# Part 1

# Trigonometric Functions

TF1.tex

**Exercise 1** *For each function, give its period.*

(a) *The period of* $\sin(x)$ *is* $\boxed{2\pi}$.

(b) *The period of* $\cos(2x)$ *is* $\boxed{\pi}$.

(c) *The period of* $\tan(\pi x)$ *is* $\boxed{1}$.

(d) *The period of* $3\sin(3x) + 3$ *is* $\boxed{\frac{2\pi}{3}}$.

(e) *The period of* $\cos\left(\frac{x}{2}\right) - 7$ *is* $\boxed{4\pi}$.

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TF2.tex

**Exercise 2** *For each function, give its range. Remember to use* `oo` *or* `infty` *when necessary to input the infinity symbol* $\infty$.

(a) *The range of* $7\sin(5x-3)$ *is* $[\boxed{-7}, \boxed{7}]$.

(b) *The range of* $\tan\left(\frac{x}{4}\right)$ *is* $(\boxed{-\infty}, \boxed{\infty})$.

(c) *The range of* $2\cos(x-\pi) - 2$ *is* $[\boxed{-4}, \boxed{0}]$.

(d) *The range of* $3\csc(x)$ *is* $(\boxed{-\infty}, \boxed{-3}] \cup [\boxed{3}, \boxed{\infty})$.

(e) *The range of* $2\sec(x) + 1$ *is* $(\boxed{-\infty}, \boxed{-1}] \cup [\boxed{3}, \boxed{\infty})$.

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**Exercise 3** *For each function, give its amplitude.*

(a) *The amplitude of* $\pi\cos(2x)$ *is* $\boxed{\pi}$.

(b) *The amplitude of* $\sin(5\pi x)$ *is* $\boxed{1}$.

(c) *The amplitude of* $\frac{5}{2}\cos(x) - 2\pi$ *is* $\boxed{\frac{5}{2}}$.

 

(d) *The amplitude of* $\frac{\pi}{2}\cos(x-\pi)+\frac{\pi}{6}$ *is* $\boxed{\frac{\pi}{2}}$.

(e) *The amplitude of* $-7\sin(x-2)$ *is* $\boxed{7}$.

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**Exercise 4** *Select all the functions that are equal to the sine function* $\sin(x)$*. Remember that in order for two functions to be equal, they must have the same domain and range.*

***Select All Correct Answers:***

(a) $\cos(x)$

(b) $\cos\left(x+\frac{\pi}{2}\right)$

(c) $\cos\left(x-\frac{\pi}{2}\right)$ ✓

(d) $\sin(x+\pi)$

(e) $\sin(x+4\pi)$ ✓

(f) $\tan(x)\cos(x)$

(g) $\cot(x)\sin(x)$

(h) $-\sin(-x)$ ✓

(i) $\sin(-x)$

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`TF4.tex`

In this exercise, we will find the solutions to the trigonometric equation $\cos(3x)=\frac{1}{2}$ that lie in the interval $[0,2\pi)$.

To start, make the substitution $u=3x$ and find solutions to the equation $\cos(u)=\frac{1}{2}$ for $u$ in the interval $[0,2\pi)$.

The solutions to $\cos(u)=\frac{1}{2}$ lying in the interval $[0,2\pi)$ are $u=\frac{\pi}{3}$ and $u=\frac{5\pi}{3}$.

**Exercise 5** *The period of the cosine function is* $2\pi$*, so all real solutions to* $\cos(u)=\frac{1}{2}$ *are of the form* $u=\frac{\pi}{3}+\boxed{2\pi}k$ *or* $u=\frac{5\pi}{3}+\boxed{2\pi}k$ *for some integer* $k$.

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**Exercise 5.1** *Undoing our substitution $u = 3x$, we find that all real solutions to* $\cos(3x) = \frac{1}{2}$ *are of the form* $x = \boxed{\frac{\pi}{9}} + \boxed{\frac{2\pi}{3}}k$ *or* $x = \boxed{\frac{5\pi}{9}} + \boxed{\frac{2\pi}{3}}k$ *for some integer $k$.*

**Exercise 5.1.1** *There are* $\boxed{6}$ *solutions to* $\cos(3x) = \frac{1}{2}$ *that lie in the interval* $[0, 2\pi)$.

**Exercise 5.1.1.1** *The solutions to* $\cos(3x) = \frac{1}{2}$ *that lie in the interval* $[0, 2\pi)$ *are (in increasing order)* $\boxed{\frac{\pi}{9}}$, $\boxed{\frac{5\pi}{9}}$, $\boxed{\frac{7\pi}{9}}$, $\boxed{\frac{11\pi}{9}}$, $\boxed{\frac{13\pi}{9}}$, *and* $\boxed{\frac{17\pi}{9}}$.

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In this exercise, we will find the solutions to the trigonometric equation $\tan(2x-\pi) = 1$ that lie in the interval $[0, 2\pi)$.

To start, make the substitution $u = 2x - \pi$ and find solutions to the equation $\tan(u) = 1$ for $u$ in the interval $[0, 2\pi)$.

The solutions to $\tan(u) = 1$ lying in the interval $[0, 2\pi)$ are $u = \frac{\pi}{4}$ and $u = \frac{5\pi}{4}$.

**Exercise 6** *The period of the tangent function is $\pi$, so all real solutions to* $\tan(u) = 1$ *are of the form* $u = \frac{\pi}{4} + \boxed{\pi}k$ *for some integer $k$.*

**Exercise 6.1** *Undoing our substitution $u = 2x - \pi$, we find that all real solutions to* $\tan(2x - \pi) = 1$ *are of the form* $x = \boxed{\frac{5\pi}{8}} + \boxed{\frac{\pi}{2}}k$ *for some integer $k$.*

**Exercise 6.1.1** *There are* $\boxed{4}$ *solutions to* $\tan(2x - \pi) = 1$ *that lie in the interval* $[0, 2\pi)$.

**Exercise 6.1.1.1** *The solutions to* $\tan(2x - \pi) = 1$ *that lie in the interval* $[0, 2\pi)$ *are (in increasing order)* $\boxed{\frac{\pi}{8}}$, $\boxed{\frac{5\pi}{8}}$, $\boxed{\frac{9\pi}{8}}$, *and* $\boxed{\frac{13\pi}{8}}$.


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**Exercise 7** *In this exercise, we will find properties of the function $h$ defined by $h(x) = \dfrac{1}{4}\cos(4x) - 3$.*

(a) *The domain of $h$ is*

***Multiple Choice:***

(i) $(-\infty, 4) \cup (4, \infty)$.

(ii) $\left[-\dfrac{13}{4}, -\dfrac{11}{4}\right]$.

(iii) $(-\infty, \infty)$. ✓

(iv) $(-\dfrac{13}{4}, -\dfrac{11}{4})$.

(v) $(-\infty, 4] \cup [4, \infty)$.

(b) *The range of $h$ is*

***Multiple Choice:***

(i) $(-\infty, 4) \cup (4, \infty)$

(ii) $\left[-\dfrac{13}{4}, -\dfrac{11}{4}\right]$ ✓

(iii) $(-\infty, \infty)$

(iv) $(-\dfrac{13}{4}, -\dfrac{11}{4})$

(v) $(-\infty, 4] \cup [4, \infty)$

(c) *The period of $h$ is* $\boxed{\dfrac{\pi}{2}}$.

(d) *The function $h$ is*

***Multiple Choice:***

(i) *odd.*

(ii) *even.* ✓


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(iii) *odd and even.*

(iv) *neither odd nor even.*

(e) *The amplitude of* $h$ *is* $\boxed{\dfrac{1}{4}}$.

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For each equation below, determine how many solutions lie in the interval $[0, 2\pi)$ and list them in increasing order, if applicable.

**Exercise 8** *The equation* $\csc(\pi x) = 0$ *has* $\boxed{0}$ *solution(s) in the interval* $[0, 2\pi)$.

**Exercise 9** *The equation* $\sin(5x) = 0$ *has* $\boxed{10}$ *solution(s) in the interval* $[0, 2\pi)$.

**Exercise 9.1** *The solutions are* $\boxed{0}$, $\boxed{\dfrac{\pi}{5}}$, $\boxed{\dfrac{2\pi}{5}}$, $\boxed{\dfrac{3\pi}{5}}$, $\boxed{\dfrac{4\pi}{5}}$, $\boxed{\pi}$, $\boxed{\dfrac{6\pi}{5}}$, $\boxed{\dfrac{7\pi}{5}}$, $\boxed{\dfrac{8\pi}{5}}$, *and* $\boxed{\dfrac{9\pi}{5}}$.

**Exercise 10** *The equation* $\sec(3x) = \sqrt{2}$ *has* $\boxed{6}$ *solution(s) in the interval* $[0, 2\pi)$.

**Exercise 10.1** *The solutions are* $\boxed{\dfrac{\pi}{12}}$, $\boxed{\dfrac{7\pi}{12}}$, $\boxed{\dfrac{3\pi}{4}}$, $\boxed{\dfrac{5\pi}{4}}$, $\boxed{\dfrac{17\pi}{12}}$, *and* $\boxed{\dfrac{23\pi}{12}}$.

**Exercise 11** *The equation* $\sin\left(\dfrac{x}{3}\right) = \dfrac{\sqrt{2}}{2}$ *has* $\boxed{1}$ *solution(s) in the interval* $[0, 2\pi)$.

**Exercise 11.1** *The solution is* $\boxed{\dfrac{3\pi}{4}}$.

 