## The binomial theorem and $\pi$

In this activity we investigate a generalization of the binomial theorem and its connection to an approximation of  $\pi$ .

**Question 1** The binomial theorem states

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k.$$

Why would we be interested in this?

Question 2 Newton says

$$(1+x)^r = \sum_{k=0}^{\infty} \left( \frac{x^k}{k!} \prod_{\ell=0}^{k-1} (r-\ell) \right).$$

How did Newton come up with this? Hint: Calculus!

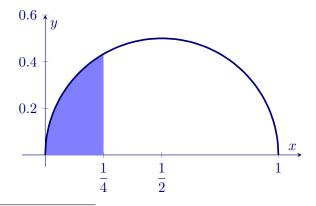
Now we're going to use this to approximate  $\pi$ .

**Question 3** Come up with a function of x for the semicircle of radius 1/2 centered at (1/2,0).

**Question 4** Use Newton's binomial theorem to show your function above is equal to:

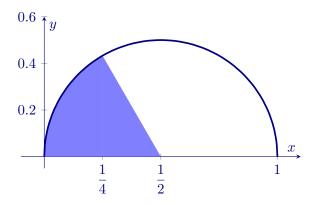
$$x^{1/2} - \frac{x^{3/2}}{2} - \frac{x^{5/2}}{8} - \frac{x^{7/2}}{16} - \frac{5x^{9/2}}{128} - \frac{7x^{11/2}}{256} - \cdots$$

**Question 5** Use calculus to compute the area of the shaded region:

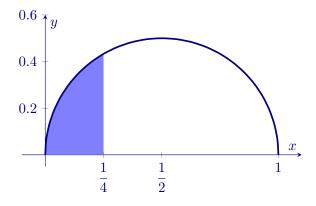


Learning outcomes: Author(s):

Question 6 Use proportional reasoning to compute the area of the sector below:



**Question 7** Use the previous problem, along with the area of a certain 30-60-90 right triangle to give a different computation of the area below.



**Question 8** Use your work from above to give an approximation of  $\pi$ .