

Problem 1: Calculate and sketch the autocorrelation functions for each of the following AR(1) models. Plot for sufficient lags that the autocorrelation function has nearly died out.

(a) $\phi = 0.6$; (b) $\phi = -0.6$; (c) $\phi = 0.3$; and (d) $\phi = 0.95$.

Problem 2: Use the recursive formula to obtain the autocorrelation function of the following AR(2) models. In each case, (i) specify whether the roots of the characteristics equations are real or complex, and (ii) If they are complex, find the damping factor and frequency.

(a) $\phi_1 = 0.6$ and $\phi_2 = 0.3$

(b) $\phi_1 = -0.4$ and $\phi_2 = 0.5$

(c) $\phi_1 = -0.5$ and $\phi_2 = -0.6$

(d) $\phi_1 = -1$ and $\phi_2 = -0.6$

Problem 3: Sketch the autocorrelation function for each of the following ARMA models:

(a) ARMA (1,1) with $\phi = 0.7$ and $\theta = 0.4$

(b) ARMA (1,1) with $\phi = 0.7$ and $\theta = -0.4$

Problem 4: For the following ARMA(1,2) model, $Y_t = 0.8Y_{t-1} + \epsilon_t + 0.7\epsilon_{t-1} + 0.6\epsilon_{t-2}$. Show that $\rho_k = 0.8\rho_{k-1}$ for $k > 2$ and that $\rho_2 = 0.8\rho_1 + 0.6\sigma^2/\gamma_0$.

Problem 5: Consider a process that satisfies the zero-mean stationary AR(1) equation $Y_t = \phi Y_{t-1} + \epsilon_t$ with $|\phi| < 1$. Let $c \neq 0$ and define $W_t = Y_t + c\phi^t$.

(a) Show that $E(W_t) = c\phi^t$

(b) Show that $\{W_t\}$ satisfies the stationary AR(1) equation $W_t = \phi W_{t-1} + \epsilon_t$. Is $\{W_t\}$ stationary?

Problem 6: The TSA library contains the data set `prescrip` which lists monthly prescription costs for the months 8/1986-3/1992. The data are from the state of New Jersey prescription drug program and are the cost per prescription claim during this time period.

(a) Construct a time series plot. Describe the appearance of the series.

(b) Use the command `diff` to calculate the first differences $\nabla X_t = X_t - X_{t-1}$.

Plot those differences and comment on the graph.

(c) Use all the model diagnostic check on the differences. Comments?

Do the following in the textbook:

3.6 Page 95; 3.7 Page 67; 3.8 Page 67; 2.3 Page 68.