

Estimating Pi

Question 1 List as many ways as you can think of for estimating the value of π .

Draw a (fairly large) circle on a blank sheet of paper. We'll think of this as a unit circle.

Problem 2 Divide the unit circle into $2^2 = 4$ equal wedges each with its vertex at the center of the circle O . On each wedge, call the two corners of the wedge that lie on the circle A and B_2 . Let \mathcal{A}_2 denote the area of the triangle $\triangle OAB_2$ and let θ_2 denote the measure of the angle at O . Explain how to estimate the area of the circle with triangle $\triangle OAB_2$. What is your estimate?

Problem 3 Divide the unit circle into $2^3 = 8$ equal wedges each with its vertex at the center of the circle O . On each wedge, call the two corners of the wedge that lie on the circle A and B_3 . Let \mathcal{A}_3 denote the area of the triangle $\triangle OAB_3$ and let θ_3 denote the measure of the angle at O . Explain how to estimate the area of the circle with triangle $\triangle OAB_3$. What information do you need to know to actually do this computation?

Problem 4 Given an angle θ , explain the relation of $\sin(\theta)$ and $\cos(\theta)$ to the unit circle. How could these values help with the calculation described above?

Problem 5 Divide the unit circle into 2^n equal wedges each with its vertex at the center of the circle O . On each wedge, call the two corners of the wedge that lie on the circle A and B_n . Let \mathcal{A}_n denote the area of the triangle $\triangle OAB_n$ and let θ_n denote the measure of the angle at O . Explain why someone would be interested in the value of:

$$\sin\left(\frac{\theta_n}{2}\right)$$

Learning outcomes:
Author(s):

Problem 6 Recalling that:

$$\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 - \cos(\theta)}{2}} \quad \text{and} \quad \cos(\theta)^2 + \sin(\theta)^2 = 1$$

Explain why:

$$2\mathcal{A}_{n+1} = \sqrt{\frac{1 - \sqrt{1 - (2\mathcal{A}_n)^2}}{2}}$$

Problem 7 Let's fill out the following table (a calculator will help!):

n	\mathcal{A}_n	Approx. Area	$\sqrt{1 - (2\mathcal{A}_n)^2}$	$\frac{1 - \sqrt{1 - (2\mathcal{A}_n)^2}}{2}$	$2\mathcal{A}_{n+1} = \sqrt{\frac{1 - \sqrt{1 - (2\mathcal{A}_n)^2}}{2}}$
2					
3					
4					
5					
6					
7					
8					

What do you notice?