MBMT Tiebreaker Round — Erdös

March 9, 2025

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **3** questions. You will have **15** minutes to complete the round.

For each of the 3 problems, you may submit an answer up to **3 times, whenever you want,** as long as it is during the round. **Only the last submission to each problem will be counted.**

In order to submit an answer, write your answer on the paper slip with the corresponding problem number. Then, fold the slip in half and place it on the grader's desk behind you.

Your placement will be determined first by the number of problems you solve and then by time of last correct counted submission to break any remaining ties. Please write your answers in a reasonably simplified form.

1. The numbers a, b, c, d, e, f, g, h, and i are the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9, not necessarily in that order. Given that $a+b=16$, $\frac{c}{d}=4$, and $e+f+g=8$, find hi.

Proposed by Ivy Guo.

Answer: $\boxed{30}$

Solution: Since $a+b=16$, a and b must be 7 and 9 in some order. Since $\frac{c}{d}=4$, either $c=4$ and $d=1$, or $c=8$ and $d=2$.

If $c=4$ and $d=1$, then the smallest $e+f+g$ could be is $2+3+5>8$. Therefore, we must have $c=8$ and $d=2$. Then, e, f, and g must be 1, 3, and 4 in some order, leaving h and i to be 5 and 6, so $hi=30$.

2. Regular hexagon $ABCDEF$ has sides of length 8. Point B' is the midpoint of side AB, and a regular hexagon $AB'C'D'E'F'$ is drawn inside $ABCDEF$. What is the area of quadrilateral $E'D'DE$?

Proposed by Olivia Guo.

Answer: $\boxed{24\sqrt{3}}$

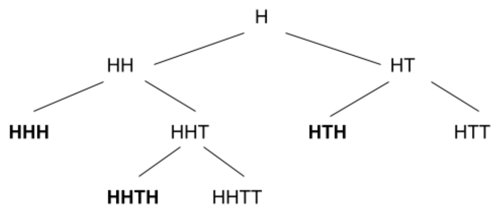
Solution: The quadrilateral is a trapezoid with side $D'E'$ lying on diagonal CF of hexagon $ABCDEF$. The upper base has length 4 and the lower base has length 8. The height is that of an equilateral triangle with side 8, which is $4\sqrt{3}$. Combining these lengths gives an area of $\left(\frac{8+4}{2}\right)\cdot 4\sqrt{3}=24\sqrt{3}$.

3. A fair coin is flipped repeatedly. What is the probability the sequence HTH appears before the sequence HHH?

Proposed by Ivy Guo.

Answer: $\boxed{\frac{3}{5}}$

Solution: If we ever get two T's in a row, it's equivalent to starting over. Consider the following diagram:



The bolded strings are end-states. The probability of reaching any given state on the fourth row is half the probability of reaching any given state on the third row, so the probability of ending with HTH is $\frac{1.5}{2.5} = \frac{3}{5}$.