6.24 Simulate an MA(1) time series with $\theta = 0.7$, with

(a) $n = 24$, and estimate ρ_1 with r_1;

(b) $n = 60$, and estimate ρ_1 with r_1;

(c) $n = 120$, and estimate ρ_1 with r_1.

(d) For each of the series in parts (a), (b), and (c), compare the estimated values of ρ_1 with the theoretical value. Use Exhibit 6.2 on page 112, to quantify the comparisons. In general, describe how the precision of the estimate varies with the sample size.

6.25 Simulate an AR(1) time series of length $n = 36$ with $\phi = 0.7$.

(a) Calculate and plot the theoretical autocorrelation function for this model. Plot sufficient lags until the correlations are negligible.

(b) Calculate and plot the sample ACF for your simulated series. How well do the values and patterns match the theoretical ACF from part (a)?

(c) What are the theoretical partial autocorrelations for this model?

(d) Calculate and plot the sample ACF for your simulated series. How well do the values and patterns match the theoretical ACF from part (a)? Use the large-sample standard errors reported in Exhibit 6.1 on page 111, to quantify your answer.

(e) Calculate and plot the sample PACF for your simulated series. How well do the values and patterns match the theoretical PACF from part (c)? Use the large-sample standard errors reported on page 115 to quantify your answer.

6.26 Simulate an MA(1) time series of length $n = 48$ with $\theta = 0.5$.

(a) What are the theoretical autocorrelations for this model?

(b) Calculate and plot the sample ACF for your simulated series. How well do the values and patterns match the theoretical ACF from part (a)?

(c) Calculate and plot the theoretical partial autocorrelation function for this model. Plot sufficient lags until the correlations are negligible. (Hint: See Equation (6.2.6) on page 114.)

(d) Calculate and plot the sample PACF for your simulated series. How well do the values and patterns match the theoretical PACF from part (c)?

6.27 Simulate an AR(2) time series of length $n = 72$ with $\phi_1 = 0.7$ and $\phi_2 = -0.4$.

(a) Calculate and plot the theoretical autocorrelation function for this model. Plot sufficient lags until the correlations are negligible.

(b) Calculate and plot the sample ACF for your simulated series. How well do the values and patterns match the theoretical ACF from part (a)?

(c) What are the theoretical partial autocorrelations for this model?

(d) Calculate and plot the sample ACF for your simulated series. How well do the values and patterns match the theoretical ACF from part (a)?

(e) Calculate and plot the sample PACF for your simulated series. How well do the values and patterns match the theoretical PACF from part (c)?

6.28 Simulate an MA(2) time series of length $n = 36$ with $\theta_1 = 0.7$ and $\theta_2 = -0.4$.

(a) What are the theoretical autocorrelations for this model?

(b) Calculate and plot the sample ACF for your simulated series. How well do the values and patterns match the theoretical ACF from part (a)?

(c) Plot the theoretical partial autocorrelation function for this model. Plot sufficient lags until the correlations are negligible. (We do not have a formula for this PACF. Instead, perform a very large sample simulation, say $n = 1000$, for this model and calculate and plot the sample PACF for this simulation.)

(d) Calculate and plot the sample PACF for your simulated series of part (a). How well do the values and patterns match the "theoretical" PACF from part (c)?

6.29 Simulate a mixed ARMA(1,1) model of length $n = 60$ with $\phi = 0.4$ and $\theta = 0.6$.

(a) Calculate and plot the theoretical autocorrelation function for this model. Plot sufficient lags until the correlations are negligible.

(b) Calculate and plot the sample ACF for your simulated series. How well do the values and patterns match the theoretical ACF from part (a)?

(c) Calculate and interpret the sample EACF for this series. Does the EACF help you specify the correct orders for the model?

(d) Repeat parts (b) and (c) with a new simulation using the same parameter values and sample size.

(e) Repeat parts (b) and (c) with a new simulation using the same parameter values but sample size $n = 36$.

(f) Repeat parts (b) and (c) with a new simulation using the same parameter values but sample size $n = 120$.

6.30 Simulate a mixed ARMA(1,1) model of length $n = 100$ with $\phi = 0.8$ and $\theta = 0.4$.

(a) Calculate and plot the theoretical autocorrelation function for this model. Plot sufficient lags until the correlations are negligible.

(b) Calculate and plot the sample ACF for your simulated series. How well do the values and patterns match the theoretical ACF from part (a)?

(c) Calculate and interpret the sample EACF for this series. Does the EACF help you specify the correct orders for the model?

(d) Repeat parts (b) and (c) with a new simulation using the same parameter values and sample size.

(e) Repeat parts (b) and (c) with a new simulation using the same parameter values but sample size $n = 48$.

(f) Repeat parts (b) and (c) with a new simulation using the same parameter values but sample size $n = 200$.

6.31 Simulate a nonstationary time series with $n = 60$ according to the model ARIMA(0,1,1) with $\theta = 0.8$.

(a) Perform the (augmented) Dickey-Fuller test on the series with $k = 0$ in Equation (6.4.1) on page 128. (With $k = 0$, this is the Dickey-Fuller test and is not augmented.) Comment on the results.

(b) Perform the augmented Dickey-Fuller test on the series with k chosen by the software—that is, the "best" value for k. Comment on the results.

(c) Repeat parts (a) and (b) but use the differences of the simulated series. Comment on the results. (Here, of course, you should reject the unit root hypothesis.)

6.32 Simulate a stationary time series of length $n = 36$ according to an AR(1) model with $\phi = 0.95$. This model is stationary, but just barely so. With such a series and a short history, it will be difficult if not impossible to distinguish between stationary and nonstationary with a unit root.

(a) Plot the series and calculate the sample ACF and PACF and describe what you see.

(b) Perform the (augmented) Dickey-Fuller test on the series with $k = 0$ in Equation (6.4.1) on page 128. (With $k = 0$ this is the Dickey-Fuller test and is not augmented.) Comment on the results.

(c) Perform the augmented Dickey-Fuller test on the series with k chosen by the software—that is, the "best" value for k. Comment on the results.

(d) Repeat parts (a), (b), and (c) but with a new simulation with $n = 100$.

6.33 The data file named deere1 contains 82 consecutive values for the amount of deviation (in 0.000025 inch units) from a specified target value that an industrial machining process at Deere & Co. produced under certain specified operating conditions.

(a) Display the time series plot of this series and comment on any unusual points.

(b) Calculate the sample ACF for this series and comment on the results.

(c) Now replace the unusual value by a much more typical value and recalculate the sample ACF. Comment on the change from what you saw in part (b).

(d) Calculate the sample PACF based on the revised series that you used in part (c). What model would you specify for the revised series? (Later we will investigate other ways to handle outliers in time series modeling.)

6.34 The data file named deere2 contains 102 consecutive values for the amount of deviation (in 0.0000025 inch units) from a specified target value that another industrial machining process produced at Deere & Co.

(a) Display the time series plot of this series and comment on its appearance. Would a stationary model seem to be appropriate?

(b) Display the sample ACF and PACF for this series and select tentative orders for an ARMA model for the series.

6.35 The data file named deere3 contains 57 consecutive measurements recorded from a complex machine tool at Deere & Co. The values given are deviations from a target value in units of ten millionths of an inch. The process employs a control mechanism that resets some of the parameters of the machine tool depending on the magnitude of deviation from target of the last item produced.

(a) Display the time series plot of this series and comment on its appearance. Would a stationary model be appropriate here?

(b) Display the sample ACF and PACF for this series and select tentative orders for an ARMA model for the series.

6.36 The data file named robot contains a time series obtained from an industrial robot. The robot was put through a sequence of maneuvers, and the distance from a desired ending point was recorded in inches. This was repeated 324 times to form the time series.

(a) Display the time series plot of the data. Based on this information, do these data appear to come from a stationary or nonstationary process?

(b) Calculate and plot the sample ACF and PACF for these data. Based on this additional information, do these data appear to come from a stationary or nonstationary process?

(c) Calculate and interpret the sample EACF.

(d) Use the best subsets ARMA approach to specify a model for these data. Compare these results with what you discovered in parts (a), (b), and (c).

6.37 Calculate and interpret the sample EACF for the logarithms of the Los Angeles rainfall series. The data are in the file named larain. Do the results confirm that the logs are white noise?

6.38 Calculate and interpret the sample EACF for the color property time series. The data are in the color file. Does the sample EACF suggest the same model that was specified by looking at the sample PACF?

6.39 The data file named days contains accounting data from the Winegard Co. of Burlington, Iowa. The data are the number of days until Winegard receives payment for 130 consecutive orders from a particular distributor of Winegard products. (The name of the distributor must remain anonymous for confidentiality reasons.)

(a) Plot the time series, and comment on the display. Are there any unusual values?

(b) Calculate the sample ACF and PACF for this series.

(c) Now replace each of the unusual values with a value of 35 days—much more typical values—and repeat the calculation of the sample ACF and PACF. What ARMA model would you specify for this series after removing the outliers? (Later we will investigate other ways to handle outliers in time series modeling.)