An Alternative Model of Metcalfe's Law for Valuing Bitcoin

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Abstract

This short paper presents a new model of the market capitalization of Bitcoin that builds upon a standard model based upon Metcalfe's Law. The model incorporates the logistic diffusion of the innovation, which could be extended to capture population and economic factors. This model appears to have some improved efficacy over the standard model. Using this model, some areas for future research are briefly discussed.

Keywords

Cryptocurrency, Metcalfe's Law, Diffusion of Innovation

JEL Codes

C5, C22, G1

1. INTRODUCTION

The diffusion of Bitcoin technology has been astounding. Yet, traditional models of fiat currencies fail to explain its valuation. One new contribution, offered by Peterson (2017) proposes a model that is a function of the network effect using Metcalfe's Law, which is formally defined in Shapiro and Varian (1999). In this short paper, we augment Peterson's model to make two contributions. One, the new model for valuing cryptocurrencies incorporates Rogers (1962) diffusion of innovation, which captures population parameters and growth rates. This model appears to have some improved efficacy. Two, we make some brief theoretical conjectures about the ties of the model parameters to macro-economic factors for future research.

2. BRIEF LITERATURE REVIEW

Peterson has provided an excellent literature review on Bitcoin, including Grinburg (2012) and Ciain's (2016) discussion of the impact of macro-factors on the price. However, most of their citations have not (yet) found their way into the academic literature, notably for example Hayes' (2016) cost of production model, Kristoufek's (2015) wavelet coherence analysis, and (we add) Wang and Vergne's (2017) factor model. Peterson (2017) itself has only recently been uploaded to SSRN and not published in an academic journal, as of the writing of this paper. This is a very new area of inquiry. In any case, this paper very narrowly addresses Peterson's standard model and proposes a small variation.

Metcalfe's Law M describes the value of a network as being proportional to the square of the number of users n. The network becomes more valuable as n increases according to $M = n(n-1)/2$. Peterson (2017) presents an approach to valuing Bitcoin by applying Metcalfe's Law. In their model, they use a Gompertz sigmoid function to model the number of Bitcoins over time

$b_t = b_{t-1} \cdot \ln(B / b_{t-1})$, where B is the maximum number of Bitcoins. Their final model for the value V of a Bitcoin is expressed in (1).

$$V = A \cdot \left(M \cdot \frac{1}{\ln(B/b_{t-1})} \right) \tag{1}$$

Using empirical Bitcoin price data Y_t and the number of Bitcoin wallets as a proxy for n_t, they arrive at the value of Metcalfe's constant of proportionality $A \equiv \beta$ through a linear regression according to equation (2).

$$\ln(Y_t) = 0 + \beta \cdot \left(\frac{\ln(B/b_t)}{\ln(M_t)} \right) + \varepsilon_t \tag{2}$$

The model presented in the paper makes two changes to Peterson's model. One, we model b_t using a bounded exponential function. Two, we model the growth of n_t as a logistic function. The enables forecasting both the number of Bitcoins and the diffusion, something not possible under their model.

3. MODEL

To begin, the possible number of Bitcoins B is capped at 21,000,000. Over time, we assume the number of Bitcoins b_t follows the pattern of bounded exponential growth as in equation (3), where η is the optimized rate parameter. Peterson states that "as of 2017, the rate of new bitcoin creation is approximately 60 per hour, creating near-perfect price inelasticity of supply."

$$\hat{b}_t = B(1 - e^{-\eta t}) \tag{3}$$

We assume the number of users n_t follows a logistic, sigmoid function as in equation (4), which follows Rogers (1962) for the diffusion of innovations. The logistic function requires some upper asymptote, or saturation level, which is maximum number of users N. In (4), the optimized parameters φ and v work to together to define the rate of diffusion.

$$\hat{n}_t = \frac{N}{1+\phi e^{-\nu t}} \tag{4}$$

Then, similar to Peterson, the log of the market capitalization of Bitcoin V_t follows the linear regression equation (5), again where $A \equiv \beta$.

$$\ln(V_t) = 0 + \beta \cdot \left(\frac{\ln(\hat{n}_t)}{\ln(\hat{b}_t)} \right) + \varepsilon_t \tag{5}$$

In the following section, we calibrate the model using empirical data.

4. EMPIRICAL RESULTS

Using data from Blockchain.info, we fit the model of the number of Bitcoins over time b_t in (3) to the monthly, historical data on the actual number of Bitcoins since January, 2009. In Figure 1 (and also Figures 2 and 4), the empirical data is represented by the solid line and the model is represented by the dashed line. The optimized value of η that minimizes the sum of the squared errors is 0.0142.

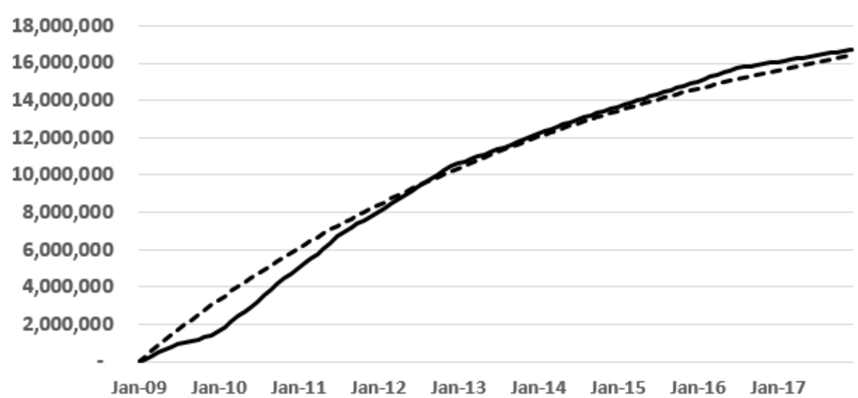

Figure 1: Model Fit to Number of Bitcoins

As a proxy for diffusion, we follow Peterson (2017) and use the number of Bitcoin wallets in existence since November, 2011. In (4), the maximum diffusion N is set to 1,000,000,000. While this is arbitrary, it attempts to represent some economically sensible upper bound on diffusion, which we will address later. The optimized values of φ and ν that minimize the sum of the squared errors are 3202.37 and 0.0578, respectively. The model and the data are shown in Figure 2.

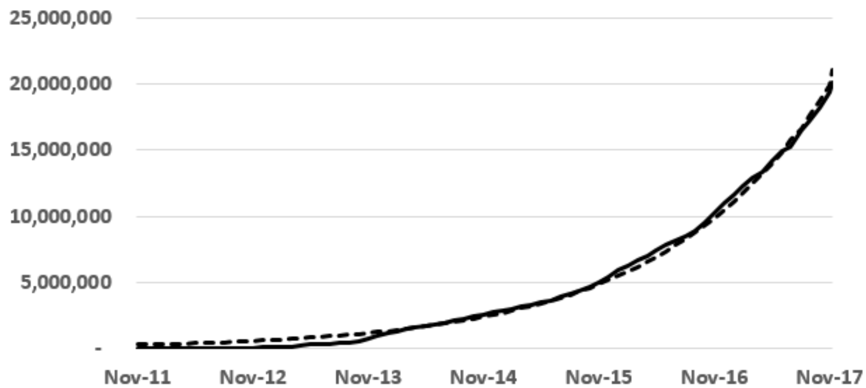

Figure 2: Model Fit to Number of Bitcoin Wallets

As a frame of reference, Figure 3 depicts the growth in the market capitalization of Bitcoin since November, 2011. In (5), we take the natural log of the market capitalization data and fit the regression line.

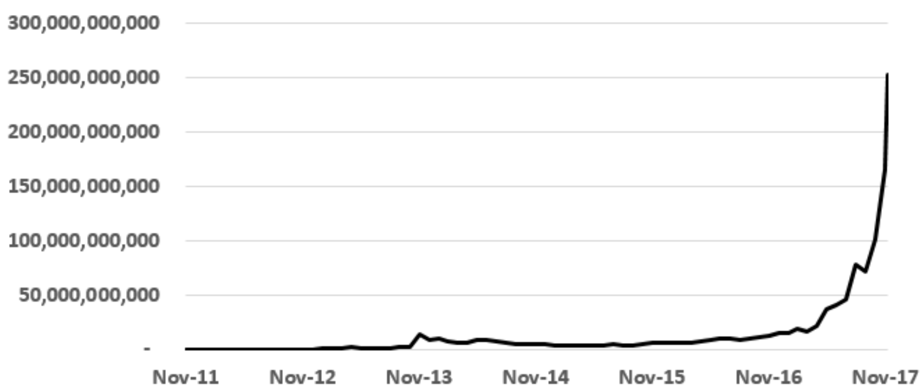

Figure 3: Bitcoin Market Capitalization

The regression model in (5) holds the intercept equal to zero and finds the least-squares coefficient (X Variable 1 in Table 1) as the estimate of *A*, which is 24.1969. Figure 4 show the data and the regression line.

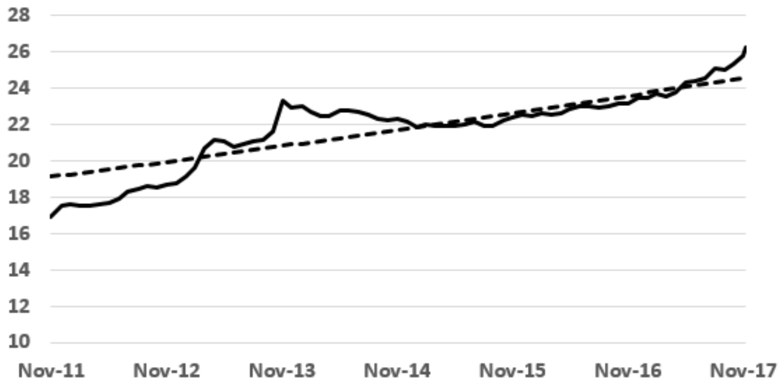

Figure 4: Regression Fit to Log of Bitcoin Market Capitalization

Table 1 presents the regression output. The R-square value is extremely high at 0.9977, which improves upon Peterson's results. Nevertheless, we can notice in Figure 4 some misalignment, particularly between November, 2011 and November 2014. Peterson (2017) provides an excellent overview of Gandal et al.'s (2018) findings, arguing that price manipulation during these periods may be the best explanation for serial deviations from Metcalfe's law, which otherwise appears to have strong explanatory power. Additionally, it is important to point out that Peterson's (2017) caveats regarding the exclusion of "zombie Bitcoins" and the use of wallets as a proxy for *n* also apply to the model in this paper.

Regression Statistics	
Multiple R	0.9988
R Square	0.9977
Adjusted R Square	0.9840
Standard Error	1.0585
Observations	74

ANOVA

	df	SS	MS	F	Significance F
Regression	1	35361.59	35361.59	31536.16	0.00
Residual	73	81.7851	1.1203		
Total	74	35443.38			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	**0**	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
X Variable 1	**24.19691**	0.1362	177.6602	0.0000	24.92547	24.4684	23.92547	24.4684

Table 1: Regression Output

5. DISCUSSION

The model presented incorporates the logistic representation of the diffusion of Bitcoin. Metcalfe's Law alone measures network capacity as the maximum number of possible paired connections. But certainly, the Bitcoin network cannot grow to infinity. At some point diffusion must be bounded by human population, which itself is growing at an exponential rate of between 1% and 1.5% annually. Presumably, the population in Figure 5 is subject itself (eventually) to some carrying capacity, some upper bound, and several models exist. The goal of this paper is not to examine the impact of long-term population trends nor diffusion of Bitcoin in various geographical regions, but rather to simply introduce a model that allows for such analysis and demonstrate its efficacy.

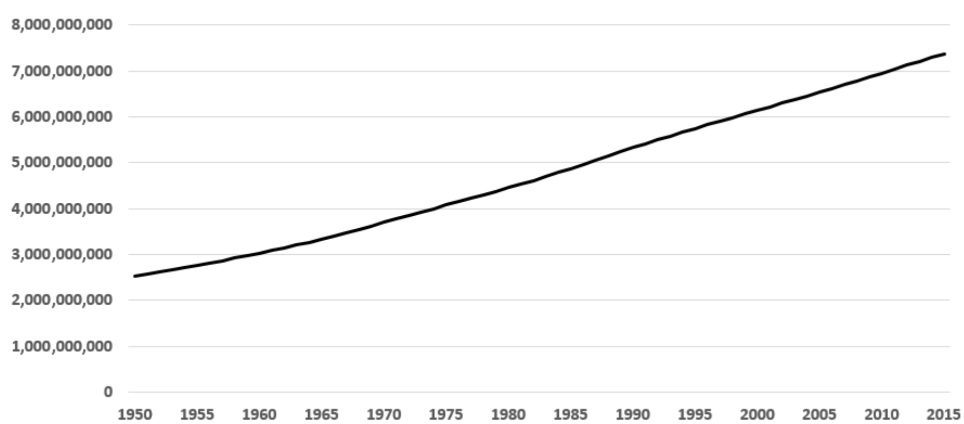

Figure 5: World Population Growth

Further research into the diffusion in various geographical or economic regions may shed light on the upper asymptote *N*. For example, diffusion in emerging economies may be a function of the level of confidence in the local fiat currency. Likewise, diffusion may be a function of idiosyncratic transaction fees among currency pairs. In developed economies, diffusion among income strata may depend on tax implications of cryptocurrency transactions.

To understand future growth in Bitcoin's market capitalization, these areas all present fertile opportunities for new research.

6. CONCLUSION

This short paper presented a new model for the market capitalization of Bitcoin that is based upon Metcalfe's Law. As in incremental step on Peterson (2017), both the number of Bitcoins and the diffusion are modeled. The model presented fits the empirical data well. This opens new areas to incorporate population growth and economic variables into the theoretical model.

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