MBMT Number Theory Round — Weierstrass

March 9, 2025

Full Name _____

Student ID Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

_____ **1.** What is the second-smallest positive integer that is a multiple of both 4 and 6?

_____ **2.** Stephen correctly computes the product of the first four prime numbers. Ricky also attempts to compute the product of the first four prime numbers, but he mistakenly believes that the first prime number is 1, not 2. What is the positive difference between Stephen's and Ricky's calculations?

_____ **3.** Alice's answer to her math homework has been eaten by her pet ants, who only eat their favorite digit. Her answer is now $7X91X8$ where X is a missing digit. If Alice remembers that her answer was divisible by 12, what digit did the ants eat?

_____ **4.** Shriyan divides his favorite three-digit number by 2, 3, 4, 8, 9, and 11 and gets a remainder of 1 each time. What is Shriyan's favorite three-digit number?

_____ **5.** Three *consecutive* nonzero digits are taken, and the 6 numbers formed by permuting the digits are added. What is the largest integer that must divide the sum?

_____ **6.** Let $\lfloor x \rfloor$ represent the largest integer less than or equal to x. There exists a unique 5-digit positive integer n such that the sum of its digits is 20 and

$$\left\lfloor \frac{n}{10} \right\rfloor + \left\lfloor \frac{n}{100} \right\rfloor + \left\lfloor \frac{n}{1000} \right\rfloor + \left\lfloor \frac{n}{10000} \right\rfloor = 2025$$

What is the product of the digits of n?

_____ **7.** Let $m, n > 2025$ be prime numbers such that $m = n + 2$. What is the remainder when mn is divided by 36?

_____ **8.** Define S_n as the sum of all positive integers less than or equal to n that are relatively prime to n (their greatest common factor with n is equal to 1). What is S_{210}?