Pythagorean means

In this activity we explore the three different means of the ancient Greeks.

The arithmetic mean

The *arithmetic mean* is the good-old mean that we are all familiar with.

Question 1 *What is the mean that we are all familiar with? Explain how to compute the mean of $a_1, a_2, \ldots, a_n$. Give some examples.*

The geometric mean

The *geometric mean* is a bit different. The geometric mean of $a_1, a_2, \ldots, a_n$ is given by:

$$\left(\prod_{i=1}^{n} a_i\right)^{1/n}$$

Question 2 *Explain an analogy between the arithmetic mean and the geometric mean.*

Question 3 *Can you explain the geometric mean in terms of geometry? First do it for 2 numbers. Next do it for three.*

The harmonic mean

The *harmonic mean* might be the most mysterious of all. The harmonic mean of $a_1, a_2, \ldots, a_n$ is given by:

$$\frac{n}{\sum_{i=1}^{n} \frac{1}{a_i}}$$

Exploration 4 *Can you find a connection between the harmonic mean and music?*

Question 5 *In the United States, the fuel efficiency of a car is usually given in the units:*

$$\frac{\textit{miles}}{\textit{gallon}}$$

However, in Europe, the fuel efficiency of a car is usually given in the units:

$$\frac{\textit{liters}}{100\text{km}}$$

Learning outcomes:
Author(s):

Give some examples of fuel efficiency (both efficient and inefficient) with each set of units.

Question 6 *Now suppose that a car gets* $60\dfrac{\textit{miles}}{\textit{gallon}}$ *and another car gets* $20\dfrac{\textit{miles}}{\textit{gallon}}$. *What is the average fuel efficiency?*

Question 7 *Now suppose that a car gets* $4\dfrac{\textit{liters}}{100\text{km}}$ *and another car gets* $20\dfrac{\textit{liters}}{100\text{km}}$. *What is the average fuel efficiency?*

Exploration 8 *Compare your answers to the last two questions. Something fishy is going on, what is it?*