Problemas con ecuaciones exponenciales y Logaritmos

Juanito compró una casa dando 1.5 millones de pesos de enganche y le dejaron pagar el restante (4 millones de pesos a pagos anuales de
si la tasa de interés es de 22% anual

María compró la casa de al lado con el mismo valor de 5.5 millones de pesos pero ella no pagó nada de enganche. Sin embargo a ella le dieron el crédito con la forma de crecimiento que aún pagando su pago anual de 720000 pesos ella terminará pagando aún así un 105% del precio original

¿Quién paga más en total y cuál es la diferencia?

$V_f = C(1+i)^t$ ← t: num de periodos

V_f: valor final; C: enganche capital; i: tasa de interés

$$V_p = C\,\frac{1-(1+i)^{-n}}{i}$$

$4_M = 580000\,(1+0.22)^2$

$$n = \frac{\ln\left(\frac{4_M}{580}\right)}{\ln(1.22)}$$

$$V_p = C \quad 4 = 0.58 \quad \frac{1-(1+0.22)^{-1}}{0.22}$$

$$-n = \ln\left(1-\frac{(0.22)(4)}{0.22}\right)$$

$$\frac{(1.22)^n}{0.22} = \frac{1}{0.22} - \frac{1}{0.58}$$

$$\ln(1.22)^{-n} = \ln\left(1-\frac{(0.22)(4)}{0.58}\right)$$

$$n = -\frac{\ln\left(1-\frac{0.22(4)}{0.58}\right)}{\ln(1.22)}$$

$n = 6.93$ /

$V_f = C(1+i)^n$

$= 1.5(1+0.22)^7$

$V_f = 6.03$ millones /

$$\left(\sqrt[3]{\frac{3^8}{3^{-2}}}\right)\left(\sqrt[3]{\frac{4^{-9}}{12^4}}\right)$$

$$\sqrt[3]{\frac{3^8}{3^{-2}} \times \frac{4^9}{12^{11}}}$$

$$\sqrt[3]{3^{10}\,\frac{4^{-9}}{3^4 \times 4^4}}$$

$$\sqrt[3]{3^6 \times \frac{1}{4^{13}}}$$

$$3^2 \sqrt[3]{\frac{1}{4^{13}}}$$

$$9 \times \frac{1}{\sqrt[3]{4^{13}}}$$

$$9 \times \frac{1}{4^4 \sqrt[3]{4}}$$

$$\frac{9}{4^4 \sqrt[3]{4}}$$

$$\frac{9\sqrt[3]{2}}{512} = 0.022$$

$$\left(\sqrt[5]{\frac{2^9}{3^2}}\right)\left(\sqrt[3]{\frac{4^9}{12^2}}\right)$$

$$\sqrt[5]{\frac{2^9}{9}} \quad \sqrt[3]{\frac{4^9}{3^2 \times 4^2}}$$

$$\sqrt[15]{\left(\frac{2^9}{9}\right)^3} \quad \sqrt[3]{\frac{4^7}{9}}$$

$$\sqrt[15]{\left(\frac{2^9}{9}\right)^3 \times \left(\frac{4^7}{9}\right)^5}$$

$$\sqrt[15]{\frac{2^{27}}{729} \times \frac{4^{35}}{59049}}$$

$$\sqrt[15]{\frac{2^{27} \times 4^{35}}{43046721}}$$

$$\sqrt[15]{\frac{2^{97} \times 2^{70}}{43046721}}$$

$$\sqrt[15]{\frac{2^{97}}{43046721}}$$

$$\frac{\sqrt[15]{2^{97}}}{\sqrt[15]{43046721}}$$

$$\frac{2^6 \times \sqrt[15]{2^7}}{3^{15}\sqrt{3}}$$

$$\frac{64\sqrt[15]{128}}{3^{15}\sqrt{3}}$$

$$\frac{64\sqrt[15]{128 \times 3^{14}}}{9}$$

$$= 27{,}398708$$

Regla de la cadena

$$\frac{d}{dx}\left(\log_5(\sin(x^3-x^2+2x))\right)$$

$$=\frac{1}{\ln(5)}\frac{d}{dx}\left(\ln\left(\sin(x^3-x^2+2x)\right)\right)$$

$$=\frac{1}{\sin(x^3-x^2+2x)}\frac{d}{dx}\left(\sin(x^3-x^2+2x)\right)$$

$$=\frac{1}{\ln(5)}\cdot\frac{1}{\sin(x^3-x^2+2x)}\cos(x^3-x^2+2x)(3x^2-2x+2)$$

$$=\cot\frac{(x^3-x^2+2x)(3x^2-2x+2)}{\ln(5)}$$

$$\frac{d}{dx}\left(\ln(\sin((x-4)^3)\right)$$

$$\frac{d}{dx}=\frac{1}{\sin((x-4)^3)}\frac{d}{dx}\left(\sin((x-4)^3)\right)$$

$$\frac{1}{\sin((x-4)^3)}\cos((x-4^3)\cdot 3(x-4^2$$

$$=3\cot((x-4)^3)(x-4)^2$$

Ley del cociente derivadas

$$\frac{d}{dx}\cot(x) \qquad \frac{ab - ba}{b^2}$$

$$= \frac{\cos(x)}{\operatorname{sen}(x)}$$

$$= \frac{-\operatorname{sen}(x)\operatorname{sen}(x) - \cos(x)\cos(x)}{\operatorname{Sen}^2(x)}$$

$$= \frac{-\operatorname{sen}^2(x) - \cos^2(x)}{\operatorname{Sen}^2(x)}$$

$$\frac{-1\,(\operatorname{Sen}^2(x) + \cos^2(x))}{\operatorname{Sen}^2(x)}$$

$$\frac{d}{dx} = \frac{-1}{\operatorname{sen}^2(x)} = \csc^2$$

Ecuaciones exponenciales y logarítmicas

$7^{x+1} = 7^{5x-2}$

$x+1 = 5x-2$

$1+2 = 5x-x$

$3 = 4x$

$x = \frac{3}{4}$

$10^{3+2x} = 2^{x} \cdot 256$

$(3+2x)\ln 10 = x\ln(2) + \ln(256)$

$x = \frac{\ln \frac{32}{125}}{\ln(50)}$

$\log_5(x) + \log_5(x^2) = \log_5(25) + 10$

$\log_5(x) + 2\log_5(|x|) = \log_5(5^2) + 10$

$\log_5(x + 2\log_5(x) = 2 + 10$

$3\log_5(x) = 12$

$\log_5(x) = 4$

$x = 5^4$

$x = 625$

$\log_5(x) + \log_5(x^2) = \log_5 + 10$

$\log_5(x) + 2\log_5(x) = \log_5(5^2) + 10$

$\log_5(x) + 2\log_5(x) = 2 + 10$

$3\log_5(x) = 4$

$x = 5^4$

$x = 625$

Ley del producto de derivadas

$$\frac{d}{dx}\left(\cos(x)\cdot\log(x)\right)$$

$$\frac{d}{dx}(\cos(x))\ln(x)+\frac{d}{dx}(\ln(x))\cos(x)$$

$$(\cos(x)) = -\sin(x)$$

$$(\ln(x)) = \frac{1}{x}$$

$$=(-\sin(x))\ln(x)+\frac{1}{x}\cos(x)$$

$$=-\sin(x)\ln(x)+\frac{\cos(x)}{x}$$

Límites

$\lim_{x \to 5} (x^2 + 4x - 5)$

$(5^2 + 4(5) - 5)$

$(25 + 20 - 5)$

$(45 - 5)$

$(\lim_{x \to 5} = 40$

$\lim_{x \to 5} (x^2 \cdot 4x - 5)$

$(5^2 \cdot 4(5) - 5)$

$(25(20) - 5)$

$(500 - 5) = 495$

Derivadas simples

$\frac{d}{dx}(5\cos(x) + 8x^3 - \log_5(x))$

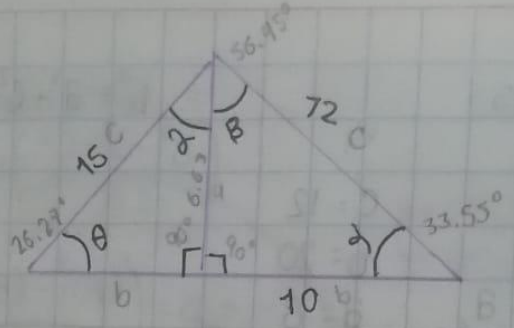
$-5\operatorname{sen}(x) + 24x^2 - \frac{1}{x \cdot \ln(5)}$

$\frac{d}{dx}(\tan(x) - 3x^{\frac{3}{5}} - 8^x)$

$\frac{1}{\cos^2(x)} \left[-3\right] \left[\frac{3}{5} x^{\frac{3}{5}-1}\right]$ $\quad \frac{3}{5} - \frac{1}{1} = \frac{3-5}{5} = \frac{-2}{5}$

$\frac{3}{1} \quad \frac{3}{5} = \frac{-9}{5}$

$\frac{1}{\cos^2(x)} - \frac{9}{5x^{\frac{2}{5}}} - 8x \cdot \ln(8)$



$c^2 = a^2 + b^2$

$a = \sqrt{c^2 - b^2}$

$a = \sqrt{(12)^2 - (10)^2}$

$a = \sqrt{144 - 100}$

$a = \sqrt{44} = 6.63$

$b = \sqrt{c^2 - a^2}$

$b = \sqrt{(15)^2 - (6.63)^2}$

$b = \sqrt{225 - 43.9569}$

$b = \sqrt{181.0431}$

$b = 13.45$

$\cos^{-1} \frac{10}{12} = 33.55°$ /

$\beta = 90 + 33.55 = 123.55$

$180 - 123.55 = 56.45°$ /

$\cos^{-1} \frac{13.45}{15} = 26.27°$ /

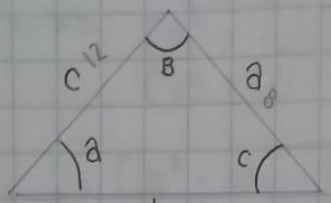
$\alpha = 90 + 26.27 = 116.27$

$180 - 116.27 = 63.73$ /

Ley de cosenos

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$\left(\frac{b^2 - a^2 - c^2}{-2ac}\right)\cos^{-1} = B$$

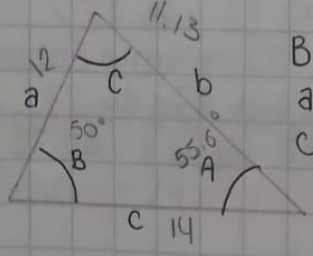


$c = 12$
$b = 20$
$a = 8$

$$B = \left(\frac{(20)^2 - (8)^2 - (12)^2}{-2(8)(12)}\right)\cos^{-1}$$

$$B = \left(\frac{400 - 64 - 144}{-192}\right)\cos^{-1}$$

$$B = \left(\frac{192}{-192}\right)\cos^{-1} = 180\ ?$$



$B = 50°$
$a = 12$
$c = 14$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$b = \sqrt{(12)^2 + (14)^2 - 2(12)(14) \cdot \cos B}$$

$$b = \sqrt{(144 + 196) - 336 \times 0.6428}$$

$$b = 11.1366$$

$$\frac{\sin A}{a} \quad \frac{\sin B}{b}$$

$$\frac{\sin A}{12} = \frac{\sin(50°)}{11.13}$$

$$\frac{\sin A}{12} = \frac{0.7660}{11.13}$$

$$\sin A = 0.8254$$

$$\sin A = 55.6327°$$

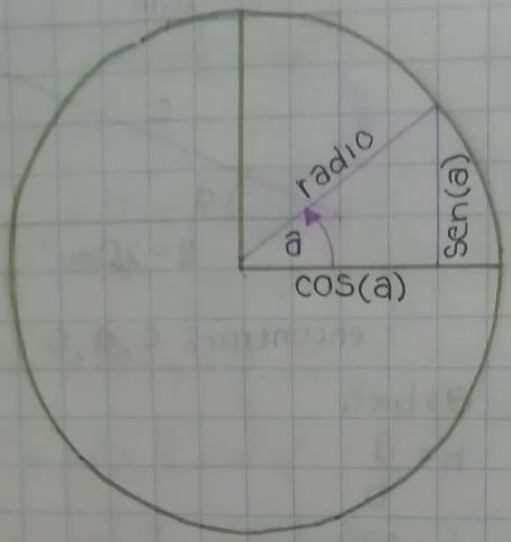
$C = 180 - 50 - 55.63$

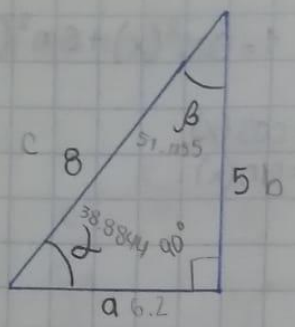
$C = 74.36°$

Saca la identidad

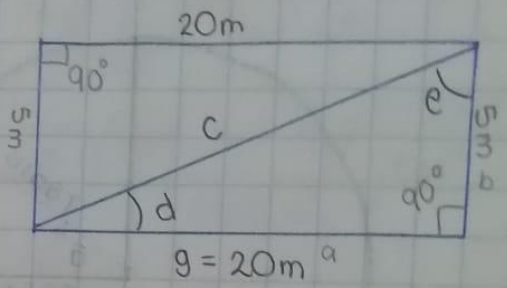
$1 = \cos^2(x) + \sin^2(x)$

$x = \cos(x)$
$y = \sin(x)$





área del rectangulo = 100 m²



encontrar: c, d, e

$c^2 = a^2 + b^2$

$a = \sqrt{c^2 - b^2}$

$a = \sqrt{(8)^2 - (5)^2}$

$a = \sqrt{64 - 25}$

$a = \sqrt{39}$

$a = 6.2449$

ángulo α:

$\text{Tan}^{-1} \frac{5}{6.2} = 38.8844$

ángulo β:

$\text{Tan}^{-1} \frac{6.2}{5} = 51.1155$

$a = b \times h$

$h = \frac{a}{b}$

$h = \frac{100}{20}$

$h = 5$

$c^2 = a^2 + b^2$

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{(20)^2 + (5)^2}$

$c = \sqrt{400 + 25}$

$c = \sqrt{425}$

$c = 20.6155$

$\measuredangle d \quad \text{Tan}^{-1} \frac{Co}{Ca} = \frac{5}{20} = 14.0362°$

$\measuredangle e \quad \text{Cos}^{-1} \frac{Ca}{h} = \frac{5}{20.6155} = 75.9637°$

Multiplicación Algebráica

$15xy^2w^3 \cdot 18xy$

$270x^2y^3w^3$

$10xy^2w^3 \cdot 18xy\left(\frac{5xy}{6wy^2}\right)$

$10xy^2w^3 \cdot 18xy\left(\frac{5}{6}xwy\right)$

$10xy^2w^3 \cdot 18xy \cdot \frac{5}{6}xwy$

$180x^2y^3w^3 + \frac{15}{6}x^2w^4y^3$

$xw^{\frac{3}{4}}y \cdot 18x^{-2}y\left(\frac{wy}{6w^{-2}y^2}\right)$

$xw^{\frac{3}{4}}y \cdot 18x^{-2}y\left(\frac{wy}{6w^2y^2}\right)$

$\frac{1}{x}w^{\frac{3}{4}}y \cdot 18 \cdot \frac{wy}{6w^2y^2}$

$\frac{3yw^{\frac{3}{4}+1}}{xw^2}$

$\frac{1 \cdot wyw^{\frac{3}{4}}y^2 \cdot 18}{x \cdot 6w^2y^2}$

$\frac{3y}{xw-(\frac{3}{4}+1)+2}$

$\frac{1 \cdot wyw^{\frac{3}{4}} \cdot 18}{6xw^2}$

$\frac{3y}{xw^{\frac{1}{4}}}$

$\frac{18yw^{\frac{3}{4}+1}}{xw^2}$

Potencias de 10

$8 \times 10^{-4} = .0008$

$25.2 \times 10^{-5} = .000252$

$12.4 \times 10^{-14} = .00000000000124$

Expresiones algebraicas

$4 + y + w = 4 + y + w$

$4 + 2xy + w^2 = 4 + 2xy + w^2$

$4 + w = 4 + w$

Suma y resta algebráica

$4xw + 10xw - 10xw^2 + 5xy^2 + 12 - 8$

$14xw - 10xw^2 + 5xy^2 + 4$

$w + 11w - 5xy^2 + 12xy^2 + 12 - 8$

$12w + 7xy^2 + 4$

División de fracciones

$$\frac{\frac{5}{4}}{4.8\left(\frac{3}{7}\right)} \qquad \frac{\frac{5}{4}}{\frac{32}{1}\left(\frac{3}{7}\right)} = \frac{\frac{5}{4}}{\frac{96}{7}} = \frac{35}{384}$$

Suma y resta de fracciones

$$\frac{5}{4} + \frac{8}{10} - \frac{3}{7} + \frac{15}{14}$$

$$\frac{5}{4} + \frac{8}{10} = \frac{50+32}{40} = \frac{82}{40} = \frac{41}{20}$$

$$\frac{41}{20} - \frac{3}{7} = \frac{287-60}{140} = \frac{227}{140}$$

$$\frac{227}{140} + \frac{15}{14} = \frac{3178+2100}{1960} = \frac{5278}{1960}$$

$$\frac{377}{140}$$

$$\frac{2}{8} - \frac{8}{1} + \frac{12}{17} + \frac{5}{24}$$

$$\frac{2}{8} - \frac{8}{1} = \frac{2-64}{8} = \frac{-62}{8} = \frac{-31}{4}$$

$$\frac{-31}{4} + \frac{12}{17} = \frac{-527-48}{108} = \frac{-575}{108}$$

$\frac{377}{140}$

$\frac{474}{68}$

Tipos de Números

$\{1,2,3,4,5\}$ enteros $\{2.5, 8, \frac{2}{7}, \sqrt{25}\}$ racionales

Fracciones

Parte de arriba de la fracción: numerador
Parte de abajo de la fracción: denominador

Multiplicación de fracciones

$$\frac{4}{5}\,6 \cdot \frac{8}{9} \cdot \frac{2}{11} \cdot \frac{12}{17}$$

$$\frac{4}{5} \cdot \frac{6}{1} \cdot \frac{8}{9} \cdot \frac{2}{11} \cdot \frac{12}{17} = \frac{4608}{8415} = \frac{1536}{2805} = \frac{512}{935}$$

Leyes de los signos

$(+)(-)(+)(-)\frac{(-)}{(+)}$

$(+)(-)(+)(-)(-)$

$(-)(-)(-)$

$(+)(-)$

$=(-)$

$(80)(-4)^5(10)(-20)^2\left(\frac{(-2)^5}{(5)}\right)$

$(80)(-1024)(10)(400)\left(\frac{32}{5}\right)$

$-81920\,(4000)\,(6.4)$

$-327,680,000\,(6.4)$

$-2,097,152,000$

Potencias y raíces (Leyes de los exponentes)

$\sqrt[3]{\frac{3^8}{3^{-2}}}$ $\qquad \frac{\sqrt[3]{3^8}}{\sqrt[3]{3^{-2}}} = 3^{\frac{8}{3}}$

$= 3^{\frac{-2}{3}} = \frac{1}{\sqrt[3]{3^2}}$

Operaciones básicas y Jerarquía de operaciones

$$8+5-6\times10\div10*\left(\frac{5+4}{8^2}\right)$$

$$8+5-6\times10\div10*\frac{9}{64}$$

$$8+5-6\times10\div10\cdot\frac{9}{64}$$

$8+5-60\div10\cdot0.1406$

$8+5-6\,(0.1406)$

$8+5-0.8436$

$13-0.8436$

12.1564

$$18^2-15^3-8\times10\div(8+10)*\frac{5+4}{(10-4)^2}$$

$$324-225-8\times10\div(18)*\frac{5+4}{36}$$

$$324-225-80\div18\left(\frac{9}{36}\right)$$

$324-225-4.4+0.25$

$99-4.4+0.25$

94.85