Volatility in the Cryptocurrency Market

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Abstract:

How do cryptocurrency prices evolve? Is there any interdependence among cryptocurrency returns and/or volatilities? Are there any return spillovers and volatility spillovers between the cryptocurrency market and other financial markets? To answer these questions, we use GARCH-in-mean models to examine the relationship between volatility and returns of leading cryptocurrencies, to investigate spillovers within the cryptocurrency market, and also from the cryptocurrency market to other financial markets. Overall, we find statistically significant transmission of shocks and volatilities among the leading cryptocurrencies. We also find statistically significant spillover effects from the cryptocurrency market to other financial markets in the United States, as well as in other leading economies (Germany, the United Kingdom, and Japan).

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1 Introduction

Cryptocurrencies have received a great deal of attention in the news as of late. The total market capitalization of cryptocurrency has grown stunningly. In January 2017, the market capitalization of all cryptocurrencies was approximately $18 billion. As of January 2018, the total market capitalization was approximately $599 billion. Despite the exponential growth of Bitcoin and other cryptocurrencies, the cryptocurrency market is rather young (Bitcoin was created in 2009, but active trade only started in 2013) and therefore is still mostly unexplored (see Caporale and Plastun (2017)). So, it is important to understand how the cryptocurrency ecosystem works. One of the key issues yet to be analyzed is the spillover effects within the cryptocurrency market and from the cryptocurrency market to other financial markets. Using GARCH-in-mean models, we analyze the price evolution of the major cryptocurrencies, and investigate spillovers within the cryptocurrency market as well as across other financial markets.

Investigating volatility connectedness and spillovers among cryptocurrencies contributes to understanding the information transmission mechanism in the cryptocurrency market, thus providing useful information for market participants (e.g., investors and miners). Theories and empirical work focusing on cryptocurrency volatility spillovers and related information transmission mechanisms can be divided into two groups. The first one views the spillover transmission mechanism of cryptocurrency through the correlation with economic fundamentals and global capital allocation. Schilling and Uhlig (2018) obtain a fundamental pricing equation, which in its simplest form implies that Bitcoin prices form a martingale. Using asymmetric GARCH models, Bouri *et al.* (2017), Baur and Dimpful (2018), and Stavroyiannis (2018) investigate the response of the conditional variance to past positive and negative shocks and find an inverted leverage effect. The other group holds the view that the cryptocurrency market is inefficient and investors will seek speculation or hedging opportunities in a certain cryptocurrency by assessing the performance of other cryptocurrencies, thereby causing contagious comovement of cryptocurrency returns through a correlated information channel. The existence of bubbles in cryptocurrencies has been examined by Fry and Cheah (2016). Corbet *et al.* (2018) examines the potential market manipulation in cryptocurrency cross-correlations and market interdependencies. Despite the growing interest in cryptocurrency as a digital asset, the current economics and finance literature is still lacking empirical evidence on its diversification, hedging and safe haven properties against other assets such as bonds and stocks. This feature, combined with the fact that the cryptocurrency market is a very young market, makes it particularly interesting to examine spillover effects within the cryptocurrency market as well as across other financial markets.

This paper contributes to the cryptocurrency literature in several ways. It fills the gap in univariate GARCH-in-mean modelling of cryptocurrency returns which allows the risk premium to be affected by the changing conditional variance directly. Secondly, we study the volatility transmission among the three leading cryptocurrencies to examine their exposure to common market innovation shocks using a trivariate GARCH-in-mean BEKK model. Third, this is also the first attempt to study the volatility spillovers from the cryptocurrency market to other financial markets in the context of a trivariate GARCH-in-mean BEKK model. We investigate the exposure of the cryptocurrency to common stock market and macroeconomic factors in leading economies.

The remainder of the paper is organized as follows. Section 2 reviews the development of the cryptocurrency market. Section 3 discusses the univariate GARCH modeling of Bitcoin, Ethereum, and Litecoin. Section 4 estimates a trivariate GARCH-in-mean BEKK model to explore the interdependence of Bitcoin, Ethereum, and Litecoin. Section 5 estimates the interconnectedness between the cryptocurrencies market and volatility transmission across the stock and bond markets in the United States. Section 6 examines the spillover effects of the cryptocurrency market across international financial markets. The final section concludes the paper and discusses the policy implications.

2 Basic Cryptocurrency Facts

Bitcoin and most other cryptocurrencies do not require a central authority to validate and settle transactions. Instead, these currencies use only cryptography (and an internal incentive system) to control transactions, manage the supply, and prevent fraud. Payments are validated by a decentralized network. Once confirmed, all transactions are stored digitally and recorded in a public "blockchain," which can be thought of as an accounting system. Although the Bitcoin blockchain is limited in size and frequency on the amount of transactions it can handle, the introduction of the Lightening Network to the blockchain makes payments faster and cheaper, and it is in growing use and capacity.

Through innovations in its technical design, the cryptocurrency offers the potential to disrupt payment systems and traditional currencies. In the United States, cryptocurrency exchange regulations are in an uncertain legal territory. The Internal Revenue Service regards cryptocurrencies as electronic assets subject to tax on capital gains. Although cryptocurrency is not an official legal tender in the United States, an increasing number of companies are accepting it as a form of payment for goods and services every day. Major businesses include Paypal, Microsoft, Expedia, Uber, Airbnb and Ebay. In Germany, cryptocurrency is not just a product, but is absolutely a legal tender; an order by the Federal Ministry of Finance allows making purchases with digital currencies without taxation, since virtual currency is considered an equivalent to fiat. Moreover, in the European Union member states, gains in cryptocurrency investments are not subject to value added tax due to a 2015 decision of the European Court of Justice.

Although cryptocurrency shows great promise to become more integrated into international finance and payment systems, the cryptocurrency market is extremely volatile and many purchases of cryptocurrency have been seen as raw speculation. The average daily price amplitude of cryptocurrency is up to 10 times higher than that in the money market. This phenomenon was illustrated in the Great Crypto Boom of December 2017, when Bitcoin prices had increased by about $2,700\%$ with a record high price of $\$19,891$. At the same year, some cryptocurrencies had achieved even higher growth than Bitcoin. With the sell-off of most cryptocurrencies that started in January 2018, the price of Bitcoin fell by about 65 percent in one month. Subsequently, nearly all other cryptocurrencies which also peaked from December 2017 through January 2018, then followed Bitcoin. The market capitalization of cryptocurrencies declined by at least 342 billion US dollars in the first quarter of 2018, the largest loss in cryptocurrencies up to that date. Moreover, more than 900 cryptocurrencies deceased due to fraud, heck and scam (see http://www.deadcoins.com). Despite the burst bubbles, however, widely recognized institutions are actively involved in the cryptocurrency market since 2018. Some of the largest financial institutions such as Fidelity, ICE, and Nasdaq have continued to strengthen the infrastructure supporting cryptocurrencies as an asset class. It shows that institutional investors are opening up to the cryptocurrency sector and are becoming more comfortable with this type of asset. In February 2019, the entrance of public pensions into the cryptocurrency sector furthered fueled the confidence of other institutional investors in the traditional financial sector.

The disruption that the cryptocurrency caused in the money market poses great challenges and opportunities to policy makers, economists, and investors. Legislators and economists have been debating about whether the cryptocurrencies are currencies or speculative investment assets. On the one hand, cryptocurrency serves as a medium of exchange based on a decentralized network. On the other hand, the highly volatile rate of return seems to fulfill similar functions as other more traditional assets. Is cryptocurrency a form of a currency, an asset, or a completely different instrument? How is cryptocurrency affected by other financial markets? One way to understand what cryptocurrencies represent is to investigate the comovement of their returns with other classes of assets, in other words, to assess how investors and markets value cryptocurrencies and other financial assets. This paper provides a sense of what capabilities cryptocurrency might have in the market for risk management and portfolio analysis by examining the transmission or volatility spillover effects within the cryptocurrency market as well as across the financial markets. Particularly, we use a trivariate VARMA GARCH-in-mean BEKK model to study the interdependence of the cryptocurrency market, stock market, and bond market, with the objective of removing some of the widespread mystery surrounding the cryptocurrency.

3 The Data

To investigate the price evolution and spillover effects of cryptocurrencies, we use daily time series data from 2015:8:7 to 2019:4:27 (a total of 1360 observations) of three major cryptocurrencies, Bitcoin, Ethereum, and Litecoin. The data source is CoinMarket (https://coinmarketcap.com/). As shown in Figure 1, the three together captured more than 40% of the cryptocurrency market. Although the Bitcoin and Litecoin data goes back to 2013:4:28, we begin our analysis in 2015:8:7, because the second largest component of the cryptocurrency market, Ethereum, was not market capitalized until then. While digital currencies were proposed as early as the 1980s, Bitcoin was the first to catch on. The total value of all Bitcoins in circulation today is around \$135 billion (CoinMarketCap, 2019:4), and its massive increase has inspired scores of competing cryptocurrencies that follow a similar design. The number of cryptocurrencies has increased from approximately 80 in January 2014 to 2,112 by April 2019. Despite the huge increase in the market capitalization of other cryptocurrencies, their markets are still very thin. Thus, Bitcoin still has a strong dominance under the competition from rival cryptocurrencies.

Cryptocurrency returns are computed based on the cryptocurrency market price p_t. The cryptocurrency return at time t is then calculated as

$$r_t = \log p_t - \log p_{t-1}$$

where $r_t = b_t$, e_t, l_t, representing the returns of Bitcoin, Ethereum and Litecoin, respectively. Table 1 presents the descriptive statistics for the logarithmic prices as well as for the returns of cryptocurrencies. The mean returns for the three cryptocurrencies are ranging from a minimum of 0.002 (for Bitcoin and Litecoin) to a maximum of 0.003 (for Ethereum). Moreover, the Bitcoin return series is the least volatile series with a standard deviation of 0.039, while the Ethereum return series can be considered as the most volatile series of the three cryptocurrencies with a standard deviation of 0.076. Figures 2 to 4 provide a visual perspective on the volatility of the return series over the sample period. Indeed, the price movements of the cryptocurrencies have been quite spectacular. For example, as shown in Figure 2, Bitcoin prices rose by more than 20 times in 2017, but by December 2018, Bitcoin fell by over 80% from its peak. We observe similar huge swings of the other two major cryptocurrencies, Ethereum and Litecoin, in Figures 3 and 4.

Based on the estimated skewness statistics, the Bitcoin and Ethereum return series are skewed to the left, while the Litecoin return series is skewed to the right. As expected with any high frequency financial return series, the value of kurtosis for all of the return series of cryptocurrencies indicates a typical leptokurtic distribution, meaning that the return series is more peaked around the mean with thicker tails compared to the normal distribution. Furthermore, the Jarque-Bera statistics and corresponding p-values reinforce the above findings by rejecting the null hypothesis of normality at the 1 percent level of significance.

The logarithmic prices and returns of the three cryptocurrencies are highly correlated as shown in Table 2. The estimated correlation coefficients of the pairwise logarithmic prices are all greater than 0.9, with the highest correlation (0.984) between Ethereum and Litecoin, and the lowest (0.937) between Bitcoin and Ethereum. The returns of the cryptocurrencies are also positively correlated, with the highest correlation coefficient of approximately 0.624 between Litecoin and Bitcoin, and the lowest between Ethereum and Litecoin with a coefficient of 0.377. Figure 5 confirms this by showing a close comovement of the cryptocurrency prices. To determine whether these correlations are statistically significant, we follow Pindyck and Rotemberg (1990) and perform a likelihood ratio test of the hypothesis that the correlation matrix is equal to the identity matrix. We reject the null hypothesis that the correlation matrix is an identity matrix. The strong correlation between cryptocurrencies is to be expected since they are all exposed to the same market innovations and macroeconomic shocks. The resulting collinearity tends to weaken the individual impact of these variables.

The first step in volatility modeling is to test for the presence of a stochastic trend (a unit root) in the autoregressive representation of each individual series. Thus, we conduct a set of unit root and stationary tests of the logarithmic prices of each cryptocurrency. Both the Augmented Dickey Fuller (ADF) test (see Dickey and Fuller (1981)) and the Dickey Fuller GLS test (see Elliott *et al.* (1996)) cannot reject the null hypothesis of the presence of a unit root as shown in panel A of Table 3, suggesting that all three logarithmic price series are nonstationary. We select the optimal lag length in each test by the Bayesian information criterion (BIC) with a maximum lag length of 4. Moreover, given that unit root tests have low power against trend stationary alternatives, we also use the KPSS test (see Kwiatkowski *et al.* (1992)) to test the null hypothesis of stationarity around a trend. As shown in panel A of Table 3, the null hypothesis of trend stationary is rejected at the 5 percent statistical significance level. We thus conclude that each of the three cryptocurrency logarithmic price series is nonstationary.

Nelson and Plosser (1982) argue that although most macroeconomic and financial time series have a unit root (a stochastic trend), the first difference of a time series is stationary. Therefore, we repeat the unit root and stationarity tests using the first differences of the logarithms of the series, which are the returns of the cryptocurrencies. As shown in panel B of Table 3, the null hypotheses of the ADF and DF-GLS tests are rejected, and the null hypothesis of the KPSS test cannot be rejected, suggesting that the returns of cryptocurrencies are stationary. Therefore, the logged cryptocurrency prices are integrated of order one, I(1), and the cryptocurrency returns are integrated of order zero, I(0).

4 Univariate GARCH Modeling

This section explores the return evolutions of Bitcoin, Ethereum, and Litecoin using the univariate GARCH-in-mean models (see Bollerslev (1986)) which allow the simultaneous modeling of both the first and second moments of the return series. As we have discussed, the degree of uncertainty in cryptocurrency returns varies dramatically over time, suggesting that the compensation required by risk averse economic agents for holding these cryptocurrencies also varies accordingly. We examine the possibility of risk being an explanation for the higher returns of cryptocurrency using the GARCH-in-mean model accordingly.

One of the key postulates is that time varying risk premia on different cryptocurrencies can be well modeled as unanticipated shocks and are measured by the conditional variances of the one period holding yields. The autoregressive conditional heteroscedasticity (ARCH) model, introduced by Engle (1982), explicitly models time varying conditional variances by relating them to variables known from previous periods. In its standard form, the ARCH model expresses the conditional variance as a linear function of past squared innovations; in markets where prices follow a martingale, price changes reflect innovations. The ARCH model is used to provide a rich class of possible parameterizations of heteroscedasticity. This paper first introduces the GARCH-in-mean model to allow the conditional variance of cryptocurrency to affect the mean cryptocurrency return. In this way changing conditional variances directly affect the expected return on a portfolio.

As it is standard in the GARCH literature, the conditional variance (covariance under the bivariate setting) is the proxy for the market risk. If the market risk is priced, the conditional variance (covariance) will be positively correlated with the market (portfolio) return. The slope of the return-variance relationship is the proxy for the risk premium. The time-series approach links daily returns with daily volatility over a long sample period (1360 observations in our study) which involves a long series of return-volatility data pairs to generate the return-risk regression slope. It should hence have stronger statistical power to track down the true relationship.

To model the return evolution of each cryptocurrency, the lag error terms are added to the mean equation to filter out possible first-order serial correlation. As shown in Figure 2, visually there is a structural break in December 2017 for all the three cryptocurrencies. In the context of the GARCH model, we perform the Andrews and Ploberger (1994) and the Andrews-Quandt structural break tests for a single structural break at an unknown point. We detect a structural break in 2017:12:7 for Bitcoin, 2017:12:21 for Ethereum, and 2017:12:13 for Litecoin. To capture the structural break, we introduce a dummy variable in the mean equation. We use the Akaike Information Criterion (AIC) to select the best fitted model. We find that an ARMA(2,2) with a GARCH(2,2) and a structural break model yields the lowest AIC value for all the three cryptocurrencies. Therefore, the univariate GARCH-in-mean model for each cryptocurrency is specified as

$$\begin{aligned} r_t &= \alpha_0 + \alpha_1 r_{t-1} + \alpha_2 r_{t-2} + \alpha_3 \epsilon_{t-1} + \alpha_4 \epsilon_{t-2} + \alpha_5 D + \alpha_6 h_t + \epsilon_t \\ \epsilon_t | \Phi_{t-1} &\sim N(0, h_t) \\ h_t &= \beta_0 + \beta_1 \epsilon_{t-1}^2 + \beta_2 \epsilon_{t-2}^2 + \beta_3 D + \beta_4 h_{t-1} + \beta_5 h_{t-2} + \beta_6 \epsilon_{t-1}^2 I_{t-1} + \beta_7 \epsilon_{t-2}^2 I_{t-2} \end{aligned}$$

where r_t is the cryptocurrency rate of return, h_t is the variance of ϵ_t conditional upon the information set Φ_{t-1}, and D is the dummy variable. The conditional variance is used here as a proxy for the market risk anticipated by investors. To capture the leverage effects in cryptocurrency return volatility, we include the GJR asymmetry coefficient of Glosten *et al.* (1993), $\epsilon_{t-i}^2 I_{t-i}$, which represents the disproportionate response of the variance to unexpected price decreases.

Since standardized residuals are usually not normally distributed (see Bollerslev *et al.* (1988)), quasi-maximum likelihood estimation is used. Bollerslev and Wooldridge (2007) have shown that the quasi-maximum likelihood estimator's asymptotic standard errors are valid under non-normality. All the estimations are performed in RATS 10.0.

4.1 Bitcoin

The empirical estimates of the univariate GARCH-in-mean model of Bitcoin are reported in Table 4. In general, there is a strong ARCH effect. One possible explanation for the prominence of ARCH effects is of course the presence of a serially correlated news arrival process, as discussed by Diebold and Nerlove (1989) and Gallant *et al.* (1988). In a related context Engle *et al.* (1990) have shown how the actual market mechanisms may themselves result in very different temporal dependence in the volatility of transactions prices, with a particular automated trade execution system inducing a very high degree of persistence in the variance process.

We did not find statistically significant GARCH-in-mean effects in the mean equation of Bitcoin. It could be because Bitcoin is integrated with other cryptocurrencies and financial markets, and thus exposed to the market innovations and macroeconomic shocks. The resulting collinearity could weaken the individual impact of the volatility of Bitcoin on its price. Our investigations in the next two sections confirm this, and we will discuss it in more details later. One of the GJR asymmetry coefficients, $\epsilon_{t-1}^2 I_{t-1}$, is statistically significant, suggesting there is a leverage effect in the dynamics of Bitcoin returns. Panel C of Table 4 reports the log-likelihood values and diagnostic test statistics for the standardized residuals, $\hat{\epsilon}_t = \epsilon_t / \sqrt{h_t}$. The descriptive statistics for $\hat{\epsilon}_t$ reveal a distribution that is very close to normal. The Ljung-Box Q test statistic tests the null hypothesis that the residuals are independently distributed, and the McLeod-Li Q^2 test statistics tests the null hypothesis that the squared residuals are independently distributed. Both the Q and Q^2 statistics are reported for 8 lags. Both diagnostic tests suggest that the standardized residuals are serially uncorrelated. We perform the Andrews and Ploberger (1994) and the Andrews-Quandt structural break tests for a single structural break at an unknown point within the sample, and find no structural break. Overall, the diagnostic tests show that the GARCH-in-mean model is correctly specified.

4.2 Ethereum

The empirical estimates of the price evolution of Ethereum are reported in Table 4 and all the coefficients in the mean equation are statistically significant. The constant term is -0.010 and statistically significant. Bollerslev *et al.* (1988) argue that the negative expected excess return on the market portfolio may be attributed to the preferential tax treatment on capital gains. Because of a lower tax on capital gains of the cryptocurrency as we discussed in section 2, investors have the incentive to hold the market portfolio even when its gross expected excess return is negative. The negative intercept could also be an artifact of approximating a nonlinear relation with a linear function. It is plausible that the negative intercept may imply that the true relation between the expected market risk premium and its ex ante variance is in fact convex, and trying to fit a straight line between them may result in the line intersecting the negative segment of the y-axis.

The dummy variable in the mean equation is positive and statistically significant with a coefficient of 0.008, suggesting that the mean return before the crash is 0.8% higher than the daily return over the whole sample period. The positive GARCH-in-mean coefficient suggests that investors are risk averse and that the higher Ethereum volatility is rewarded with higher returns. Furthermore, the risk premium is more than two times of the variance of the return, which is quite substantial, indicating stronger risk aversion by the investors in this market. We do not find statistically significant leverage effects in the Ethereum returns. None of the diagnostic tests of the standardized residuals suggest serial correlation. Overall, the diagnostic tests indicate that the model is correctly specified.

4.3 Litecoin

We find a moderate ARCH effect in the Litecoin price evolution as shown in Table 4. The GARCH-in-mean coefficient is statistically significant, suggesting that Litecoin volatility has an impact on the direction and magnitude of the Litecoin price. We also find statistically significant GARCH effects and leverage effect in the variance equation. We conduct rigorous diagnostic tests as we did in the previous section, all of the diagnostic tests suggest that the standardized residuals are serially uncorrelated. Overall, the model is correctly specified.

From our analysis so far, we can see that forecasting in the cryptocurrency market is less certain and speculation in the cryptocurrency market is risky. Risk premia are therefore adjusted to induce investors to absorb the greater uncertainty associated with holding the highly risky cryptocurrency. Our results show that among all the three leading cryptocurrencies, Ethereum has the highest excess return. Although it is not clear why this is the case, it can be consistent with the consumption-based capital asset pricing model (CCAPM). As shown in Jagannathan and Wang (2007), the expected return of cryptocurrency asset i at time t, $Er_{i,t+j}$, under the linear version of the CCAPM, is equal to $\lambda_{cj}\beta_{icj}$. β_{icj} is the consumption beta, and $\beta_{icj} = \text{cov}(r_{i,t+j}, c_{t+1}/c_t)/\text{var}(c_{t+1}/c_t)$. Therefore, when the rate of return of a particular cryptocurrency correlates more with the investor's consumption growth, the cryptocurrency is weaker in hedging the investor's consumption risk, thus requiring a higher risk premium for a representative investor to willingly hold it. On the other hand, $\lambda_{cj} = \gamma \text{var}(c_{t+1}/c_t)/(1 - \gamma E(c_{t+1}/c_t))$, where γ is the coefficient of relative risk aversion (RRA) and is typically assumed to be constant. However, if RRA is variable, for example, increasing in wealth, λ could be neutral or increasing in wealth as well.

5 Cryptocurrency Market Spillovers

This section tests whether there are any spillover effects within the cryptocurrency market. We test whether cryptocurrency prices commonly react to innovations and/or economic news that the market is sensitive to. Daily experience seems to support the view that individual cryptocurrency prices are influenced by a wide variety of unanticipated events and that some events have a more pervasive effect on cryptocurrency prices than do others. The comovement of the cryptocurrency prices suggest the presence of common underlying exogenous influences. We examine whether common underlying shocks, such as innovations in the cryptocurrency market, are risks that are rewarded in the cryptocurrency market.

We test the significance and importance of fluctuations over time in cryptocurrency return volatility as a determinant of cryptocurrency prices. Such a relationship is implied by conventional theories of the effects of risk on asset returns; these models generally assume that risk averse traders respond to perceived risk, which is proxied by the volatility of the cryptocurrency market. The empirical model allows joint estimation of the relationship between volatility and returns and how past information is related to perceived volatility. In effect, the model imposes rationality on the variance forecasts of market participants. This is accomplished by estimating a multivariate GARCH-in-mean model (see Engle (2001)), in which the variance appears in the conditional mean specification. Thus, our approach avoids the arbitrariness of the conventional tests by using the data to specify the variance forecast model.

We use a trivariate vector autoregressive moving average (VARMA) GARCH-in-mean BEKK model (see Engle and Kroner (1995) for more details), to model Bitcoin, Ethereum, and Litecoin returns and volatilities as a system. We have stated the CAPM in terms of conditional moments since these reflect the information set available to agents at the time the portfolio decisions are made. The conditional covariance matrix of a set of asset returns is allowed to vary over time following the GARCH process (see Engle (1982) and Bollerslev (1986)). This essentially assumes that agents update their estimates of the means and covariances of returns each period using the newly revealed surprises in last period's asset returns. Thus, agents learn about changes in the covariance matrix only from information on returns. For the mean equation, to account for short-run conditional mean dynamics, we include the VARMA components, which can pick up serial correlation in the reduced form errors due, for example, to lagged adjustment to changes in the exogenous variables. It is important to control for time dependence in the mean to avoid confusing these effects with the dependence implicit in the GARCH specification.

Let $\boldsymbol{z}_t$ be the vector of the Bitcoin, Ethereum, and Litecoin nominal log returns during period t, and consider the following trivariate vector autoregressive moving average VARMA with GARCH-in-mean specification for the mean equation

$$\boldsymbol{z}_t \;=\; \mathbf{a}+\boldsymbol{\Gamma}\boldsymbol{z}_{t-1}+\boldsymbol{\Psi}\sqrt{\boldsymbol{h}_t}+\boldsymbol{\Theta}\boldsymbol{e}_{t-1}+\boldsymbol{\epsilon}_t,$$

$$\boldsymbol{\epsilon}_t|\,\Omega_{t-1} \;\sim\; (\mathbf{0},\,\boldsymbol{H}_t)\,, \quad \boldsymbol{H}_t=\begin{bmatrix} h_{b,t} & h_{be,t} & h_{bl,t} \\ h_{eb,t} & h_{e,t} & h_{el,t} \\ h_{lb,t} & h_{le,t} & h_{l,t} \end{bmatrix}$$

where Ω_{t-1} is the information set available in period $t-1$ and

$$\boldsymbol{z}_t=\begin{bmatrix} b_t \\ e_t \\ l_t \end{bmatrix};\quad \boldsymbol{\epsilon}_t=\begin{bmatrix} \epsilon_{b,t} \\ \epsilon_{e,t} \\ \epsilon_{l,t} \end{bmatrix};\quad \boldsymbol{h}_t=\begin{bmatrix} h_{b,t} \\ h_{e,t} \\ h_{l,t} \end{bmatrix};$$

$$\boldsymbol{\Gamma}=\begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{bmatrix};\quad \boldsymbol{\Psi}=\begin{bmatrix} \psi_{11} & \psi_{12} & \psi_{13} \\ \psi_{21} & \psi_{22} & \psi_{23} \\ \psi_{31} & \psi_{32} & \psi_{33} \end{bmatrix};\quad \boldsymbol{\Theta}=\begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \\ \theta_{31} & \theta_{32} & \theta_{33} \end{bmatrix}.$$

We use the asymmetric version of the BEKK model, introduced by Grier *et al.* (2004), for the variance equation. We choose the BEKK(1,1) specification, which is a multivariate extension of GARCH(1,1). Thus, the variance equation is

$$\boldsymbol{H}_t=\boldsymbol{C}'\boldsymbol{C}+\boldsymbol{B}'\boldsymbol{H}_{t-1}\boldsymbol{B}+\boldsymbol{A}'\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}'_{t-1}\boldsymbol{A}+\boldsymbol{D}'_{t-1}\boldsymbol{u}_{t-1}\boldsymbol{u}'_{t-1}\boldsymbol{D}$$

where $\boldsymbol{C}$, $\boldsymbol{B}$, $\boldsymbol{A}$, and $\boldsymbol{D}$ are 3×3 matrices with $\boldsymbol{C}$ being a triangular matrix to ensure positive definiteness of $\boldsymbol{H}$. This specification allows past volatilities, $\boldsymbol{H}_{t-1}$, as well as lagged values of $\boldsymbol{\epsilon}\boldsymbol{\epsilon}'$ to show up in estimating current volatility. The asymmetry vector is defined as $\boldsymbol{u}_{t-1}=\boldsymbol{\epsilon}_{t-1}\circ I_{\boldsymbol{\epsilon}<0}\boldsymbol{\epsilon}_{t-1}$. Since the $\boldsymbol{H}$ matrix is symmetric, the variance equation produces six unique equations modeling the dynamic variances of the cryptocurrency returns as well as the covariances between them.

5.1 Empirical Evidence

We use the quasi-Maximum Likelihood method to estimate the trivariate VARMA BEKK model of Bitcoin, Ethereum, and Litecoin. We obtain the initial conditions by performing several iterations using the simplex algorithm and use the BFGS (Broyden, Fletcher, Goldfarb and Shanno) estimation algorithm to maximize the non-linear log likelihood function. Table 5 reports the coefficients obtained, as well as key diagnostics for the standardized residuals, $\hat{z}_{jt}=\hat{\epsilon}_t/\sqrt{\hat{h}_t}$.

Based on the diagonal elements in $\boldsymbol{\Gamma}$, presented in Table 5, the own-mean spillovers, are statistically significant at the one percent level, providing evidence of an influence on current returns of each cryptocurrency market arising from their own past returns. The own-mean spillovers vary from a minimum of $\hat{\gamma}_{33}=-2.537$ (for Litecoin) to a maximum of $\hat{\gamma}_{22}=2.755$ (for Ethereum). Positive cross-mean spillover effects exist from Ethereum to Bitcoin ($\hat{\gamma}_{21}=0.158$) and Ethereum to Litecoin ($\hat{\gamma}_{23}=0.194$).

There are spillover effects in the moving average terms as well. All of the three cryptocurrencies are not only directly affected by the news from their own market (see the statistical significance of the diagonal terms in $\mathbf{\Theta}$) but also indirectly affected by news generated from the other cryptocurrencies (see the statistical significance of the off diagonal terms in $\mathbf{\Theta}$). Bitcoin and Litecoin returns have a positive relationship with shocks originating in their own market, with coefficients of $\hat{\theta}_{11} = 0.857$ and $\hat{\theta}_{33} = 2.533$, respectively, while Ethereum returns are negatively related to the shocks originated in its own market ($\hat{\theta}_{22} = -2.678$). Bitcoin receives spillover effects form both Ethereum and Litecoin. For example, an unexpected increase of one percent in the Litecoin price in the previous period is associated with a 1.867 percent increase in the Bitcoin price. Similarly, an unexpected one percent increase in the Ethereum price in the previous period is associated with a -0.149 percent decrease in the Bitcoin price. We find that Ethereum is heavily affected by news in the Bitcoin ($\hat{\theta}_{21} = 5.473$) and Litecoin ($\hat{\theta}_{31} = 37.772$) markets.

Most of the estimates of the GARCH-in-mean coefficients are not statistically significant with the exception of $\hat{\psi}_{12}$. One possible explanation for the insignificant GARCH-in-mean effect could be that the cryptocurrency market is subject to the influence of other financial markets, thus the individual explaining power of the volatility within the cryptocurrency market on the returns is weakened accordingly. We further explore this possibility in the next section. We note that the spillover effects which we find in the autoregressive coefficients, moving-average coefficients, and the GARCH-in-mean coefficients, are rather asymmetric in terms of the sign and magnitude of the coefficients especially between Bitcoin and Ethereum, as well as between Ethereum and Litecoin. This asymmetry could be found once we compare the coefficients which lay on the off-diagonal of the $\mathbf{\Gamma}$ and $\mathbf{\Theta}$ matrices. For example, $\hat{\theta}_{12}$ implies that the Bitcoin return decreases by 0.149 percent with an increase in the Ethereum price; however, an unexpected increase of Bitcoin returns increases the Ethereum return ($\hat{\theta}_{21}$). Therefore, the spillover effects of shocks between the Bitcoin and Ethereum markets are asymmetric. We observe similar asymmetry between Ethereum and Litecoin. Moreover, this pattern also exists when we compare the estimates of autoregressive coefficients, $\hat{\gamma}_{12}$ and $\hat{\gamma}_{21}$, $\hat{\gamma}_{23}$, and $\hat{\gamma}_{32}$.

Turning now to the variance equation, own-volatility shocks vary from $\hat{a}_{33}^2 = (0.253)^2$ (for Litecoin) to $\hat{a}_{22}^2 = 0.414^2$ (for Ethereum), indicating the presence of ARCH effects. This means that the past shocks arising from the Ethereum market will have the strongest impact on its own future market volatility. Based on the magnitudes of the estimated cross-volatility coefficients, $\hat{a}_{ij}, i \neq j$, innovations in all of the three cryptocurrency markets affect the volatility of all the other markets, except for Bitcoin and Litecoin. The shocks in Litecoin increase the volatility of Ethereum returns by $\hat{a}_{23}^2 = (-0.028)^2$, but do not influence the volatility of Bitcoin. Overall, it appears that the lagged cryptocurrency-specific shocks (ARCH effects) generally contribute to each other's market volatility in a recursive way.

All the estimated coefficients of the variance-covariance matrix are statistically significant, indicating the presence of high volatility persistence. The lowest value for the own-volatility spillover effect belongs to Ethereum, $\hat{b}_{22}^2 = (-0.693)^2$, and the highest one belongs to the Litecoin market, $\hat{b}_{33}^2 = 0.855^2$. This implies that the past volatility in the Litecoin market will have the strongest impact on its own future volatility compared to the other two markets. The nonzero off-diagonal coefficients of the $\boldsymbol{B}$ matrix ($\hat{b}_{ij}, i \neq j$ for all i and j) provide further evidence for the presence of high and positive volatility spillovers across these three well-integrated cryptocurrency markets. Interestingly, the past volatility of Ethereum affects Bitcoin by 0.930^2 and Litecoin by $(0.730)^2$, making it the most influential cryptocurrency among the three leading cryptocurrencies in this retrospect.

Finally, the $\boldsymbol{D}$ matrix presents the asymmetric ARCH effects in the cryptocurrency markets. The diagonal coefficients of the $\boldsymbol{D}$ matrix suggest that negative shocks (bad news) to cryptocurrency returns are associated with more volatility in all the cryptocurrency markets, compared with positive shocks (good news).

Panel B of Table 5 presents the normality test on the standardized residuals of the model. Both the Q statistics and the adjusted Q^2 statistics show that the null hypothesis of no autocorrelation cannot be rejected at the 1 percent level for various lags of up to 25. Thus, we conclude that there is no significant amount of serial correlation left in the system residuals compared to the original return series. This provides further support for the VARMA BEKK model as it absorbs a great deal of inertia of the ARCH and GARCH effects present in the original return series of the cryptocurrencies.

Overall, the trivariate VARMA BEKK model shows significant spillover effects among the three cryptocurrencies, including spillovers from surprise return changes in one cryptocurrency to the return volatility of another cryptocurrency.

6 Spillovers across Financial Markets

So far we discussed internal cryptocurrency interdependencies. Now we turn to an investigation of interdependence between the cryptocurrency market and other financial markets. As already noted, the cryptocurrency markets have grown steadily in both volume and value in recent years. This growth has raised the risks of the financial system. We will explore the diversification and hedging benefits of cryptocurrencies through an examinination of their spillover effects across financial markets. We use the same GARCH framework to study the conditional volatility dynamics along with interlinkages and conditional correlations between the cryptocurrency and stock and bond markets in order to obtain a better understanding of what elements of the financial markets cryptocurrency is sensitive to. Also, if cryptocurrency is uncorrelated with other types of assets, then it is still an important feature in an era of globalization in which correlations increased dramatically among most asset types, since cryptocurrency could be a safe haven in the asset optimization process.

Another major thrust of our effort is to examine the relation between non-cryptocurrency economic variables and cryptocurrency returns. However, because of the smoothing and averaging characteristics of most macroeconomic time series, in short holding periods, these series cannot be expected to capture all the information available to the market in the same period. Stock prices, on the other hand, respond very quickly to public information. The effect of this is to guarantee that cryptocurrency market returns will be, at best, weakly related and very noisy relative to innovations in macroeconomic factors. Consequently, the main focus of this section is to apply the GARCH framework to answer the following questions: i) how do cryptocurrency returns behave compared to the stock and bond markets? and ii) does cryptocurrency have any hedging capabilities as the traditional financial assets do, with its price reacting to and influencing other assets in financial markets?

We use an interest rate series and a stock market index to investigate the spillover effects of the cryptocurrency market to other financial markets. For the interest rate series (i_t), we use the 3-month Treasury bill rate, obtained from the Federal Reserve Economic Database (FRED) maintained by the Federal Reserve Bank of St. Louis. The SP 500 index (s_t) is obtained from the New York Exchange. The series for the cryptocurrency index (c_t) that we use is the Cryptocurrency Index 30 (see https://cci30.com/). Launched on January 1, 2017, this price index is a weighted average of the 30 largest cryptocurrencies by market capitalization, and therefore measures the overall growth, daily and long-term movement of the blockchain sector. The cruptocurrency and stock markets returns are calculated as before as $r_t = 100 \times (\log p_t - \log p_{t-1})$, where $r_t = c_t$, s_t, representing the returns of Cryptocurrency Index 30 and SP500, respectively. We use the 3-month Treasury bill rate as the interest rate series in our empirical analysis.

Table 6 shows the statistics of daily returns of cryptocurrencies, stocks, and interest rate. All the returns are nonnegative. The volatility of cryptocurrency is more than five times higher than that of traditional asset classes, with a standard deviation of 5.0 percent. It can also be noticed that all the financial returns are leptokurtic, with cryptocurrency exhibiting the highest excess kurtosis. Moreover, the returns of both cryptocurrency and stocks are negatively skewed, indicating that the two have a longer left tail, and that large negative returns are more common than large positive returns. The departure from normality for all the three return series is also confirmed by the Jarque-Bera test statistic, which rejects the null hypothesis of normally distributed returns for all the three return series.

Figures 6 and 7 visually show that the Cryptocurrency Index 30 is much more volatile than the SP500 and the interest rate. Cryptocurrency Index 30 dived in early 2018; in contrast, the returns of SP500 and the 3-month Treasury bill are steadily rising during the whole sample period. Table 7 shows the positive correlation between cryptocurrencies and financial markets. Moreover, DF-GLS and KPSS tests show a unit root in the logarithm of the indexes. Thus, in our specification we use the returns of the Cryptocurrency Index 30 and SP500 and the interest rate (results available upon request). In the following section we estimate the return evolution and spillover effects across the cryptocurrency market and the U.S. stock and bond markets using the trivariate VARMA BEKK model discussed in the previous section.

6.1 Empirical Evidence for the United States

The main limitation the literature has faced in measuring the propagation channels of the spillover effects has been the endogeneity of cryptocurrency prices, even at daily frequencies. Clearly, if the cryptocurrency is well integrated in the financial market, macroeconomic shocks such as shocks to productivity, monetary policy, inflation expectations, and risk premia have an effect on all cryptocurrency prices. In this section, we estimate the propagation of shocks by modeling the returns of cryptocurrency, stocks, and bonds with a trivariate VARMA BEKK model, and then estimate the contemporaneous financial transmission coefficients.

The structure of the model assumes that contemporaneous and lagged shocks in the stock and bond markets can affect cryptocurrency returns, with the residuals as the unanticipated innovations in the economic factors. It helps us investigate if cryptocurrency has any possibilities in risk management and portfolio analysis, thus providing further information regarding the capabilities of cryptocurrency in the financial marketplace. When stocks or bonds exhibit extreme negative returns, investors may buy more cryptocurrency and thus bid up the price of cryptocurrency. Thus cryptocurrency could serve as a potential hedge. If the price of cryptocurrency is not affected, then it suggests investors neither purchase nor sell cryptocurrency in such adverse market conditions, and cryptocurrency could serve as a potential safe haven. Whether cryptocurrency is a hedge or a safe haven asset for stocks and bonds is tested via the coefficients in the $\mathbf{\Gamma}$, $\mathbf{\Theta}$, and $\mathbf{\Psi}$ matrices in Table 8. For example, if $\hat{\gamma}_{12}$ and $\hat{\gamma}_{13}$ are zero or negative, it implies that cryptocurrency is a hedge for stocks (bonds) since the assets are uncorrelated or negatively correlated with each other on average.

In the mean equation, $\hat{\gamma}_{12} = 6.233$ and it is statistically significant, suggesting that the return of cryptocurrency is positively affected by the returns in the stock market, thus creating a possibility for investors to diversify their market portfolio. It indicates that when the returns in the stock market increase, investors tend to increase the investment on cryptocurrency to optimize their portfolio, and thus bid up the price of cryptocurrency. On the other hand, cryptocurrency can be used as a safe haven against the bond market as $\hat{\gamma}_{13}$ is not statistically significant, suggesting that when the returns in the bond market are extremely low, investors could use cryptocurrency as safe haven since its expected return is not affected by the bond market.

When we look at the moving average coefficients in the matrix $\mathbf{\Theta}$, $\hat{\theta}_{12} = -6.245$ suggesting that past shocks in the stock markets have spillover effects on the cryptocurrency market. However, the shocks in the cryptocurrency market do not spillover to the traditional markets, as $\hat{\theta}_{21}$ and $\hat{\theta}_{31}$ are not statistically significant. For the GARCH-in-mean terms, we notice that there is a negative risk premium on the stock market ($\hat{\psi}_{12} = -1.112$). One possible explanation for the negative risk premia could be the dominance of income effects over substitution effects of stocks, leading to a positive relationship between the return and volatility. In general, we find evidence of the shock transmission effects between the cryptocurrency market and the stock markets. Particularly, we find that the conditional covariances with stocks are variable over time and are a statistically significant determinant of the time-varying cryptocurrency risk premia.

In the variance equation, when we look at the impact of the financial markets on the cryptocurrency market, the shocks in the bond market have a very slight effect on volatility in the cryptocurrency market ($\hat{a}_{13} = -0.001$). However, the shock ($\hat{a}_{12}$) and volatility ($\hat{b}_{12}$) in the stock market do not affect the volatility in the cryptocurrency market. Overall, cryptocurrency seems to possess allied hedging capabilities against the stock market and is a safe haven for the bond market. Attracting market and economic influence, cryptocurrency may become a more balanced investment vehicle, driven both internally and externally.

The preference of investors for cryptocurrency can be explained by the fact that, unlike conventional currencies, cryptocurrency is fully decentralized and independent of any central authorities; if the financial system is not working well or is under threat, investors seek refuge in cryptocurrency, which is independent from the financial system and its underlying technology. According to Ciaian *et al.* (2016), cryptocurrency also has an investment attractiveness that is reflected in its increasing acceptance and trust, Moreover, the decreasing transaction costs and uncertainty for investors increase investment demand for cryptocurrency.

On the demand side, changes in cryptocurrency prices can induce changes in the marginal utility of real wealth, which perhaps is measured by real consumption changes, and will influence the pricing of the cryptocurrency; such effects could also show up as unanticipated changes in risk premia. On the production side, changes in the expected level of real production would affect the current real value of cryptocurrency. Insofar as the risk premium does not capture cryptocurrency mining production uncertainty, innovations in the rate of productive activity should have an influence on cryptocurrency returns through their impact on cash flows.

6.2 Robustness

Bitcoin has dominated more than a third of the cryptocurrency market over the years. Since its volatility and risk premium are of great interest to market participants, as well as policymakers, we take a further look at the spillover effects of Bitcoin on the financial markets using the same trivariate VARMA BEKK model. Overall, as shown in Table 9, we find strong bi-directional transmission and volatility linkages between Bitcoin and the fiancial markets, providing evidence that time-varying conditional correlations between the Bitcoin and other financial markets exist. Moreover, the magnitude of spillover effects between the Bitcoin market and the other financial markets is greater than that of the cryptocurrency index.

We find significant mean spillover effects across all the three markets as shown in Table 9. In the mean equation, the autoregressive coefficients in matrix $\mathbf{\Theta}$ and the moving average coefficients along the diagonal of the $\mathbf{\Theta}$ matrix are moderate and statistically significant, suggesting that the Bitcoin return, stock return, and interest rate series are consistent with a typical ARMA process. The off-diagonal elements of the $\mathbf{\Theta}$ matrix indicate the spillover effects of past shocks across the three markets. There is evidence of shock spillovers from each of the financial markets to the Bitcoin market. Particularly, an unexpected shock in the stock market in the previous period could increase the Bitcoin price by 23.611 percent, and an unexpected shock in the bond market will increase the Bitcoin price by 5.347 percent. It suggests that a positive shock to the traditional financial market may make investors more risk seeking and invest in alternative assets like Bitcoin, thus bidding up the price of Bitcoin. However, past Bitcoin price shocks affect the traditional financial market negatively ($\hat{\theta}_{21} = -1.416$ with a p-value of 0.000 and $\hat{\theta}_{31} = -0.091$ with a p-value of 0.000).

None of the coefficients in the GARCH-in-mean $\mathbf{\Psi}$ matrix are statistically significant except for $\hat{\psi}_{13} = 0.001$, suggesting that the expected interest rate is affected by the volatility in the Bitcoin market. We note that the spillover effects which we find in the autoregressive coefficients, moving-average coefficients, and the GARCH-in-mean coefficients, are rather asymmetric in terms of the magnitude of the coefficients. The shocks in the stock market have a stronger impact on the cryptocurrency market than the bond market, since $|\hat{\gamma}_{12}| > |\hat{\gamma}_{13}|$ and $\left|\hat{\theta}_{12}\right| > \left|\hat{\theta}_{13}\right|$.

Regarding volatility linkages, the "own-market" coefficients of Bitcoin, $\hat{a}_{11}$ and $\hat{b}_{11}$, are statistically significant and the estimates suggest a high degree of persistence. We find that previous bond market shocks have an impact on current Bitcoin volatility ($\hat{a}_{13} = -0.001$ with a p-value of 0.001), but the volatility and shock in the stock market do not have a statistically significant impact on the conditional volatility of Bitcoin. We also find that the volatility and shock in the Bitcoin market will increase the volatility in the bond market in a statistically significant way.

Our results point to additional risk management capabilities of the Bitcoin, that it may have some risk management capabilities against traditional financial assets. We find that shocks in the traditional financial markets are generally more influential on Bitcoin than on an average cryptocurrency. A plausible explanation for why the spillover effects vary is in part the integration of different cryptocurrencies with the financial market. Bitcoin is the most widely accepted and trusted cryptocurrency in the cryptocurrency market. As the other cryptocurrencies are getting more and more mature, their interconnectedness with the economy and financial markets may increase as it is now the case of Bitcoin.

7 International Evidence

We have estimated the financial transmission among cryptocurrency, equity, and bond markets in the United States. Now we use the same framework to analyze the nature of financial integration and the transmission channels within some other leading economies in the world: Germany, the United Kingdom, and Japan.

The purpose of this section is to determine the reduced form of the spillover effects between the cryptocurrency market and the domestic equities and interest rate markets, using a joint estimation technique in the context of a parameterized model of conditional variances. Particulary, the stock market equation may be interpreted as a proxy of domestic demand, in that a positive demand shock at home raises domestic equity prices. Alternatively, changes in equity prices may also be explained by supply shocks, such as productivity changes. The short-term interest rate can essentially be interpreted as the market's expectation about the course of monetary policy in the short to medium term. Of course, these interpretations are not clear-cut and may not exclude alternative interpretations and explanations.

The data covers the period from August 7, 2015 to April 27, 2019. The data source for all financial market series is Datastream. We use the 3-month government bond interest rate for the interest rate series. We use the DAX30 index, FTSE100 index, and NIKKEI 225 index for the equity markets in Germany, U.K., and Japan, respectively. All of the series exhibit the typical characteristics of heteroskedasticity, skewness, and excess kurtosis. The summary statistics in Table 10 show that all series have a kurtosis value in excess of that in a normal distribution. The Jarque-Bera test shows that all return series depart from the normal distribution. Table 11 clearly shows that there are correlations among cryptocurrency prices and the international financial markets in all three countries. Interestingly, the cryptocurrency returns are positively related with the interest rate in the U.K., but negatively related with the interest rate in Germany and Japan. Results from two-unit root tests, Augmented Dickey Fuller (ADF) and Phillips Perron (PP), indicate that all return series are stationary (these results are available upon request). There is also evidence of significant conditional heteroscedasticity, as suggested by the ARCH test.

The estimation results of the trivariate VARMA BEKK model in Tables 12 to 14 show that there are statistically significant spillover effects between cryptocurrency and the traditional financial markets in all the three leading economies. Furthermore, the linkages between the stock market and cryptocurrency market are stronger than those between the bond market and the cryptocurrency market.

For the spillover effects from financial markets to the cryptocurrency market, the past returns and shocks in the stock market have statistically significant spillover effects on the cryptocurrency market in all the three countries as can be seen in $\hat{\gamma}_{12}$ and $\hat{\theta}_{12}$ in Tables 12 to 14. For the spillover effects from the bond market to the cryptocurrency market, only in Germany, there are moderate spillover effects from the bond market to the cryptocurrency market as shown in Table 12 that $\hat{\gamma}_{31} = -7.056$ and $\hat{\theta}_{31} = -5.616$, but the shock spillover effects from the stock market are much stronger than those of the bond market, as $\left|\hat{\theta}_{12}\right| > \left|\hat{\theta}_{13}\right|$. The integration of the interest rate and cryptocurrency returns in Germany could be because of the fact that cryptocurrency is a legal tender in Germany, thus it is potentially subject to the market's expectation about the course of monetary policy in the short to medium term as reflected in the interest rate.

There are moderate spillover effects from the cryptocurrency market to the stock markets in Germany and the U.K. as shown in Tables 12 and 13 with $\hat{\gamma}_{21} = 1.514$ and -0.322, $\hat{\theta}_{21} = -1.514$ and 0.322, respectively. The spillover effects from the cryptocurrency market to the bond market are not statistically significant. Interestingly, it seems that the bond market in Japan is insulated from the cryptocurrency market, since none of $\hat{\gamma}_{13}$, $\hat{\theta}_{13}$ and $\hat{\psi}_{13}$ is statistically significant as shown Table 14.

In general, the results show that shocks in the stock market are more influential on cryptocurrency markets than those in the bond market in all the countries we study. Particularly, spillover effects from the cryptocurrency market to the equity market are most pronounced in Germany, and there are no statistically significant spillovers effects from the cryptocurrency market to the bond market. A plausible explanation for why the cryptocurrency effects vary across countries is in part that the integration of cryptocurrency varies in different countries; the United States is the largest economy in the world and the cryptocurrency is most used in the United States. Cryptocurrency is a medium of exchange recognized by the government in Germany and not subject to any tax on its capital gains. As the cryptocurrency market is growing more and more mature, and recognized by more countries, the interconnectedness of cryptocurrency with international financial markets may increase as it is now the case of Bitcoin in the United States. The integration of cryptocurrencies with the financial markets has increased as the cryptocurrency market evolved. When facing macroeconomic uncertainty, investors may choose a suitable cryptocurrency to adjust their asset portfolio based on their risk preference. In particular, as the cryptocurrency market capitalization increases, investors employ cryptocurrencies as a hedge against stocks.

8 Conclusion

The price of cryptocurrency and its return volatility are of great concern to market participants and policymakers. Being able to accurately forecast the volatility and predict its spillover effects carries direct implications for transactions in the market. Motivated by these considerations, we estimate univariate and multivariate volatility models for the returns of three cryptocurrencies, Bitcoin, Ethereum, and Litecoin using the most recent data. We use a trivariate VARMA BEKK model to identify the role of cryptocurrency in the financial market context, and contribute to the understanding of price and volatility in cryptocurrency market and its interdependence with other financial markets.

The internal cryptocurrency market study results provide strong evidence supporting the progress of cryptocurrency market integration and further support the findings by earlier studies on interdependencies within the cryptocurrency market. The cross financial markets study suggests that cryptocurrency can combine some of the advantages of both stocks and bonds in the financial markets and therefore be a useful tool for portfolio management, risk analysis, and market sentiment analysis. We find that the linkages between the cryptocurrency market and other financial markets are stronger in the countries where cryptocurrencies are more accepted and used. Attracting market and economic influence, cryptocurrency may become a more balanced investment vehicle, driven both internally and externally. It is interesting to observe that the established risk-reward mechanism of traditional financial markets seems to hold even for a highly dynamic and evolving cryptocurrency market. Most aspects of cryptocurrency are similar to financial assets as they react to similar variables in the GARCH models and possess similar hedging capabilities when react to good and bad news.

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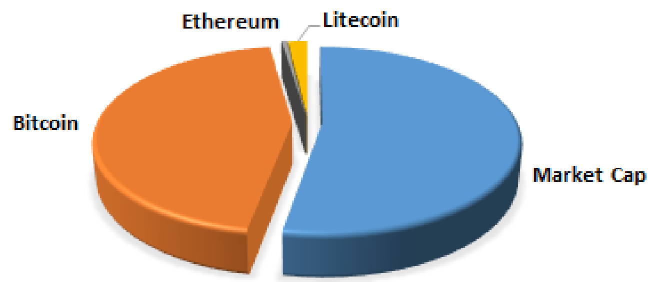
Figure 1. Market share of leading cryptocurrencies

2015:8:7



2019:4:27

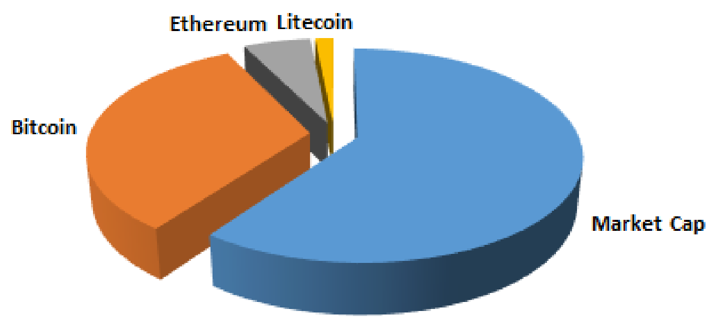


Table 2. Contemporaneous correlations of leading cryptocurrencies

	A. Logarithmic prices			B. Returns		
	Bitcoin	Ethereum	Litecoin	Bitcoin	Ethereum	Litecoin
Bitcoin	1	0.937	0.947	1	0.385	0.624
Ethereum	0.937	1	0.984	0.385	1	0.377
Litecoin	0.947	0.984	1	0.624	0.377	1

Sample period: Daily data 2015:8:7-2019:4:27 (T=1360).

Figure 2. Bitcoin price and return

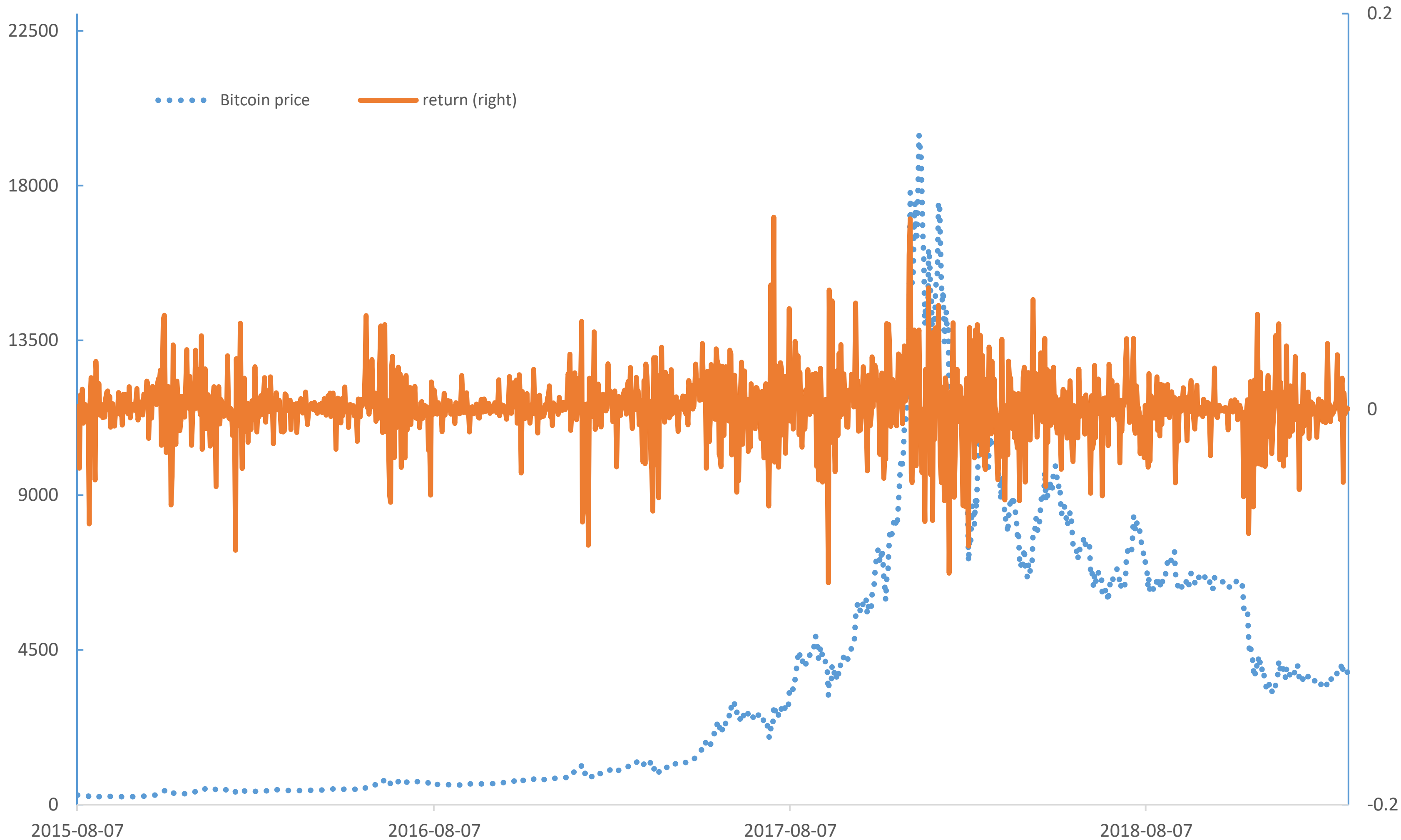


Figure 3. Ethereum price and return

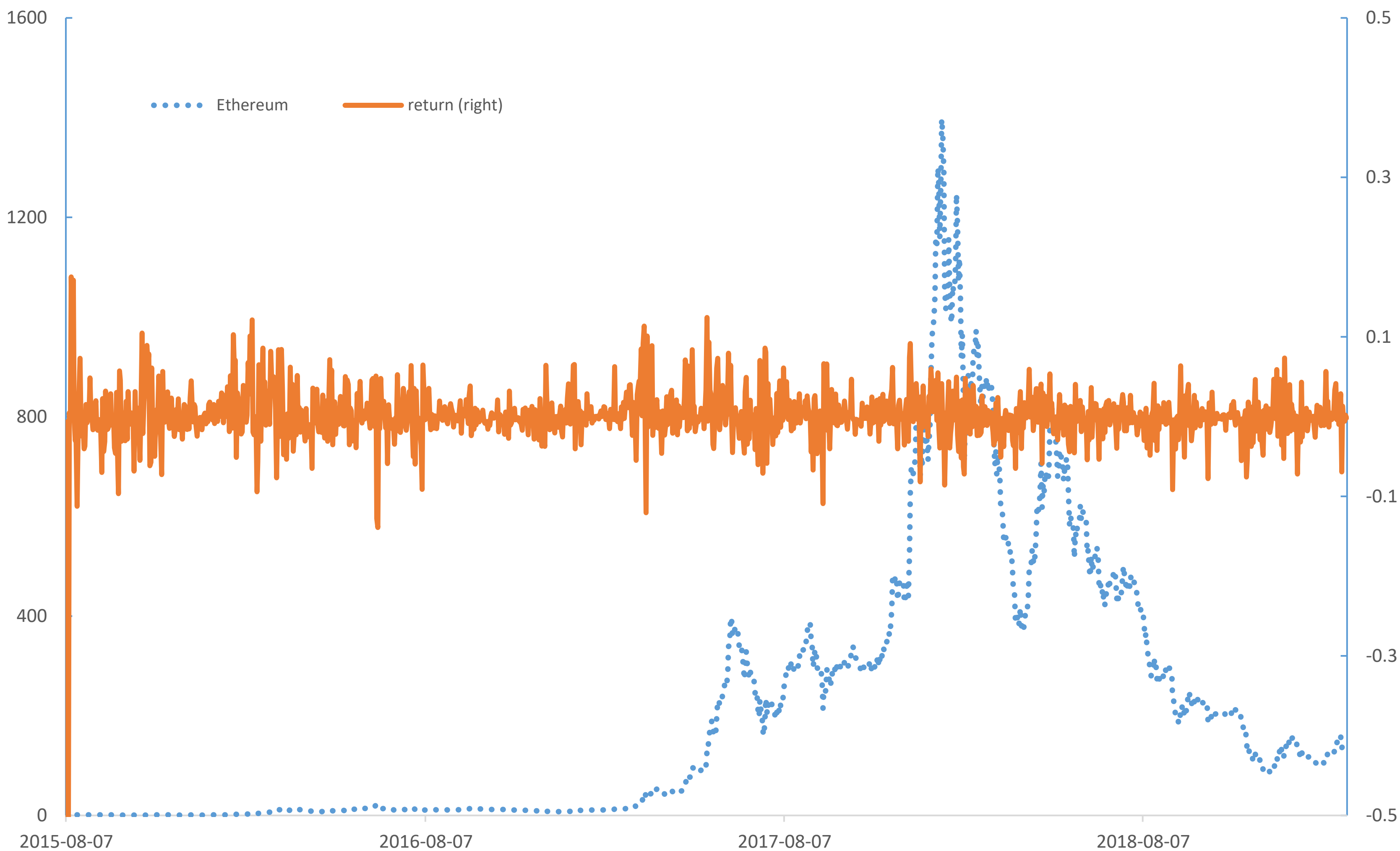


Figure 4. Litecoin Price and Return

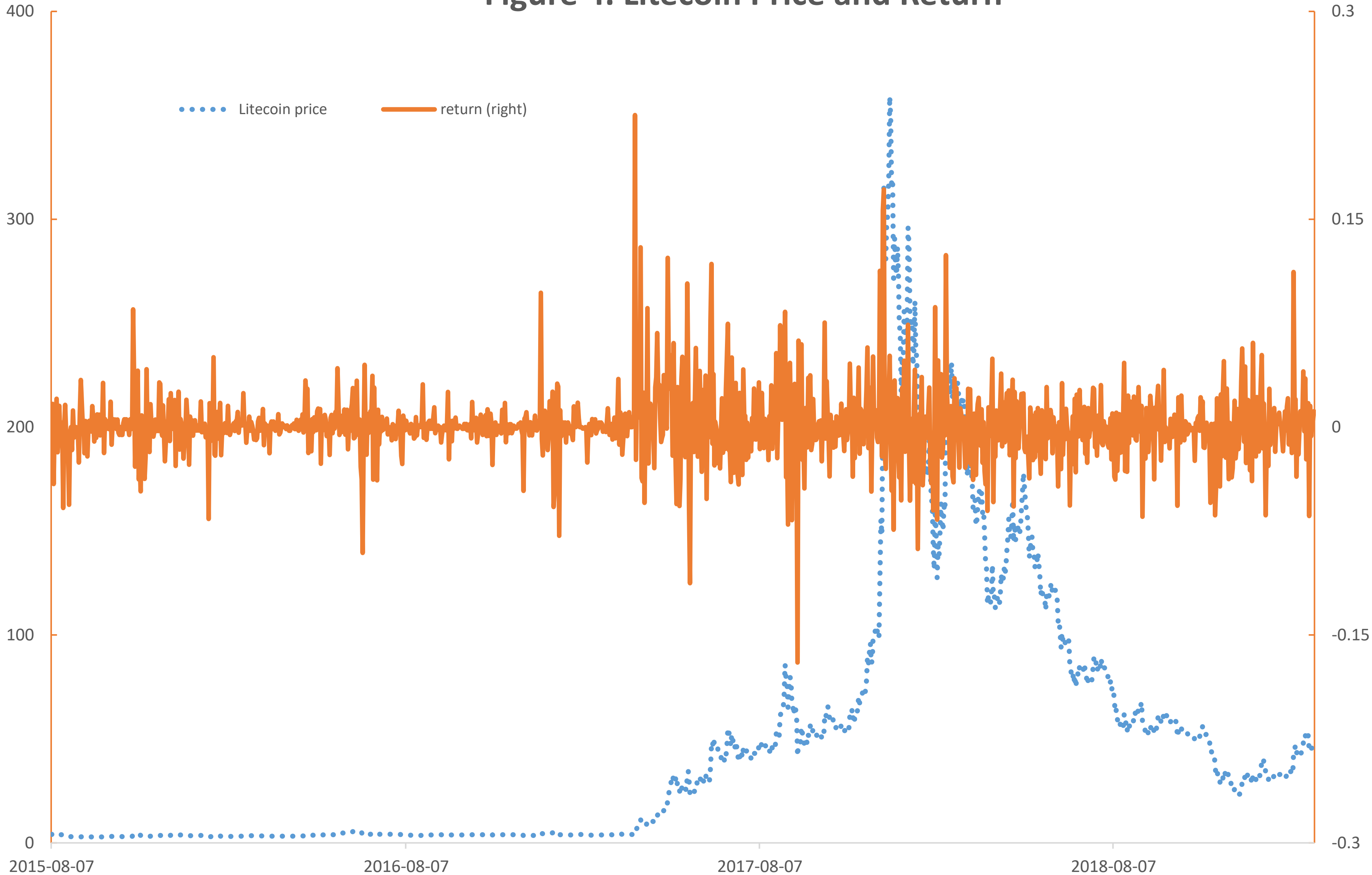


Table 2. Contemporaneous correlations of leading cryptocurrencies

	A. Logarithmic prices			B. Returns		
	Bitcoin	Ethereum	Litecoin	Bitcoin	Ethereum	Litecoin
Bitcoin	1	0.937	0.947	1	0.385	0.624
Ethereum	0.937	1	0.984	0.385	1	0.377
Litecoin	0.947	0.984	1	0.624	0.377	1

Sample period: Daily data 2015:8:7-2019:4:27 (T=1360).

Figure 5. Prices of Bitcoin, Ethereum and Litecoin

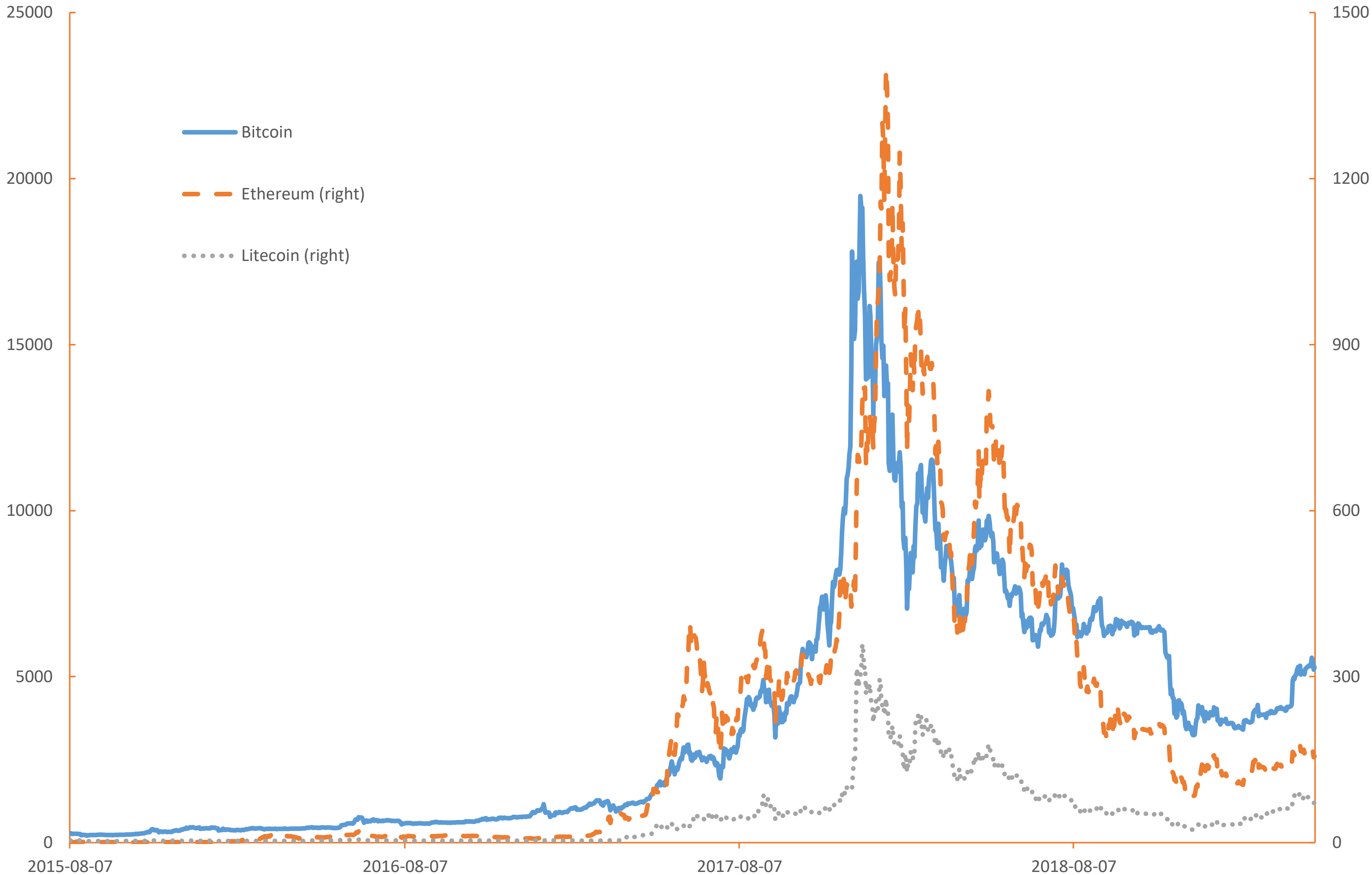


Table 3. Unit root and stationary tests

Series	ADF	PP	KPSS	Decision
		A. Logarithmic prices		
Bitcoin	-0.737	-0.767	3.886	$I(1)$
Ethereum	-0.593	-0.667	4.409	$I(1)$
Litecoin	-1.206	-1.236	3.198	$I(1)$
		B. Returns		
Bitcoin	-36.520	-36.565	0.154	$I(0)$
Ethereum	-35.076	-35.112	0.158	$I(0)$
Litecoin	-36.369	-36.416	0.193	$I(0)$

Sample period: Daily data 2015:8:7-2019:4:27 (T=1360). The 1% and 5% critical values are -3.970 and -3.415 for the ADF test, for -3.970 and -3.416 for the PP test, and 0.216 and 0.146 for the KPSS test, respectively.

Table 4. Univariate GARCH-in-mean models of Bitcoin, Ethereum and Litecoin

Coefficients	Bitcoin	Ethereum	Litecoin
	A. Conditional mean equation		
constant	0.001 (0.897)	-0.010 (0.004)	0.001 (0.902)
r_{t-1}	0.522 (0.000)	0.106 (0.001)	-0.717 (0.005)
r_{t-2}	-0.834 (0.000)	0.000 (0.000)	0.039 (0.875)
ϵ_{t-1}	-0.496 (0.000)	-0.086 (0.045)	0.676 (0.010)
ϵ_{t-2}	0.838 (0.000)	0.209 (0.000)	0.039 (0.874)
Dummy	0.004 (0.105)	0.008 (0.010)	0.004 (0.401)
h_t	-0.577 (0.592)	2.408 (0.005)	-1.435 (0.007)
	B. Conditional variance equation		
constant	0.000 (0.000)	0.001 (0.000)	0.000 (0.000)
ϵ_{t-1}	0.221 (0.000)	0.242 (0.000)	0.098 (0.000)
ϵ_{t-2}	0.135 (0.000)	0.196 (0.000)	0.049 (0.000)
h_{t-1}	-0.089 (0.000)	-0.224 (0.000)	-0.055 (0.000)
h_{t-2}	0.781 (0.000)	0.651 (0.000)	0.925 (0.000)
$\epsilon^2_{t-1}I_{t-1}$	-0.129 (0.000)	-0.067 (0.142)	-0.100 (0.000)
$\epsilon^2_{t-2}I_{t-2}$	0.005 (0.834)	0.007 (0.854)	-0.019 (0.224)
Dummy	-0.000 (0.000)	-0.000 (0.376)	-0.000 (0.088)
	C. Standardized residual diagnostics		
$\hat{\epsilon}$ mean	0.037	-0.003	0.032
$\hat{\epsilon}$ standard deviation	1.000	1.002	0.999
$Jarque-Bara$	406.409 (0.000)	387.051 (0.000)	402.390 (0.000)
$Q(8)$	11.930 (0.154)	13.030 (0.222)	12.960 (0.226)
$Q^2(8)$	4.642 (0.914)	2.158 (0.995)	1.427 (0.999)
Log likelihood	2703.740	1921.582	2127.278
AIC	-3.963	-2.810	-3.113

Sample period: Daily data 2015:8:7-2019:4:27 (T=1360). Numbers in parentheses are p-values.

Table 5. Spillover effects within the cryptocurrency market

A. Conditional mean equation

$$\mathbf{a}=\begin{bmatrix} 0.717\ (0.001) \\ 9.918\ (0.001) \\ 0.949\ (0.001) \end{bmatrix};\ \mathbf{\Gamma}=\begin{bmatrix} -0.867\ (0.000) & 0.158\ (0.000) & -1.866\ (0.000) \\ -5.529\ (0.000) & 2.755\ (0.000) & -37.821\ (0.000) \\ -0.580\ (0.000) & 0.194\ (0.000) & -2.537\ (0.000) \end{bmatrix};$$

$$\mathbf{\Psi}=\begin{bmatrix} 0.046\ (0.247) & 0.014\ (0.004) & -0.027\ (0.218) \\ -0.272\ (0.420) & 0.250\ (0.124) & -0.198\ (0.475) \\ -0.020\ (0.531) & 0.019\ (0.161) & -0.020\ (0.411) \end{bmatrix};\ \mathbf{\Theta}=\begin{bmatrix} 0.857\ (0.000) & -0.149\ (0.000) & 1.867\ (0.000) \\ 5.473\ (0.000) & -2.678\ (0.000) & 37.772\ (0.000) \\ 0.577\ (0.000) & -0.189\ (0.000) & 2.533\ (0.000) \end{bmatrix}.$$

B. Residual diagnostics

	Mean	Variance	$Q(5)$	$Q^2(5)$	$Q(25)$	$Q^2(25)$
z_{b_t}	−0.004	0.971	9.540 (0.089)	6.513 (0.259)	40.570 (0.025)	13.719 (0.966)
z_{e_t}	−0.020	0.974	5.530 (0.354)	2.378 (0.795)	31.730 (0.166)	15.161 (0.938)
z_{l_t}	−0.028	0.972	1.460 (0.918)	0.744 (0.980)	22.880 (0.584)	10.263 (0.996)

C. Conditional variance-covariance structure

$$\boldsymbol{C}=\begin{bmatrix} 0.440\ (0.000) & 0.980\ (0.000) & 0.276\ (0.010) \\ & 0.197\ (0.577) & -0.232\ (0.002) \\ & & 0.018\ (0.858) \end{bmatrix};\ \boldsymbol{B}=\begin{bmatrix} 0.849\ (0.000) & 0.930\ (0.000) & -0.135\ (0.000) \\ 0.168\ (0.000) & -0.693\ (0.000) & 0.238\ (0.000) \\ -0.073\ (0.000) & 0.730\ (0.000) & 0.855\ (0.000) \end{bmatrix};$$

$$\boldsymbol{A}=\begin{bmatrix} 0.301\ (0.000) & -0.117\ (0.010) & -0.016\ (0.664) \\ -0.023\ (0.016) & 0.414\ (0.000) & -0.028\ (0.029) \\ 0.000\ (0.993) & -0.060\ (0.048) & 0.253\ (0.000) \end{bmatrix};\ \boldsymbol{D}=\begin{bmatrix} -0.231\ (0.000) & -0.149\ (0.148) & -0.556\ (0.000) \\ 0.058\ (0.000) & -0.196\ (0.006) & 0.102\ (0.000) \\ 0.198\ (0.000) & 0.409\ (0.000) & 0.431\ (0.000) \end{bmatrix}.$$

Model: Trivariate VARMA, GARCH-in-mean, asymmetric BEKK model for Bitcoin, Ethereum and Litecoin. Sample period: 2015:8:7 to 2019:4:27.(T=1360). Numbers in parentheses are p-values.

Table 6. Summary Statistics of cryptocurrency and US financial markets

	Mean	Standard deviation	Skewness	Kurtosis	Normality
			A. Log levels		
Crypto Index	6.930	1.662	−0.113 (0.159)	−1.486 (0.000)	87.349 (0.000)
SP500	7.781	0.129	−0.165 (0.040)	−1.292 (0.000)	68.749 (0.000)
			B. Returns		
Crypto Index	0.368	5.045	−0.610 (0.000)	4.714 (0.000)	915.777 (0.000)
SP500	0.037	0.876	−0.511 (0.000)	4.259 (0.000)	740.908 (0.000)
Interest rate	1.070	0.807	0.373 (0.000)	−1.328 (0.000)	89.745 (0.000)

Sample period: Daily data 2015:8:7-2019:4:27 (T=928).

Figure 6. Cryptocurrency Index 30 and SP 500

Figure 7. Cryptocurrency Index 30 and interest rate

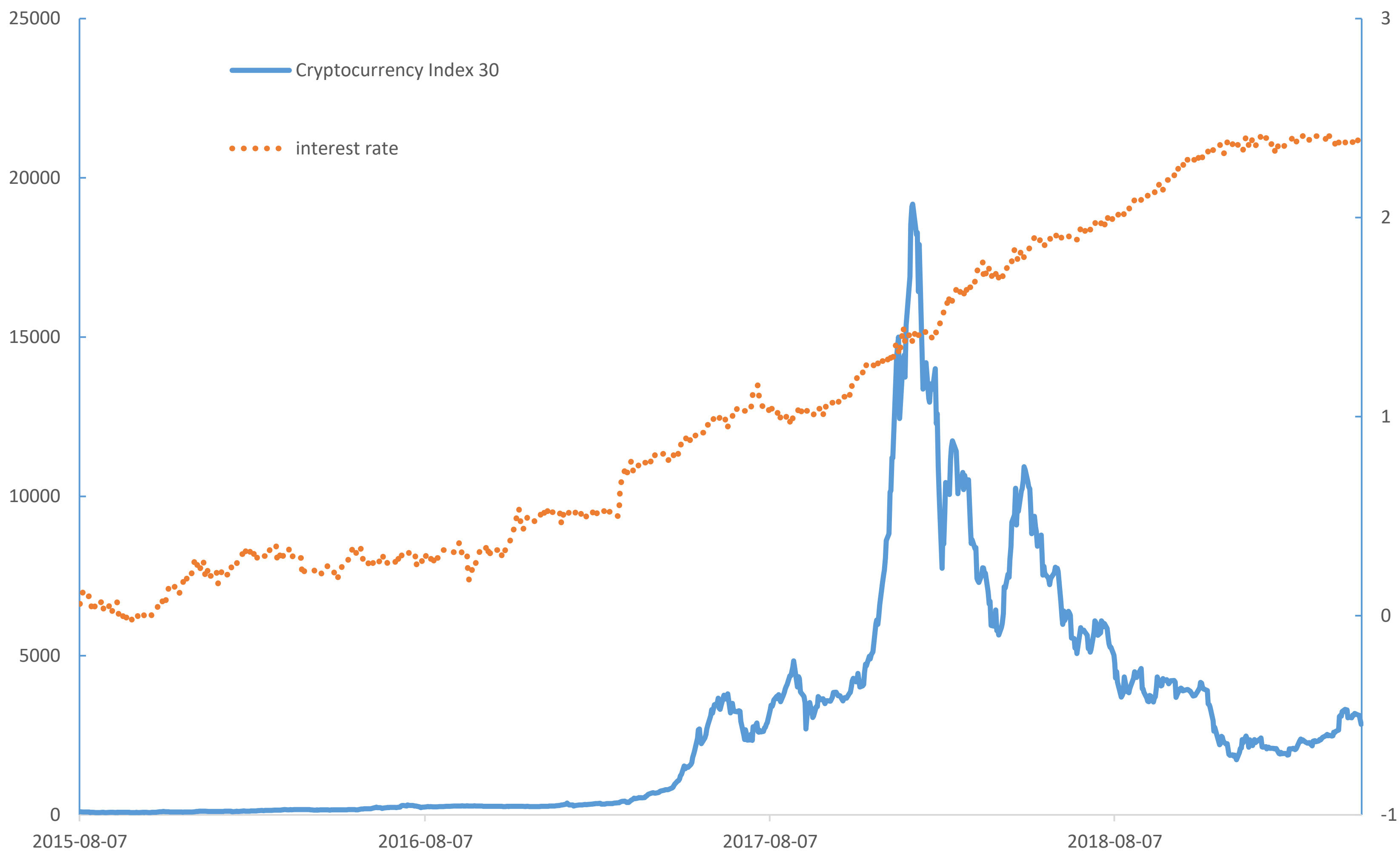


Table 7. Contemporaneous correlations of cryptocurrency and US financial markets

	A. Levels			B. Returns		
	Crypto Index	SP500	Interest rate	Crypto Index	SP500	Interest rate
Crypto Index	1	0.976	0.892	1	0.073	0.018
SP500	0.976	1	0.808	0.073	1	0.035
Interest rate	0.892	0.808	1	0.018	0.035	1

Sample period: Daily data 2015:8:7-2019:4:27 (T=928). The Crypto Index and the SP500 are in logarithmic levels.

Table 8. Cryptocurrency Spillover effects to US financial markets

A. Conditional mean equation

$$\mathbf{a} = \begin{bmatrix} 0.110\ (0.929) \\ -0.018\ (0.950) \\ 0.021\ (0.139) \end{bmatrix}; \mathbf{\Gamma} = \begin{bmatrix} 0.452\ (0.484) & 6.233\ (0.009) & -0.305\ (0.405) \\ -0.067\ (0.638) & -1.156\ (0.075) & -0.020\ (0.789) \\ -0.019\ (0.117) & -0.093\ (0.036) & 0.992\ (0.000) \end{bmatrix};$$

$$\mathbf{\Psi} = \begin{bmatrix} 0.153\ (0.190) & -1.112\ (0.021) & -0.590\ (0.993) \\ -0.014\ (0.453) & 0.223\ (0.074) & 5.439\ (0.693) \\ 0.002\ (0.031) & 0.003\ (0.679) & -0.579\ (0.349) \end{bmatrix}; \mathbf{\Theta} = \begin{bmatrix} -0.454\ (0.482) & -6.245\ (0.008) & 4.261\ (0.248) \\ 0.068\ (0.632) & 1.163\ (0.073) & 0.649\ (0.328) \\ 0.019\ (0.114) & 0.093\ (0.038) & -0.115\ (0.000) \end{bmatrix}$$

B. Residual diagnostics

	Mean	Variance	$Q(5)$	$Q^2(5)$	$Q(25)$	$Q^2(25)$
z_{c_t}	0.052	0.991	21.890 (0.001)	1.580 (0.904)	50.340 (0.002)	18.265 (0.831)
z_{s_t}	−0.042	1.003	1.550 (0.907)	2.153 (0.828)	16.190 (0.909)	10.725 (0.994)
z_{i_t}	0.013	0.984	40.380 (0.000)	21.953 (0.001)	149.280 (0.000)	39.831 (0.030)

C. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} 0.420\ (0.001) & 0.066\ (0.210) & -0.008\ (0.016) \\ & 0.170\ (0.000) & 0.008\ (0.029) \\ & & -0.000\ (1.000) \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 0.923\ (0.000) & -0.000\ (0.807) & -0.000\ (0.000) \\ -0.120\ (0.086) & 0.891\ (0.000) & 0.001\ (0.192) \\ 17.745\ (0.001) & -2.504\ (0.126) & 0.777\ (0.000) \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} -0.379\ (0.000) & -0.007\ (0.173) & -0.001\ (0.000) \\ -0.192\ (0.469) & -0.042\ (0.538) & 0.005\ (0.000) \\ 2.457\ (0.672) & 1.578\ (0.139) & -0.069\ (0.259) \end{bmatrix}; \boldsymbol{D} = \begin{bmatrix} 0.031\ (0.552) & 0.010\ (0.047) & -0.000\ (0.399) \\ 0.412\ (0.035) & 0.504\ (0.000) & 0.000\ (0.812) \\ -0.822\ (0.935) & 0.542\ (0.789) & -0.077\ (0.509) \end{bmatrix}.$$

Model: Trivariate VARMA, Garch-in-mean, asymmetric BEKK model for Cryptocurrency Index 30, SP 500 and interest rate in the US. Sample period: 2015:8:7 to 2019:4:27 (T=928). Numbers in parentheses are p-values.

Table 9. Bitcoin Spillover effects to US financial markets

A. Conditional mean equation

$$\mathbf{a} = \begin{bmatrix} 3.675\ (0.000) \\ -0.784\ (0.000) \\ -0.067\ (0.000) \end{bmatrix}; \mathbf{\Gamma} = \begin{bmatrix} -5.504\ (0.000) & 23.611\ (0.000) & -1.775\ (0.000) \\ 1.418\ (0.000) & 5.924\ (0.000) & 0.388\ (0.000) \\ 0.090\ (0.000) & 0.412\ (0.000) & 1.025\ (0.000) \end{bmatrix};$$

$$\mathbf{\Psi} = \begin{bmatrix} 0.019\ (0.465) & 0.177\ (0.282) & -0.828\ (0.849) \\ -0.006\ (0.331) & -0.046\ (0.250) & 0.269\ (0.796) \\ 0.001\ (0.063) & 0.000\ (0.832) & 0.473\ (0.012) \end{bmatrix}; \mathbf{\Theta} = \begin{bmatrix} 5.494\ (0.000) & 23.628\ (0.000) & 5.347\ (0.000) \\ -1.416\ (0.000) & -5.927\ (0.000) & -1.479\ (0.005) \\ -0.091\ (0.000) & -0.412\ (0.000) & -0.200\ (0.000) \end{bmatrix}$$

B. Residual diagnostics

	Mean	Variance	$Q(5)$	$Q^2(5)$	$Q(25)$	$Q^2(25)$
z_{b_t}	0.031	0.984	17.640 (0.003)	3.019 (0.697)	29.720 (0.235)	14.540 (0.951)
z_{s_t}	−0.001	0.991	1.210 (0.943)	1.780 (0.879)	15.610 (0.926)	9.860 (0.997)
z_{i_t}	−0.010	0.986	10.830 (0.055)	8.610 (0.126)	39.290 (0.034)	25.443 (0.438)

C. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} 0.885\ (0.000) & -0.005\ (0.911) & 0.007\ (0.010) \\ & 0.103\ (0.030) & -0.004\ (0.619) \\ & & -0.015\ (0.000) \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 0.900\ (0.000) & -0.003\ (0.254) & -0.000\ (0.353) \\ -0.016\ (0.803) & 0.885\ (0.000) & -0.000\ (0.746) \\ -24.963\ (0.000) & -7.574\ (0.000) & -0.024\ (0.885) \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} 0.372\ (0.000) & -0.003\ (0.545) & -0.001\ (0.001) \\ 0.437\ (0.001) & 0.112\ (0.022) & 0.002\ (0.031) \\ -10.038\ (0.032) & 1.670\ (0.055) & 0.401\ (0.000) \end{bmatrix}; \boldsymbol{D} = \begin{bmatrix} -0.090\ (0.170) & -0.021\ (0.000) & -0.001\ (0.014) \\ 0.092\ (0.618) & -0.555\ (0.000) & 0.000\ (0.814) \\ -5.860\ (0.521) & 0.548\ (0.687) & -0.425\ (0.000) \end{bmatrix}.$$

Model: Trivariate VARMA, GARCH-in-mean, asymmetric BEKK model for Bitcoin, SP 500 and interest rate in the US.
Sample period: 2015:8:7 to 2019:4:27 (T=928). Numbers in parentheses are p-values.

Table 10. Summary Statistics of International Financial Markets

	Mean	Standard deviation	Skewness	Kurtosis	Normality
Germany					
A. Logarithmic levels					
Crypto Index	6.889	1.675	−0.061 (0.454)	−1.504 (0.000)	86.978 (0.000)
DAX30	9.343	0.101	−0.311 (0.000)	−1.069 (0.000)	58.392 (0.000)
B. Returns					
Crypto Index	0.358	5.040	−0.591 (0.000)	4.644 (0.000)	876.578 (0.000)
DAX30	−0.001	1.115	−0.447 (0.000)	3.195 (0.000)	420.166 (0.000)
Interest rate	−0.672	0.182	0.406 (0.000)	−0.238 (0.143)	27.315 (0.000)
United Kingdom					
A. Logarithmic levels					
Crypto Index	6.924	1.661	−0.113 (0.156)	−1.486 (0.000)	88.540 (0.000)
FTSE100	8.849	0.080	−0.722 (0.000)	−0.641 (0.000)	97.822 (0.000)
B. Returns					
Crypto Index	0.365	5.043	−0.590 (0.000)	4.557 (0.000)	866.878 (0.000)
FTSE100	0.010	0.894	−0.166 (0.038)	2.705 (0.000)	290.542 (0.000)
Interest rate	0.424	0.188	0.324 (0.000)	−1.060 (0.000)	60.407 (0.000)
Japan					
A. Logarithmic levels					
Crypto Index	6.919	1.666	−0.100 (0.224)	−1.500 (0.000)	85.680 (0.000)
NIKKEI225	9.887	0.117	−0.292 (0.000)	−1.016 (0.000)	51.417 (0.000)
B. Returns					
Crypto Index	0.395	5.208	−0.229 (0.005)	6.339 (0.000)	1509.593 (0.000)
NIKKEI225	0.006	1.330	−0.294 (0.000)	5.727 (0.000)	1238.858 (0.000)
Interest rate	−0.163	0.101	−0.327 (0.000)	−0.086 (0.299)	16.326 (0.000)

Sample period: Daily data 2015:8:7-2019:4:27 (Germany: T=917, UK: T=940, Japan: T=898).

Table 11. Contemporaneous correlations of cryptocurrency and international financial markets

	A. Levels			B. Returns		
Germany						
	Crypto Index	DAX30	Interest rate	Crypto Index	DAX30	Interest rate
Crypto Index	1	0.974	−0.966	1	0.041	−0.096
DAX30	0.974	1	-0.967	0.041	1	−0.013
Interest rate	-0.966	−0.967	1	−0.096	-0.013	1
United Kingdom						
	Crypto Index	FTSE100	Interest rate	Crypto Index	FTSE100	Interest rate
Crypto Index	1	0.974	0.905	1	0.039	0.027
FTSE100	0.974	1	0.914	0.039	1	0.007
Interest rate	0.905	0.914	1	0.027	0.007	1
Japan						
	Crypto Index	Nikkei225	Interest rate	Crypto Index	Nikkei225	Interest rate
Crypto Index	1	0.974	−0.833	1	0.028	−0.063
Nikkei225	0.974	1	-0.850	0.028	1	−0.021
Interest rate	-0.834	−0.850	1	−0.063	-0.021	1

Sample period: Daily data 2015:8:7-2019:4:27 (Germany: $T = 917$, UK: $T = 940$, Japan: $T = 898$). The Crypto Index and the stock indices are in logarithmic levels.

Table 12. Cryptocurrency spillover effects to German financial markets

A. Conditional mean equation

$$\mathbf{a} = \begin{bmatrix} -8.331 & (0.031) \\ 2.796 & (0.050) \\ 0.001 & (0.917) \end{bmatrix}; \boldsymbol{\Gamma} = \begin{bmatrix} -3.193\ (0.000) & -6.837\ (0.000) & -7.056\ (0.016) \\ 1.514\ (0.000) & 3.000\ (0.000) & 2.347\ (0.030) \\ -0.000\ (0.988) & -0.001\ (0.965) & 0.994\ (0.000) \end{bmatrix};$$

$$\boldsymbol{\Psi} = \begin{bmatrix} 0.028\ (0.652) & 3.958\ (0.052) & 6.448\ (0.160) \\ -0.016\ (0.489) & -1.316\ (0.081) & -2.376\ (0.196) \\ 0.000\ (0.534) & 0.000\ (0.996) & -0.269\ (0.003) \end{bmatrix}; \boldsymbol{\Theta} = \begin{bmatrix} 3.192\ (0.000) & 7.027\ (0.000) & 5.616\ (0.055) \\ -1.514\ (0.000) & -3.073\ (0.000) & -2.149\ (0.106) \\ -0.000\ (0.983) & 0.001\ (0.930) & -0.275\ (0.000) \end{bmatrix}$$

B. Residual diagnostics

	Mean	Variance	$Q(5)$	$Q^2(5)$	$Q(25)$	$Q^2(25)$
z_{c_t}	0.050	0.983	21.940 (0.001)	0.874 (0.972)	60.350 (0.000)	20.765 (0.706)
z_{s_t}	−0.018	1.004	5.810 (0.325)	4.209 (0.520)	25.570 (0.378)	13.317 (0.972)
z_{i_t}	0.057	0.985	2.130 (0.831)	1.466 (0.917)	17.420 (0.866)	34.046 (0.107)

C. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} 0.114\ (0.259) & 0.085\ (0.441) & -1.548\ (0.031) \\ & 0.100\ (0.261) & 0.707\ (0.649) \\ & & 0.005\ (0.999) \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 0.945\ (0.000) & 0.001\ (0.370) & -0.006\ (0.537) \\ 0.007\ (0.756) & 0.959\ (0.000) & -0.030\ (0.674) \\ 0.025\ (0.448) & 0.014\ (0.116) & 0.619\ (0.000) \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} -0.332\ (0.000) & -0.000\ (0.865) & 0.007\ (0.747) \\ 0.121\ (0.148) & 0.125\ (0.001) & -0.217\ (0.095) \\ 0.023\ (0.567) & -0.006\ (0.547) & 0.600\ (0.000) \end{bmatrix}; \boldsymbol{D} = \begin{bmatrix} -0.054\ (0.339) & -0.003\ (0.717) & 0.009\ (0.768) \\ 0.188\ (0.133) & -0.295\ (0.000) & -0.373\ (0.050) \\ 0.198\ (0.001) & -0.002\ (0.874) & 0.005\ (0.958) \end{bmatrix}.$$

Model: Trivariate VARMA, GARCH-in-mean, asymmetric BEKK model for Cryptocurrency Index 30, DAX30 and interest rate in Germany. Sample period: 2015:8:7 to 2019:4:27 (T=917). Numbers in parentheses are p-values.

Table 13. Cryptocurrency spillover effects to British financial markets

A. Conditional mean equation

$$\mathbf{a} = \begin{bmatrix} -1.282\ (0.017) \\ 0.103\ (0.012) \\ -0.002\ (0.362) \end{bmatrix}; \boldsymbol{\Gamma} = \begin{bmatrix} 5.119\ (0.000) & 66.295\ (0.000) & -1.993\ (0.122) \\ -0.322\ (0.000) & -4.012\ (0.000) & 0.145\ (0.115) \\ 0.001\ (0.772) & -0.041\ (0.461) & 1.008\ (0.000) \end{bmatrix};$$

$$\boldsymbol{\Psi} = \begin{bmatrix} 0.078\ (0.458) & 1.949\ (0.002) & 19.661\ (0.042) \\ -0.007\ (0.367) & -0.146\ (0.002) & -1.401\ (0.048) \\ 0.001\ (0.157) & -0.004\ (0.053) & -0.030\ (0.638) \end{bmatrix}; \boldsymbol{\Theta} = \begin{bmatrix} -5.117\ (0.000) & -66.106\ (0.000) & 1.328\ (0.696) \\ 0.322\ (0.000) & 4.002\ (0.000) & -0.021\ (0.930) \\ -0.001\ (0.774) & 0.041\ (0.466) & -0.392\ (0.000) \end{bmatrix}$$

B. Residual diagnostics

	Mean	Variance	$Q(5)$	$Q^2(5)$	$Q(25)$	$Q^2(25)$
z_{c_t}	0.049	0.963	18.050 (0.003)	1.876 (0.866)	58.320 (0.000)	18.366 (0.826)
z_{s_t}	0.007	0.995	4.870 (0.432)	1.530 (0.910)	21.620 (0.658)	13.282 (0.973)
z_{i_t}	−0.053	0.954	16.610 (0.005)	19.012 (0.002)	67.590 (0.000)	23.970 (0.521)

C. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} 0.456\ (0.001) & 0.097\ (0.107) & 0.004\ (0.109) \\ & -0.126\ (0.003) & -0.002\ (0.481) \\ & & 0.000\ (0.962) \end{bmatrix}; \ \boldsymbol{B} = \begin{bmatrix} 0.932\ (0.000) & 0.019\ (0.029) & -0.000\ (0.231) \\ 0.232\ (0.413) & -0.932\ (0.000) & 0.000\ (0.945) \\ 15.325\ (0.002) & 1.268\ (0.671) & 0.828\ (0.000) \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} -0.354\ (0.000) & -0.001\ (0.772) & 0.000\ (0.697) \\ -0.142\ (0.198) & 0.048\ (0.301) & 0.002\ (0.000) \\ -26.120\ (0.000) & -1.913\ (0.023) & 0.456\ (0.000) \end{bmatrix}; \boldsymbol{D} = \begin{bmatrix} -0.043\ (0.410) & 0.007\ (0.127) & -0.0000\ (0.001) \\ -0.101\ (0.491) & 0.414\ (0.000) & 0.000\ (0.917) \\ 7.699\ (0.351) & -2.246\ (0.253) & -0.393\ (0.000) \end{bmatrix}.$$

Model: Trivariate VARMA, GARCH-in-mean, asymmetric BEKK model for Cryptocurrency Index 30, FTSE 100 and interest rate in UK. Sample period: 2015:8:7 to 2019:4:27 (T=940). Numbers in parentheses are p-values.

Table 14. Cryptocurrency spillover effects to Japanese financial markets

A. Conditional mean equation

$$\mathbf{a} = \begin{bmatrix} 0.278\ (0.834) \\ 0.334\ (0.042) \\ -0.004\ (0.390) \end{bmatrix}; \mathbf{\Gamma} = \begin{bmatrix} -1.022\ (0.012) & -4.570\ (0.076) & 1.437\ (0.601) \\ 0.023\ (0.786) & -0.048\ (0.916) & 0.473\ (0.251) \\ -0.001\ 0.720) & -0.007\ (0.622) & 0.985\ (0.000) \end{bmatrix};$$

$$\mathbf{\Psi} = \begin{bmatrix} 0.016\ (0.901) & 0.797\ (0.284) & -21.865\ (0.403) \\ -0.038\ (0.021) & 0.224\ (0.152) & -11.069\ (0.051) \\ 0.000\ (0.855) & 0.001\ (0.808) & 0.020\ (0.8897) \end{bmatrix}; \mathbf{\Theta} = \begin{bmatrix} 0.984\ (0.017) & 4.681\ (0.072) & 4.095\ (0.258) \\ -0.014\ (0.870) & 0.034\ (0.941) & -1.816\ (0.031) \\ 0.001\ (0.716) & 0.006\ (0.682) & -0.252\ (0.000) \end{bmatrix}$$

B. Residual diagnostics

	Mean	Variance	$Q(5)$	$Q^2(5)$	$Q(25)$	$Q^2(25)$
z_{c_t}	0.074	0.993	28.740 (0.000)	1.738 (0.884)	73.620 (0.000)	10.471 (0.995)
z_{s_t}	−0.028	0.971	4.760 (0.445)	4.506 (0.479)	26.600 (0.376)	12.404 (0.983)
z_{i_t}	−0.004	0.967	4.190 (0.523)	7.731 (0.172)	17.800 (0.317)	14.929 (0.943)

C. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} 0.548\ (0.000) & 0.131\ (0.005) & -0.001\ (0.285) \\ & 0.095\ (0.198) & -0.001\ (0.272) \\ & & 0.000\ (1.000) \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 0.912\ (0.000) & -0.002\ (0.344) & -0.000\ (0.186) \\ -0.036\ (0.182) & 0.951\ (0.000) & 0.001\ (0.000) \\ 1.233\ (0.059) & -1.118\ (0.000) & 0.981\ (0.000) \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} 0.453\ (0.000) & 0.011\ (0.058) & 0.000\ (0.292) \\ -0.073\ (0.302) & -0.085\ (0.009) & 0.000\ (0.372) \\ -1.378\ (0.478) & -2.992\ (0.003) & 0.101\ (0.002) \end{bmatrix}; \boldsymbol{D} = \begin{bmatrix} -0.049\ (0.327) & 0.010\ (0.053) & -0.000\ (0.475) \\ -0.033\ (0.701) & 0.321\ (0.000) & -0.002\ (0.000) \\ -0.854\ (0.662) & 3.088\ (0.002) & 0.161\ (0.000) \end{bmatrix}.$$

Model: Trivariate VARMA, GARCH-in-mean, asymmetric BEKK model for Cryptocurrency Index 30, NIKKEI 225 and interest rate in Japan. Sample period: 2015:8:7 to 2019:4:27 (T=898). Numbers in parentheses are p-values.