

Comprehensive Study Notes on Elementary Mathematics: A Student's Guide

Welcome to your comprehensive study guide for elementary mathematics! These notes are designed to be a complete, self-contained resource to help you prepare for your exams. The content is structured topic-by-topic—covering numbers, shapes, measurement, data handling, and algebra—to make your learning and revision sessions focused and effective. Let's get started!

1. Unit 5: Numbers and Operations on Numbers

1.1. The Concept of Numbers

Introduction to Number Systems

Numbers are essential tools we use every day for quantification—counting items, measuring distance, telling time, and so much more. While we use a standard system today, different civilizations throughout history developed their own number systems, like the Roman system (I, V, X). These early systems were often difficult to use for basic calculations like addition and subtraction, which led to the development of the more efficient decimal system we use now.

The Decimal System: Place Value vs. Face Value

The decimal system is built on two key ideas: face value and place value. Let's break this down with a simple analogy.

- **Face Value:** This is the digit's name. A '2' is always a '2', and a '6' is always a '6'. It's the intrinsic value of the digit itself.
- **Place Value:** This is the digit's job title. Its value depends on its position in the number (e.g., the unit's place, ten's place, hundred's place).

Let's use the number 26 to see this in action:

- The digit 6 is in the unit's place. Its face value is 6, and its "job" is to be 6 ones, so its place value is also 6.
- The digit 2 is in the ten's place. Its face value is 2, but its "job" is to be 2 tens, so its place value is 20.

1.2. Sets of Numbers

Whole Numbers

Whole numbers are simply the set of counting numbers, but with zero included (0, 1, 2, 3...). The smallest whole number is 0. The counting number 1 has no predecessor in the set of natural numbers (counting numbers).

Integers

Integers came about from the need to describe situations with opposite characteristics, like a profit versus a loss, or moving upward versus downward.

Measurement with Opposite Characters	Balancing Position
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Profit - Loss	No Profit nor Loss
Asset - Liability	No Asset nor Liability
Credit - Debit	No Credit nor Debit
Upward - Downward	Neither Upward nor Downward

This "balancing position" is represented by zero.

- **Integers** are the set of all whole numbers and their opposites: ...-3, -2, -1, 0, +1, +2, +3....
- **Positive Integers** are +1, +2, +3....
- **Negative Integers** are -1, -2, -3....
- For every positive integer $+p$, there is an opposite negative integer $-p$ so that when you add them together, they balance out to zero: $(+p) + (-p) = 0$.

Representing Integers on a Number Line A number line gives us a great visual for integers. To draw one, simply draw a line, mark a point for zero (0), and then mark the positive integers at equal spaces to the right and the negative integers at equal spaces to the left.

Think about it: Why is -5 less than -2, even though 5 is greater than 2? The number line shows us that numbers further to the left are always smaller.

Rational Numbers (Fractions)

Rational numbers, which you probably know as fractions, were created to represent parts of a whole object. Every fraction has two parts:

- **Numerator:** The top number, which tells us how many parts we have.
- **Denominator:** The bottom number, which tells us how many equal parts the whole was divided into.

Here are the main types of fractions:

- **Proper Fraction:** The numerator is less than the denominator (e.g., $3/5$). It represents a quantity less than one whole.
- **Improper Fraction:** The numerator is greater than the denominator (e.g., $7/5$). It represents a quantity greater than one whole.
- **Mixed Number:** A combination of a whole number and a proper fraction (e.g., $2\frac{3}{7}$). Mixed numbers are just another way to write improper fractions and can be converted back and forth.
- **Unit Fraction:** A fraction where the numerator is 1 (e.g., $1/9$).

Standard Form of a Rational Number A rational number is in its standard form when it is written as p/q , where p and q have no common factors other than 1, and the denominator q is positive ($q > 0$).

1.3. Properties of Operations on Number Sets

Operations on Whole Numbers

Addition and multiplication with whole numbers follow a few predictable rules, or properties.

Property	Description	Example
Closure Property	The sum or product of any two whole numbers is always another whole number.	$5 + 3 = 8$
Commutative Property	You can add or multiply two whole numbers in any order and get the same result.	$5 \times 4 = 4 \times 5$
Associative Property	When adding or multiplying three numbers, it doesn't matter how you group them.	$(2+3)+4 = 2+(3+4)$
Additive Identity	Adding 0 to any whole number doesn't change its value. 0 is the additive identity.	$p + 0 = p$

Operations on Integers

Integers follow all the same addition properties as whole numbers, but they add one more important property:

- **Existence of Additive Inverse:** For any integer $+p$, there's an opposite integer $-p$ that brings it back to zero: $(+p) + (-p) = 0$.

This new property changes how we think about subtraction. Subtracting an integer is the same as adding its opposite (its additive inverse). $p - q = p + (-q)$ For example, $(+5) - (+8)$ is the same as $(+5) + (-8)$, which equals -3.

Operations on Rational Numbers

- **Properties of Addition:** Rational numbers follow the Closure, Commutative, and Associative properties for addition. They also have an **Additive Identity (0)** and an **Additive Inverse** for every number.
- **Properties of Multiplication:** Rational numbers follow the Closure, Commutative, and Associative properties for multiplication. They also have a **Multiplicative Identity (1)**, a **Multiplicative Inverse** (the reciprocal) for every non-zero number, and **Multiplication distributes over addition**.

A quick summary: Notice how as we move from Whole Numbers to Integers and then to Rational Numbers, the number sets retain the previous properties while adding new ones like the 'additive inverse' and 'multiplicative inverse'!

1.4. Factors, Multiples, and Primes

Factors and Prime Numbers

- A **Factor** of a number is any natural number that divides into it exactly, with no remainder. The factors of 12 are 1, 2, 3, 4, 6, and 12.
- A **Prime Number** is a natural number that has exactly two factors: 1 and itself. Examples include 2, 3, 5, 7, and 11.

Related Terms

- **Co-Primes (or Mutually Primes):** These are two natural numbers that do not have any common factor other than 1. For example, (8, 27) are co-primes (even if each of them is composite). This is a key point—the numbers themselves don't have to be prime!
- **Twin Primes:** This is a pair of prime numbers whose difference is exactly 2. Examples include (3, 5), (11, 13), and (17, 19).

Highest Common Factor (H.C.F.)

The H.C.F. of two or more numbers is the largest number that is a factor of all of them. Here are two ways to find it:

1. **Prime Factorization Method:** Break down each number into its prime factors. The H.C.F. is the product of the lowest powers of all the common prime factors.
2. **Continued Division Method:** Divide the larger number by the smaller one. Then, divide the first divisor by the remainder. Keep repeating this process until the remainder is 0. The last divisor you used is the H.C.F.

Lowest Common Multiple (L.C.M.)

The L.C.M. of two or more numbers is the smallest number that is a multiple of all of them.

- **Prime Factorization Method:** Break down each number into its prime factors. The L.C.M. is the product of the highest power of all prime factors that appear in any of the numbers. For example, for 12 ($2^2 \times 3$) and 18 (2×3^2), the L.C.M. includes the highest power of 2 (which is 2^2) and the highest power of 3 (which is 3^2). So, L.C.M. = $2^2 \times 3^2 = 36$.

Relation between L.C.M. and H.C.F.

For any two numbers, there's a handy formula that connects their H.C.F. and L.C.M.: **Product of two numbers = (H.C.F. of the numbers) × (L.C.M. of the numbers)**

1.5. Decimals and Arithmetic Applications

Decimal Numbers

- **Terminating Decimals:** These are decimals that have a finite number of digits (e.g., 0.75). They end.
- **Non-terminating and Recurring Decimals:** These decimals go on forever, but they have a digit or a group of digits that repeats indefinitely (e.g., 0.333...). We often use a dot or a bar to show the repeating part: 0.3 or 0.23. The dot above a digit (or a bar over a group of digits) indicates that it is the part that repeats forever.
- **Non-terminating and Non-recurring Decimals:** These decimals go on forever without any repeating pattern (e.g., π or 3.2010010001...).

The key takeaway is that both terminating decimals and non-terminating recurring decimals represent rational numbers.

How to convert a recurring decimal into a fraction (e.g., $x = 0.\overline{232323}$):

1. Let $x = 0.\overline{232323}$...

2. Since two digits are repeating, multiply both sides by 100: $100x = 23.232323\dots$
3. Subtract the first equation from the second: $100x - x = 23.232323\dots - 0.232323\dots$
4. This simplifies to $99x = 23$.
5. Solve for x : $x = 23/99$.

Applications in Arithmetic

The Unitary Method This method is all about figuring out relationships between quantities.

- **Direct Variation:** Two quantities vary directly if when one increases, the other increases proportionally. For example, the more pens you buy, the higher the total cost.
- **Inverse Variation:** Two quantities vary inversely if when one increases, the other decreases proportionally. For example, the more workers you have on a job, the less time it takes to complete.

Profit and Loss

- **Profit = S.P. – C.P.** (Selling Price – Cost Price)
- **Loss = C.P. – S.P.** (Cost Price – Selling Price)
- **Profit % = (Profit / C.P.) × 100**
- **Loss % = (Loss / C.P.) × 100**

Simple Interest

- **Principal (P):** The initial amount of money borrowed or invested.
 - **Interest (I):** The extra money paid for using the principal.
 - **Time (t):** The duration for which the money is borrowed/lent.
 - **Rate (r):** The interest on 100 units of currency for one year (usually given as a percentage).
 - **Amount (A):** The total money paid back, which is the Principal plus the Interest.
 - **$I = (P \times t \times r) / 100$**
 - **$A = P + I$**
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2. Unit 6: Shapes and Spatial Understanding

2.1. Fundamental Concepts

2-D and 3-D Shapes

A **2-D shape** (two-dimensional) is a flat figure that can be drawn on a piece of paper. It only has two dimensions, like length and width. A **3-D object** (three-dimensional), like a brick or a ball, exists in the real world and has extensions in three directions: length, width, and height.

Basic Geometric Figures

- **Point:** A simple dot on a piece of paper that marks a specific location.
- **Line:** A perfectly straight path that extends forever in both directions. We draw arrows on the ends to show it's infinite.
- **Line Segment:** A part of a line that has two specific endpoints. Its length is the distance between those two points.
- **Ray:** A part of a line that starts at one point and extends infinitely in only one direction.

2.2. Common 2-D Shapes

Quadrilaterals

A quadrilateral is any polygon with four sides. Here are some special types:

- **Rhombus:** A quadrilateral where all four sides are of equal length.
- **Rectangle:** A parallelogram that has one right angle (which implies all four angles are right angles).
- **Square:** A quadrilateral where all sides are equal length and all angles are right angles.

Circle

- **Circle:** Imagine a fixed point on a piece of paper (the **centre**). A **circle** is every single point that is the exact same distance (the **radius**) away from that centre.
- **Centre:** The fixed point in the middle.
- **Radius:** The fixed distance from the centre to any point on the circle.
- **Chord:** A line segment that connects any two points on the circle.
- **Diameter:** A special chord that passes directly through the centre of the circle. It is the longest possible chord.

2.3. Congruence, Similarity, and Symmetry

Congruence of Triangles

Congruent figures are identical twins—they have the exact same shape and the exact same size. For two triangles to be congruent, they must meet one of these four conditions:

1. **S-A-S (Side-Angle-Side):** This means two triangles are congruent if we can show two of their sides, and the specific angle *between* those two sides, are equal.
2. **S-S-S (Side-Side-Side):** This is straightforward—if all three corresponding sides of two triangles are equal in length, the triangles are congruent.
3. **A-A-S (Angle-Angle-Side):** Two triangles are congruent if two of their angles and a corresponding (non-included) side are equal.
4. **RHS (Right-Hypotenuse-Side):** This is a special case for right-angled triangles. They are congruent if their hypotenuses are equal and one other corresponding side is equal.

Similarity

Similar figures have the same shape but can be different sizes. Think of two maps of India drawn at different scales—they show the same country but one is much larger than the other. They are similar.

Symmetry

- **Line Symmetry (Reflection):** A figure has line symmetry if you can draw a line through it that divides it into two perfect mirror-image halves. If you were to fold the figure along this line, the two parts would match up exactly. This line is called the **line (or axis) of symmetry**.
- **Rotational Symmetry:** A figure has rotational symmetry if it looks the same after being rotated less than a full 360° around a central point. A windmill is a great example. The point it rotates around is the **point of symmetry**, and the number of times it looks identical during a full turn is the **order of rotational symmetry**.

2.4. Three-Dimensional (3-D) Bodies

Here are some common 3-D shapes and the key formulas you'll need for them.

- **Cuboid:** A box-like shape with 6 rectangular faces, 12 edges, and 8 vertices.
 - **Volume (V) = length × breadth × height ($l \times b \times h$)**
- **Cube:** A special cuboid where the length, breadth, and height are all equal.
 - **Volume (V) = side³ (l^3)**
- **Prism:** A body with two identical polygonal bases and rectangular faces connecting them.
 - **Lateral Surface Area = Perimeter of base × height**
 - **Total Surface Area = Lateral Surface Area + 2 × Area of base**
 - **Volume = Area of base × height**
- **Cylinder:** A shape with two parallel circular bases and a single curved face.
 - **Curved Surface Area = $2\pi rh$**
 - **Volume = $\pi r^2 h$**
- **Cone:** A shape with a circular base that tapers to a single point (the vertex).
 - **Slant height (l) = $\sqrt{h^2 + r^2}$**
 - **Total Surface Area = $\pi r(l + r)$**
- **Sphere:** A perfectly round shape, like a ball, with one continuous curved face.
 - **Surface Area = $4\pi r^2$**
 - **Volume = $(4/3)\pi r^3$**

Tutor's Tip: In these formulas, the symbol π (pi) is a special mathematical constant. For calculations, you can almost always use the approximations $\pi \approx 22/7$ or $\pi \approx 3.14$.

3. Unit 7: Measures and Measurements

3.1. Fundamental Concepts of Measurement

Measurement as Comparison

At its core, measurement is a process of comparison. When we measure the length of a plot of land, we are comparing it to a known length, like a meter scale. We are asking, "How many of this known unit fit into the thing I'm measuring?"

Non-standard vs. Standard Units

We use different kinds of units to measure things. Let's compare them.

Feature	Non-Standard Units	Standard Units
Origin	Evolved for immediate or local needs.	Evolved through common agreement and scientific refinement.
Consistency	Varies by person, situation, or time.	Fixed and understood globally.
Examples	Handful, spoonful, foot, cubit, pace, fathom.	Meter, gram, second, litre.
Usefulness	Works well for personal/local tasks (e.g., cooking).	Essential for accuracy, science, and global communication.

3.2. Measuring Area and Volume

Area of Plane Figures

- **Area** is the measure of how much a flat (plane) figure spreads out. It's the amount of surface it covers.
- The formula for the area of a rectangle is: **Area = length × breadth ($l \times b$)**.
- **Area of Irregular Figures:** To estimate the area of an irregular shape like a leaf, you can place it on centimeter graph paper. Count the squares it covers using this simple rule: count any square that is half-covered or more as a full square, and ignore any square that is less than half-covered.
- **Units of Land Measure:** For large areas of land, we use standard units like the **Hectare** (equal to $10,000 m^2$) and the **Are** (equal to $100 m^2$). The relationship is simple: 1 hectare = 100 are.

Volume of Solid Objects

- The formula for the volume of a cuboid is: **Volume = $l \times b \times h$** .
- **Volume vs. Capacity:** These terms are related but different. **Volume** is the amount of space an object takes up. **Capacity** is the amount of a substance (like a liquid) that a container can hold.

3.3. The Metric System (SI Units)

Introduction

The Metric system, officially known as the **International System of Units (SI)**, is the globally accepted standard for measurement. Its great advantage is that it is a decimal-based system, meaning all units are related by powers of 10, making conversions very easy.

Structure of Units

The system uses standard prefixes to show multiples or fractions of a base unit. Here's how it works for the most common measurements:

Prefix	Multiplier	Length (meter)	Capacity (litre)	Mass (gram)
Kilo-	1000	Kilometre (km)	Kilolitre (kl)	Kilogram (kg)
Hecto-	100	Hectometre (hm)	Hectolitre (hl)	Hectogram (hg)
Deca-	10	Decametre (dam)	Decalitre (dal)	Decagram (dag)
(Base Unit)	1	Metre (m)	Litre (l)	Gram (g)
Deci-	1/10	Decimetre (dm)	Decilitre (dl)	Decigram (dg)
Centi-	1/100	Centimetre (cm)	Centilitre (cl)	Centigram (cg)
Milli-	1/1000	Millimetre (mm)	Millilitre (ml)	Milligram (mg)

Larger Units of Mass

For weighing very heavy items, we use larger units: **quintal (100 kg)** and **metric ton (1000 kg)**.

3.4. Measurement of Time

Units of Time

You're likely familiar with these: 60 seconds = 1 minute, 60 minutes = 1 hour, 24 hours = 1 day, and 365 days = 1 year.

Leap Year

A year that is divisible by 4 is a leap year (it has 366 days, with 29 days in February). There's one exception: a century year (like 1800 or 1900) is *not* a leap year unless it is also divisible by 400 (like the year 2000).

Clock Time

- **12-hour clock:** This system uses a.m. (ante meridiem, meaning "before midday") for the time between midnight and noon, and p.m. (post meridiem, "after midday") for the time between noon and midnight.
- **24-hour clock:** Used by railways, airlines, and the military for clarity, this system runs from 00:00 (midnight) to 23:59. It avoids any confusion by not using a.m. or p.m.

4. Unit 8: Data Handling

4.1. Collection and Organization of Data

Data Collection

- **Data:** A collection of numbers gathered to provide some information.
- **Primary Source:** Data you collect yourself, directly from the source (e.g., surveying your classmates about their favorite fruit).
- **Secondary Source:** Data you get from existing records that someone else collected (e.g., using a government census report).
- **Raw Data:** The information you've collected before it has been organized in any way.

Tabular Representation

- **Frequency Distribution Table:** A simple table is a great way to organize raw data. It usually has columns for the item being counted, tally marks to keep track as you count, and a final frequency column with the total count.
- **Grouped Frequency Distribution:** When you have a large amount of data, it's useful to group it into ranges called **Class Intervals (C.I.)**. For example, you might group test scores into intervals like 60-69, 70-79, etc. Each interval has a lower limit (e.g., 60) and an upper limit (e.g., 69).

4.2. Pictorial Representation of Data

- **Pictograph:** This chart uses picture symbols to represent data. It's visually appealing but can be time-consuming to draw, and using parts of symbols can sometimes be misleading.
- **Bar Graph:** This graph uses rectangular bars of uniform width to show data. The length of the bar represents the frequency, and there are equal gaps between the bars.
 - **Steps to draw a bar graph:**
 1. Draw a horizontal line (x-axis) and a vertical line (y-axis).
 2. On the horizontal line, mark points at equal intervals for each item you are graphing.
 3. On the vertical line, choose a suitable scale and mark the numbers for your frequencies.
 4. Draw bars (columns) of equal width above each item mark on the horizontal axis, up to the height that corresponds to its frequency.
 5. Ensure all bars are shaded or colored consistently.
- **Histogram:** This looks like a bar graph but is used specifically for grouped data. The key difference is that there are **no gaps** between the bars, because they represent continuous intervals.
- **Pie Chart (or Pie Graph):** This is a circular graph where the whole circle represents the total amount, and the different "slices" (sectors) represent parts of the whole.

- To figure out the size of each slice, you calculate its central angle with this formula: **Central Angle (θ) = (Frequency of item / Total Frequency) × 360°**
- **Steps to draw a pie chart:**
 1. Calculate the central angle for each item using the formula above.
 2. Draw a circle of a suitable radius using a compass.
 3. Use a protractor to draw the sectors inside the circle, one by one, corresponding to each angle you calculated.
 4. Shade or color each sector differently and label it.

4.3. Analysis of Data: Central Tendency and Variability

Measures of Central Tendency

This is a single value that attempts to describe a "typical" or "central" entry of a data set.

- **Mean:** This is the most common type of average.
 - **Formula:** Mean = (Sum of all observations) / (Number of observations)
- **Median:** This is the middle value when the data is arranged in order (from smallest to largest or vice-versa). If there's an even number of observations, the median is the average of the two middle values.
- **Mode:** This is simply the value that appears most often in the data.

Measures of Variability (Dispersion)

This tells us how spread out or scattered the data points are.

- **Range:** The simplest measure of spread.
 - **Formula:** Range = Largest observation – Smallest observation
 - **Interquartile Range:**
 - **Quartiles** are points that divide the ordered data into four equal parts. Q1 is the 25th percentile, Q2 is the median, and Q3 is the 75th percentile.
 - **Formula:** Interquartile Range = Q3 – Q1 (This shows the spread of the middle 50% of the data).
 - **Standard Deviation:** This is the most widely used and stable measure of variability. It's essentially the average distance of each data point from the mean. A larger standard deviation means the data is more spread out, while a smaller one means the data is clustered closely around the mean.
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5. Unit 9: Algebra as Generalized Arithmetic

5.1. Introduction to Algebra

Core Concept

Algebra is a powerful extension of arithmetic. It generalizes arithmetic by using symbols (usually letters), called variables, to represent numbers. This allows us to describe rules and relationships in a general way.

Variables and Constants

- **Variable:** A symbol (like x or y) that can stand for any numerical value.
- **Constant:** A symbol that has a fixed numerical value (like 5 or -10).

Algebraic Expressions

An algebraic expression is a combination of constants and variables connected by arithmetic operations like addition, subtraction, multiplication, or division (e.g., $4x - 7$).

5.2. Components of an Algebraic Expression

- **Term:** A term is a part of an expression separated by '+' or '-' signs. In $4x - 7$, the terms are $4x$ and -7 .

Classification of Expressions

- **Monomial:** An expression with only one term (e.g., $7xy$).
- **Binomial:** An expression with two unlike terms (e.g., $x + y$).
- **Trinomial:** An expression with three unlike terms (e.g., $2a - 5b + 3c$).
- **Polynomial:** This is the general name for any expression with one or more terms.

Factors and Coefficients

- **Factors:** The numbers or variables that are multiplied together to create a term. In the term $2ab$, the factors are 2, a , and b .
- **Coefficient:** The numerical factor of a term. In $2ab$, the coefficient is 2.

Like and Unlike Terms

- **Like Terms:** Terms that have the exact same algebraic factors (the same variables raised to the same powers). For example, $5xy$ and $-2xy$ are like terms.
- **Unlike Terms:** Terms that have different algebraic factors. For example, $5xy$ and $5x^2y$ are unlike terms.

Here's a simple way to think about it: You can only add or subtract **Like Terms**. Think of it like fruit: you can add 5 apples and 2 apples to get 7 apples. But you can't add 5 apples and 2 oranges to get '7 apple-oranges.' $5xy$ and $-2xy$ are like terms (apples), but $5xy$ and $5x^2y$ are unlike terms (apples and oranges).

5.3. Operations on Algebraic Expressions

- **Addition and Subtraction:** The golden rule is that you can only add or subtract like terms. You do this by combining their coefficients.
- **Multiplication:**

- **Monomial by Polynomial:** Use the distributive law. The monomial outside the parentheses multiplies each term inside the polynomial.
- **Polynomial by Polynomial:** You must multiply each term in the first polynomial by each term in the second polynomial.
- **Division:**
 - **Polynomial by Monomial:** Divide each term of the polynomial by the monomial.
 - **Polynomial by Polynomial:** This usually requires a long division process, similar to the one used in arithmetic.

5.4. Linear Equations

Definition

A **Linear Equation** is an equation where the highest power of the variable is 1. Its general form is $ax + b = 0$. It's called "linear" because if you were to graph it, it would form a straight line.

Solving Linear Equations

Solving an equation means finding the value of the unknown variable that makes the statement true.

- **Balancing Method:** The key principle is to keep the equation balanced. Whatever you do to one side of the equation, you must do to the other. You can add, subtract, multiply, or divide by the same non-zero quantity on both sides without changing the equality.
- **Transposition Method:** This is a shortcut for the balancing method. You can move a term from one side of the equation to the other by simply changing its sign (e.g., a +5 on one side becomes a -5 on the other).

Application to Word Problems

Here is a five-step plan for tackling word problems with algebra:

1. **Understand the problem:** Read it carefully to figure out what you know and what you need to find.
2. **Choose a variable:** Use a letter (like x) to represent the unknown quantity.
3. **Write an equation:** Translate the problem's conditions into a mathematical equation.
4. **Solve the equation:** Use the methods above to find the value of your variable.
5. **Verify the solution:** Check your answer by plugging it back into the original problem to make sure it makes sense.