Estimating Pi

Question 1 *List as many ways as you can think of for estimating the value of π.*

Draw a (fairly large) circle on a blank sheet of paper. We'll think of this as a unit circle.

Problem 2 *Divide the unit circle into $2^2 = 4$ equal wedges each with its vertex at the center of the circle O. On each wedge, call the two corners of the wedge that lie on the circle A and B_2. Let $\mathcal{A}_2$ denote the area of the triangle $\triangle OAB_2$ and let θ_2 denote the measure of the angle at O. Explain how to estimate the area of the circle with triangle $\triangle OAB_2$. What is your estimate?*

Problem 3 *Divide the unit circle into $2^3 = 8$ equal wedges each with its vertex at the center of the circle O. On each wedge, call the two corners of the wedge that lie on the circle A and B_3. Let $\mathcal{A}_3$ denote the area of the triangle $\triangle OAB_3$ and let θ_3 denote the measure of the angle at O. Explain how to estimate the area of the circle with triangle $\triangle OAB_3$. What information do you need to know to actually do this computation?*

Problem 4 *Given an angle θ, explain the relation of $\sin(\theta)$ and $\cos(\theta)$ to the unit circle. How could these values help with the calculation described above?*

Problem 5 *Divide the unit circle into 2^n equal wedges each with its vertex at the center of the circle O. On each wedge, call the two corners of the wedge that lie on the circle A and B_n. Let $\mathcal{A}_n$ denote the area of the triangle $\triangle OAB_n$ and let θ_n denote the measure of the angle at O. Explain why someone would be interested in the value of:*

$$\sin\left(\frac{\theta_n}{2}\right)$$

Problem 6 *Recalling that:*

$$\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos(\theta)}{2}} \qquad \textit{and} \qquad \cos(\theta)^2 + \sin(\theta)^2 = 1$$

Explain why:

$$2\mathcal{A}_{n+1} = \sqrt{\frac{1-\sqrt{1-(2\mathcal{A}_n)^2}}{2}}$$

Learning outcomes:
Author(s):

Problem 7 *Let's fill out the following table (a calculator will help!):*

n	$\mathcal{A}_n$	*Approx. Area*	$\sqrt{1-(2\mathcal{A}_n)^2}$	$\dfrac{1-\sqrt{1-(2\mathcal{A}_n)^2}}{2}$	$2\mathcal{A}_{n+1} = \sqrt{\dfrac{1-\sqrt{1-(2\mathcal{A}_n)^2}}{2}}$
2					
3					
4					
5					
6					
7					
8					

What do you notice?