

# Pythagorean means

*In this activity we explore the three different means of the ancient Greeks.*

## The arithmetic mean

The *arithmetic mean* is the good-old mean that we are all familiar with.

**Question 1** *What is the mean that we are all familiar with? Explain how to compute the mean of  $a_1, a_2, \dots, a_n$ . Give some examples.*

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## The geometric mean

The *geometric mean* is a bit different. The geometric mean of  $a_1, a_2, \dots, a_n$  is given by:

$$\left( \prod_{i=1}^n a_i \right)^{1/n}$$

**Question 2** *Explain an analogy between the arithmetic mean and the geometric mean.*

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**Question 3** *Can you explain the geometric mean in terms of geometry? First do it for 2 numbers. Next do it for three.*

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## The harmonic mean

The *harmonic mean* might be the most mysterious of all. The harmonic mean of  $a_1, a_2, \dots, a_n$  is given by:

$$\frac{n}{\sum_{i=1}^n \frac{1}{a_i}}$$

**Exploration 4** *Can you find a connection between the harmonic mean and music?*

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**Question 5** *In the United States, the fuel efficiency of a car is usually given in the units:*

$$\frac{\text{miles}}{\text{gallon}}$$

*However, in Europe, the fuel efficiency of a car is usually given in the units:*

$$\frac{\text{liters}}{100\text{km}}$$

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Learning outcomes:  
Author(s):

Give some examples of fuel efficiency (both efficient and inefficient) with each set of units.

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**Question 6** Now suppose that a car gets  $60 \frac{\text{miles}}{\text{gallon}}$  and another car gets  $20 \frac{\text{miles}}{\text{gallon}}$ . What is the average fuel efficiency?

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**Question 7** Now suppose that a car gets  $4 \frac{\text{liters}}{100\text{km}}$  and another car gets  $20 \frac{\text{liters}}{100\text{km}}$ . What is the average fuel efficiency?

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**Exploration 8** Compare your answers to the last two questions. Something fishy is going on, what is it?

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