

The binomial theorem and π

In this activity we investigate a generalization of the binomial theorem and its connection to an approximation of π .

Question 1 The binomial theorem states

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k.$$

Why would we be interested in this?

Question 2 Newton says

$$(1 + x)^r = \sum_{k=0}^{\infty} \left(\frac{x^k}{k!} \prod_{m=0}^{k-1} (r - m) \right).$$

How did Newton come up with this? Hint: Calculus!

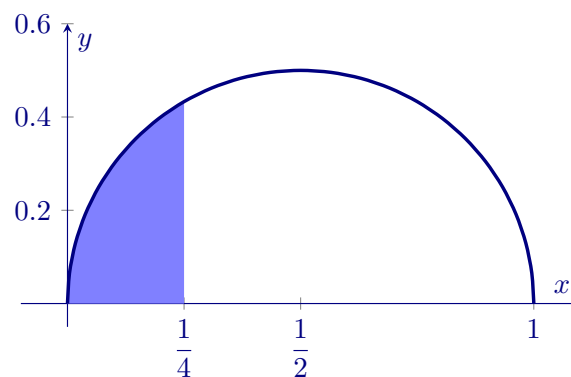
Now we're going to use this to approximate π .

Question 3 Come up with a function of x for the semicircle of radius $1/2$ centered at $(1/2, 0)$.

Question 4 Use Newton's binomial theorem to show your function above is equal to:

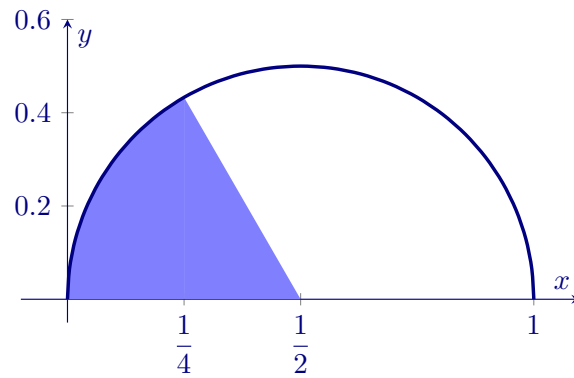
$$x^{1/2} - \frac{x^{3/2}}{2} - \frac{x^{5/2}}{8} - \frac{x^{7/2}}{16} - \frac{5x^{9/2}}{128} - \frac{7x^{11/2}}{256} - \dots$$

Question 5 Use calculus to compute the area of the shaded region:

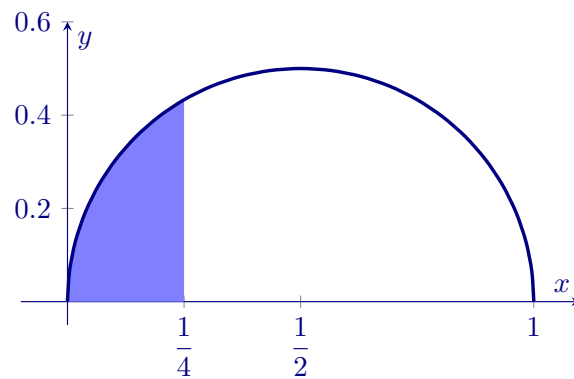


Author(s):

Question 6 Use proportional reasoning to compute the area of the sector below:



Question 7 Use the previous problem, along with the area of a certain 30-60-90 right triangle to give a different computation of the area below.



Question 8 Use your work from above to give an approximation of π .