

Euler and Fermat

Question 1 Remind me: what are the necessary ingredients for a proof by induction?

Our goal is to prove the following theorem:

Theorem 1. Suppose that a is an even number and p a prime which does not divide a . Suppose that p does divide $a^{2^n} + 1$. Then p is of the form $p = 2^{n+1}k + 1$ for some positive integer k .

In order to prove this theorem, we'll need the so-called "Little Fermat Theorem". You can find Euler's proof of this theorem in your text.

Theorem 2 (Little Fermat Theorem). Let a be a whole number and p a prime which does not divide a . Then p divides $a^{p-1} - 1$.

Question 2 Remind me: what is the definition of "divides"?

Question 3 Prove that Theorem 1 is true in the case that $n = 0$.

Question 4 Suppose that A is any whole number, and that you divide A by some number C . What are the possible remainders? What are the possibilities for how you could write A as related to a multiple of C ?

Question 5 Repeat question 4, but related to Theorem 1 and the case $n = 1$. What are the possibilities for the prime p when you divide by $2^2 = 4$? Eliminate all but two of these cases.

Question 6 Use proof by contradiction to eliminate the case you don't want.

Question 7 Repeat questions 5 and 6, but for the case $n = 2$. If you're confident you understand what's going on, move to the next question!

Question 8 Prove Theorem 1.

Question 9 Euler used Theorem 1 to prove that $2^{2^5} + 1$ is not prime. How did he do this? Check his work. Could you use his method to prove that $2^{2^6} + 1$ is not prime?

Learning outcomes:
Author(s):