Assignment 12

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Problem 2

See the implementation in function `tabular_td_lambda` in `monte/assignment12/td_lambda.py`.

Problem 3

We wish to prove that the MC error is the sum of discounted TD errors:

$$G_t - V(S_t) = \sum_{u=t}^{T-1} \gamma^{u-t} \cdot (R_{u+1} + \gamma \cdot V(S_{u+1}) - V(S_u)) \tag{1}$$

We proceed by backwards induction. Note that since

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots + \gamma^{T-t-1} R_T \tag{2}$$

we have

$$\begin{aligned} G_{T-1} - V(S_{T-1}) &= R_T - V(S_{T-1}) \\ &= \gamma^0 (R_T + \gamma V(S_T) - V(S_{T-1})) \end{aligned}$$

since $V(S_T) = 0$. Having established the above equality for $t = T - 1$, we assume its truth for arbitrary $t + 1 > 0$ and show that it holds for t as well. Calculating,

$$\begin{aligned} G_t - V_(S_t) &= R_{t+1} + \gamma G_{t+1} - V(S_t) && (3) \\ &= \gamma(G_{t+1} - V(S_{t+1})) + R_{t+1} + \gamma V(S_{t+1}) - V(S_t) && (4) \\ &= \gamma V(S_{t+1}) - V(S_t) + \sum_{u=t+1}^{T-1} \gamma^{u-t-1} (R_{u+1} + \gamma V(S_{u+1}) - V(S_u)) && (5) \\ &= \sum_{u=t}^{T-1} \gamma^{u-t} \cdot (R_{u+1} + \gamma \cdot V(S_{u+1}) - V(S_u)), && (6) \end{aligned}$$

establishing the claim.