The binomial theorem and π

In this activity we investigate a generalization of the binomial theorem and its connection to an approximation of π.

Question 1 *The binomial theorem states*

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k.$$

Why would we be interested in this?

Question 2 *Newton says*

$$(1+x)^r = \sum_{k=0}^{\infty} \left(\frac{x^k}{k!} \prod_{\ell=0}^{k-1} (r-\ell) \right).$$

How did Newton come up with this? Hint: Calculus!

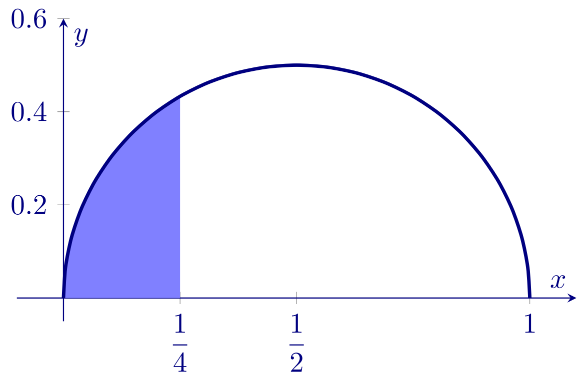
Now we're going to use this to approximate π.

Question 3 *Come up with a function of x for the semicircle of radius $1/2$ centered at $(1/2, 0)$.*

Question 4 *Use Newton's binomial theorem to show your function above is equal to:*

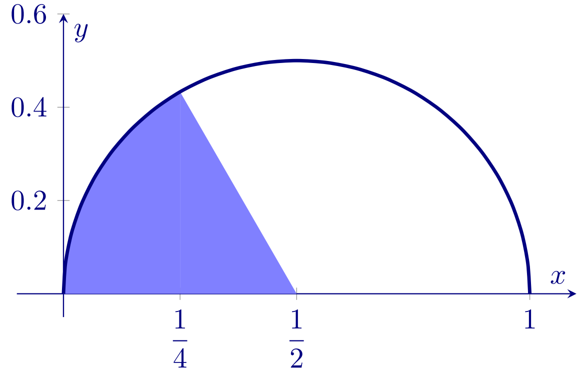
$$x^{1/2} - \frac{x^{3/2}}{2} - \frac{x^{5/2}}{8} - \frac{x^{7/2}}{16} - \frac{5x^{9/2}}{128} - \frac{7x^{11/2}}{256} - \cdots$$

Question 5 *Use calculus to compute the area of the shaded region:*

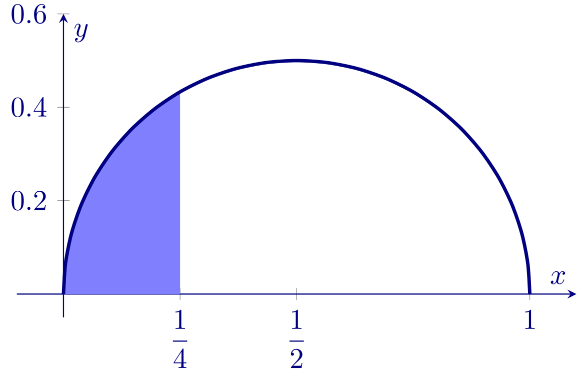


Learning outcomes:
Author(s):

Question 6 *Use proportional reasoning to compute the area of the sector below:*



Question 7 *Use the previous problem, along with the area of a certain 30-60-90 right triangle to give a different computation of the area below.*



Question 8 *Use your work from above to give an approximation of π.*