

# Estimating Pi

**Question 1** List as many ways as you can think of for estimating the value of  $\pi$ .

Draw a (fairly large) circle on a blank sheet of paper. We'll think of this as a unit circle.

**Problem 2** Divide the unit circle into  $2^2 = 4$  equal wedges each with its vertex at the center of the circle  $O$ . On each wedge, call the two corners of the wedge that lie on the circle  $A$  and  $B_2$ . Let  $\mathcal{A}_2$  denote the area of the triangle  $\triangle OAB_2$  and let  $\theta_2$  denote the measure of the angle at  $O$ . Explain how to estimate the area of the circle with triangle  $\triangle OAB_2$ . What is your estimate?

**Problem 3** Divide the unit circle into  $2^3 = 8$  equal wedges each with its vertex at the center of the circle  $O$ . On each wedge, call the two corners of the wedge that lie on the circle  $A$  and  $B_3$ . Let  $\mathcal{A}_3$  denote the area of the triangle  $\triangle OAB_3$  and let  $\theta_3$  denote the measure of the angle at  $O$ . Explain how to estimate the area of the circle with triangle  $\triangle OAB_3$ . What information do you need to know to actually do this computation?

**Problem 4** Given an angle  $\theta$ , explain the relation of  $\sin(\theta)$  and  $\cos(\theta)$  to the unit circle. How could these values help with the calculation described above?

**Problem 5** Divide the unit circle into  $2^n$  equal wedges each with its vertex at the center of the circle  $O$ . On each wedge, call the two corners of the wedge that lie on the circle  $A$  and  $B_n$ . Let  $\mathcal{A}_n$  denote the area of the triangle  $\triangle OAB_n$  and let  $\theta_n$  denote the measure of the angle at  $O$ . Explain why someone would be interested in the value of:

$$\sin\left(\frac{\theta_n}{2}\right)$$

Learning outcomes:  
Author(s):

**Problem 6** Recalling that:

$$\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 - \cos(\theta)}{2}} \quad \text{and} \quad \cos(\theta)^2 + \sin(\theta)^2 = 1$$

Explain why:

$$2\mathcal{A}_{n+1} = \sqrt{\frac{1 - \sqrt{1 - (2\mathcal{A}_n)^2}}{2}}$$

**Problem 7** Let's fill out the following table (a calculator will help!):

$n$	$\mathcal{A}_n$	Approx. Area	$\sqrt{1 - (2\mathcal{A}_n)^2}$	$\frac{1 - \sqrt{1 - (2\mathcal{A}_n)^2}}{2}$	$2\mathcal{A}_{n+1} = \sqrt{\frac{1 - \sqrt{1 - (2\mathcal{A}_n)^2}}{2}}$
2					
3					
4					
5					
6					
7					
8					

What do you notice?