



សាកលវិទ្យាល័យភូមិន្ទភ្នំពេញ
មហាវិទ្យាល័យវិទ្យាសាស្ត្រ
ជេម៉ាតិកម្ម គណិតវិទ្យា

ប្រលងបញ្ចប់ឆមាសទី 1
ពីខែ កញ្ញា 2023 ដល់ ខែ មករា 2024
ឆ្នាំទី 3 មុខវិជ្ជា ពិជគណិតលីនេអ៊ែរ 1
រយៈពេល 2 ម៉ោង

Let V be the set of 2×2 matrices of the form

$$\begin{bmatrix} a & b \\ -b & c \end{bmatrix} \text{ where } a, b, c \in \mathbb{R}$$

1. Prove that V is a real vector subspace of $\text{Mat}_{2 \times 2} \mathbb{R}$ and

$$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

is a basis of V (this is the natural basis of V). Extend this basis to a basis of $\text{Mat}_{2 \times 2} \mathbb{R}$.

Consider a linear map $f : V \rightarrow \mathbb{R}_2[X]$ given by a matrix

$$A = \begin{bmatrix} 4 & 3 & 8 \\ 4 & 3 & 8 \\ 6 & 3 & 13 \end{bmatrix}$$

relative to the bases

$$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} \right\} \text{ of } V$$

and

$$\{1 - X - 2X^2, 1 - 2X - 2X^2, -1 + 2X + 3X^2\} \text{ of } \mathbb{R}_2[X]$$

2. What is the matrix B of f relative to the natural bases of V and of $\mathbb{R}_2[X]$?
Deduce the expression of $f(v)$ for any $v \in V$.
3. Determine a basis of $\text{Im} f$ and of $\text{Ker} f$.
4. Determine the normal form N of the matrix B .
5. Find a basis \mathcal{B}_V of V and a basis \mathcal{B}_R of $\mathbb{R}_2[X]$ for which the matrix of f is N .

"Just believe in yourself. Even if you don't,
pretend that you do and, at some point, you will."
— Venus Williams