# Formal Structures in Systems Architecture

## Towards Ontology-Informed Mathematical Foundations

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Section 1

# Introduction

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## Motivation

### Definition

A *system architecture* is a conceptual model defining the structure, behavior, and views of a given system[1]. It is the language in which requirements and specifications are declared and verified.

Hannu Jaakkola and Bernhard Thalheim. (2011) "Architecture-driven modelling methodologies." In: Proceedings of the 2011 conference on Information Modelling and Knowledge Bases XXII. Anneli Heimbürger et al. (eds). IOS Press. p. 98

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### Motivation

- Complex systems (or systems-of-systems) are ubiquitous, and undergoing dramatic transformation.
- Evolution of complex systems is slow, difficult, and often ad hoc.
- An abundance of data and mathematical tools can enable novel architectural reasoning for societal good.

# Introduction

## Programmatic Perspective

### Methodology

1. Develop a rich ontology for describing systems.
2. Define foundational mathematical structures grounded in this ontology.
3. Develop mathematical tools to ...

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## Programmatic Overview

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### Envisioned Utility

- validate system requirements from primitive data,
- detect, avoid, and analyze anomalous or adverse behavior,
- predict novel capabilities,
- rapidly evolve complex systems in a *safe*, *ethical*, and *interpretable* manner.

# Brief Literature Review

**Previous Work** – Joint between NASA LaRC and JHU APL.


1 M.M., Samantha Jarvis, Nelson Niu, Angeline Aguinaldo, Amanda Hicks, and Ian Levitt. *Formal Structures in Systems Ontology towards Air Traffic Management Architectures*. NASA Technical Memorandum, 2025. Report no. NASA/TM-20250010771.


**Sample of Related Work**


2 Patrick Schultz and David I. Spivak. *Temporal Type Theory: A Topos-Theoretic Approach to Systems and Behavior*. Birkhäuser, 2019. DOI 10.1007/978-3-030-00704-1.

3 Patrick Schultz, David I. Spivak, and Christina Vasilakopoulou. *Dynamical Systems and Sheaves*. arXiv preprint, 2016. arXiv:1609.08086. DOI 10.48550/arXiv.1609.08086.

4 Gioele Zardini, David I. Spivak, Andrea Censi, and Emilio Frazzoli. *A Compositional Sheaf-Theoretic Framework for Event-Based Systems*. arXiv preprint, 2021. arXiv:2101.10485. DOI 10.48550/arXiv.2101.10485.

5 Sophie Libkind and David Jaz Myers. *Towards a double operadic theory of systems*. arXiv preprint, 2025. arXiv:2505.18329. DOI 10.48550/arXiv.2505.18329.

# Section 2

# Ontological Foundations

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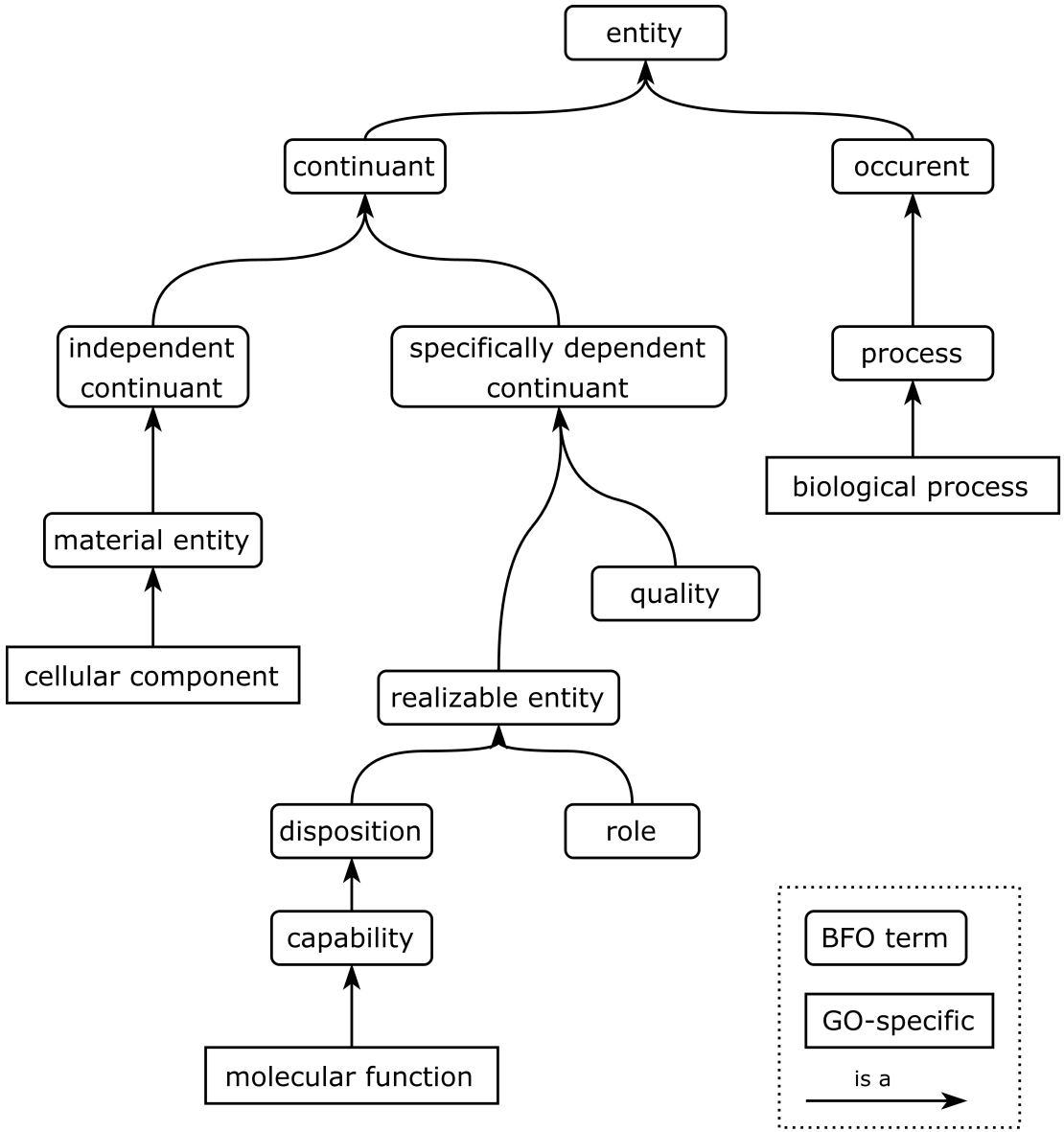
## The Basic Formal Ontology (BFO)

Figure: A sample of terms from BFO. The vocabulary is further connected by relations not shown here, e.g., every process has participating independent continuants.



# Ontological Foundations

## The Gene Ontology (GO) as Inspiration

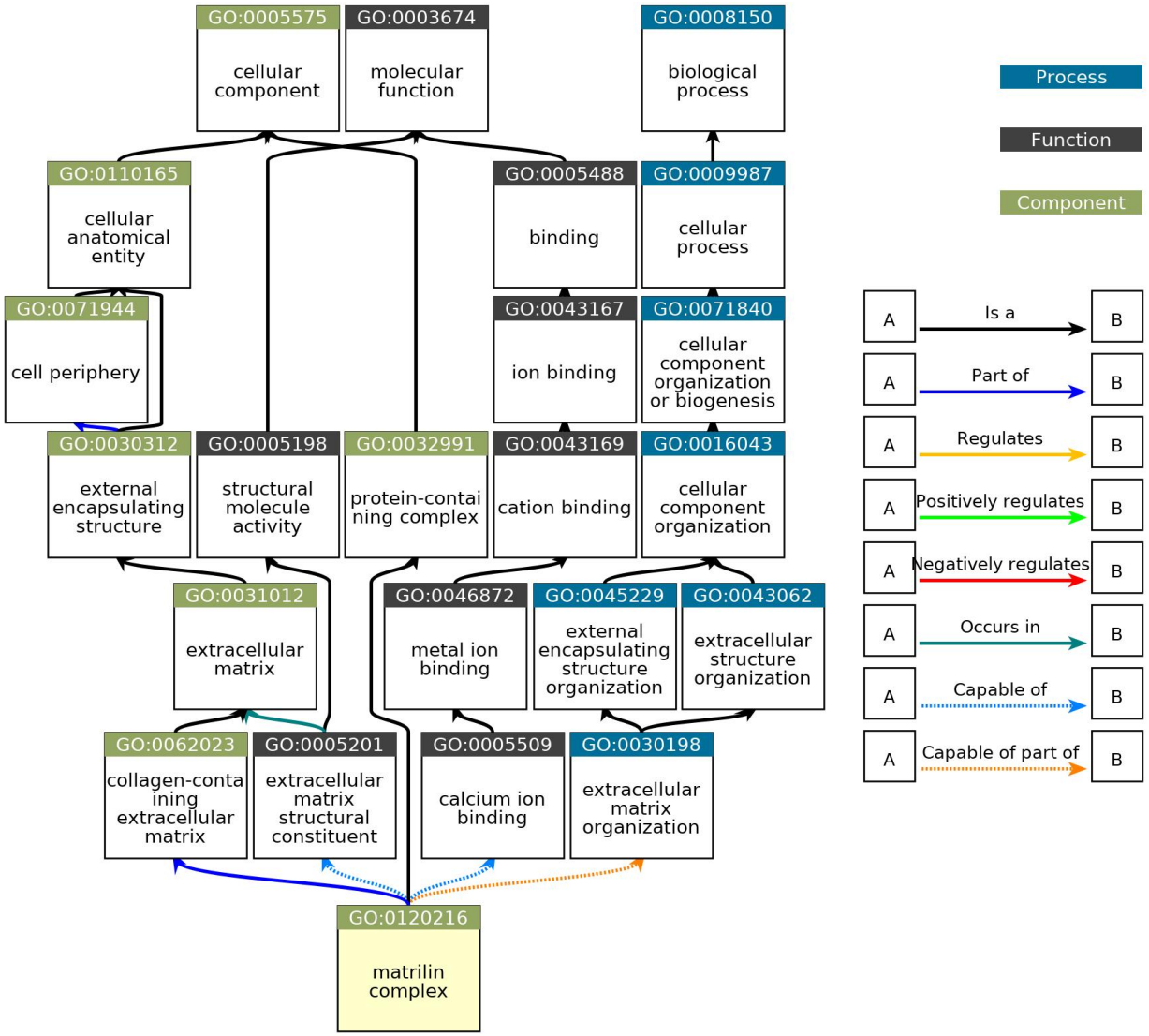



Figure: A sample of terms from the Gene Ontology and their relation to *matrilin complex* (bottom), a type of cellular component.

Sourced from EMBL's European Bioinformatics Institute, CC0, via Wikimedia Commons

# Ontological Foundations

## The Systems Ontology (SO)

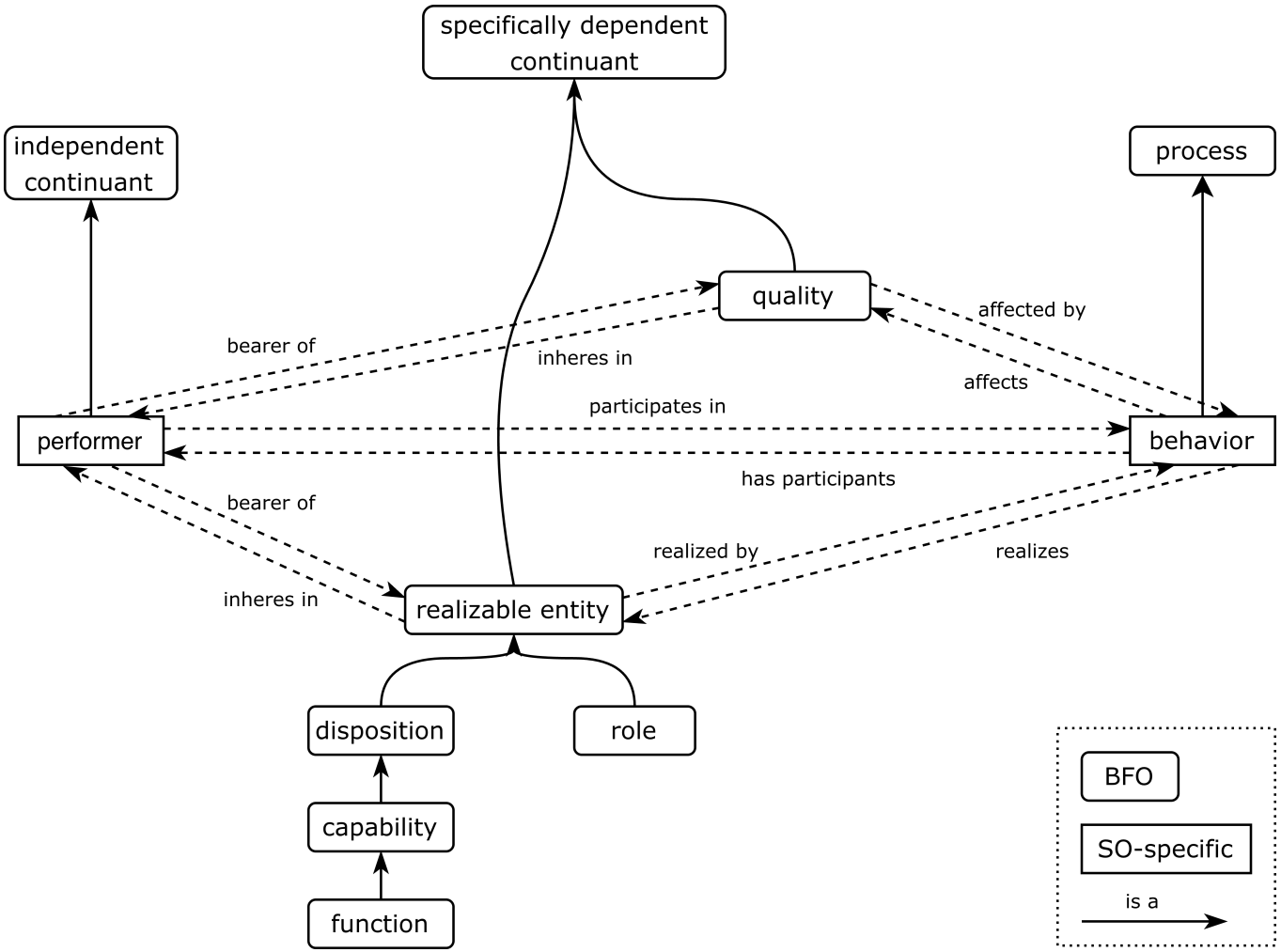


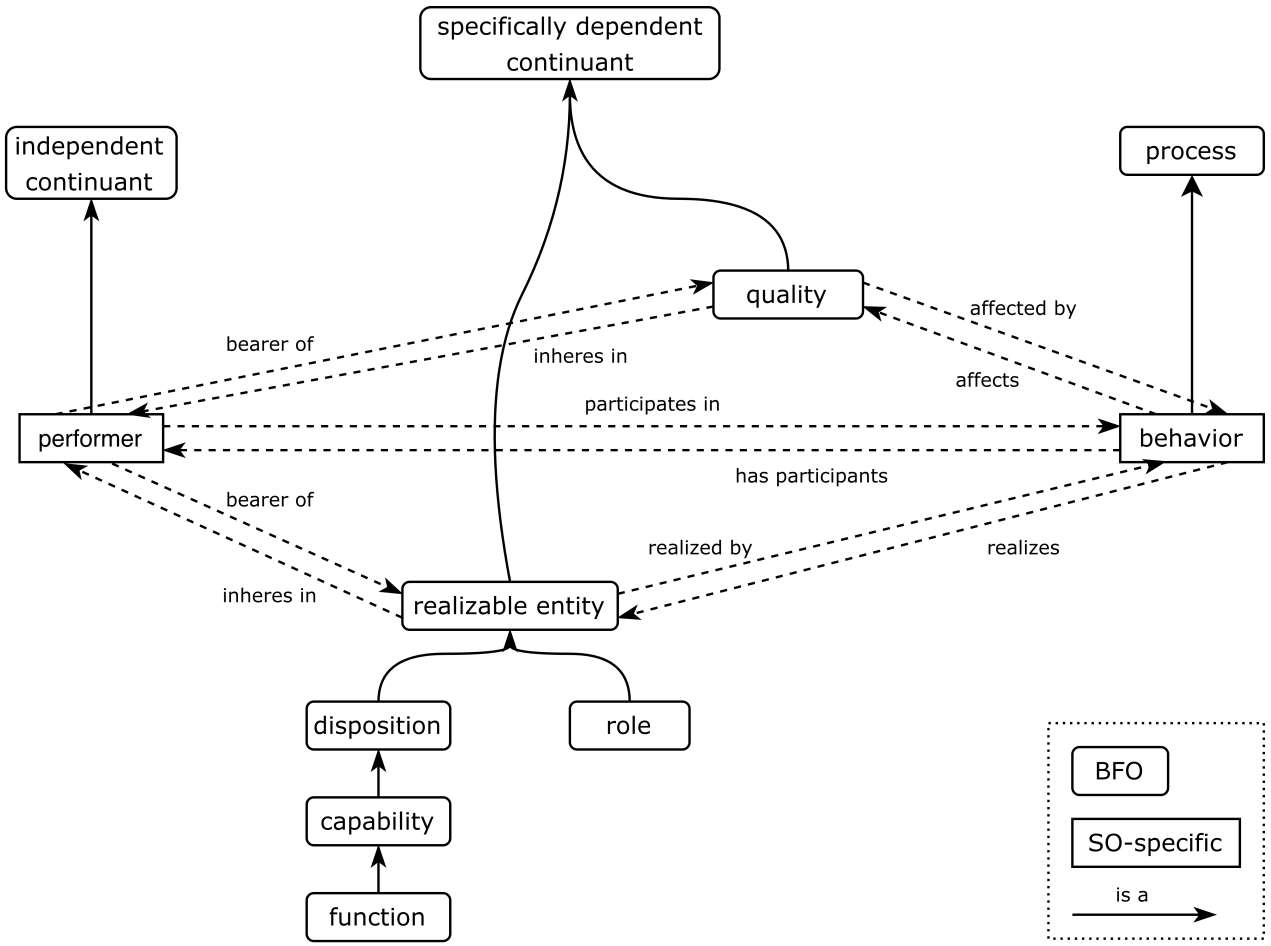

Figure: The core terms and relations of SO, including the central realization pattern (lower dashed triangle).

# Ontological Foundations

## The Systems Ontology (SO)

### Interpretation

Systems Ontology is a lightweight *framework* more than a complete ontology in itself.



# Ontological Foundations

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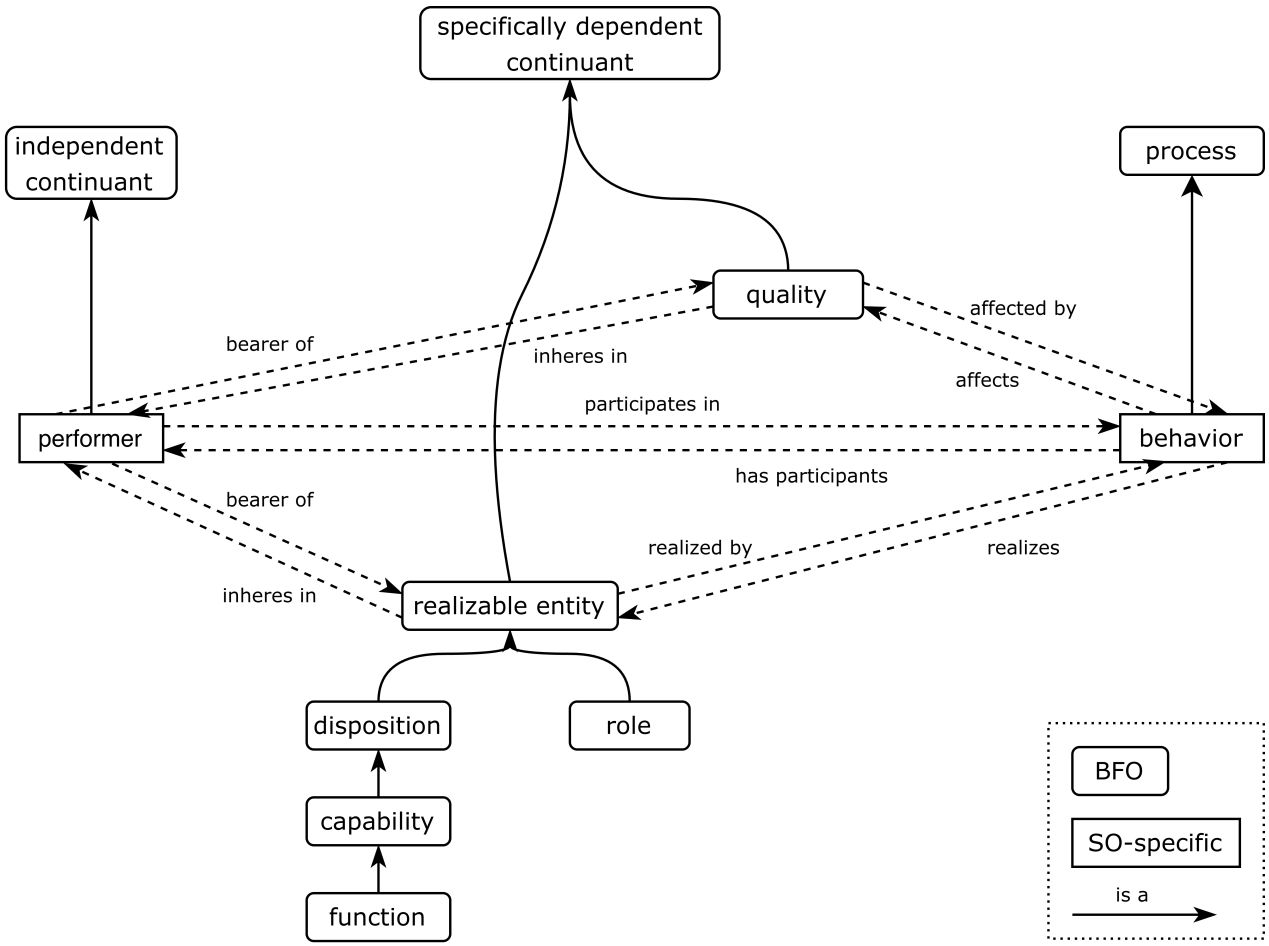
### Intended Utility

A double-categorical schema to build domain-specific system ontologies.

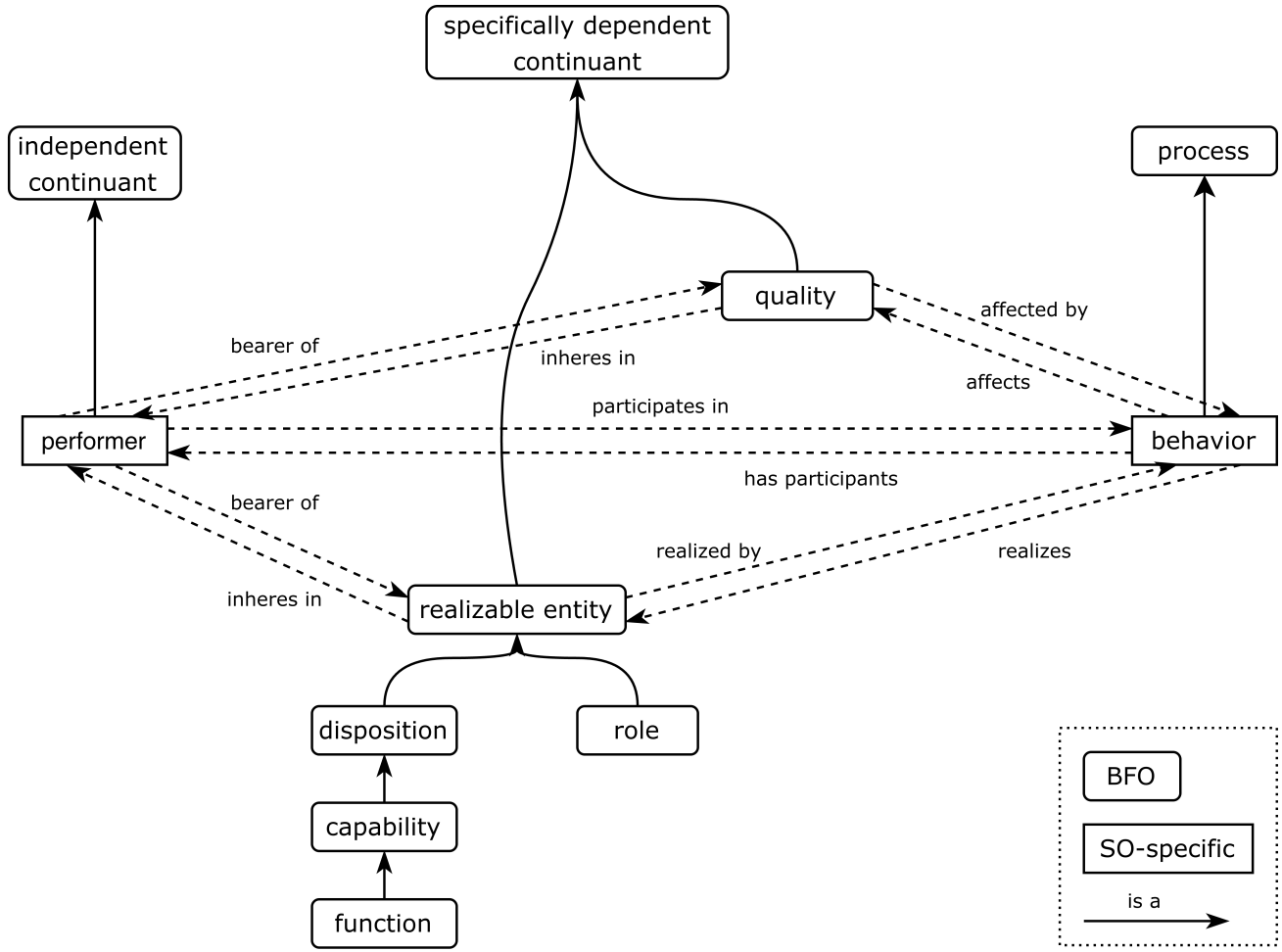


# Ontological Foundations

## The Systems Ontology (SO)

### Examples of Possible Performer Types

- pilot or plane,
- computer,
- measurement tool,
- cellular component (GO),
- any other class of physical components deemed relevant to the system,

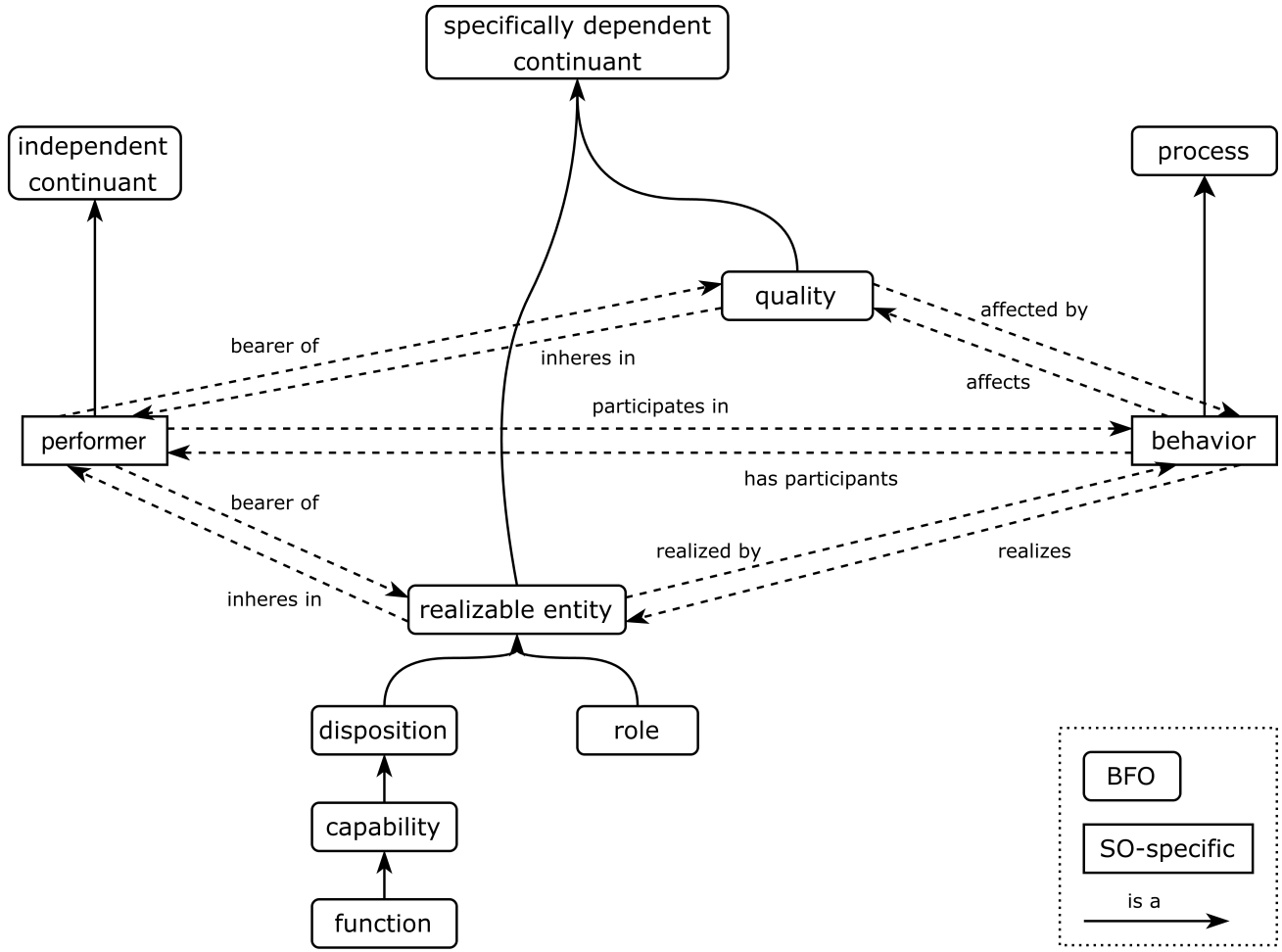


# Ontological Foundations

## The Systems Ontology (SO)

### Examples of Possible Behavior Types

- take off,
- high-altitude maneuver,
- photovoltaic conversion,
- signal transduction (GO),
- DNA replication (GO)



# Ontological Foundations

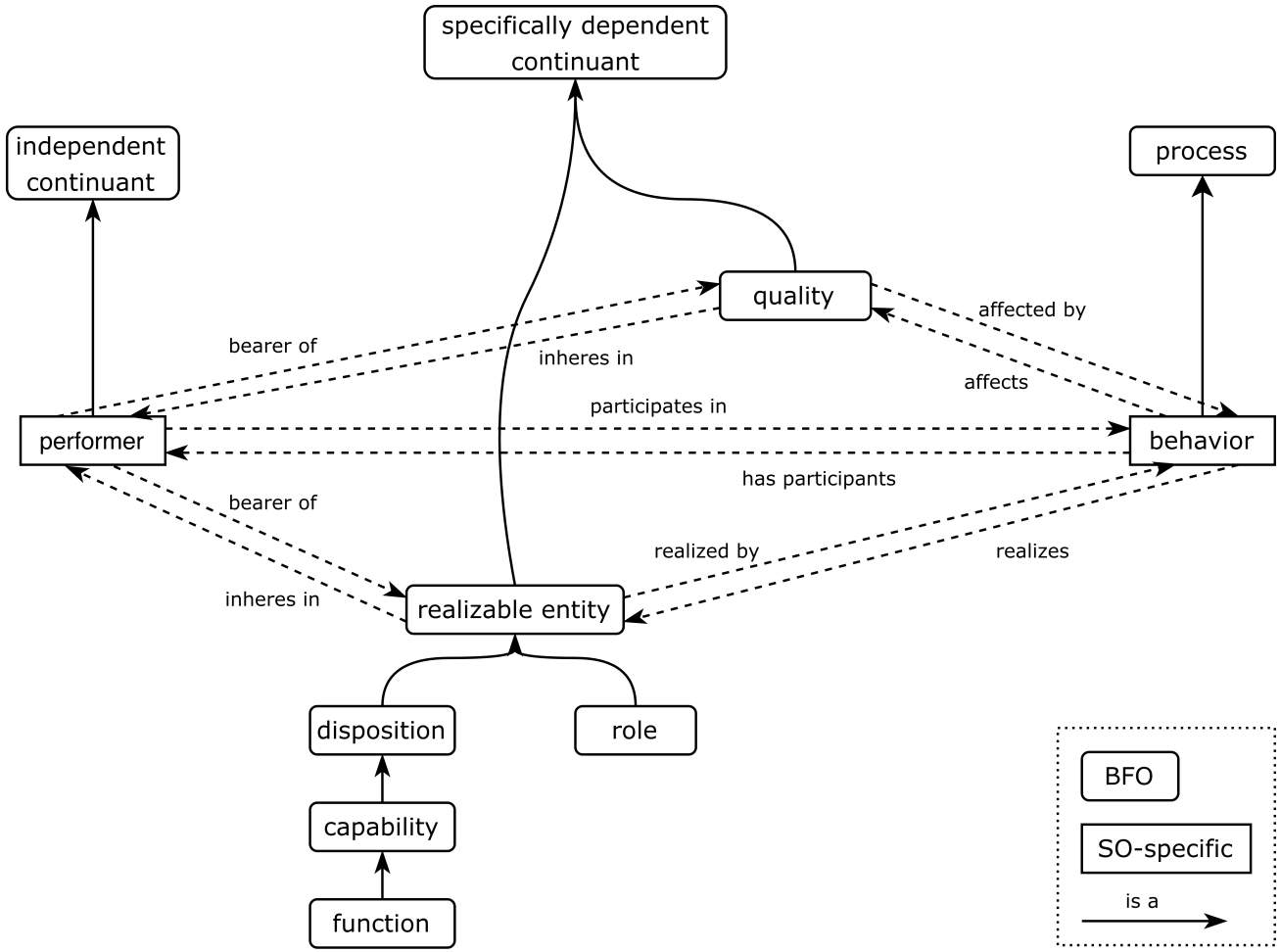
## The Systems Ontology (SO)

### Examples of Possible (Universal) Qualities

- temperature,
- voltage,
- protein configuration,
- location and velocity (although not standard BFO qualities).

### Definition

A *quality space* $S_Q$ for a quality $Q$ is a set of possible values that a measurement of $Q$ can take, e.g., if $Q = \text{temperature}$, $S_Q$ may be $\mathbb{R}_{\geq 0}$ interpreted in units of Kelvin.

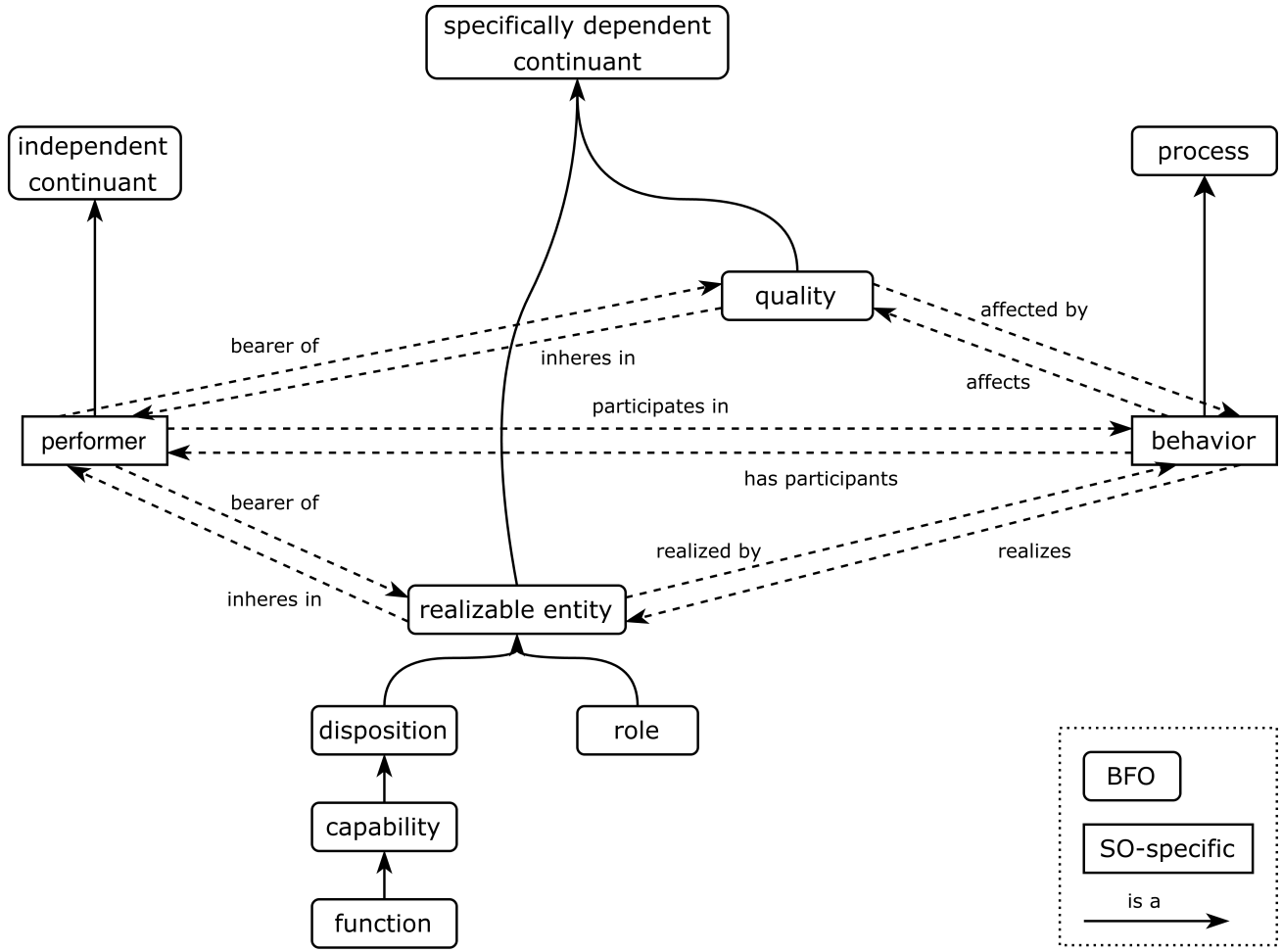


# Ontological Foundations

## The Systems Ontology (SO)

### Examples of Possible Capability Types

- emergency water landing,
- *safe* emergency water landing,
- photovoltaic conversion,
- photovoltaic conversion with efficiency $\geq e$.
- signaling receptor binding (GO).



# Section 3

# Ontology-Informed Mathematical Structures

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## The Performer Hierarchy

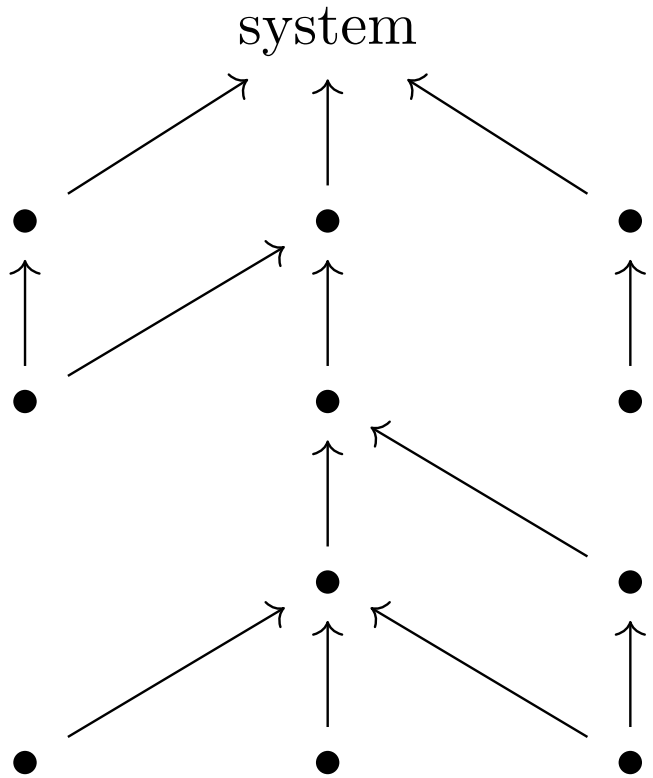
### Definition

Let **P** denote the poset (or lattice, or frame), whose objects are those performers in the system with a morphism $p \to q$ if and only if $p$ is *physically contained in* $q$.

All posets are treated as sites with the *minimal coverage* where $(p_i \to p)_i$ covers $p$ if and only if for all minimal $q \to p$, $q \to p_i$ for some $i$. Here $q$ is minimal if $q' \to q$ implies $q' = q$ or $\emptyset$.

### Remark

While every $p \in \mathbf{P}$ will have a declared type $P \xrightarrow{\text{is a}} \boxed{\text{performer}}$, the structure **P** is distinct from $\downarrow \boxed{\text{performer}}$.



# Ontology-Informed Mathematical Structures

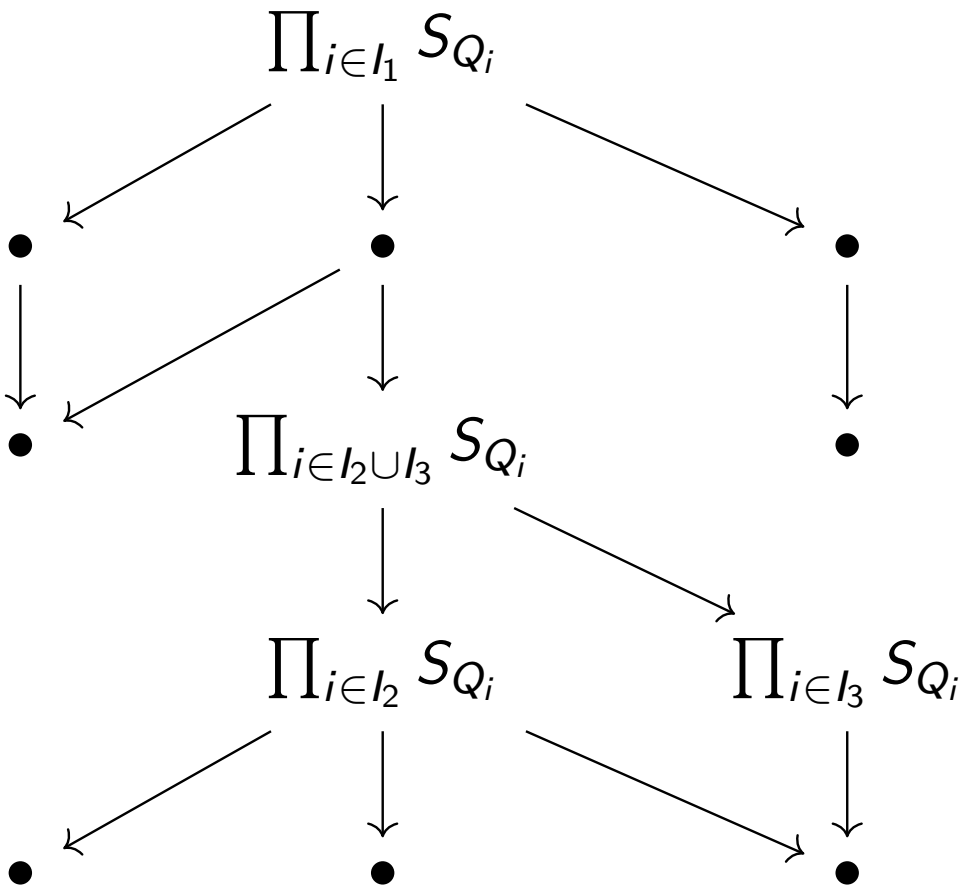
## Quality Sheaves and their Morphisms

### Definition

Let $\mathscr{Q} : \mathbf{P}^{\mathrm{op}} \to \mathbf{Set}$ be a sheaf declaring quality spaces[1], and thus qualities, for each performer.

### Remark

$\mathscr{Q}$ defines a state space $\mathscr{Q}(\mathrm{system})$ and records how these states descend to local regions of the system.




[1] For each (universal) quality $Q \xrightarrow{\text{is a}} \boxed{\text{quality}}$, there are many possible "quality spaces" $S_Q$ which are sets of possible measurement values for $Q$ each with respect to a different unit of measurement.

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### Covariant Grothendieck Construction

$(\mathbf{P}, \mathscr{Q}) \in \int^{\mathrm{cov}}_{\mathbf{FinPos}^{\mathrm{op}}} \mathrm{Sh}(-)$ where

$$\mathrm{Sh}(-) : \mathbf{FinPos}^{\mathrm{op}} \to \mathbf{Cat}$$
$$\mathbf{P} \mapsto \mathrm{Sh}(\mathbf{P})$$
$$(\mathbf{P} \xrightarrow{f} \mathbf{P}') \mapsto f_*$$

A *quality sheaf morphism*

$$(\rho, \alpha) : (\mathbf{P}, \mathscr{Q}) \to (\mathbf{P}', \mathscr{Q}')$$

is then a pair

$$\rho : \mathbf{P}' \to \mathbf{P} \quad \text{in } \mathbf{FinPos}$$
$$\alpha : \rho_* \mathscr{Q} \to \mathscr{Q}' \quad \text{in } \mathrm{Sh}(\mathbf{P}')$$

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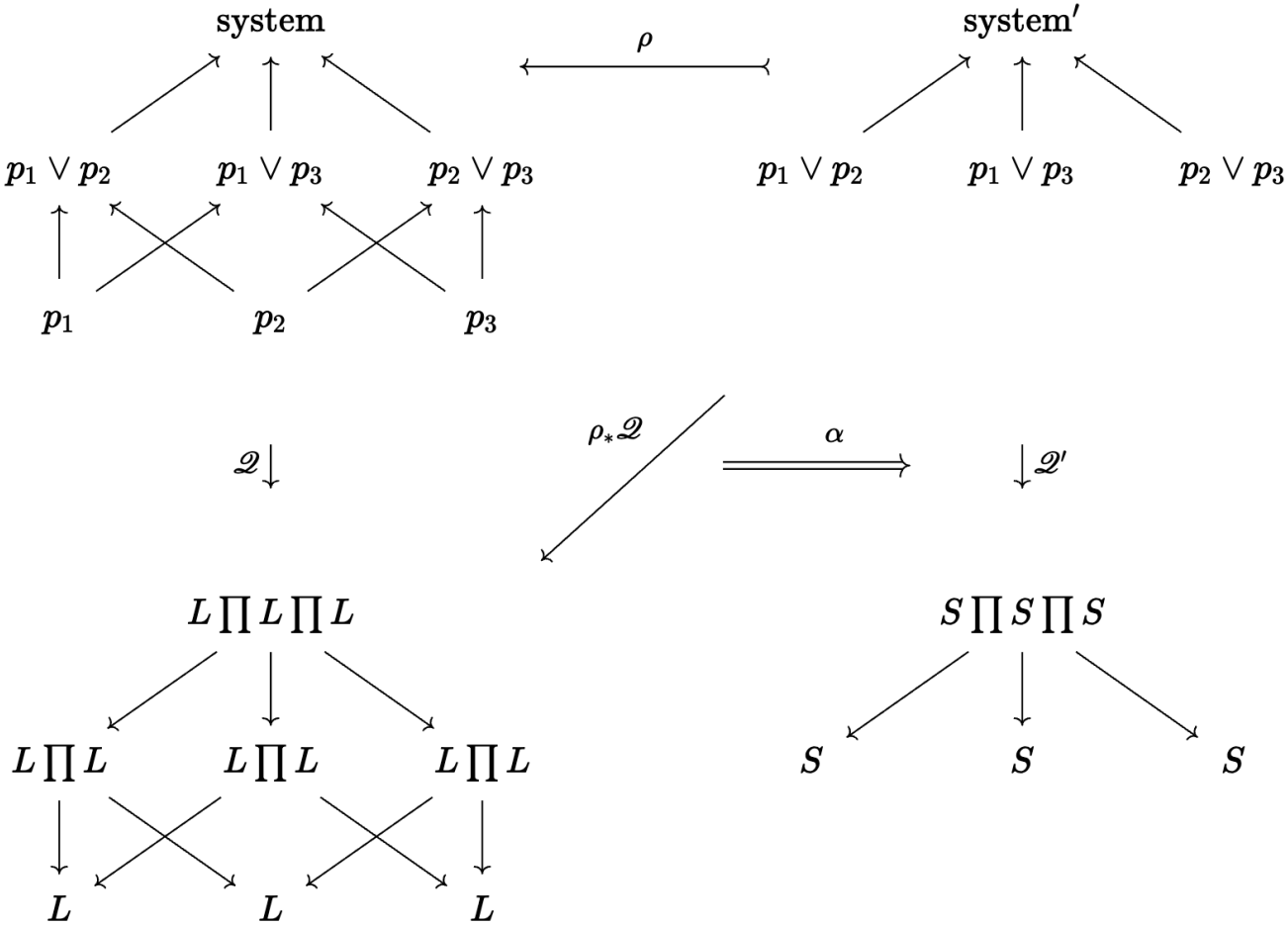


Figure: Depiction of a quality sheaf morphism enabling abstraction. One interpretation of this morphism is as follows: the performers $p_i$ denote airplanes in an airspace system, the quality space $L$ is for the *aircraft location* quality, while $S = \{\text{unsafe}, \text{safe}\}$ is for the pairwise *seperation safety* quality.

# Ontology-Informed Mathematical Structures

## Behaviors & Capabilities

### Guiding Principle

While behaviors and capabilities are ontologically distinct, they should be represented in the same mathematical language as a transformation of a quality sheaf.

# Behaviors & Capabilities

## Integrating Diverse Representations

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### Difficulty

There are many disparate means of modeling processes:

- natural language specification (assumptions and guarantees),
- black box relation,
- wiring diagram,
- dynamical system, etc.

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### Primary Issue

Capabilities often expressed in natural language using an abstract quality sheaf $\mathscr{Q}'$ while raw system data and behavioral reasoning (e.g., nontrivial concurrency) often requires a highly specific and time-dependent representation, $\mathscr{Q}(\text{system})^{\mathbb{R}}$.

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### Future Work

Integrate the quality sheaf framework with behavioral theories such as Temporal Type Theory [Schultz, Spivak] or DOTS [Libkind, Jaz Myers] to enable passage from low-level behaviors to the high-level capabilities that they realize.

# References


1. M.M., Samantha Jarvis, Nelson Niu, Angeline Aguinaldo, Amanda Hicks, and Ian Levitt. *Formal Structures in Systems Ontology towards Air Traffic Management Architectures*. NASA Technical Memorandum, 2025. Report no. NASA/TM-20250010771. https://ntrs.nasa.gov/citations/20250010771
2. Robert Arp, Barry Smith, and Andrew D. Spear. *Building Ontologies with Basic Formal Ontology*. The MIT Press, 2015. ISBN 9780262527811. https://mitpress.mit.edu/9780262527811/building-ontologies-with-basic-formal-ontology/
3. Evan Patterson. *Knowledge Representation in Bicategories of Relations*. arXiv preprint, 2017. arXiv:1706.00526. DOI 10.48550/arXiv.1706.00526. https://arxiv.org/abs/1706.00526
4. Alexandru E. Stanculescu. *On calculus of functors in model categories*. arXiv preprint, 2012. arXiv:1208.1919. DOI 10.48550/arXiv.1208.1919. https://arxiv.org/abs/1208.1919
5. Patrick Schultz and David I. Spivak. *Temporal Type Theory: A Topos-Theoretic Approach to Systems and Behavior*. Birkhäuser, 2019. DOI 10.1007/978-3-030-00704-1. https://link.springer.com/book/10.1007/978-3-030-00704-1
6. Patrick Schultz, David I. Spivak, and Christina Vasilakopoulou. *Dynamical Systems and Sheaves*. arXiv preprint, 2016. arXiv:1609.08086. DOI 10.48550/arXiv.1609.08086. https://arxiv.org/abs/1609.08086
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