

# **MBMT Weierstrass Guts Round – Set 1**

March 9, 2025

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1. [3] What is  $2025^{2^0^2^5}$ ?
2. [3] Bob eats 5 bananas for every 2 bananas Kevin eats. If Bob and Kevin eat 42 bananas together, how many more bananas did Bob eat than Kevin?
3. [3] Which non-zero digit  $x$  satisfies the property that the three-digit number  $\overline{x01}$  is divisible by 7?
4. [3] Two trucks, one of length 7 feet and the other of length 11 feet, are passing each other in opposite directions. If the first truck is driving at 6 feet per second and 2 seconds elapse from when the trucks begin passing each other to when they are fully past each other, how fast is the second truck traveling, in feet per second?
5. [3] Arjun is on an elevator starting on the 1st floor. If the elevator moves up at a constant speed, what is the ratio of the time it takes him to go to the 25th floor to the time it takes him to go to the 5th floor?

# **MBMT Weierstrass Guts Round – Set 2**

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# MBMT Weierstrass Guts Round – Set 2

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6. [4] What is the maximum number of intersection points between two squares?
  
7. [4]  $x$  and  $y$  are real numbers. Gloria the Cicada wants to estimate the value of  $x - y$ , so she rounds  $x$  down by 0.05 and  $y$  up by 0.02. What is the positive difference between Gloria the Cicada's estimated value and her actual value?
  
8. [4] Jason draws a rocket. Sadly, he is very bad at drawing, so his rocket simply consists of an isosceles triangle sharing its base with one side of a rectangle. Given that each side of the rocket is an integer, if the rectangle has perimeter 40 and the isosceles triangle has perimeter 24, what is the largest possible perimeter of Jason's rocket?
  
9. [4] Kian is buying pizza. A pizza with a diameter of 5 inches costs him 100 dollars. How much should a pizza with the same thickness and a diameter of 12 inches cost to have the same amount of pizza per dollar?
  
10. [4] Evan was biking to RTC, but  $\frac{2}{3}$  of the way there, his bike broke down, so he walked the rest of the way there. If he walked for  $\frac{2}{3}$  of the total time, how many times faster is his biking speed than his walking speed?

# **MBMT Weierstrass Guts Round – Set 3**

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# MBMT Weierstrass Guts Round – Set 3

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11. [5] A hexagon is inscribed in a circle. Yunyi draws two triangles, using each vertex of the hexagon exactly once. What is the probability that the two triangles intersect?
  
12. [5] Peter and Caleb are trading business cards. Caleb has triple Peter’s number of cards. Peter then gives Caleb 3 cards, and Caleb now has quadruple Peter’s number of cards. How many cards do they have in total?
  
13. [5] Jimmy’s least favorite number is between 0 and 64. He decides to add up all the numbers from 0 to 64, leaving out his least favorite number. If he correctly finds the sum to be 2025, what is Jimmy’s least favorite number?
  
14. [5] If  $x^2 = (7 \cdot 9 + 1)(13 \cdot 15 + 1)(19 \cdot 21 + 1)$ , what is the value of  $x$ ?
  
15. [5] Yunyi has a cube and notices that its volume and surface area have the same numerical value  $a$ . Kian has a sphere and also notices that its volume and surface area have the same numerical value  $b$ . What is  $\frac{a}{b}$ ?

# **MBMT Weierstrass Guts Round – Set 4**

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# MBMT Weierstrass Guts Round – Set 4

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16. [7] A frog is at the point  $(0, 0)$  on the coordinate plane and spots a fly at the point  $(4, 2)$ . At each step, if the frog is at the point  $(x, y)$ , it can move to the points  $(x + 1, y)$ ,  $(x, y + 1)$ , or  $(x + 1, y + 1)$ . How many sequences of moves are there for the frog to reach the fly?
17. [7] Leroy the Leg Lover lords over his legendary legion of leggy pets. Soggy squids have 10 legs each and outstanding octopi have 8 legs each. If Leroy has more squids than octopi, and his leggy total tallies to 134, how many animals does he have?
18. [7] A circle with unknown radius passes through vertices  $A$  and  $B$  of rectangle  $ABCD$ . The circle is tangent to side  $DC$  and intersects side  $BC$  at  $E$  so that  $BE = 10$  and  $CE = 8$ . What is the length of side  $AB$ ?
19. [7] What is the 100th smallest positive integer that contains only odd digits?
20. [7] Triangle  $ABC$  has  $AB = 6$ ,  $BC = 8$ , and  $AC = 10$ . If Alice chooses a random point within the triangle, what is the probability that the point lies in the incircle of  $ABC$ ?

# **MBMT Weierstrass Guts Round – Set 5**

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# MBMT Weierstrass Guts Round – Set 5

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21. [9] For a positive integer  $n$ , let  $f(n)$  represent the largest prime number that divides  $n!$ . What is the sum of  $f(n)$  as  $n$  goes from 2 to 13, inclusive?
22. [9] Stephen rolls a fair six sided die. He then randomly paints a number of faces of a cube corresponding to the number shown on the die (if he rolls a 4, he paints 4 random faces). Stephen then cuts the cube into 64 smaller cubes and picks one at random to roll. What is the probability that the top face of the small cube is painted?
23. [9] Ivy has a triangle with sides of length 13, 14, and 15. Olivia comes and makes another triangle whose side lengths are the lengths of the medians of Ivy's triangle. What is the area of Olivia's triangle?
24. [9] Alex picks two distinct primes  $p_1$  and  $p_2$ , then makes a list of all positive integers greater than 1 whose prime factorizations contain no primes other than  $p_1$  and  $p_2$ . He then takes the reciprocal of every number on the list, and adds them together. If he gets a simplified fraction with a denominator of 2024, what is the numerator of his fraction?
25. [9] For a given value  $k$ , let  $(x, y)$  be a random point such that  $-10 \leq x, y \leq 10$  and  $x + y = k$ . Let  $f(k)$  be the expected value of  $|x| + |y|$ . If  $k$  can be any real number, what is the minimum possible value of  $f(k)$ ?

# **MBMT Weierstrass Guts Round – Set 6**

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# MBMT Weierstrass Guts Round – Set 6

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26. [12] Let  $f(x)$  be the product

$$\left(1 + \frac{1}{10^6}\right) \left(1 + \frac{2}{10^6}\right) \cdots \left(1 + \frac{x}{10^6}\right).$$

Estimate the smallest positive integer  $x$  such that  $f(x) > 10^6$ .

27. [12] Square  $A_0B_0C_0D_0$  has a side length of 1. For all positive integers  $n$ , square  $A_nB_nC_nD_n$  is formed by inscribing an equilateral triangle in square  $A_{n-1}B_{n-1}C_{n-1}D_{n-1}$  such that a vertex of the triangle coincides with a vertex of the square, and then inscribing a square in the equilateral triangle. Find

$$\sum_{i=0}^{\infty} [A_iB_iC_iD_i]$$

where  $[A_iB_iC_iD_i]$  denotes the area of square  $A_iB_iC_iD_i$ .

28. [12] 3 random integers  $a$ ,  $b$ , and  $c$  are chosen with replacement from  $-2025$  to  $2025$  inclusive. Estimate the probability that the quadratic  $ax^2 + bx + c$  has at least 1 real root.

29. [12] Yunyi has a circular table with a radius of 2025 inches. He also has an infinite number of stickers in the shape of a circle with a radius of 1 inch. If stickers can overlap and hang off the edge of the table, what is the minimum number of stickers Yunyi needs to cover the entirety of the circle's surface with stickers?

30. [12] Kian is in a room with 2025 lightbulbs which are all initially off. Every second he picks one of the lightbulbs at random and flips its state (if it was off initially, he turns it on, and vice versa). Estimate the floor of the log of the expected number of seconds it'll take for him to turn all of the lightbulbs on.