less than 128. By continuing in this manner, we can eventually arrive at a situation where it is not possible to find any directed link which leads to an improvement of the distance shown in any circle. If so, we have arrived at an optimal solution. For the example at hand, the optimal tree is the same as that shown in Fig. 2 except that the arrow from Washington to Boston is dropped and one from Chicago to Boston is inserted. The 191 at Boston is changed to 186. The values of the x_{ij} are unity along the path in the final tree from Los Angeles to Boston and are zero elsewhere. Hence the optimal path is from Los Angeles to Salt Lake City, then to Chicago, and finally to Boston.

5. THE KNAPSACK PROBLEM

In certain types of problems, we can get extreme-point solutions for which not all the values of the x_{ij} are either zero or one. When any of the x_{ij} have fractional values, the corresponding extreme points are referred to as fractional extreme points. Now an example of this occurs in the knapsack problem. In this problem a person is planning a hike and has decided not to carry more than 70 lb of different items, such as bed roll, geiger counters (these days), cans of food, etc.

We try to formulate this is mathematical terms. Let a_j be the weight of the j^{th} object and let b_j be its relative value determined by the hiker in comparison with the values of the other objects he would like to have on his trip.