

**Problem 1:** Suppose  $\epsilon_t$  is a zero mean white noise process with  $\text{Var}\epsilon_t = \sigma^2$  and consider the AR(1) with  $\phi = 0.8$ , that is  $Y_t = 0.8Y_{t-1} + \epsilon_t$ . Suppose we have observed a sample of size  $n = 200$  from the model.

- (a) Write down the approximate sampling distribution of the true lag  $k$  autocorrelation  $\rho_k$ ,  $k \in \{1, 2, 5, 10\}$ .
- (b) Use Monte Carlo simulation,  $n = 200$ , to obtain the sampling distribution of  $\rho_k$ . Assume  $\epsilon_t \sim N(0, 1)$ . Comment on what you observe.

**Problem 2:** Selecting a model.

- (a) A simulated series with  $n = 150$  observations yield the following sample autocorrelation values:  $\hat{\rho}_1 = -0.37$ ,  $\hat{\rho}_2 = 0.28$ ,  $\hat{\rho}_3 = 0.31$ ,  $\hat{\rho}_4 = -0.13$ ,  $\hat{\rho}_5 = 0.04$ , and  $\hat{\rho}_6 = -0.15$ . The remaining ones, say  $\hat{\rho}_k I\{7, 8, 9, \dots\}$  are negligible. What is the best model consistent with this information.
- (b) A simulated series with  $n = 150$  observations yield the following sample partial autocorrelation values:  $\hat{\phi}_{11} = 0.24$ ,  $\hat{\phi}_{22} = -0.81$ ,  $\hat{\phi}_{33} = 0.31$ ,  $\hat{\phi}_{44} = -0.09$ , and  $\hat{\phi}_{55} = 0.04$ , and  $\hat{\phi}_{66} = -0.02$ , and the remaining partial autocorrelation function for  $xx \in \{77, 88, \dots\}$  are negligible. What is the best model for this series.

**Problem 3:** Generate three time series each of length  $n = 200$ , with (a) AR(1),  $\phi = -0.6$ , (b) MA(1),  $\theta = 0.8$ , (c) ARMA(1, 1),  $\phi = -0.6$ ,  $\theta = 0.8$ .

For each: (i) plot the series, (ii) plot the sample ACF, PACF, EACF, and (iii) and use the R command `armasubsets` to identify the best models in term BIC.

- (b) Based on your plots and graphs: (i) do the plots agree with the theory?, (ii) does the BIC identify the best models based on simulated, else give the best three models per the BIC.

Do the following in the extra attached page, Pages 144-147.

6.27, 6.28, 6.29, 6.31, 6.33, 6.39.