

# The Three Problems of Antiquity

Hippocrates' squaring of a particular lune in the Great Theorem of our first chapter of Dunham's book is just the beginning of a discussion about three famous problems on which ancient Greek mathematicians worked. These three problems are:

- (a) Can every circle be squared using only compass and straightedge?
- (b) Can every cube be doubled using only compass and straightedge?
- (c) Can every angle be trisected using only compass and straightedge?

We will find that the answer to each of these questions is actually "no", and that the negative answer is potentially the more interesting answer for this question. As the Greek mathematicians explored these problems, they also began to change the questions slightly as they asked them. Changing the question slightly in order to get a related answer is a very common mathematical method of today, and one of the best ways to begin to chip away at a difficult mathematical problem. By the time we are finished studying these problems, you should be able to talk about why each question cannot be solved with compass and straightedge alone, and you should be able to discuss at least one solution to each problem not using compass and straightedge.

In this section, we will explore briefly the solution to the circle squaring problem (its conclusion being that  $\pi$  is not constructible), and we will also look at the cube doubling problem (its conclusion being that  $\sqrt[3]{2}$  is also not constructible). In the next section, we will tackle the angle trisection problem.

## Readings

First Reading: Dunham, Chapter 1, pages 20 - 26

Second Reading: Tales of Impossibility Read at least:

- The introduction to the cube doubling problem, pages 11-14
- Chapter 5: Doubling the Cube, pages 69-80

## Questions

**Question 1** How many stories are told in the readings about how the cube doubling problem originated?

2  
given

**Question 2** What geometric subject did Menaechmus invent (or discover) while trying to duplicate the cube?

**Multiple Choice:**

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Learning outcomes:

Author(s):

See Tales of Impossibility at <https://library.ohio-state.edu/record=b8808204~S7>

- (a) *Conic sections.* ✓
  - (b) *Calculus.*
  - (c) *Formulas for volumes of pyramids.*
  - (d) *Formulas for slopes of lines.*
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