019, May 22). What is Triple Point of Water – Definition. Retrieved January 25, 2022, from Thermal Engineering Org: https://www.thermal-engineering.org/what-is-triple-point-of-water-definition/

5. Mcgill University. Winter 2022. PHYS-258-001 - Experimental Methods 2. Retrieved January 24, 2022, from MyCourses: https://mycourses2.mcgill.ca/d2l/le/lessons/557407/topics/6163533

A First Appendix: Author Contribution Statements

Monte Mahlum

M.M contributed by taking raw data, analyzing this data via Data Analysis prompts, answering lab questions, writing the Lab Report, and completing the assignment.

Evan Henderson

E.H contributed by taking raw data, analyzing this data via Data Analysis prompts, answering lab questions, writing the Lab Report, and completing the assignment.

B Second Appendix: Error Propagation

Error on triple point pressure (P_{tp}) propagated from linear fit where $m = \frac{-L}{R}$ and b are fit parameters:

$$\ln(P_{tp}) = m \cdot \left(\frac{1}{T_{tp}}\right) + b,$$

$$P_{tp} = e^{m \cdot \left(\frac{1}{T_{tp}}\right)} \cdot e^{b},$$

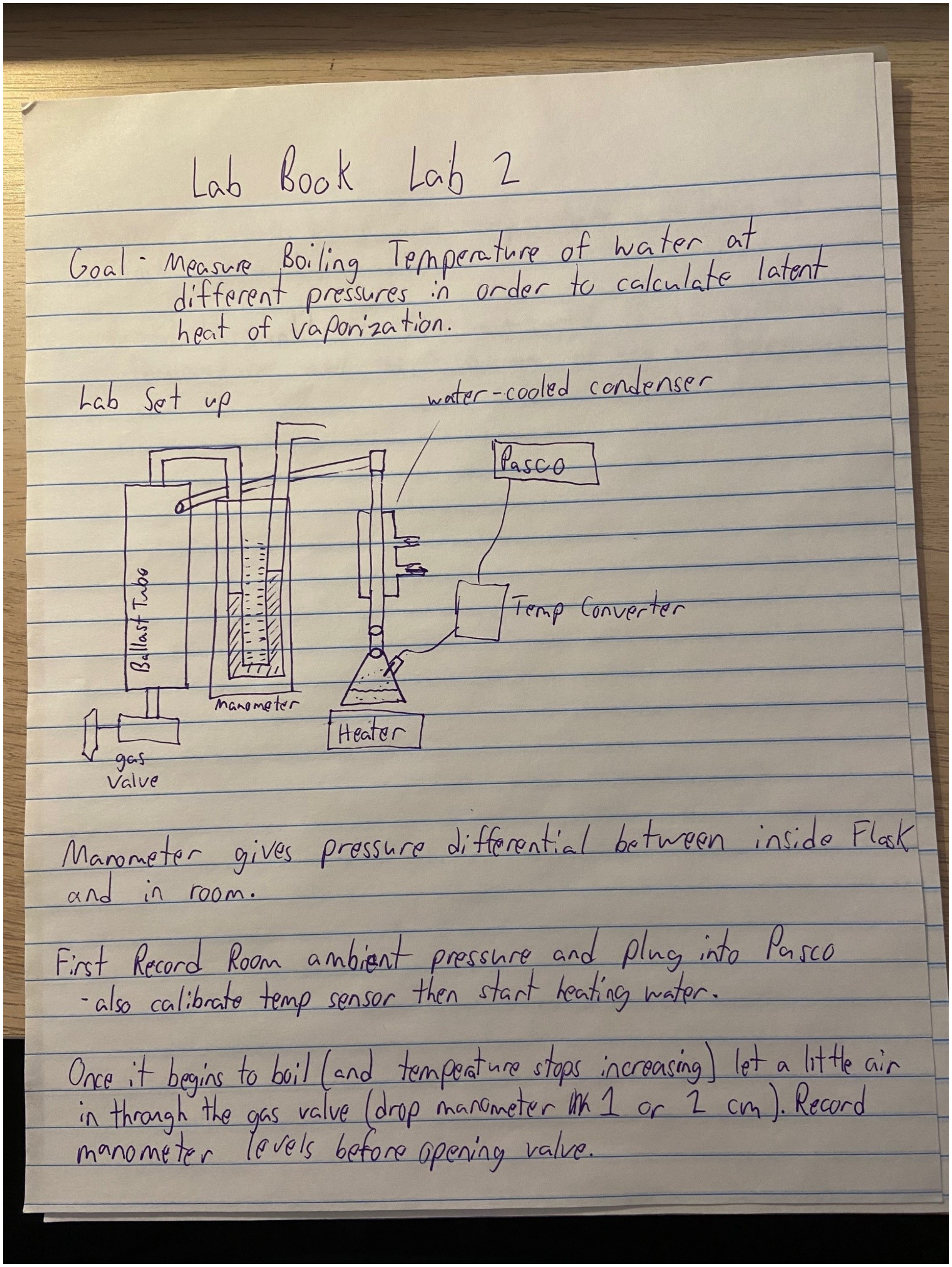
$$\sigma_{P_{tp}}^2 = \left(\frac{1}{T_{tp}} \cdot e^{m \cdot \left(\frac{1}{T_{tp}}\right)} \cdot e^{b} \cdot \sigma_m\right)^2 + \left(e^{m \cdot \left(\frac{1}{T_{tp}}\right)} \cdot e^{b} \cdot \sigma_b\right)^2.$$

Error on triple point pressure (P_{tp}) propagated from nonlinear fit where $m = \frac{-L}{R}$ and P_0 are fit parameters:

$$P_{tp} = e^{m \cdot \left(\frac{1}{T_{tp}}\right)} \cdot P_0,$$

$$\sigma_{P_{tp}}^2 = \left(\frac{1}{T_{tp}} \cdot e^{m \cdot \left(\frac{1}{T_{tp}}\right)} \cdot P_0 \cdot \sigma_m\right)^2 + \left(e^{m \cdot \left(\frac{1}{T_{tp}}\right)} \cdot \sigma_b\right)^2$$

C Third Appendix: Lab Book



Lab Book Lab 2

Goal - Measure Boiling Temperature of water at different pressures in order to calculate latent heat of vaporization.

Lab Set up

Manometer gives pressure differential between inside Flask and in room.

First Record Room ambient pressure and plug into Pasco - also calibrate temp sensor then start heating water.

Once it begins to boil (and temperature stops increasing) let a little air in through the gas valve (drop manometer ~~lik~~ 1 or 2 cm). Record manometer levels before opening valve.

Water will stop boiling, and temperature will begin to Climb again.

Repeat this until you have measured 15 ish equilibrium temperatures and the 2 column's of Hg are level.