MBMT Number Theory Round — Gödel

March 9, 2024

Full Name _____

Student ID Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

1 The seniors at Montgomery Blair High School are going on a field trip. There will be 200 students and 25 teachers on the trip. Each bus can carry 45 passengers. How many buses will be needed?

2 A number is called relatively prime to another number if they share no factors other than 1. How many positive integers less than 23 are relatively prime to 23?

3 If x, y are nonnegative integers, and $xy + x + 3y = 1$, find $x + y$.

4 A positive integer is "inspirational" if it has at least three factors and the sum of its three smallest positive factors is 12. How many inspirational numbers are less than 2024?

5 Let x equal $16^2 + 2^{16} + 4^4 + 1$, find the greatest prime factor of x.

6 What is the sum of positive integers less than 81 that do not have a "2" when expressed in base 3?

7 What is the remainder when the product of the first 2024 prime numbers is divided by 1012?

8 Evaluate $13^{11^{7^{5^{3^2}}}} \mod 17$.