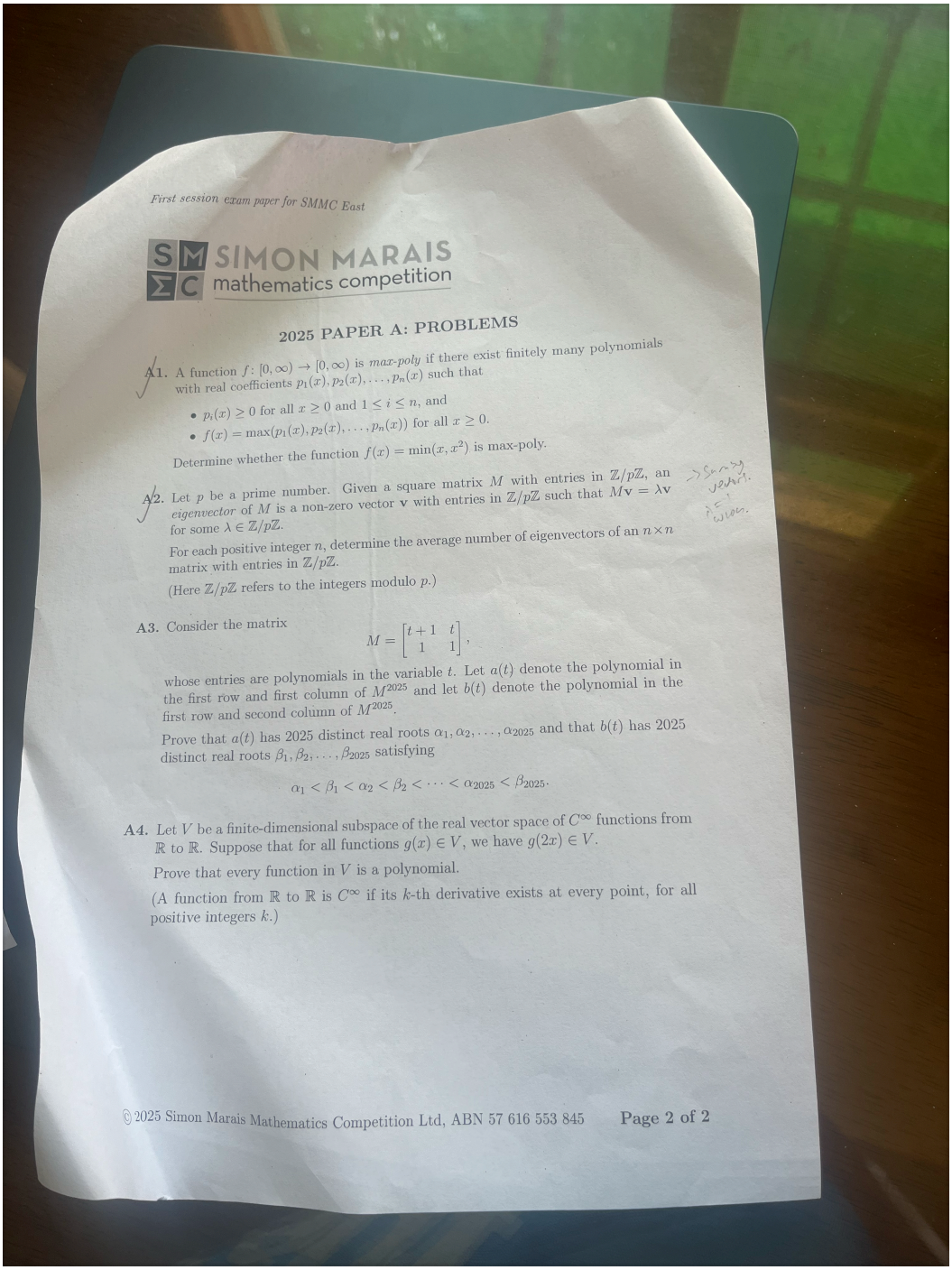
day 14

8:58 pm, 24 October 2025

Well, its been a lot of days, but here's the SMMC update:
I did 5(ish) problems. So i mean im not winning it, too bad.



First session exam paper for SMMC East

SIMON MARAIS mathematics competition

2025 PAPER A: PROBLEMS

A1. A function $f\colon [0,\infty) \to [0,\infty)$ is *max-poly* if there exist finitely many polynomials with real coefficients $p_1(x), p_2(x), \ldots, p_n(x)$ such that

- $p_i(x) \geq 0$ for all $x \geq 0$ and $1 \leq i \leq n$, and
- $f(x) = \max(p_1(x), p_2(x), \ldots, p_n(x))$ for all $x \geq 0$.

Determine whether the function $f(x) = \min(x, x^2)$ is max-poly.

A2. Let p be a prime number. Given a square matrix M with entries in $\mathbb{Z}/p\mathbb{Z}$, an *eigenvector* of M is a non-zero vector $\mathbf{v}$ with entries in $\mathbb{Z}/p\mathbb{Z}$ such that $M\mathbf{v} = \lambda\mathbf{v}$ for some $\lambda \in \mathbb{Z}/p\mathbb{Z}$.

For each positive integer n, determine the average number of eigenvectors of an $n \times n$ matrix with entries in $\mathbb{Z}/p\mathbb{Z}$.

(Here $\mathbb{Z}/p\mathbb{Z}$ refers to the integers modulo p.)

A3. Consider the matrix

$$M = \begin{bmatrix} t+1 & t \\ 1 & 1 \end{bmatrix},$$

whose entries are polynomials in the variable t. Let $a(t)$ denote the polynomial in the first row and first column of M^{2025} and let $b(t)$ denote the polynomial in the first row and second column of M^{2025}.

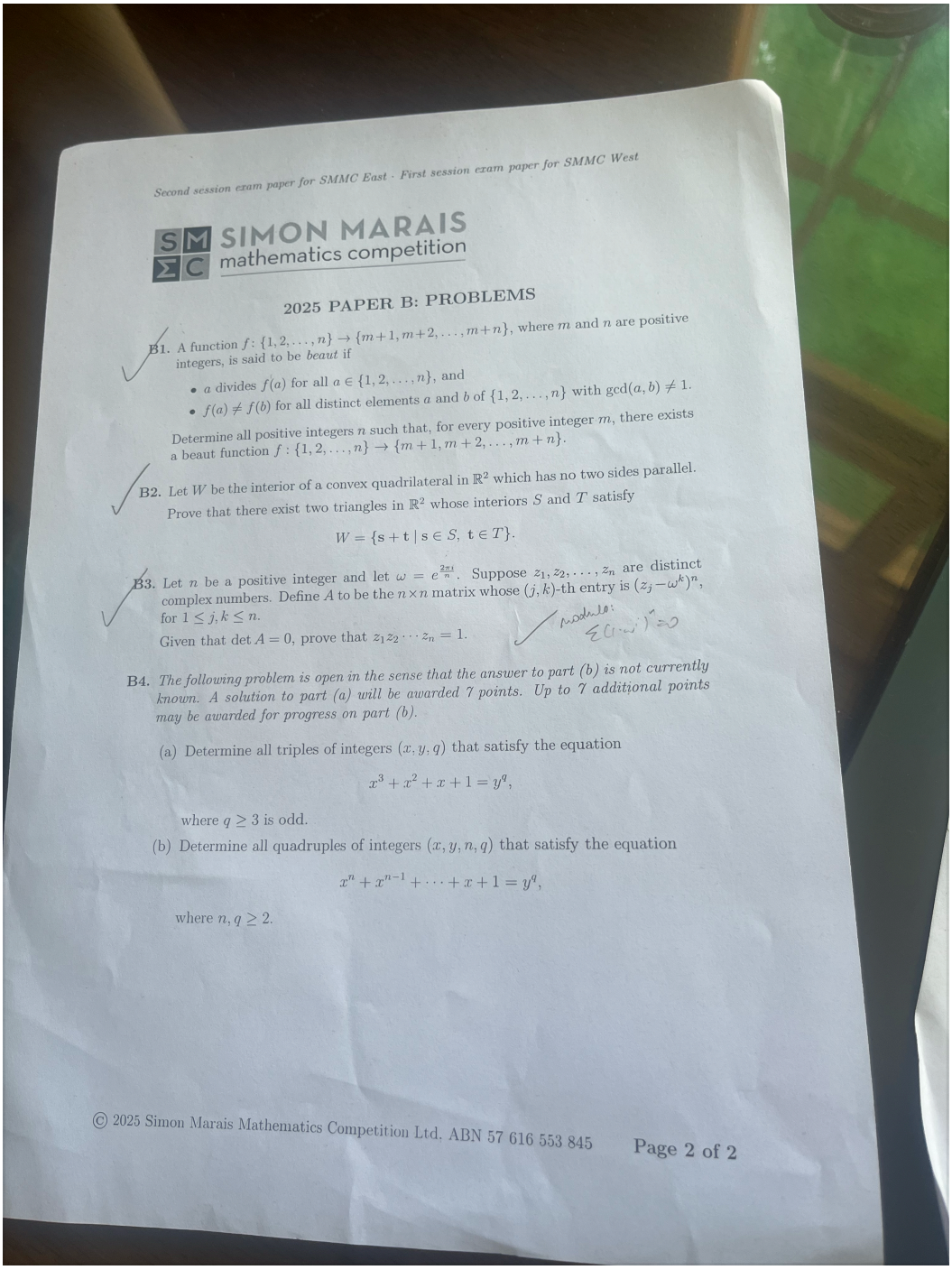
Prove that $a(t)$ has 2025 distinct real roots $\alpha_1, \alpha_2, \ldots, \alpha_{2025}$ and that $b(t)$ has 2025 distinct real roots $\beta_1, \beta_2, \ldots, \beta_{2025}$ satisfying

$$\alpha_1 < \beta_1 < \alpha_2 < \beta_2 < \cdots < \alpha_{2025} < \beta_{2025}.$$

A4. Let V be a finite-dimensional subspace of the real vector space of C^∞ functions from $\mathbb{R}$ to $\mathbb{R}$. Suppose that for all functions $g(x) \in V$, we have $g(2x) \in V$.

Prove that every function in V is a polynomial.

(A function from $\mathbb{R}$ to $\mathbb{R}$ is C^∞ if its k-th derivative exists at every point, for all positive integers k.)



Second session exam paper for SMMC East · First session exam paper for SMMC West

SIMON MARAIS mathematics competition

2025 PAPER B: PROBLEMS

B1. A function $f\colon \{1,2,\ldots,n\} \to \{m+1, m+2, \ldots, m+n\}$, where m and n are positive integers, is said to be *beaut* if

- a divides $f(a)$ for all $a \in \{1,2,\ldots,n\}$, and
- $f(a) \neq f(b)$ for all distinct elements a and b of $\{1,2,\ldots,n\}$ with $\gcd(a,b) \neq 1$.

Determine all positive integers n such that, for every positive integer m, there exists a beaut function $f : \{1,2,\ldots,n\} \to \{m+1, m+2, \ldots, m+n\}$.

B2. Let W be the interior of a convex quadrilateral in $\mathbb{R}^2$ which has no two sides parallel. Prove that there exist two triangles in $\mathbb{R}^2$ whose interiors S and T satisfy

$$W = \{\mathbf{s} + \mathbf{t} \mid \mathbf{s} \in S,\ \mathbf{t} \in T\}.$$

B3. Let n be a positive integer and let $\omega = e^{\frac{2\pi i}{n}}$. Suppose $z_1, z_2, \ldots, z_n$ are distinct complex numbers. Define A to be the $n \times n$ matrix whose (j,k)-th entry is $(z_j - \omega^k)^n$, for $1 \leq j, k \leq n$.

Given that $\det A = 0$, prove that $z_1 z_2 \cdots z_n = 1$.

B4. *The following problem is open in the sense that the answer to part (b) is not currently known. A solution to part (a) will be awarded 7 points. Up to 7 additional points may be awarded for progress on part (b).*

(a) Determine all triples of integers (x, y, q) that satisfy the equation

$$x^3 + x^2 + x + 1 = y^q,$$

where $q \geq 3$ is odd.

(b) Determine all quadruples of integers (x, y, n, q) that satisfy the equation

$$x^n + x^{n-1} + \cdots + x + 1 = y^q,$$

where $n, q \geq 2$.

This is the question paper:
These are my solutions
<attach solution pdf here>
ok but yuh, paper 1. actually fucked up, i dont know lin alg, at all, i opened the papper and straight up saw three lin alg problems. i was like wtfffff.
i was actually sad, and mad as fuck. I think this kind of messed me up cos if i stayed calm i could have probably done even more problems but wtv.
So i mean problem 1 was just absolutely trivial. Just look at the max degree in max poly
Problem 2 and problem 3 i thought about for some more time. Problem 3, i got a few ideas, some progress but not much. you know i really think the problem was just i was mad at why these niggas had to make a fucking lin alg question paper.

wtv problem 3 i didnt do much
Problem 2 after a while i got the idea, to double count by vector instead of matrices. And then i kind of got the main idea of you can basicallly choose most numbers randomly, and then the last number will be automatically fixed cos F_p is nice.
The thing that was tripping me up was what if vector has a bunch of 0's. i mean i just had to be calm lmao. wtv i wrote up tehse problems with very less time left.
problem 2 i made a small mistake when writing up, which is if k entries of the vector are zero, then the term you're supposed to get is $\binom{n}{k} \times p^{(n-k)(n)}$ but i made a small error. its ok i still consider essentially a solve. cool.
Then i went for pancaks with sagnik. We were just talking and like man im really going to be great.

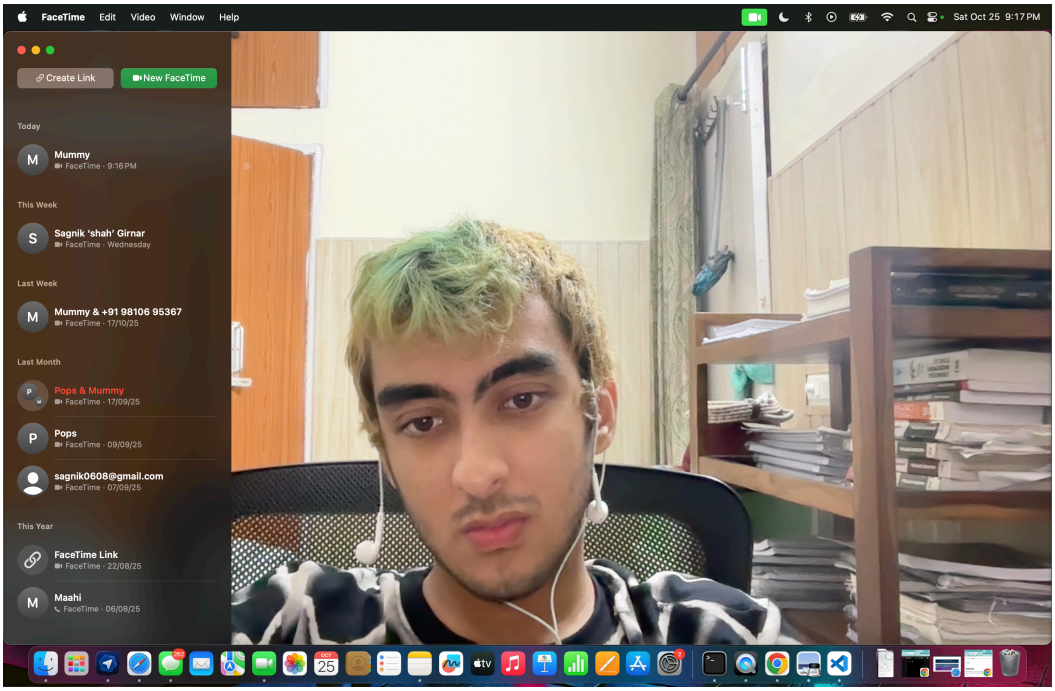
> years, however, important college accomplishments like losing your virginity have been replaced with more ambitious goals, like founding your own consulting firm.
>
> Since the universal domination of Facebook founder Mark E. Zuckerberg, originally of the class of 2006, Harvard has seen a surge in undergraduate entrepreneurs. While Harvard entrepreneurs admit that having “CEO” or “founder” written on your resume can’t hurt when applying to business school or Morgan Stanley, most of these go-getters insist that only sincere passion pushed them into the cold world of business at such a tender age.

anyways, paper 2 was much better.
For problem 1, i couldnt really think of any good ideas in the beginning at all. i thought for a while, i knew it had to be size somehow. i was like damn if m could be negative it would be so ez. then i got the m=n! idea lmao
i was feeling good
problem 2 i thought about very surgically. i was like ok both triangles must start at one vertex. the i thought about the right angle case. made a few more observations in that diagram, then i got the idea that each triangle should have one side along the quadrilateral. then i thought about the extreme case and got the construction. pretty cool.
Problem 3, i was like damn, i know 0 lin alg, dont even remember wtf a determinant is (never studied during JEE either lmao) but i still decided to think about it.
I thought a little bit about the degree stuff, and then i was like hmm obv $\prod (z_i-z_j)$ should be a factor right. So it suffices to prove that $z_1z_2...z_n-1$

is also a factor. like i thought in factorisation terms. I sort of proved the last factor must be of the form $z_1z_2 \cdots z_n-t$ (tho my reasoning for this is kinda shaky lmao) and then to compute t, you can choose $z_i=\omega^i$ and use a double summation.
kinda genius lmao.
pretty cool stuff. i mean the fact is i know im one of the best in india at math. like thats a fact. and thats off like 0 work bruh. like this is actually kinda crazy, i might be a fucking genius lmao. i can actually do any fucking thing man, maximise your potential please.
right now tho, ok the thing is, i kinda know what to do. just have fun all fucking day. that is i just need to not stop at all. just decide to not waste time at all, and then i can just do whatever i want all fucking day. this is going to be so much fun man lets go. really just dont think too much. i need to just do what i want to do and not stop at all and everything else works out.
by the end of this calendar year, i should know i am the best ever.
in 1 year, everyeone around me should know i am the best ever.
in 2 years, i should be in YC and have dropped out.
in 3 years, i should be THE whiz kid billionaire of this generation.
lets go then.
24 OCTOBER 2025
yuh honestly man its time. this is my world. witness history.
1:48 am 25 October 2025
ETH online hackathon is going on, and i wanna build something really cool. simple enough (cos i can build anything lmao) but i just need an idea. lets go.

"mony" on shuffle. JDUB in this bitch (ill explain at the end of the day)
2:38 am
let the hacking begin.
we aint gonna stop at all.
3:56
ah fuck im sick as fuck and sleepy as fuck. sleeping for a while. listen, dont

negotiate w yourself. like this is true, do not negotiate with yourself. the world is yours if you work every single moment.
8:28 PM
life can literally be a movie. this is my life. i can do whatever the fuck i want to. i'm just thinking of crypto stuff right now, i dont really care too much about the technicalities right now, i just cool ideas that excite me and we'll figure out how to build them later.
9:19 pm



account abstraction seems cool as fuck.
one click checkout is implemented already, sad. sprintcheckout.
i really wanna build something fun. btw this is perfect, im so inspired right now. im going to learn a bunch of cool math, CS, build a bunch of fun projects. do everything man. like this is my world. go be the best there has ever been bruh.
10:20
i think i wanna build something that is just fun, kinda trolly ig... let's see.