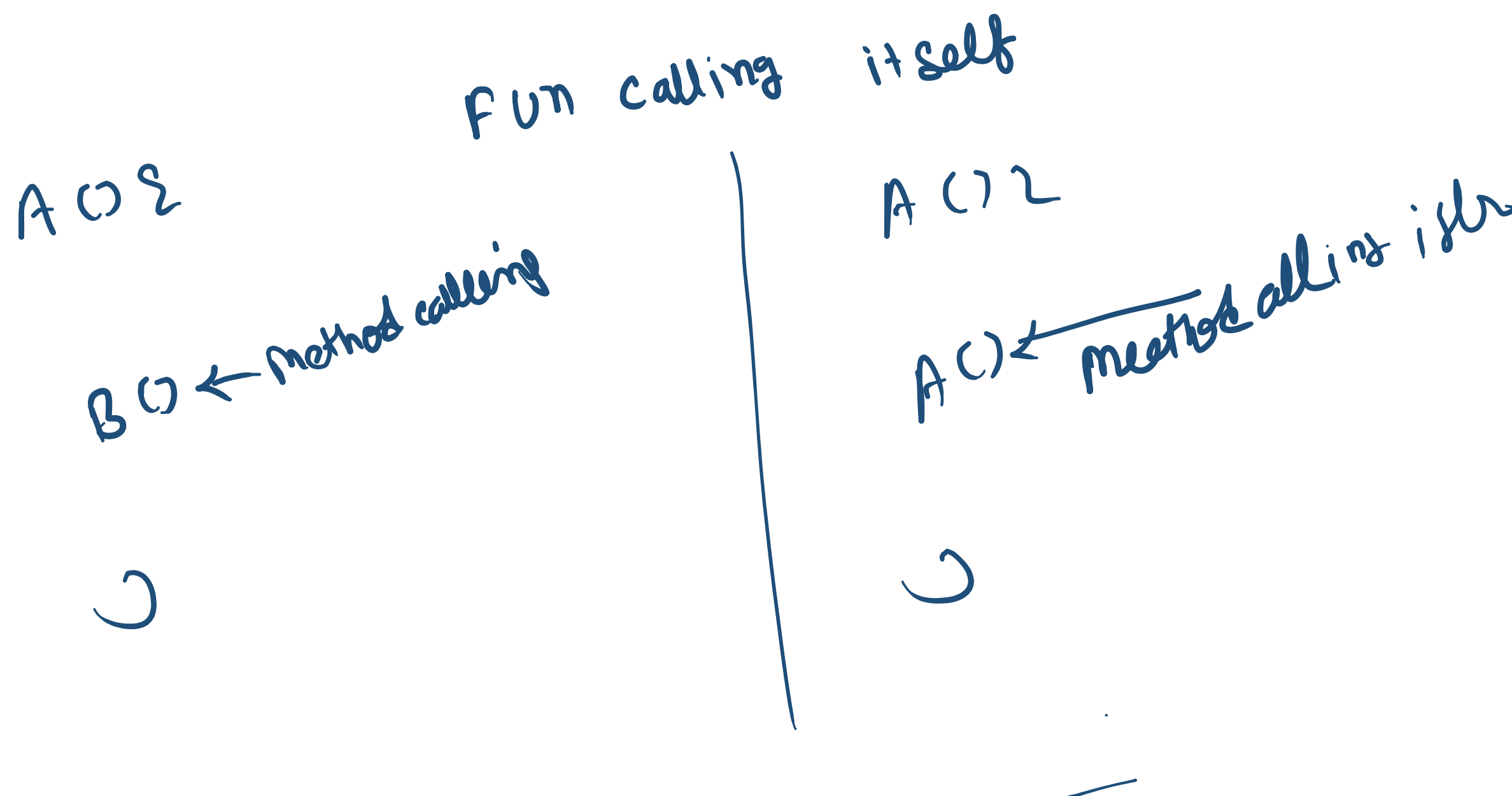


(i) Recursion \rightarrow Function calling itself



(i) function calling itself

① PMI \rightarrow Mathematical proof / using PMI

$$\sum_{i=1}^n i = 1 + 2 + 3 + 4 + \dots + n = \frac{n \times (n+1)}{2}$$

$$\sum_{i=1}^1 i = 1 = \frac{1 \times (1+1)}{2} \quad \sum_{i=1}^2 i = 1+2 = \frac{2 \times (2+1)}{2}$$

$$\sum_{i=1}^K i = 1+2+3+4+\dots+K = \frac{K \times (K+1)}{2}$$

$$\sum_{i=1}^{k+1} i = 1+2+3+4+\dots+K+K+1 = \frac{(K+1) \times (K+2)}{2}$$
$$\frac{K \times (K+1)}{2} + K+1 = \frac{(K+1)(K+2)}{2}$$

① Smaller input ✓
② Assume $n=k$ correct ✓
③ $n=k+1$ prove ✓

Base \rightarrow case

① \rightarrow B.P
② \rightarrow S.P \rightarrow assume B.P

Recursion

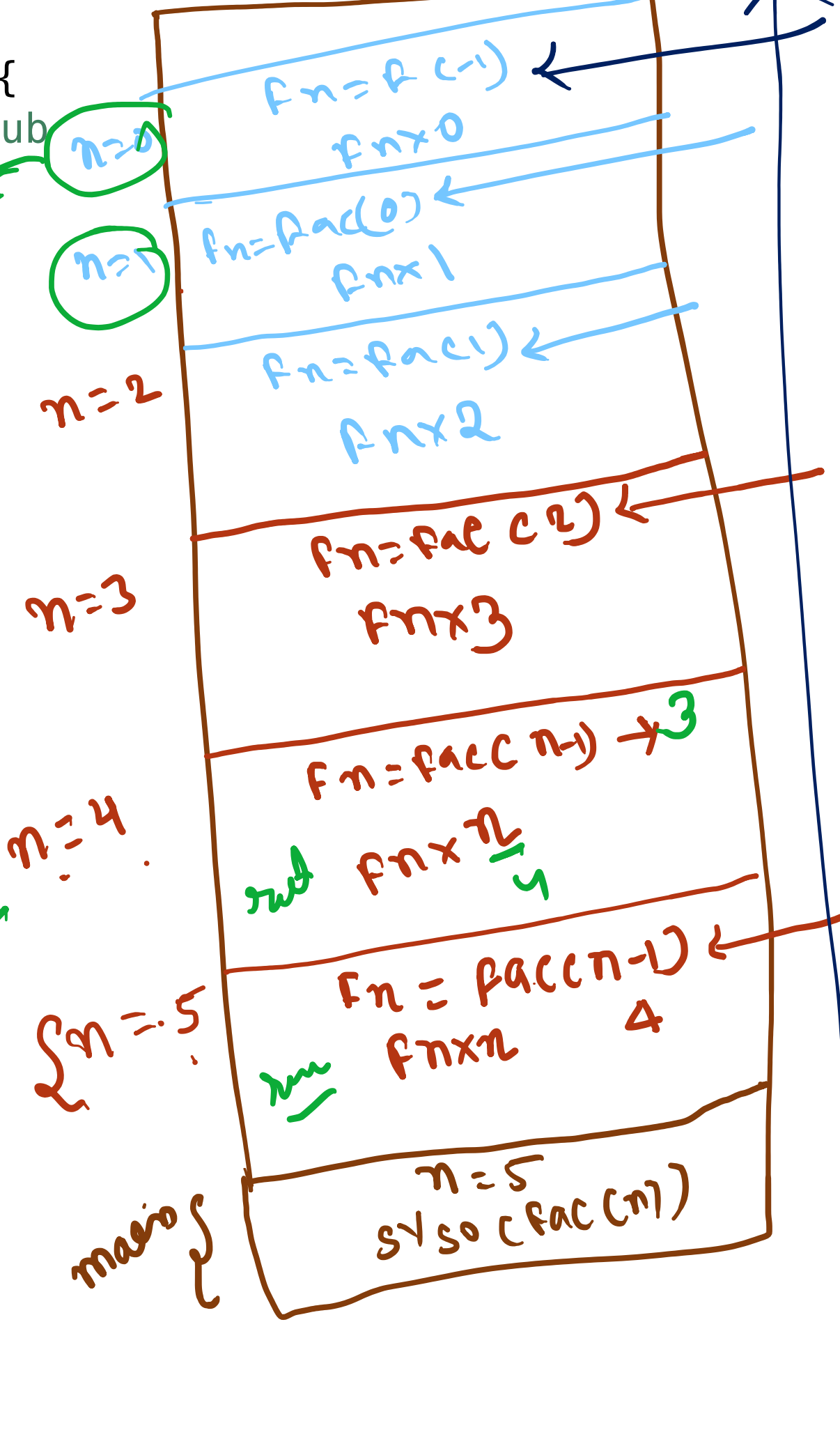
Recursion that call itself

$$9! \times 10 = 10!$$

$$5! = 4! \times 5$$
$$4! = 3! \times 4$$
$$3! = 2! \times 3$$
$$2! = 1! \times 2$$
$$1! = 0! \times 1$$

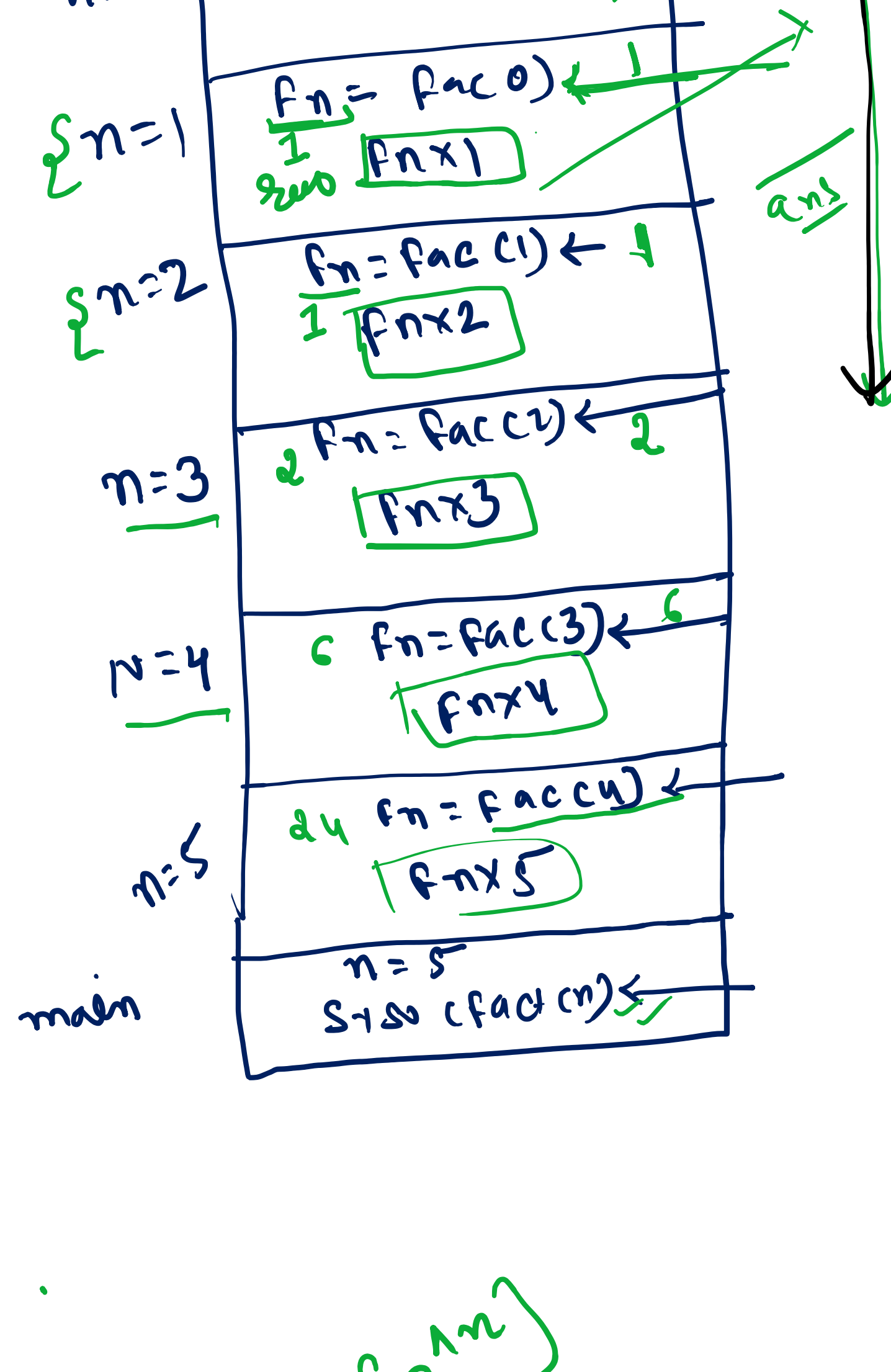
```
public static void main(String[] args) {  
    // TODO Auto-generated method stub  
    int n = 5;  
    System.out.println(fact(n));  
}
```

```
public static int fact(int n) {  
    int fn = fact(n-1); // sp  
    return fn * n;  
}
```



```
public static void main(String[] args) {  
    // TODO Auto-generated method stub  
    int n = 5;  
    System.out.println(fact(n));  
}
```

```
public static int fact(int n) {  
    // Base case  
    if (n == 0) {  
        return 1;  
    }  
    int fn = fact(n-1); // sp  
    return fn * n;  
}
```



$$(3/10) \rightarrow (3/10)$$

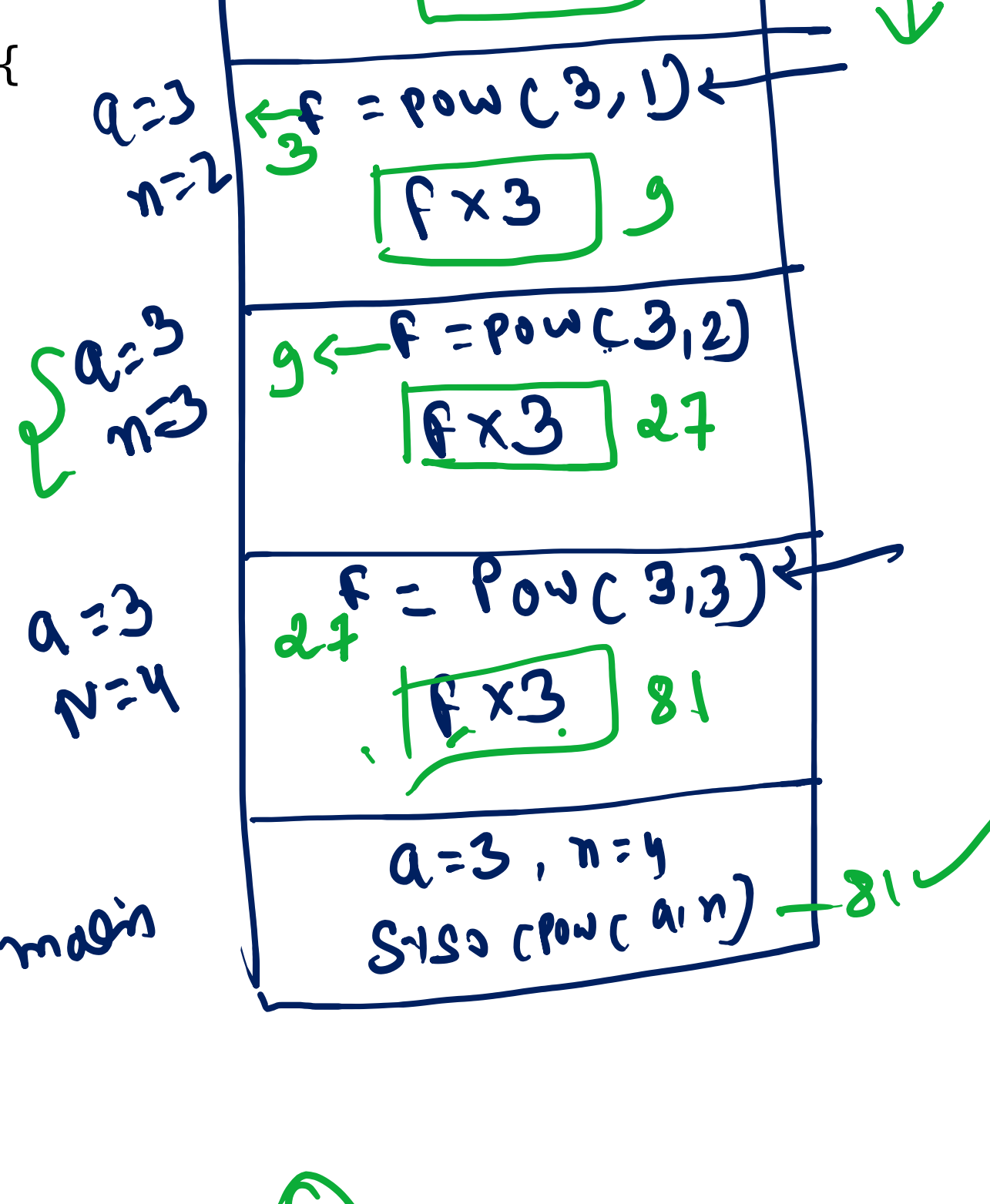
$(a, n) \rightarrow$ smaller problem $(a, n-1)$

$$a^n$$

$$\frac{a \times a \times \dots \times a}{n \text{ times}}$$

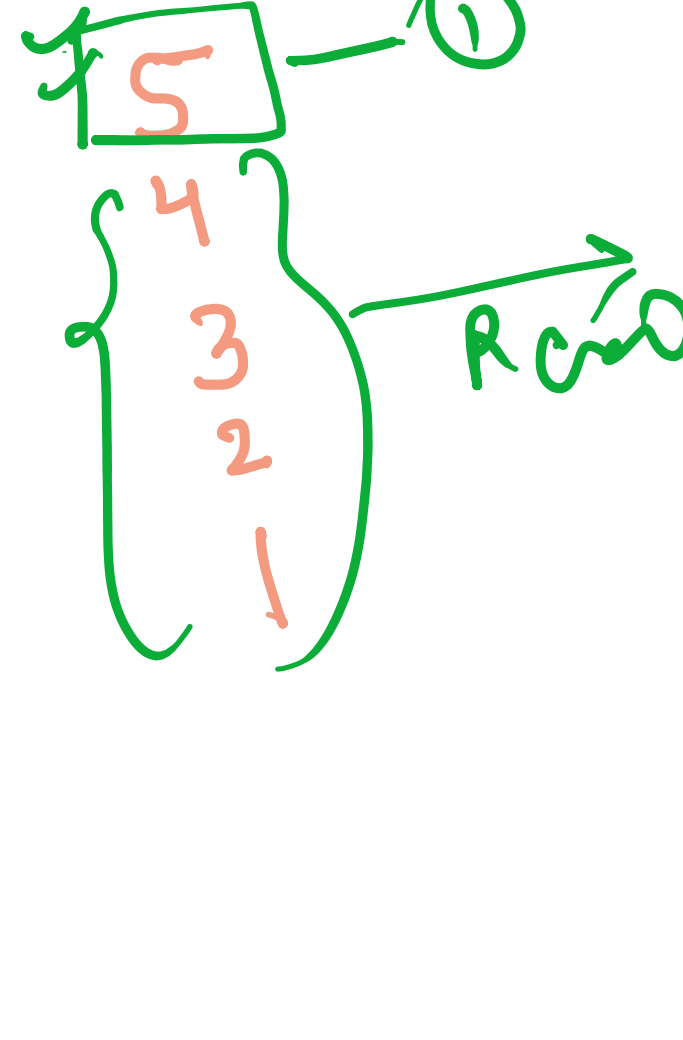
```
public static void main(String[] args) {  
    // TODO Auto-generated method stub  
    int a = 3;  
    int n = 4;  
    System.out.println(pow(a, n));  
}
```

```
public static int pow(int a, int n) {  
    if (n == 0) {  
        return 1;  
    }  
    int f = pow(a, n-1); // sp  
    return f * a;  
}
```



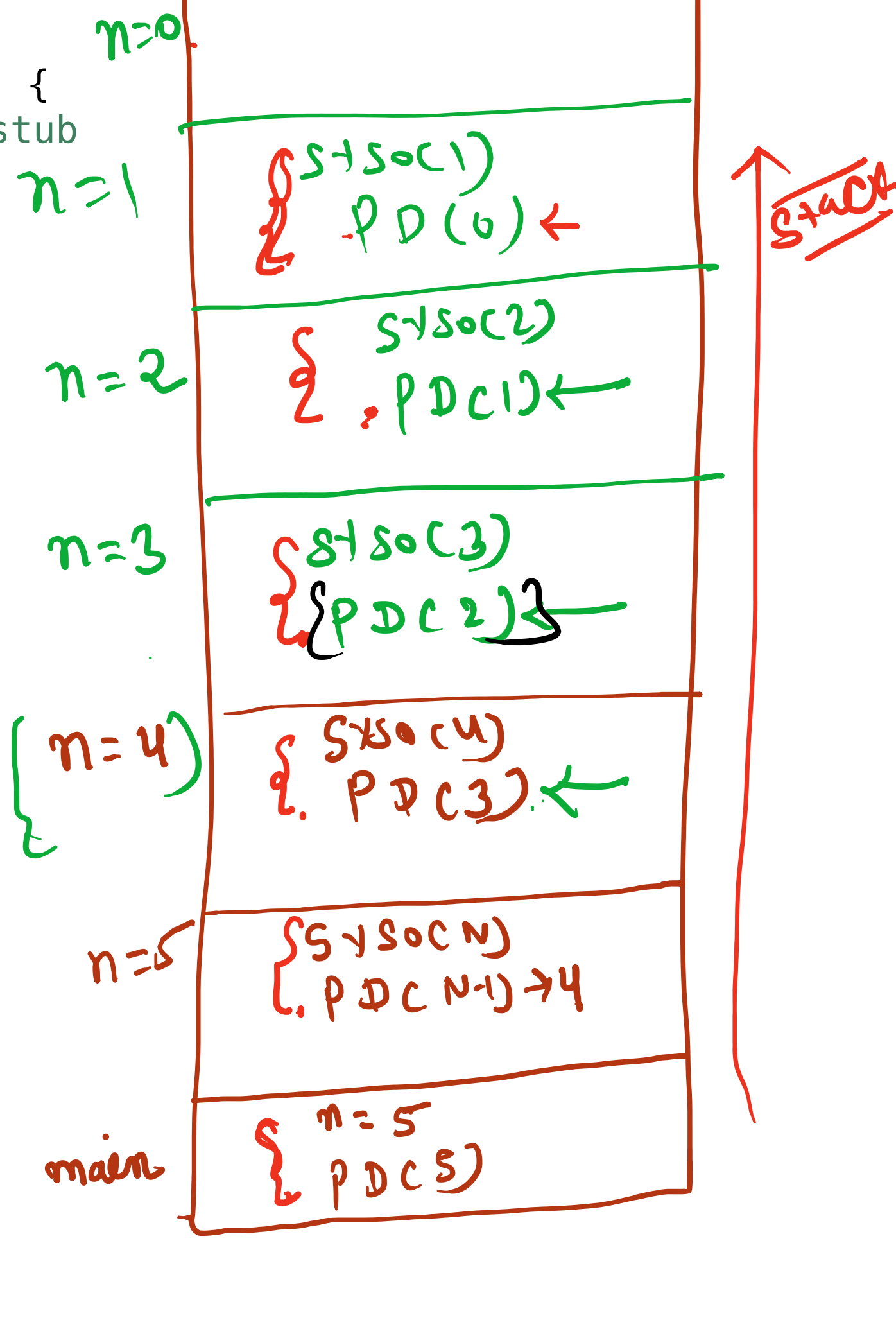
Public static void pow(int n)

N=5



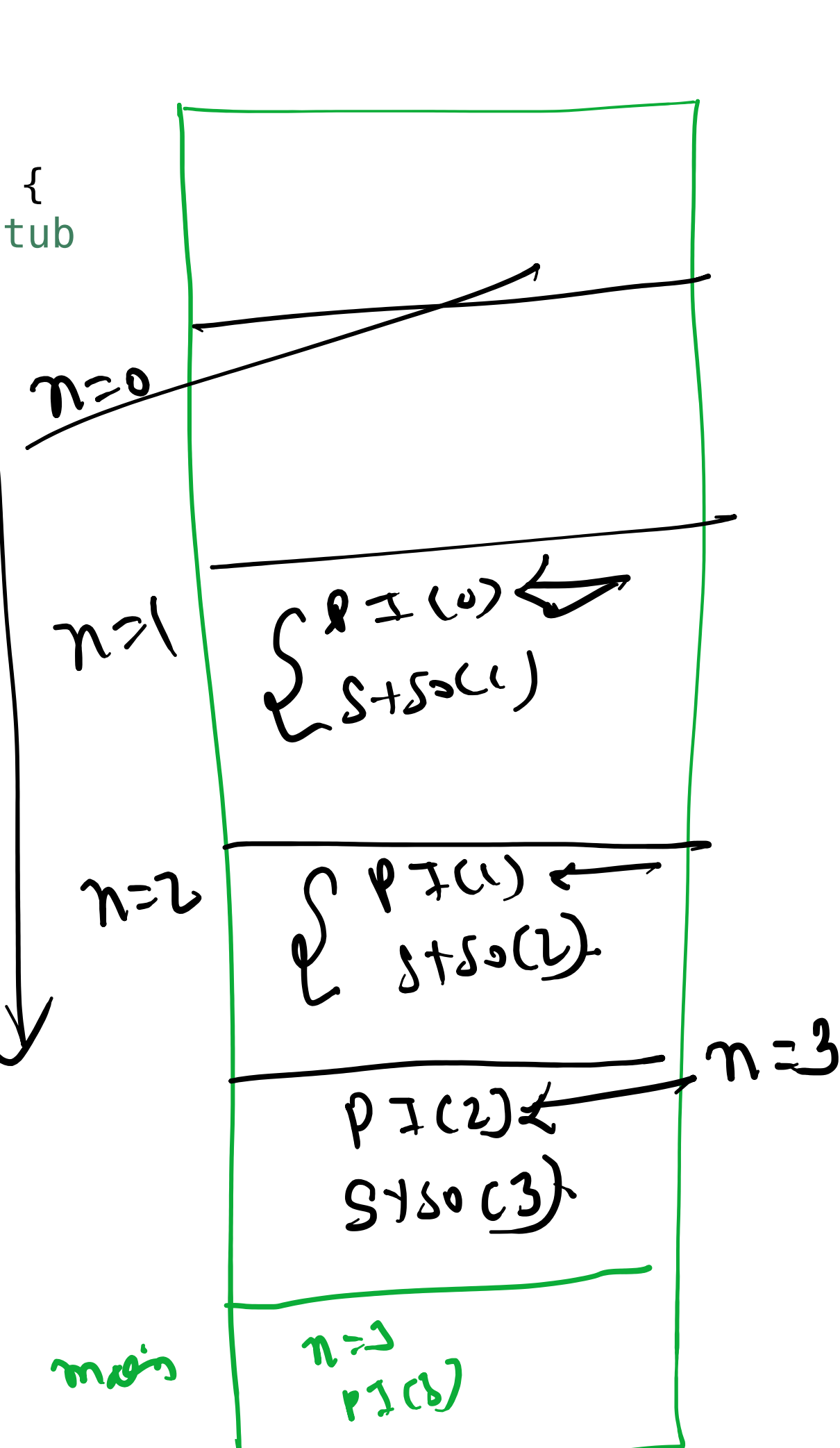
```
public static void main(String[] args) {  
    // TODO Auto-generated method stub  
    int n = 5;  
    PD(n);  
}
```

```
public static void PD(int n) {  
    if (n == 0) {  
        return;  
    }  
    System.out.println(n);  
    PD(n-1);  
}
```



```
public static void main(String[] args) {  
    // TODO Auto-generated method stub  
    int n = 5;  
    PI(n);  
}
```

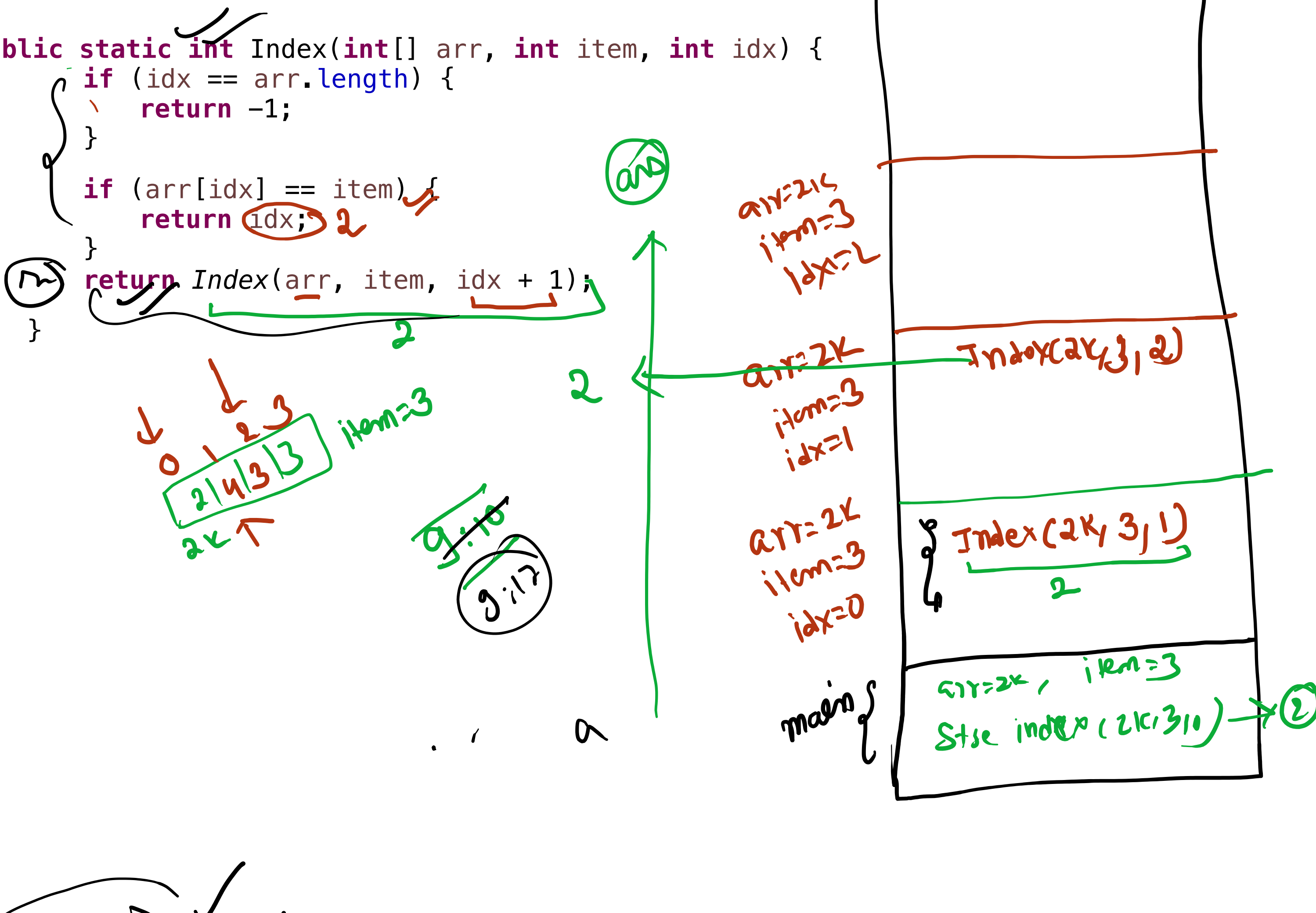
```
public static void PI(int n) {  
    if (n == 0) {  
        return;  
    }  
    PI(n-1);  
    System.out.println(n);  
}
```



Recursion is iterative

```
int[] arr = {1, 2, 5, 4, 3, 4, 7, 4, 3, 6};  
int item = 4;
```

```
public static int Index(int[] arr, int item, int idx) {  
    if (idx == arr.length) {  
        return -1;  
    }  
    if (arr[idx] == item) {  
        return idx;  
    }  
    return Index(arr, item, idx+1);  
}
```



Stack grows down

Stack grows up

fail/hot

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

```
public static void main(String[] args) {  
    // TODO Auto-generated method stub  
    int n = 5;  
    System.out.println(fact(n, 1));  
}
```

```
public static int fact(int n, int ans) {  
    // Base case  
    if (n == 0) {  
        return ans;  
    }  
    return fact(n-1, ans * n);  
}
```

