Euler and Fermat

Question 1 *Remind me: what are the necessary ingredients for a proof by induction?*

Our goal is to prove the following theorem:

Theorem 1. *Suppose that a is an even number and p a prime which does not divide a. Suppose that p does divide $a^{2^n}+1$. Then p is of the form $p = 2^{n+1}k+1$ for some positive integer k.*

In order to prove this theorem, we'll need the so-called "Little Fermat Theorem". You can find Euler's proof of this theorem in your text.

Theorem 2 (Little Fermat Theorem)**.** *Let a be a whole number and p a prime which does not divide a. Then p divides $a^{p-1}-1$.*

Question 2 *Remind me: what is the definition of "divides"?*

Question 3 *Prove that Theorem 1 is true in the case that $n = 0$.*

Question 4 *Suppose that A is any whole number, and that you divide A by some number C. What are the possible remainders? What are the possibilities for how you could write A as related to a multiple of C?*

Question 5 *Repeat question 4, but related to Theorem 1 and the case $n = 1$. What are the possibilities for the prime p when you divide by $2^2 = 4$? Eliminate all but two of these cases.*

Question 6 *Use proof by contradiction to eliminate the case you don't want.*

Question 7 *Repeat questions 5 and 6, but for the case $n = 2$. If you're confident you understand what's going on, move to the next question!*

Question 8 *Prove Theorem 1.*

Question 9 *Euler used Theorem 1 to prove that $2^{2^5}+1$ is not prime. How did he do this? Check his work. Could you use his method to prove that $2^{2^6}+1$ is not prime?*

Learning outcomes:
Author(s):