# *Gravity Corrections*

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Abstract:

A borehole was used to confirm the presence of an ore deposit at a depth of 1km in a 50x50km surveyed area. The region centering at a latitude of 49.1286 $^{o}$ north was surveyed using a moving vehicle towing a relative gravimeter stopping at regular intervals to obtain raw gravity data. Raw gravity data contains noise not associated with the geology of the region and cannot be used immediately for accurate geophysical interpretations. As a result, Matlab is used as a programming tool to perform normal gravity corrections, drift corrections, and finally elevation and terrain corrections. Once these effects are removed, a Bouguer anomaly with large relative gravity measurements is distinguishable in the eastern most region of the surveyed area. This anomaly interpreted as the ore deposit extends laterally from 0 to 12 degrees Easting, and 15 to 30 Degree Northing.

Introduction:

Raw land gravity survey data obtained using a relative gravimeter towed behind a vehicle stopping at regular intervals offers a representation of the relative gravity effect of the surveyed area, but contain noise which isn't attributed to the geology of the region. Resultantly, gravity corrections are necessary to make accurate geological and geophysical interpretations. Normal gravity correction, drift correction, elevation and terrain corrections are applied using Matlab programming software to obtain data that is representative of the true geology. Using the corrected data, the lateral extent of the ore deposit is then established. The data provided to perform these corrections are obtained from a 50x50km region centered on a latitude of 49.1286$^{o}$ north. This region is centered on an ore deposit buried at a depth of 1000m.

Background/Theory:

When investigating gravity anomalies, the measured gravity effect of a point must be contrasted with a theoretical value. The IGF formula for ground surveys is a function of latitude and calculates the theoretical gravity as a function of latitude and accounts for the effects of the earth's centrifugal acceleration in the lateral direction (1). The IGF formula implemented assumes the earth is a spheroid symmetric about the equator and gravity effect does not vary with longitude.

$$g_t(\theta) = 9.780327[1 + .0053024\sin^2(\theta) - .0000058\sin^2(2\theta) \qquad (1)$$

Earth's tidal variations caused by the moon's deformation of oceans and the sun's effects on the gravitational field around the earth causes errors to gravity data obtained over long periods of time. Therefore tide corrections are made by collecting data in a loop where the gravimeter is returned to a base station after a set amount of time. Relative gravimeters are made using zero length springs whose stiffness may change over time. Consequently, the instrument drift is corrected for by calculating the relative gravity at the beginning and end of each new loop.

In rugged terrains where there are changes in the earth's elevation, there may be variations in gravity readings owning to the change in distance from the survey point to the earth's center of mass.

Hence, the free air correction (2) is applied where G is the earth's gravitational constant, $M_e$ is the earth's mass, $R_e$ is the earth's radius, and $\Delta Z_i$ is the difference between the elevation at the survey point and the mean elevation of the surveyed area. The free air correction accounts for the distance from the earth's center of mass.

$$\Delta g_{FA} = -.3086\Delta Z \qquad (2)$$

The Bouguer plate correction (3) is used with the assumption that the material between the survey point and mean elevation (datum) is a laterally extending slab in all directions with a uniform density (2.65g/cc).

$$\Delta g_{BP} = 4\pi G\rho_B\Delta Z \qquad (3)$$

Tectonics events in active regions of the earth cause an upward pull of the earth's crust in some regions and valleys with mass defects in others. A terrain correction is carried out to correct for these effects by calculating the mass above and below relative to the Bouguer slab to find the absolute value of gravity effect (4).

$$g_z = Gm\frac{z-z_m}{r^3} \qquad (4)$$

In the corrections above we have assumed that the earth is spheroidal and symmetric about the equator with no major changes at different longitudes. However, the earth is in fact a geoid with unevenly distributed masses in its mantle and crust. To simplify this, the geoid effect is accounted for by taking the average of all the Bouguer corrected gravity values and subtracting this average from each corrected survey point.

The lateral gravity variations ($\frac{\partial g}{\partial x}$ $and$ $\frac{\partial g}{\partial y}$) are found using the expressions below, where i corresponds to the north-south coordinate, while j corresponds to the east-west coordinate.

$$\frac{\partial g}{\partial x} = \frac{g_{i,j}-g_{i,j-1}}{\Delta x} \quad (5a)$$

$$\frac{\partial g}{\partial y} = \frac{g_{i,j}-g_{i-1,j}}{\Delta y} \quad (5b)$$

The second vertical derivative of gravity with respect to depth can be found by using the survey by comparing lateral survey point readings since the LaPlace equation is satisfied by gravity.

$$\frac{\partial^2 g}{\partial z^2} = \frac{1}{\Delta x^2\Delta y^2}[2(\Delta x^2 + \Delta y^2)g_{i,j} - [\Delta x^2(g_{i+1,j} + g_{i-1,j}) + \Delta y^2(g_{i,j+1} + g_{i,j-1})]] \qquad (6)$$

Methods/Algorithm:

Matlab is used to carry out gravity corrections by first loading the provided survey data contained in the file “GOPH_547_lab_2_data_w2016_rev.mat”. The provided file contains four arrays: grav_survey_data, Xt, Yt, and Zt. The grav_survey_data array contains eight columns of different geophysical data. The first column is saved as ‘base_point’ which provides information regarding whether the survey point is at the beginning of a loop (1), end of the loop (2), or just a normal survey point. The second column of grav_survey_data defined as ‘day’ corresponds to the day of the survey. The time in a given day is set to ‘time’ and is contained in the third column of grav_survey_data. The fourth column of grav_survey_data is the ‘total time’ of the survey beginning from midnight on day 1. The fifth, sixth, and seventh columns of grav_survey_data correspond to the x,y, and z coordinates of the survey point respectively and are saved as these variables. The eighth column of grav_survey_data contains the measured gravity effect (g) in microgals, with the absolute gravity contained in the first row, and subsequent rows containing relative gravity effects. Finally, the 9$^{th}$ column of grav_survey_data contains tidal variation data (dg_tide) measured using an absolute gravimeter at regular time intervals.

Once the required variables have been initialized, g_case2 is defined and contains only relative gravity data by setting the first row of g equal to 0. Then the Matlab plot function is used with total_time as the x input, and g_case2 in the y input. The hold on function is used to superimpose tidal variation data onto the previous figure by again using the plot function with total_time as the x input and dg_tide as the y input. Then a contour plot of the survey data is created using the built in Matlab function contour with Xt, Yt, and Zt as its input. A color bar is added by using the built in function colorbar. The datum of the surveyed area is found by finding the average elevation of the studied region by first finding the mean values in the columns of Zt, and then averaging the values found in all the rows using the mean function and is set to z_datum. Then a sorted matrix x_sort is created by assigning x, y, and z to the first, second and third columns of this matrix respectively. Then a sorted matrix and its indeces are saved as x_sort and ind_sort respectively by using the built in sortrows function using x_sort as the x input, and [1,2] as its y input to sort through x, and then y. Since the gravity data is collected in loops, there are many entries at the same point, and so we the built in function unique is used with x_sort and ‘rows’ as inputs to retrieve only the unique values and are output to x_sort. The indices of these values are output to ind_uniq. Then the unique sorted grid coordinates are saved as Xg, Yg, and Zg by extracting the first, second, and third columns of x_sort respectively. The number of points in these grids are squared to find the number of rows (Nx) and columns (Ny). Then Xg, Yg, and Zg are converted to grids by using the built in reshape() Matlab function with Nx, and Ny used as the number of rows and columns. A copy of g is copied and initialized as vector g_corr. The vector g_raw is initialized and is simply g_corr.

Absolute gravity effects are computed for all points in g_raw by creating a loop that runs from i=2 to Nx. It updates the ith value of g by adding the relative gravity measurement at that ith position to g_raw(1) which is the absolute gravity measured at the base station. G_raw is then overwritten to only contain unique values by using the g_raw(ind_sort) and then g_raw(ind_uniq).  A grid is then created using the built in function reshape() as was previously done for Xg,Yg, and Zg. A contourplot is then made by using the built in function contour() with Xg, Yg, and g_raw as its inputs.

The first gravity correction applied is the normal gravity correction. It is carried out by applying (1) to obtain the IGF theoretical gravity formula gt. This correction is then implemented by simply updating the absolute gravity effect g_corr(1) to g_corr(1) minus the theoretical gravity effect. Then g_corr is saved as g_norm and a contour plot of g_norm is created similarly to g_raw in the previous section.

Next drift corrections are applied by implementing tide corrections and then instrument drift correction. Tidal drift corrections are made by creating a loop from 1 to Nx. Thisloop operates by subtracting g_corr(i) from dg_tide(i) and overwriting the g_corr(i) to this calculated value. The current value of g_corr is then saved as g_tide and a contour plot is created using the same method used for g_raw. Next instrument drift corrections are made by using a loop with range from 1 to the length of the base_point vector. In the loop an if statement checks if the position at the current ith position is at the end of the loop (base_point==2). If true, the value in g_corr(i) is saved as drift. Then dt is calculated by subtracting the value of total_time at the current position from the previous loop. This is then used to calculate the drift rate in the current loop and correct for it in g_corr. A drift corrected contour plot is finally acquired using the same method used in g_raw.

Next elevation and terrain corrections are to be made. We begin with the free air correction by creating a vector dg_FA using the expression in (2). This vector dg_FA is then added to g_corr to account for gravity variations as a result of variation in distance from the earth's center. A contour plot of the corrected data is made by copying the updated g_corr and renaming it to g_FA. The Bouger plate correction is applied using (3) to create a vector dg_BP and adding it to g_corr. Again, this updated g_corr is saved as g_elev since it accounts for Bouguer and free air effects. The final correction, terrain correction is created by first initializing a zero array, dg_terr, by using the built in function zeros() with inputs of Nx and Ny respectively. To obtain the coordinates from Xg and Yg, and z_datum, a double nested loop with the first looping from j=1 to N and the second looping from i=1 to Ny is used to initialize xi which is a row vector [Xg(i,j), Yg(i,j), z_datum]. Next the local centre of mass's coordinates are to be saved in a row vector xm by adding two loops to the ones already created the first being one that runs from k=1 to 151 and the second being n which runs from 1 to 151. Xm gets its inputs from the kth and nth coordinates of Xt, and Yt, and Zt. However, z_datum must be subtracted from Zt to make the elevations relative to the datum chosen. Next the mass above and below the datum at the current survey point grid is calculated by using and is saved as dm. Finally the absolute gravity effect at the current position is found using (4) and a previously defined Matlab function, grav_eff_point(), developed in Lab 1 and is added to dg_terr. The terrain corrected matrix dg_terr is then added to g_corr using a similar approach to g_raw. The updated g_corr is to be saved as g_terr and a contour plot is made.

The geoid effects of the surveyed area are removed by first computing the average of all values in g_corr by using the mean() function twice in the same statement with g_corr as its input and saving this variable as dg_rgnl. A double nested loop is then used to subtract dg_rgnl from g_corr and saved as g_anom. A contour plot is then made.

The gravitational derivatives with respect to x, y are found using (5a) and (5b) respectively through the implementation of a double nested loop with the outer loop having a range i=2 to Nx, and the inner loop with a range of j=2 to Ny. In the second loop, the variable dgdx is updated by using (5a) and 5(b) where g_anom(i,j) is used as the input. The second derivative of gravity with respect to depth is found similarly using (6).

Results and Discussion:

Figure 1 illustrates relative gravity measurements and tidal variations over the course of the current survey. The relative gravity initially decreases for the first 1000 seconds of surveying and then increases before tapering off. This figure also illustrates that the last surveyed area contains the largest anomaly in the map. The tidal variation shows a sinusoidal pattern of change with peaks correlating to high tides and troughs correlating to low tides caused by the variation of the moon's position relative to the earth. Additionally, the lower portions of the relative gravity curve shows a sinusoidal pattern as well which helps illustrates the effects tidal variations have on relative gravity measurements.

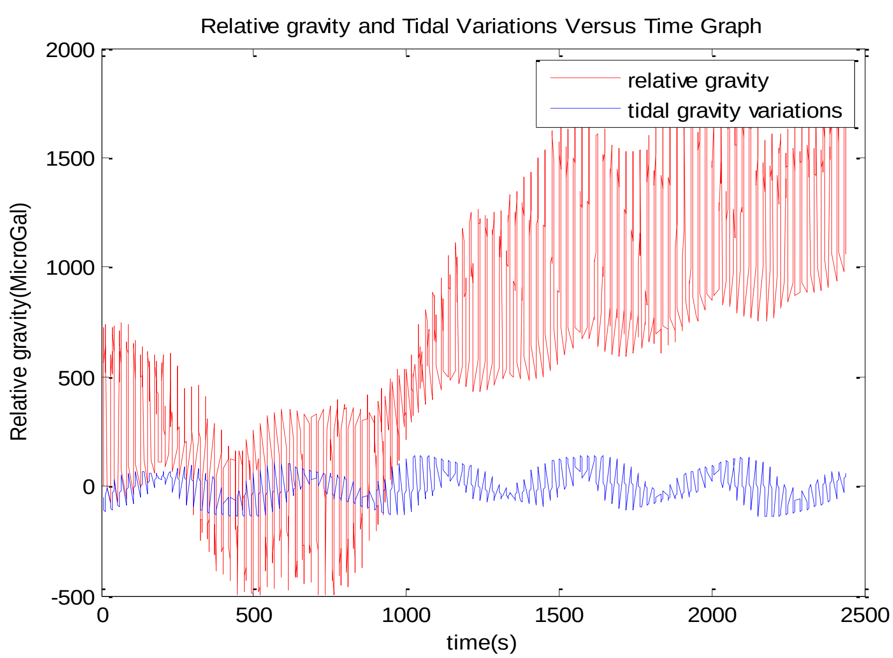


*Figure 1: Relate gravity and tidal variation data plotted against total survey time*

Figure 2 is a contour plot of the terrain surveyed data. This figure illustrates that the surveyed region primarily consists of six hills with a range in elevation from 2995m to 3005m. The mean elevation is 3000m and is chosen to be the datum.

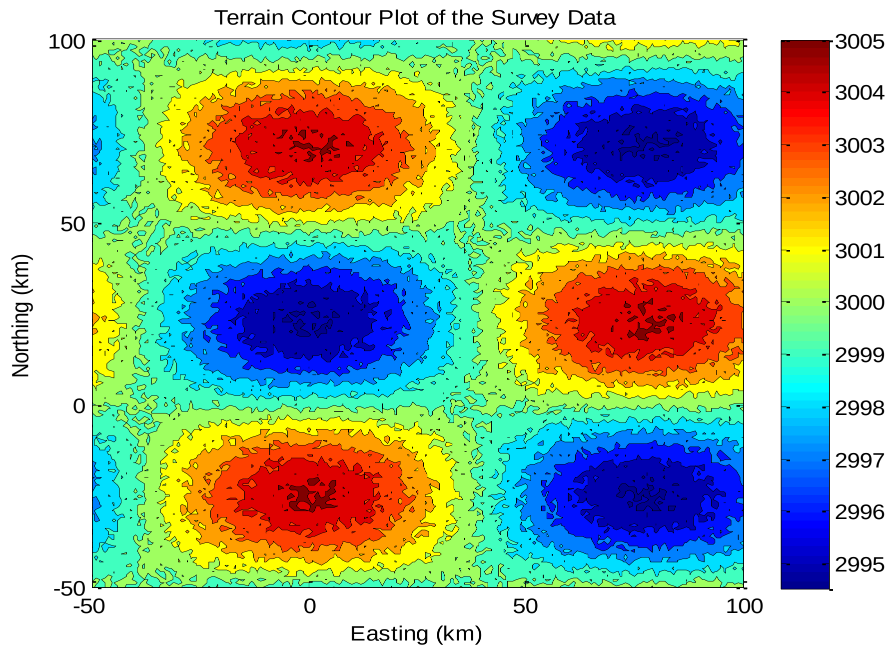


*Figure 2: A contour plot of the terrain showing different elevations in the region*

Figure 3 shows the absolute gravity of the surveyed area before any gravity corrections are made. Notably, there is a large division line at an Easting of 25m that separates the west region which is characterized by lower relative gravity readings from the east region of the surveyed area characterized by larger relative gravity readings. This may be a result of the average elevation on the west portion being higher

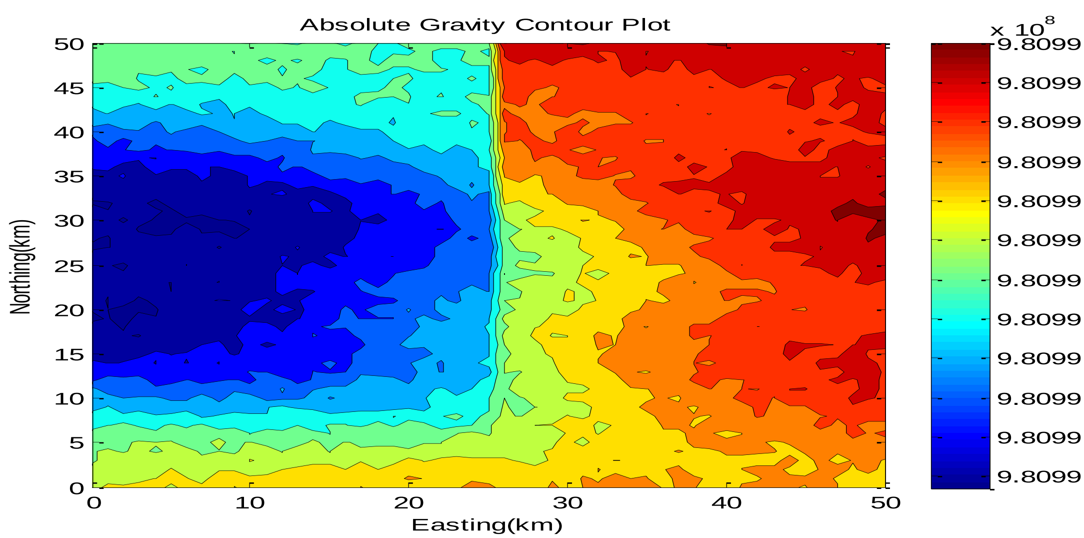


*Figure 3: Contour plot of the absolute gravity of the surveyed region before any corrections*

Figure 4 shows a contour plot of the normal gravity corrected survey data. This was acquired by computing the theoretical gravity at a latitude of 49.1286 degrees North accounting for the centrifugal force of the earth. This was then subtracted from the absolute gravity at that point. This residual value is the relative gravity measurement of the surveyed region after it has been corrected for centrifugal acceleration of the earth compared to the expected gravity measurement at the point. Other gravity errors not corrected for and an anomalous mass within the region may contribute to the discrepancy between the absolute and theoretical gravity. In contrast to figure 3, Figure 4 shows large portions owing to large relative gravity measurements. The large bisecting point is also still present in the region in both raw and normal corrected contour plots.

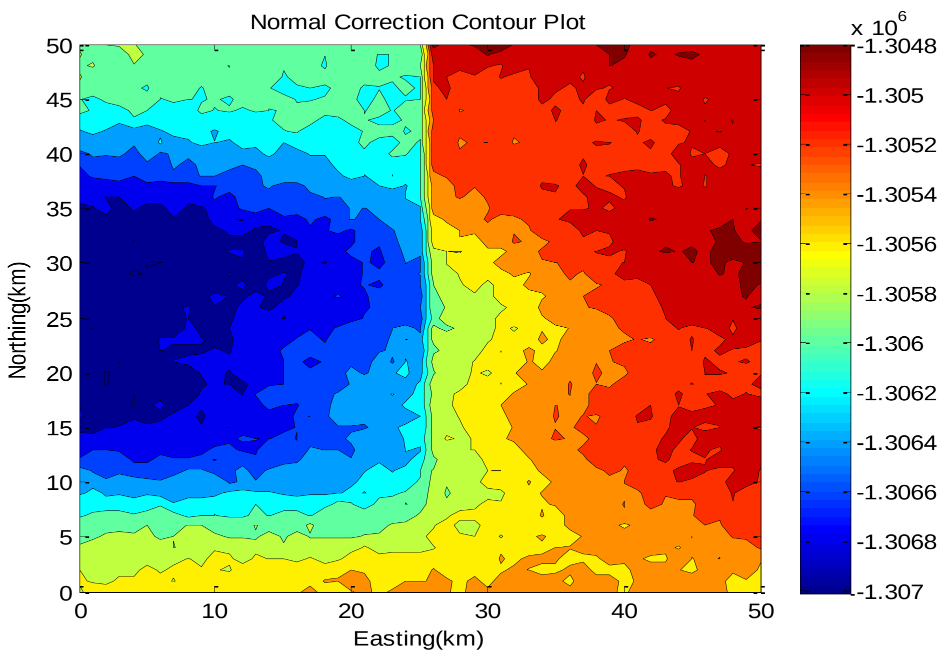


*Figure 4: Contour plot of gravity effect after correcting for centrifugal forces*

Figure 5 is a contour plot of tidal drift corrected gravity data. The tidal corrections accounted for are caused by the attractive forces of the moon and sun. However, the moon accounts for more of the gravity error due to its closer proximity to Earth. Events that cause major deformation of the ocean such as tectonic events may also cause tidal changes. In comparison to Figure 4, the tidal drift corrected contour plot contains less random anomalies and patches in the region. Additionally, the contour plot contains larger areas with higher relative gravities. Again, the bisection at easting of 25m is still present and separates the higher relative gravities on the right from the lower readings on the left.

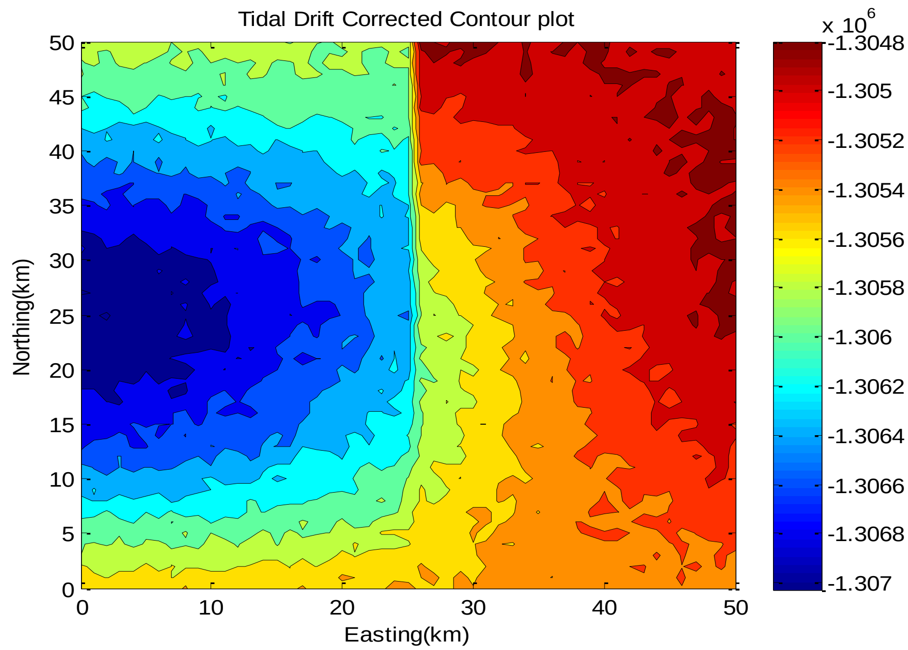


*Figure 5:: A gravity effect contour plot with time varying tidal drifts correction*

Figure 6 is a time series plot of drift corrected gravity versus total time. This drift corrected data varies less with time (one cycle in 2500s) than the uncorrected time series plot in Figure 1. This is because the drift correction corrects for instrument drift and tidal variations which are both a function of time. This remaining gravity variations with time illustrate the corrections aren't perfect.

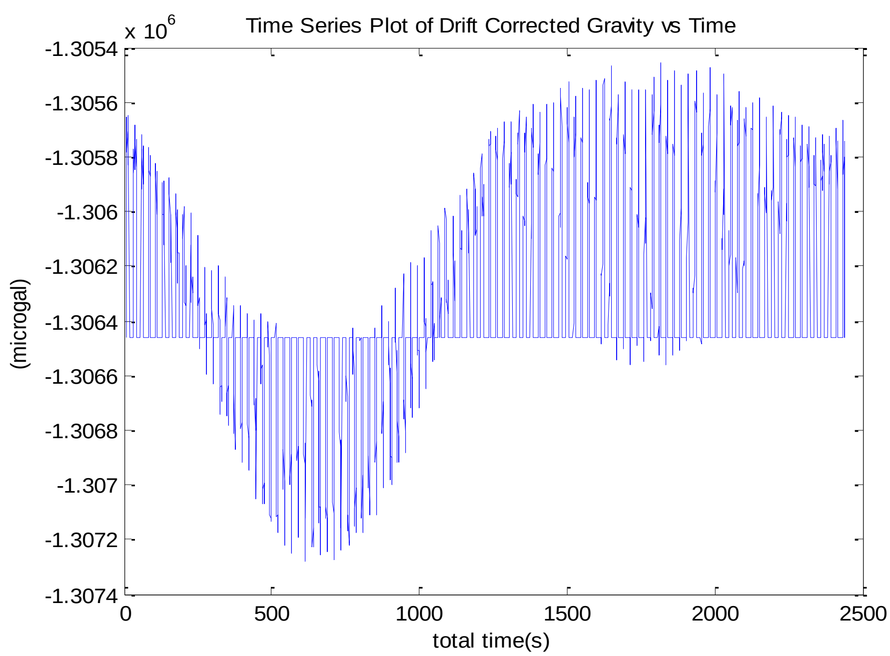


*Figure 6: Tidal drift corrected gravity data versus time graph*

Figure 7 is a contour plot containing all time varying corrections made. This plot contains instrument drift correction in addition to the previous corrections made. In comparison to Figure 6, the contour plot obtained no longer contains the large division line at an easting of 25m separating high and low relative readings. From the following outcome, we can conclude that the visible bisecting line in the previous plots was a result of instrument drift owning to the geometry and collection of the survey data (Movement of the recording vehicle). Now an anomaly with a small relative gravity of -1.3 microgals with a center at easting of 5 and northing of 25 is distinguishable and increases to the right where an anomaly with a large relative gravity of 1.3056 microgal is distinguishable. The spherical decay with increasing radius from the center of the small anomaly also suggests this data is more representative as this is how gravity data behaves around an anomaly.

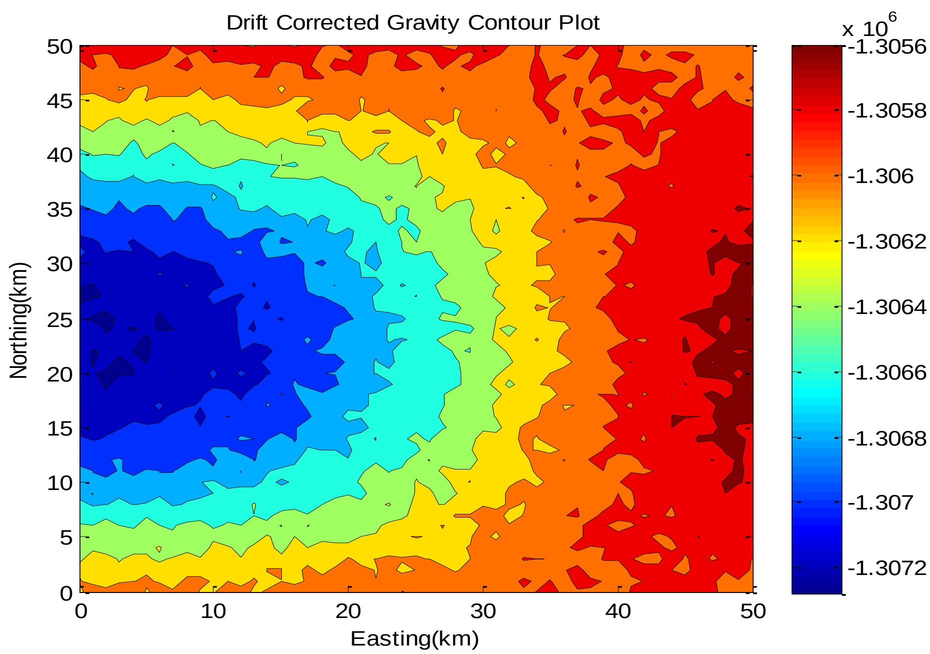


*Figure 7: Instrument drift corrected gravity contour plot.*

Figure 8 shows a free air corrected gravity contour plot. When the elevation at a survey point is above the measured datum, the free air correction decreases the relative gravity measurement in g_corr. As a result, the areas with lower mean elevations on the east portion of the surveyed data (Figure 2) increase their relative gravity, while the areas in the western region with higher mean elevation decrease their relative gravities. This phenomenon occurs because areas where Zg is greater than z_datum are located farther away from the earth's center of mass, and as a result record lower relative gravity readings. A main difference in Figure 8 with respect to previous plots is the surveyed region now contains regions with relative gravity greater than zero (excess mass). Additionally, the edges of the contours are now smoother after this correction.

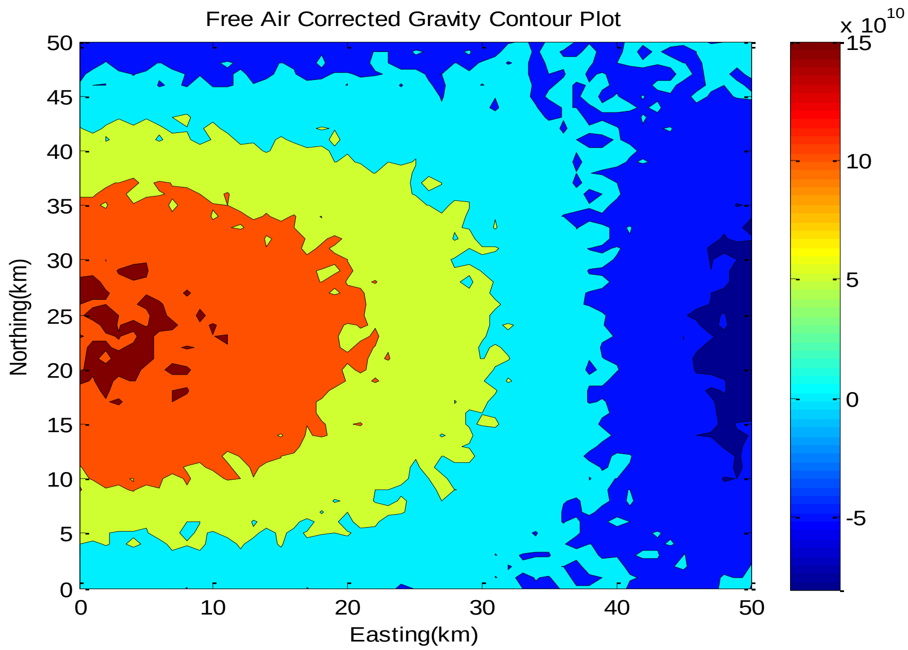


*Figure 8: A contour plot showing free air corrected gravity data*

Figure 9 shows a Bouger Plate corrected contour plot. When Zg is greater than z_datum, the Bouguer plate correction should increase g_corr. This is because the Bouguer correction accounts for the material between the datum and survey point. The relation accounting for this (1) is a function of the average density of the material found between then datum and survey point. Since density is always positive, the material above the datum will always result in a positive increase in gravity corrections. The elevation corrected in Figure 9 shows two anomalies in contrast to the previous figures. The new high relative gravity anomaly begins at 40km easting and extends from south to north.

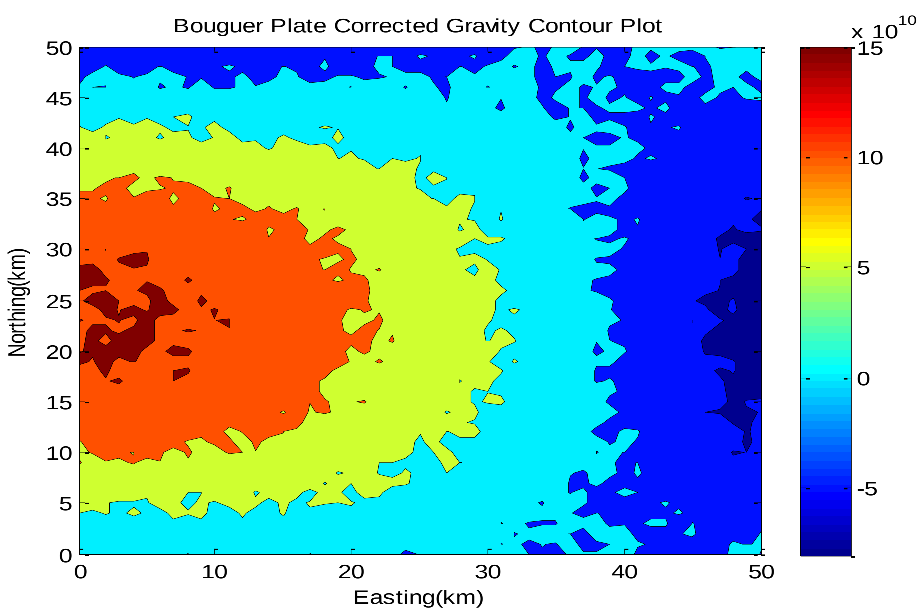


*Figure 9: A contour plot of Bouguer plate corrected contour plot using a density of2.65g/cc*

Figure 10 is a contour plot of terrain corrected data owing to the overestimation of the Bouguer plate correction in lower regions and the upward pull of high elevation areas. The gravity effect of regions with Zt greater than the datum correspond to mass pull ups (Bouguer under-correction), while valleys are mass deficiencies (Bouguer overcorrection). Looking at (3) it is evident that the Bouguer correction reduces g_corr, hence the positive gravity effect is necessary to account for this overcorrection in terrain corrections. This is why we take absolute value when computing terrain corrections. Figure 10 shows Bouguer two anomalies at Northings of 25km in the eastern and western most points of the surveyed area. The western regions contains a negative anomaly, while the eastern region contains a high relative gravity anomaly.

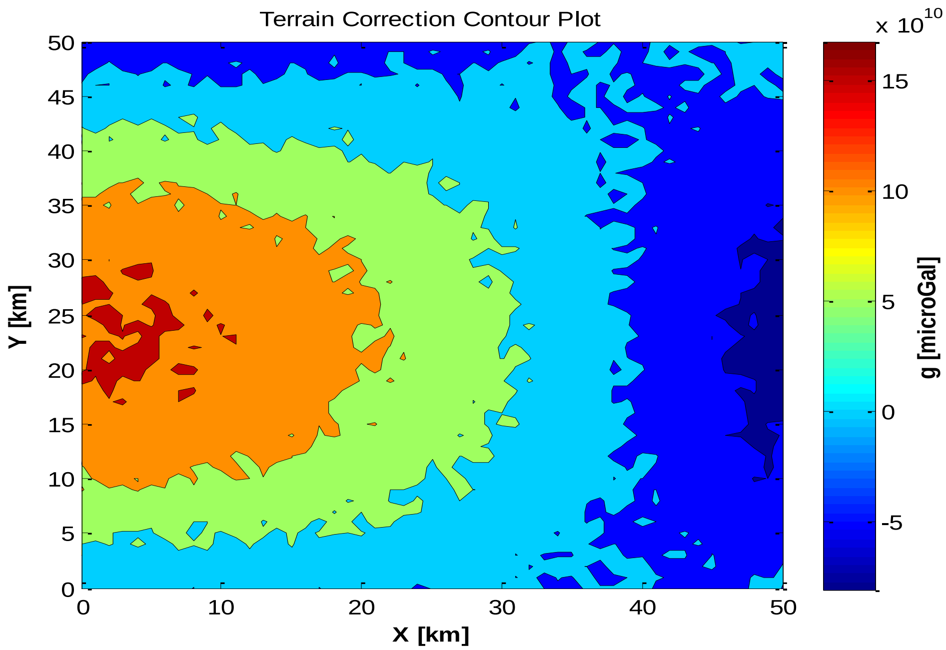


*Figure 10: Terrain corrected gravity contour plot*

Figure 11 is a contour plot of geoid corrected gravity data. In contrast to the previous figures, Figure 11 shows the decay in relative gravity radially away from the center of the anomaly with larger gravity more clearly. The contour with the highest gravity is larger, and the magnitudes of the gravity effects have been increased by a factor of 5 after the geoid effects are accounted for. Additionally, the anomaly in the western region of the surveyed area with a negative gravity is more discernable than in previous plots. The anomaly has a range of gravity between 5x10^10 µGal to 1*10^11 µGal. The anomaly of interest extends laterally from 15 to 30 degrees Northing and 0 to 12 degrees Easting.

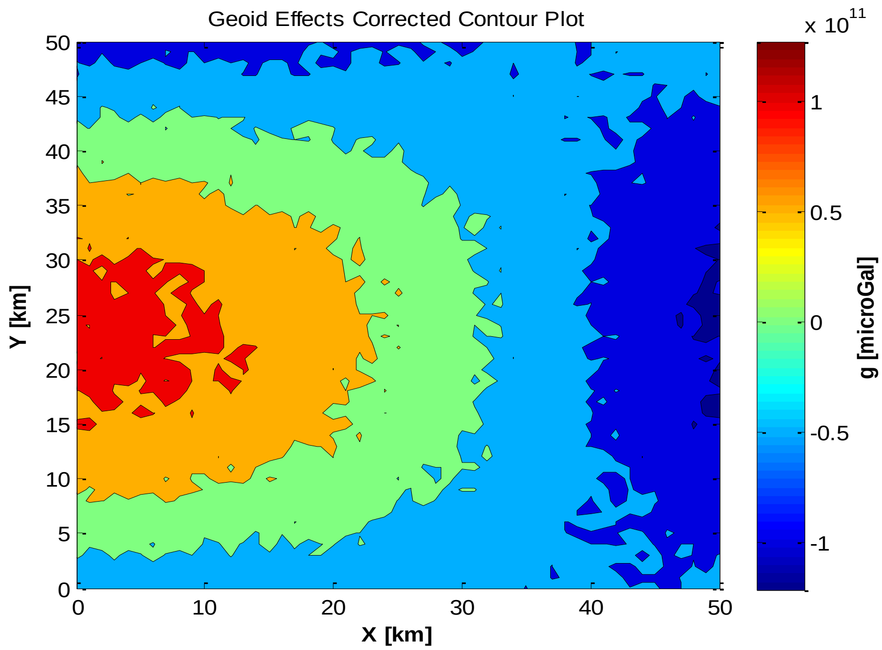


*Figure 11: A contour plot of gravity data with all gravity corrections made*

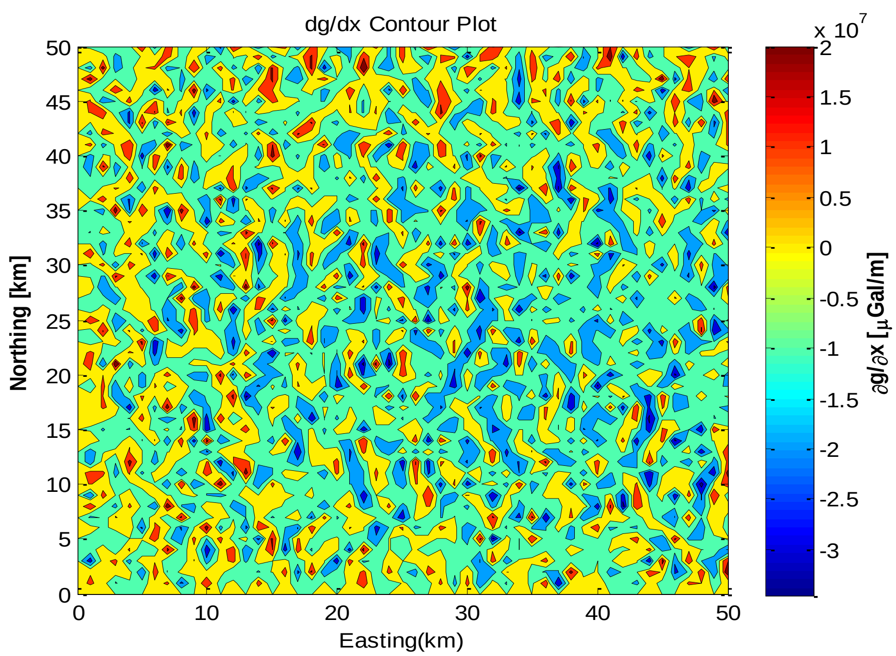


*Figure 12: A contour plot illustrating the lateral variation of data in the Easting direction*

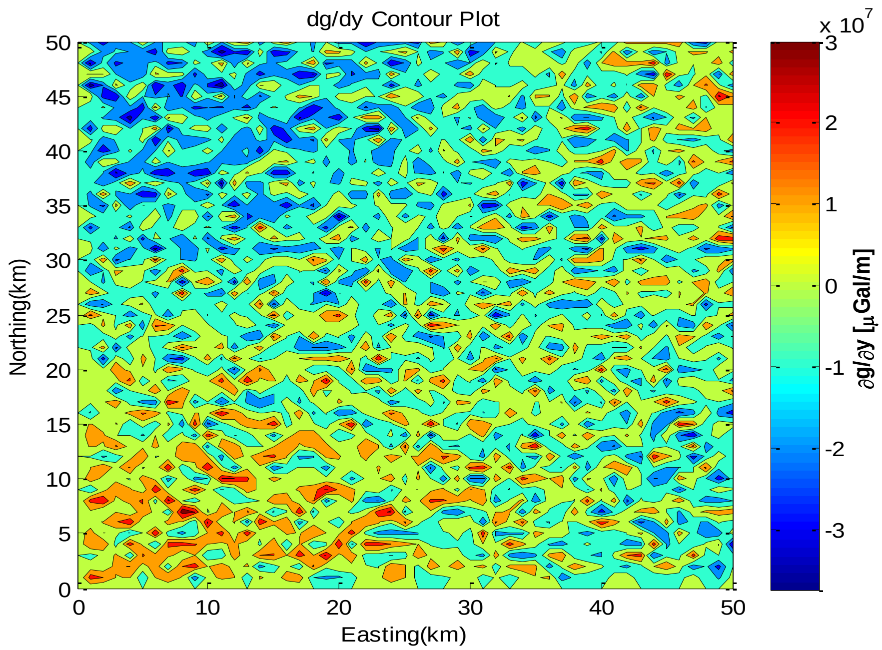


*Figure 13: A contour plot showing the rate of gravity change in the Northing direction*

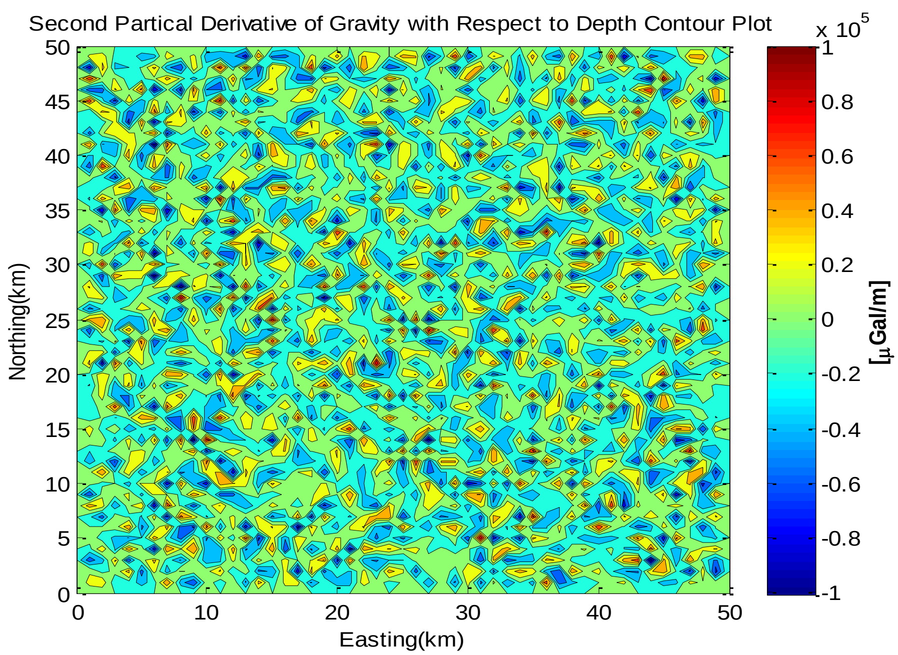


*Figure 14: A contour plot showing the second order change in gravity relative to depth*

Conclusion:

The raw gravity data obtained from a 50x50km region centered at a latitude of 49.1286 degrees north contained noise not associated with the geology of the region. Matlab was used to perform normal gravity corrections, drift corrections, as well as elevation and terrain corrections to obtain interpretable geophysical data. An ore body with gravity ranging from $5*10^{10}$ μGal to $10^{11}$ μGal was found in the contour plots. The ore body extends laterally from 0 to 12 easting and 15 to 30 northing.

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