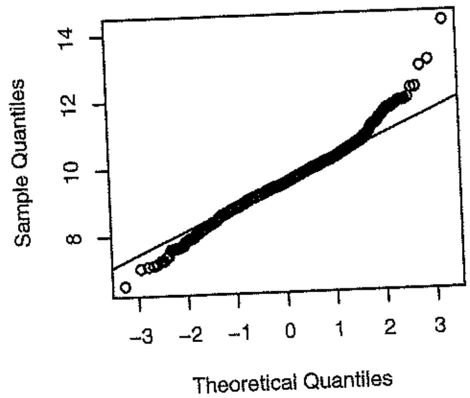
**Exhibit 7.12 Q-Q Normal Plot of Bootstrap Quasi-period Estimates**




```
> win.graph(width=2.5,height=2.5,pointsize=8)
> qqnorm(period.replace); qqline(period.replace)
```

## 7.7 Summary

This chapter delved into the estimation of the parameters of ARIMA models. We considered estimation criteria based on the method of moments, various types of least squares, and maximizing the likelihood function. The properties of the various estimators were given, and the estimators were illustrated both with simulated and actual time series data. Bootstrapping with ARIMA models was also discussed and illustrated.

## EXERCISES

**7.1** From a series of length 100, we have computed $r_1 = 0.8$, $r_2 = 0.5$, $r_3 = 0.4$, $\bar{Y} = 2$, and a sample variance of 5. If we assume that an AR(2) model with a constant term is appropriate, how can we get (simple) estimates of $\phi_1$, $\phi_2$, $\theta_0$, and $\sigma_e^2$?

**7.2** Assuming that the following data arise from a stationary process, calculate method-of-moments estimates of $\mu$, $\gamma_0$, and $\rho_1$: 6, 5, 4, 6, 4.

**7.3** If $\{Y_t\}$ satisfies an AR(1) model with $\phi$ of about 0.7, how long of a series do we need to estimate $\phi = \rho_1$ with 95% confidence that our estimation error is no more than $\pm 0.1$?

**7.4** Consider an MA(1) process for which it is *known* that the process mean is zero. Based on a series of length $n = 3$, we observe $Y_1 = 0$, $Y_2 = -1$, and $Y_3 = ½$.

**(a)** Show that the conditional least-squares estimate of $\theta$ is ½.

**(b)** Find an estimate of the noise variance. (Hint: Iterative methods are not needed in this simple case.)

**7.5** Given the data $Y_1 = 10$, $Y_2 = 9$, and $Y_3 = 9.5$, we wish to fit an IMA(1,1) model without a constant term.
**(a)** Find the conditional least squares estimate of $\theta$. (Hint: Do Exercise 7.4 first.)
**(b)** Estimate $\sigma_e^2$.

**7.6** Consider two different parameterizations of the AR(1) process with nonzero mean:

Model I. $Y_t - \mu = \phi(Y_{t-1} - \mu) + e_t$.

Model II. $Y_t = \phi Y_{t-1} + \theta_0 + e_t$.

We want to estimate $\phi$ and $\mu$ or $\phi$ and $\theta_0$ using conditional least squares conditional on $Y_1$. Show that with Model I we are led to solve nonlinear equations to obtain the estimates, while with Model II we need only solve linear equations.

**7.7** Verify Equation (7.1.4) on page 150.

**7.8** Consider an ARMA(1,1) model with $\phi = 0.5$ and $\theta = 0.45$.
**(a)** For $n = 48$, evaluate the variances and correlation of the maximum likelihood estimators of $\phi$ and $\theta$ using Equations (7.4.13) on page 161. Comment on the results.
**(b)** Repeat part (a) but now with $n = 120$. Comment on the new results.

**7.9** Simulate an MA(1) series with $\theta = 0.8$ and $n = 48$.
**(a)** Find the method-of-moments estimate of $\theta$.
**(b)** Find the conditional least squares estimate of $\theta$ and compare it with part (a).
**(c)** Find the maximum likelihood estimate of $\theta$ and compare it with parts (a) and (b).
**(d)** Repeat parts (a), (b), and (c) with a new simulated series using the same parameters and same sample size. Compare your results with your results from the first simulation.

**7.10** Simulate an MA(1) series with $\theta = -0.6$ and $n = 36$.
**(a)** Find the method-of-moments estimate of $\theta$.
**(b)** Find the conditional least squares estimate of $\theta$ and compare it with part (a).
**(c)** Find the maximum likelihood estimate of $\theta$ and compare it with parts (a) and (b).
**(d)** Repeat parts (a), (b), and (c) with a new simulated series using the same parameters and same sample size. Compare your results with your results from the first simulation.

**7.11** Simulate an MA(1) series with $\theta = -0.6$ and $n = 48$.
**(a)** Find the maximum likelihood estimate of $\theta$.
**(b)** If your software permits, repeat part (a) many times with a new simulated series using the same parameters and same sample size.
**(c)** Form the sampling distribution of the maximum likelihood estimates of $\theta$.
**(d)** Are the estimates (approximately) unbiased?
**(e)** Calculate the variance of your sampling distribution and compare it with the large-sample result in Equation (7.4.11) on page 161.

**7.12** Repeat Exercise 7.11 using a sample size of $n = 120$.

**7.13** Simulate an AR(1) series with $\phi = 0.8$ and $n = 48$.

**(a)** Find the method-of-moments estimate of $\phi$.

**(b)** Find the conditional least squares estimate of $\phi$ and compare it with part (a).

**(c)** Find the maximum likelihood estimate of $\phi$ and compare it with parts (a) and (b).

**(d)** Repeat parts (a), (b), and (c) with a new simulated series using the same parameters and same sample size. Compare your results with your results from the first simulation.

**7.14** Simulate an AR(1) series with $\phi = -0.5$ and $n = 60$.

**(a)** Find the method-of-moments estimate of $\phi$.

**(b)** Find the conditional least squares estimate of $\phi$ and compare it with part (a).

**(c)** Find the maximum likelihood estimate of $\phi$ and compare it with parts (a) and (b).

**(d)** Repeat parts (a), (b), and (c) with a new simulated series using the same parameters and same sample size. Compare your results with your results from the first simulation.

**7.15** Simulate an AR(1) series with $\phi = 0.7$ and $n = 100$.

**(a)** Find the maximum likelihood estimate of $\phi$.

**(b)** If your software permits, repeat part (a) many times with a new simulated series using the same parameters and same sample size.

**(c)** Form the sampling distribution of the maximum likelihood estimates of $\phi$.

**(d)** Are the estimates (approximately) unbiased?

**(e)** Calculate the variance of your sampling distribution and compare it with the large-sample result in Equation (7.4.9) on page 161.

**7.16** Simulate an AR(2) series with $\phi_1 = 0.6$, $\phi_2 = 0.3$, and $n = 60$.

**(a)** Find the method-of-moments estimates of $\phi_1$ and $\phi_2$.

**(b)** Find the conditional least squares estimates of $\phi_1$ and $\phi_2$ and compare them with part (a).

**(c)** Find the maximum likelihood estimates of $\phi_1$ and $\phi_2$ and compare them with parts (a) and (b).

**(d)** Repeat parts (a), (b), and (c) with a new simulated series using the same parameters and same sample size. Compare these results to your results from the first simulation.

**7.17** Simulate an ARMA(1,1) series with $\phi = 0.7$, $\theta = 0.4$, and $n = 72$.

**(a)** Find the method-of-moments estimates of $\phi$ and $\theta$.

**(b)** Find the conditional least squares estimates of $\phi$ and $\theta$ and compare them with part (a).

**(c)** Find the maximum likelihood estimates of $\phi$ and $\theta$ and compare them with parts (a) and (b).

**(d)** Repeat parts (a), (b), and (c) with a new simulated series using the same parameters and same sample size. Compare your new results with your results from the first simulation.

**7.18** Simulate an AR(1) series with $\phi = 0.6$, $n = 36$ but with error terms from a $t$-distribution with 3 degrees of freedom.

**7.26** Consider the AR(1) model specified for the color property time series displayed in Exhibit 1.3 on page 3. The data are in the file named color.

**(a)** Find the method-of-moments estimate of $\phi$.

**(b)** Find the maximum likelihood estimate of $\phi$ and compare it with part (a).

**7.27** Exhibit 6.31 on page 139 suggested specifying either an AR(1) or possibly an AR(4) model for the difference of the logarithms of the oil price series. The data are in the file named oil.price.

**(a)** Estimate both of these models using maximum likelihood and compare it with the results using the AIC criteria.

**(b)** Exhibit 6.32 on page 140 suggested specifying an MA(1) model for the difference of the logs. Estimate this model by maximum likelihood and compare to your results in part (a).

**7.28** The data file named deere3 contains 57 consecutive values from a complex machine tool at Deere & Co. The values given are deviations from a target value in units of ten millionths of an inch. The process employs a control mechanism that resets some of the parameters of the machine tool depending on the magnitude of deviation from target of the last item produced.

**(a)** Estimate the parameters of an AR(1) model for this series.

**(b)** Estimate the parameters of an AR(2) model for this series and compare the results with those in part (a).

**7.29** The data file named robot contains a time series obtained from an industrial robot. The robot was put through a sequence of maneuvers, and the distance from a desired ending point was recorded in inches. This was repeated 324 times to form the time series.

**(a)** Estimate the parameters of an AR(1) model for these data.

**(b)** Estimate the parameters of an IMA(1,1) model for these data.

**(c)** Compare the results from parts (a) and (b) in terms of AIC.

**7.30** The data file named days contains accounting data from the Winegard Co. of Burlington, Iowa. The data are the number of days until Winegard receives payment for 130 consecutive orders from a particular distributor of Winegard products. (The name of the distributor must remain anonymous for confidentiality reasons.) The time series contains outliers that are quite obvious in the time series plot.

**(a)** Replace each of the unusual values with a value of 35 days, a much more typical value, and then estimate the parameters of an MA(2) model.

**(b)** Now assume an MA(5) model and estimate the parameters. Compare these results with those obtained in part (a).

**7.31** Simulate a time series of length $n = 48$ from an AR(1) model with $\phi = 0.7$. Use that series as if it were real data. Now compare the theoretical asymptotic distribution of the estimator of $\phi$ with the distribution of the bootstrap estimator of $\phi$.

**7.32** The industrial color property time series was fitted quite well by an AR(1) model. However, the series is rather short, with $n = 35$. Compare the theoretical asymptotic distribution of the estimator of $\phi$ with the distribution of the bootstrap estimator of $\phi$. The data are in the file named color.