Vertex Form

We explore the vertex form of a quadratic.

The Vertex Form of a Quadratic

We have learned the standard form of a quadratic function's formula, which is $f(x) = ax^2 + bx + c$. we will learn another form called the vertex form.

Vertex Form of a Quadratic Function A quadratic function whose graph has vertex at the point (h, k) is given by

$$f(x) = a(x - h)^2 + k$$

Using graphing technology, consider the graphs of $f(x) = x^2 - 6x + 7$ and $g(x) = (x - 3)^2 - 2$ on the same axes.

We see only one parabola because these are two different forms of the same function. Indeed, if we convert $g(x)$ into standard form:

$$\begin{aligned} g(x) &= (x - 3)^2 - 2 \\ &= (x^2 - 6x + 9) - 2 \\ &= x^2 - 6x + 7 \end{aligned}$$

it is clear that f and g are the same function.

Graph of $f(x) = x^2 - 6x + 7$ and $g(x) = (x - 3)^2 - 2$ the graphs of the two parabolas overlap each other completely

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Learning outcomes:
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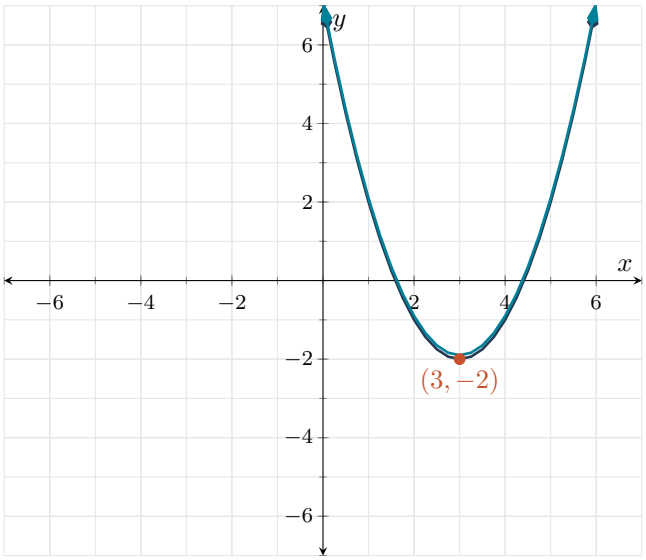
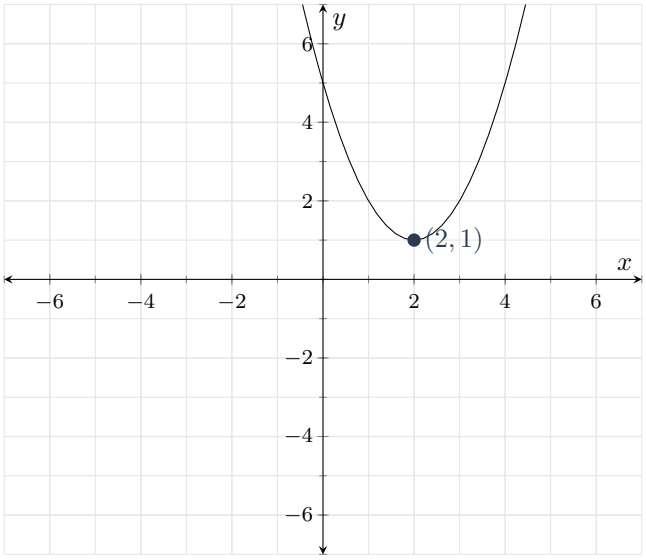
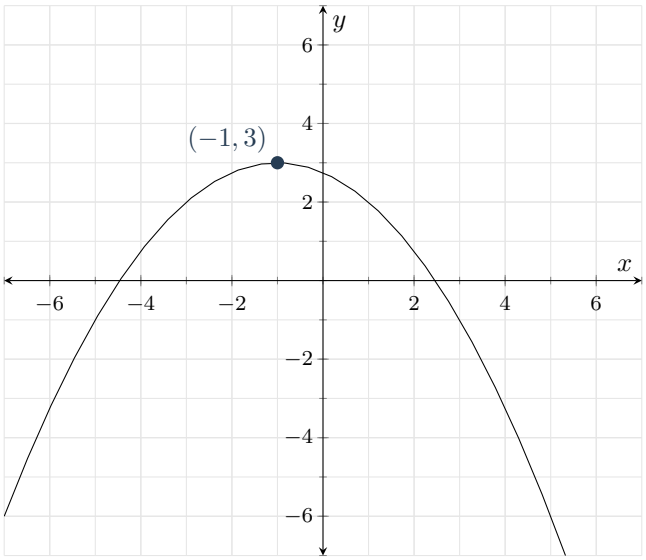


The formula given for g is said to be in vertex form because it allows us to read the vertex without doing any calculations. The vertex of the parabola is $(3, -2)$. We can see those numbers in $g(x) = (x-3)^2 - 2$. The x-value is the solution to $(x-3) = 0$, and the y-value is the constant added at the end.

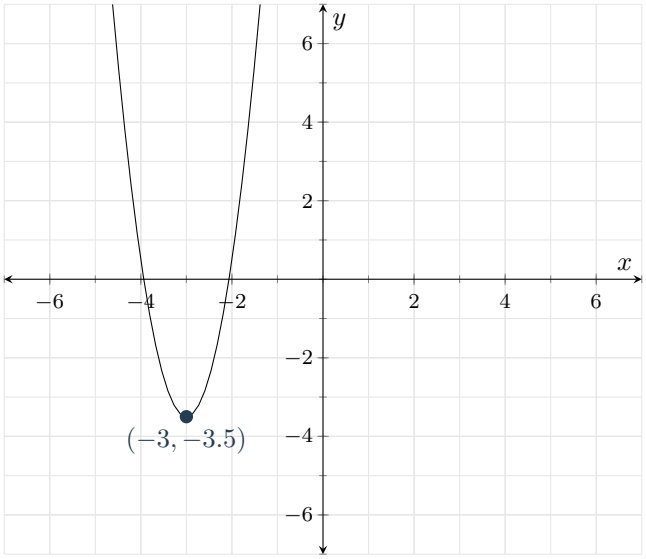
Example 1. *Here are the graphs of three more functions with formulas in vertex form. Compare each function with the vertex of its graph.*



$$r(x) = (x-2)^2 + 1$$



$$s(x) = -\frac{1}{4}(x+1)^2 + 3$$



$$t(x) = 4(x+3)^2 - 3.5$$

Notice that the x-coordinate of the vertex has the opposite sign as the value in the function formula. On the other hand, the y-coordinate of the vertex has the same sign as the value in the function formula. Let's look at an example to understand why. We will evaluate $r(2)$.

$r(2) = (2-2)^2 + 1 = 1$

The x-value is the solution to $(x - 2) = 0$, which is positive 2. When we substitute 2 for x we get the value $y = 1$. Note that these coordinates create the vertex at $(2, 1)$. Now we can define the vertex form of a quadratic function.

