MBMT Algebra Round — Weierstrass

March 9, 2025

Full Name _____

Student ID Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

_____ **1.** If $\frac{27}{45} = \sqrt{\frac{9}{x}}$, what is the value of x?

_____ **2.** Mr. Bramble teaches a World History class with 30 students. On a quiz, every student initially scored 10 points. Half of the class then retook the quiz, and each student who retook the quiz improved their score by 3 points. By how many points did the class average increase after the retakes?

_____ **3.** Farmer Yunyi has a farm with chickens and chimeras. Chickens have 1 head and 2 legs, and chimeras have 3 heads and 4 legs. If the farm has a total of 200 heads and 350 legs (not counting Farmer Yunyi), how many animals are on the farm?

_____ **4.** Pedro is trying to find rectangles with positive integer side lengths whose areas are numerically equal to their perimeters. To his surprise, he discovers that there are only two unique rectangles that have this property. What is the sum of the areas of these two rectangles?

_____ **5.** A falling ping-pong ball always bounces up to $\frac{3}{4}$ of the height it falls from. If Stanley drops a ball from 40 feet above the ground, what is the total distance the ball will travel?

_____ **6.** If $\frac{1}{x^3} + \frac{1}{y^3} = 4$ and $\frac{1}{x} + \frac{1}{y} = 2$, what is $\frac{1}{x^2} + \frac{1}{y^2}$?

_____ **7.** Michael takes the polynomial $x^{101} + 1$ and divides it by $x + 1$, giving him the polynomial $f(x)$. He then takes $f(x)$ and divides it by $x + 1$ again, giving him the polynomial $g(x)$ and the remainder s. What is the value s?

_____ **8.** A sequence of functions is defined as follows: $f_0(x) = ||x| - 1|$ and $f_n(x) = |f_{n-1}(x) - 2^n|$ for all positive integers n. Let x_n be the positive root of $f_n(x)$; in other words, $x_n > 0$ and $f_n(x_n) = 0$. What is the sum of x_n as n goes from 0 to 2025, inclusive?