Part 1

Function Transformations

FT15.tex

Exercise 1 *Given a function f, which of the following represents a vertical shift up 2 units, followed by a reflection across the y-axis?*

Multiple Choice:

(a) $y = f(-x) + 2$ ✓

(b) $y = f(2 - x)$

(c) $y = f(x) - 2$

(d) $y = 2 - f(x)$

(e) $y = -f(x - 2)$

Exercise 1.1 *If the point $(-3, 7)$ is on the graph of f, the point $(\boxed{3}, \boxed{9})$ is on the graph of the transformed function.*

Exercise 2 *Given a function f, which of the following represents a vertical shift down 3 units, followed by a vertical compression by a factor of 2?*

Multiple Choice:

(a) $y = 2f(x) - 3$

(b) $y = \frac{1}{2}f(x) - 3$

(c) $y = \frac{1}{2}(f(x) - 3)$ ✓

(d) $y = 2(f(x) - 3)$

(e) $y = f(2x - 3)$

Exercise 2.1 *If the point $(7, -1)$ is on the graph of f, the point $(\boxed{7}, \boxed{-2})$ is on the graph of the transformed function.*

Exercise 3 *Given a function f, which of the following represents a vertical compression by a factor of 2, followed by a vertical shift down 3 units?*

Multiple Choice:

(a) $y = 2f(x) - 3$

(b) $y = \frac{1}{2}f(x) - 3$ ✓

(c) $y = \frac{1}{2}(f(x) - 3)$

(d) $y = 2(f(x) - 3)$

(e) $y = f(2x - 3)$

Exercise 3.1 *If the point $(7, -1)$ is on the graph of f, the point $(\boxed{7}, \boxed{-7/2})$ is on the graph of the transformed function.*

Exercise 4 *Given a function f, which of the following represents a horizontal compression by a factor of 2, followed by a horizontal shift right 2 units?*

Multiple Choice:

(a) $y = f(2x - 2)$

(b) $y = f\left(\frac{x}{2} - 2\right)$

(c) $y = f(2x + 2)$

(d) $y = f(2(x - 2))$ ✓

(e) $y = f(2(x + 2))$

Exercise 4.1 *If the point $(2, -1)$ is on the graph of f, the point $(\boxed{3}, \boxed{-1})$ is on the graph of the transformed function.*

Exercise 5 *Given a function f, which of the following represents a horizontal shift right 2 units, followed by a horizontal compression by a factor of 2?*

Multiple Choice:

(a) $y = f(2x - 2)$ ✓

(b) $y = f\left(\frac{x}{2} - 2\right)$

(c) $y = f(2x + 2)$

(d) $y = f(2(x - 2))$

(e) $y = f(2(x + 2))$

Exercise 5.1 *If the point* $(2, -1)$ *is on the graph of* f*, the point* $(\boxed{2}, \boxed{-1})$ *is on the graph of the transformed function.*

`FT23.tex`

In this problem, we'll step through an analysis of the function f defined by $f(x) = 2 * e^{\frac{x}{2}-3} - 3$. The parent function of f is defined by $f_0(x) = e^x$. We will follow the point $(1, e)$ on the graph of e^x.

Exercise 6 *What is the first transformation applied to* f_0*?*

Multiple Choice:

(a) *Vertical stretch by a factor of 2*

(b) *Horizontal stretch by a factor of 2*

(c) *Shift right by 3 units* ✓

(d) *Shift down by 3 units*

Exercise 6.1 *The point* $(1, e)$ *on the graph of* f_0 *transforms into* $(\boxed{4}, \boxed{e})$ *on the graph of* $f_1(x) = f_0(x - 3)$*.*

Exercise 6.1.1 *What is the next transformation, applied to* f_1*?*

Multiple Choice:

(a) *Vertical stretch by a factor of 2*

(b) *Horizontal stretch by a factor of 2* ✓

(c) *Shift down by 3 units*

Exercise **6.1.1.1** *The point* $(4, e)$ *on the graph of* f_1 *transforms into* $(\boxed{8}, \boxed{e})$ *on the graph of* $f_2(x) = f_1\left(\frac{x}{2}\right)$.

Exercise **6.1.1.1.1** *What is the next transformation, applied to* f_2*?*

Multiple Choice:

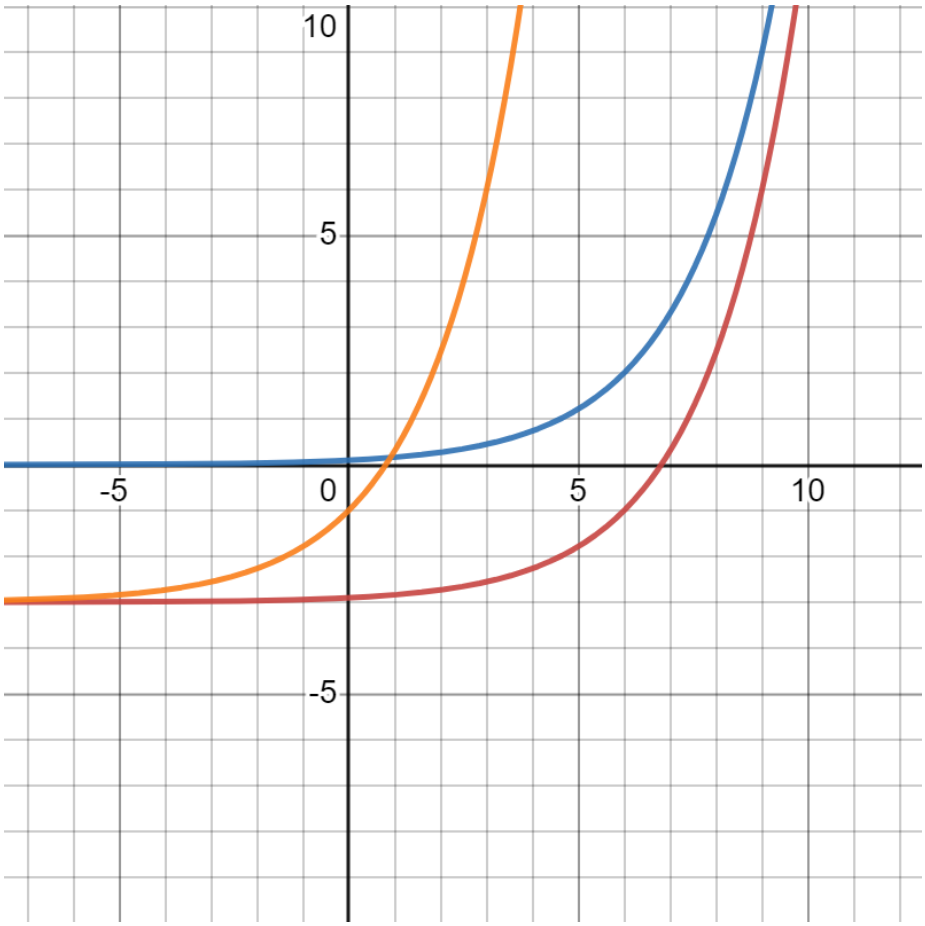
(a) *Vertical stretch by a factor of 2* ✓

(b) *Shift down by 3 units*

Exercise **6.1.1.1.1.1** *The point* $(8, e)$ *on the graph of* f_2 *transforms into* $(\boxed{8}, \boxed{2e})$ *on the graph of* $f_3(x) = 2f_2(x)$.

Exercise **6.1.1.1.1.1.1** *The last transformation, applied to* f_3*, is a shift down by 3 units. The point* $(8, 2e)$ *on the graph of* f_3 *transforms into* $(\boxed{8}, \boxed{2e-3})$ *on the graph of* $f_4(x) = f_3(x) - 3$.

Exercise **6.1.1.1.1.1.1.1** *Consider the following graphs.*



Which is the graph of f?

Multiple Choice:

(a) *Red graph* ✓

(b) *Blue graph*

(c) *Orange graph*

FT24.tex

In this problem, we'll step through an analysis of the function f defined by $f(x) = -3|2x - 5|$. The parent function of f is defined by $f_0(x) = |x|$. We will follow the point $(2, 2)$ on the graph of $|x|$.

Exercise 7 *What is the first transformation applied to f_0?*

Multiple Choice:

(a) *Vertical stretch by a factor of 3*

(b) *Horizontal compression by a factor of 2*

(c) *Shift right by 5 units* ✓

(d) *Reflection across the x-axis*

Exercise 7.1 *The point $(2, 2)$ on the graph of f_0 transforms into $(\boxed{7}, \boxed{2})$ on the graph of $f_1(x) = f_0(x - 5)$.*

Exercise 7.1.1 *What is the next transformation, applied to f_1?*

Multiple Choice:

(a) *Vertical stretch by a factor of 3*

(b) *Horizontal compression by a factor of 2* ✓

(c) *Reflection across the x-axis*

Exercise 7.1.1.1 *The point $(7, 2)$ on the graph of f_1 transforms into $(\boxed{7/2}, \boxed{2})$ on the graph of $f_2(x) = f_1(2x)$.*

Exercise 7.1.1.1.1 *What is the next transformation, applied to f_2?*

Multiple Choice:

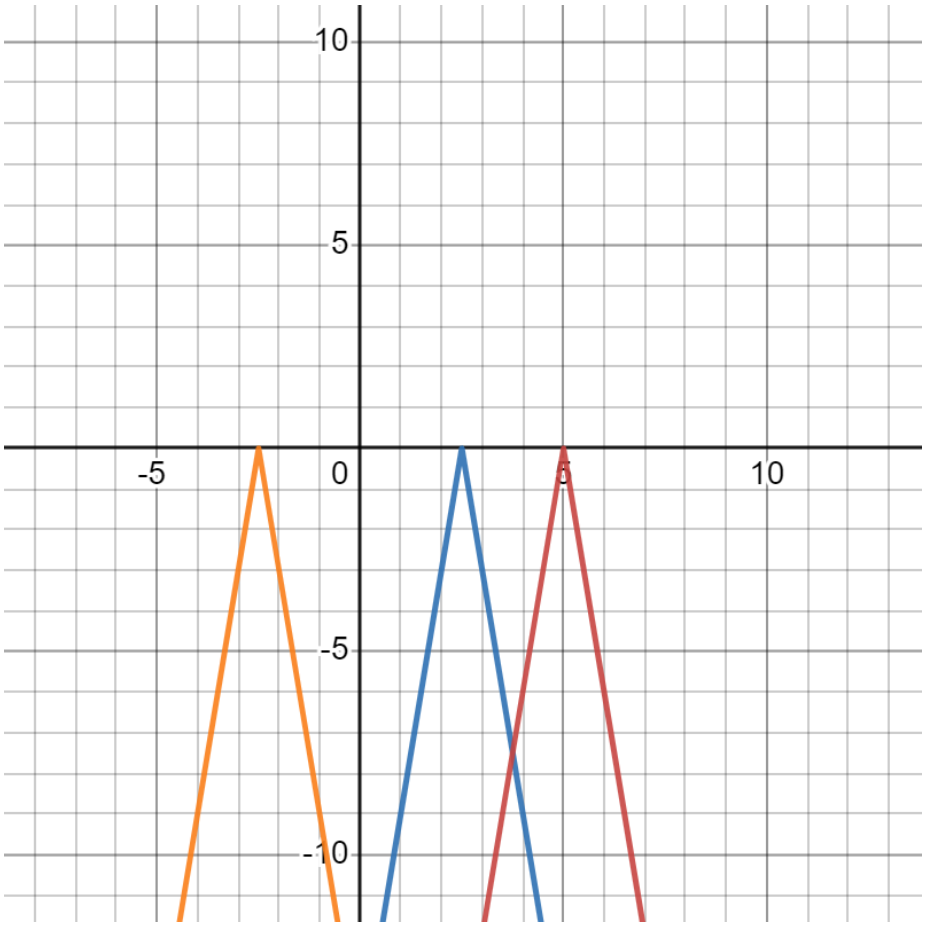
(a) *Vertical stretch by a factor of 3* ✓

(b) *Reflection across the x-axis*

Exercise 7.1.1.1.1.1 *The point $(7/2, 2)$ on the graph of f_2 transforms into $(\boxed{7/2}, \boxed{6})$ on the graph of $f_3(x) = 3f_2(x)$.*

Exercise 7.1.1.1.1.1.1 *The last transformation, applied to f_3, is a reflection across the x-axis. The point $(7/2, 6)$ on the graph of f_3 transforms into $(\boxed{7/2}, \boxed{-6})$ on the graph of $f_4(x) = -f_3(x)$.*

Exercise 7.1.1.1.1.1.1.1 *Consider the following graphs.*



Which is the graph of f?

Multiple Choice:

(a) *Red graph*

(b) *Blue graph* ✓

(c) *Orange graph*

FT16.tex

Exercise 8 *Let $f(x) = \sqrt{x}$. Find a formula for a function g whose graph is obtained from f from the given sequence of transformations. (1) horizontal compression by a factor of 2; (2) shift right 3 units; (3) shift up 1 unit*

$$g(x) = \boxed{(2(x-3))^{1/2} + 1}$$

FT18.tex

Exercise 9 *Let $f(x) = |x|$. Find a formula for a function g whose graph is obtained from f from the given sequence of transformations. (1) horizontal stretch by a factor of 3; (2) shift down 3 units; (3) shift right 1 unit; (4) vertical compression by a factor of 2*

$$g(x) = \boxed{\frac{1}{2}\left(\left|\frac{x-1}{3}\right| - 3\right)}$$

FT19.tex

Exercise 10 *Let $f(x) = \cos(x)$. Find a formula for a function g whose graph is obtained from f from the given sequence of transformations. (1) reflection across the y-axis; (2) shift left 3π units; (3) shift up 1 unit; (4) vertical stretch by a factor of 2*

$$g(x) = \boxed{2(\cos(-(x+3\pi))+1)}$$

FT17.tex

Exercise 11 *Consider the function f defined by $f(x) = 3(-x+2)^2 - 5$.*

Its parent function is $f_0(x) = \boxed{x^2}$.

Exercise 11.1 *Select all the transformations that are applied to the graph of f_0 to obtain the graph of f.*

Select All Correct Answers:

(a) *vertical shift up*

(b) *vertical shift down* ✓

(c) *vertical stretch* ✓

(d) *vertical compression*

(e) *horizontal shift left* ✓

(f) *horizontal shift right*

(g) *horizontal stretch*

(h) *horizontal compression*

(i) *reflection across x-axis*

(j) *reflection across y-axis* ✓

Exercise 11.1.1 *The vertical shift is down by $\boxed{5}$ units.*

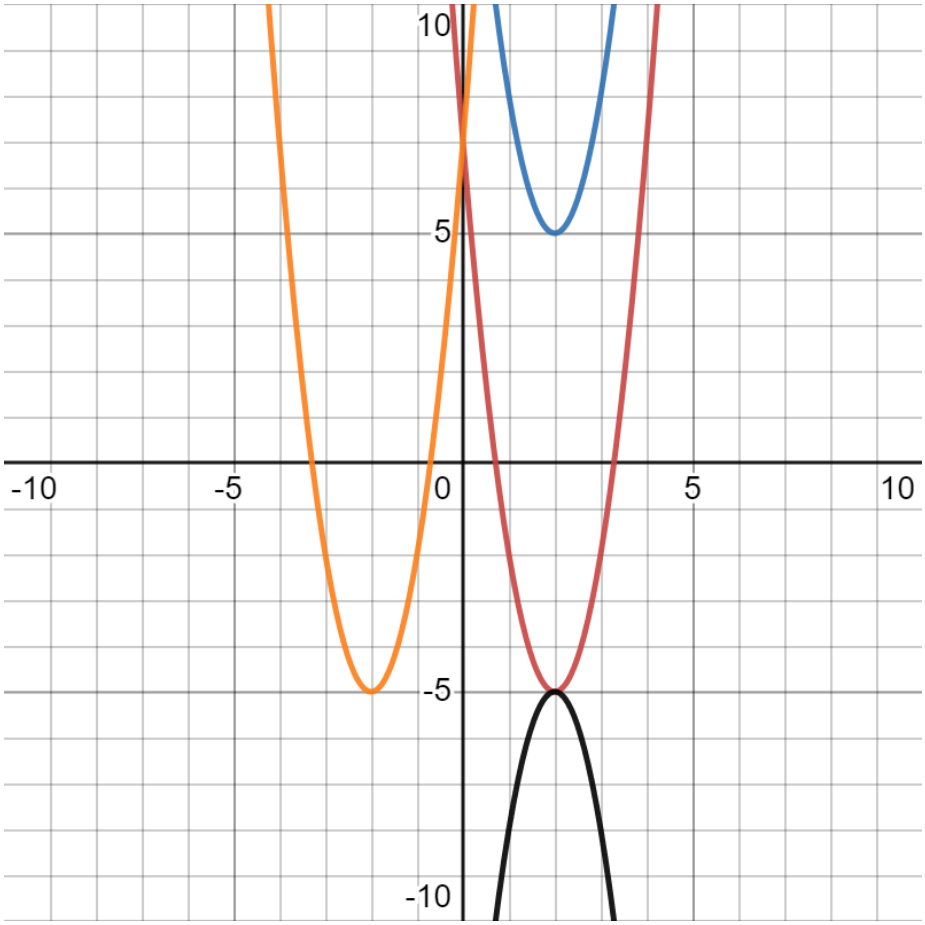
The vertical stretch is by a factor of $\boxed{3}$.

The horizontal shift is left by $\boxed{2}$ units.

Exercise 11.1.1.1 *To produce the graph of f starting with the graph of the parent function, in which order should you perform the following steps? Enter the numbers 1, 2, 3, and 4, accordingly. Use the order given in the textbook section.*

- *Vertical shift down 5 units.* $\boxed{4}$
- *Vertical stretch by a factor of 3.* $\boxed{2}$
- *Horizontal shift left 2 units.* $\boxed{1}$
- *Reflection across y-axis.* $\boxed{3}$

Which of the following graphs is the graph of $f(x)$?



Multiple Choice:

(a) *Red graph* ✓

(b) *Blue graph*

(c) *Orange graph*

(d) *Black graph*

FT20.tex

Exercise 12 *Consider the function f defined by $f(x) = -\ln(2x-1)+2$.*

Its parent function is $f_0(x) =$ $\boxed{\ln(x)}$.

Exercise 12.1 *Select all the transformations that are applied to the graph of f_0 to obtain the graph of f.*

Select All Correct Answers:

(a) *vertical shift up* ✓

(b) *vertical shift down*

(c) *vertical stretch*

(d) *vertical compression*

(e) *horizontal shift left*

(f) *horizontal shift right* ✓

(g) *horizontal stretch*

(h) *horizontal compression* ✓

(i) *reflection across x-axis* ✓

(j) *reflection across y-axis*

Exercise 12.1.1 *The vertical shift is up by* $\boxed{2}$ *units.*

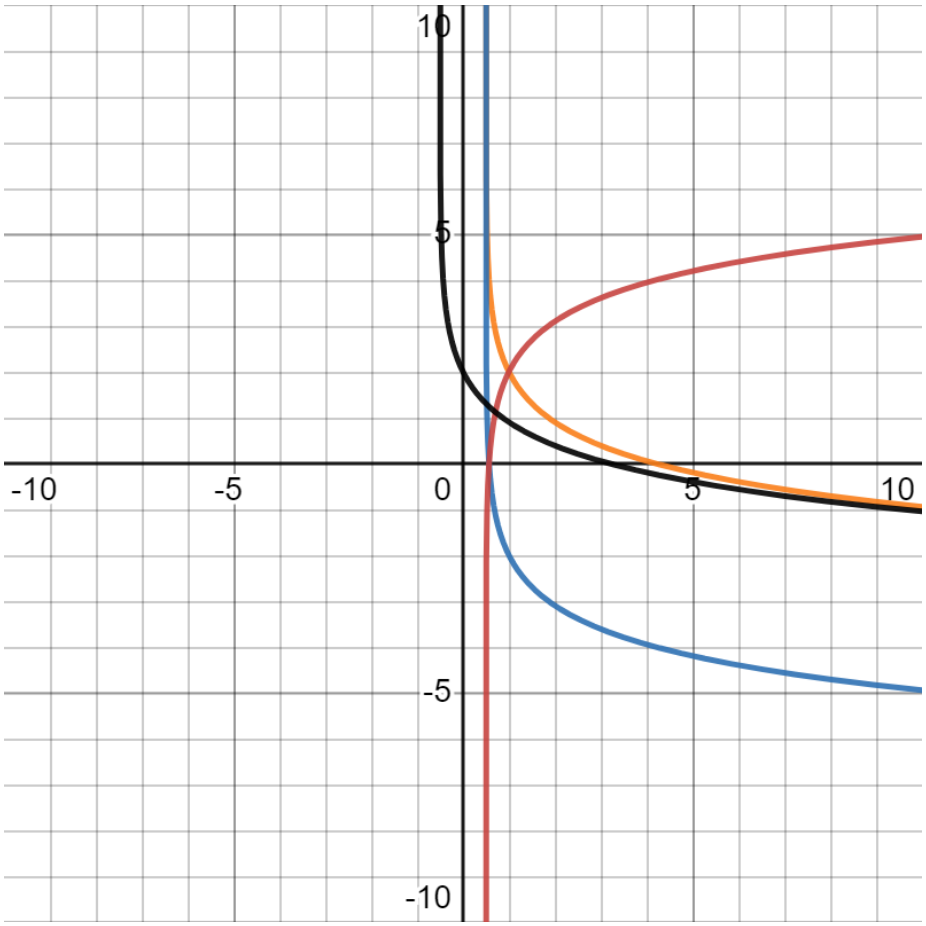
The horizontal shift is right by $\boxed{1}$ *units.*

The horizontal compression is by a factor of $\boxed{2}$.

Exercise 12.1.1.1 *To produce the graph of f starting with the graph of the parent function, in which order should you perform the following steps? Enter the numbers 1, 2, 3, and 4, accordingly. Use the order given in the textbook section.*

- *Vertical shift up 2 units.* $\boxed{4}$
- *Horizontal shift right 1 unit.* $\boxed{1}$
- *Horizontal compression by a factor of 2.* $\boxed{2}$
- *Reflection across x-axis.* $\boxed{3}$

Which of the following graphs is the graph of $f(x)$?



Multiple Choice:

(a) *Red graph*

(b) *Blue graph*

(c) *Orange graph* ✓

(d) *Black graph*

FT26.tex

Exercise 13 *Match the sequences of transformations of the function f below to their corresponding formulas.*

Sequence 1		*Sequence 2*	
(1)	*Shift up 1 unit*	*(1)*	*Shift up 1 unit*
(2)	*Shift left 1 unit*	*(2)*	*Shift right 1 unit*
(3)	*Vertical stretch by a factor of 2*	*(3)*	*Vertical stretch by a factor of 2*

Sequence 3		*Sequence 4*	
(1)	*Shift left 2 units*	*(1)*	*Shift left 2 units*
(2)	*Reflection across the x-axis*	*(2)*	*Reflection across the y-axis*
(3)	*Shift up 2 units*	*(3)*	*Shift up 2 units*

(a) *The function defined by $2(f(x-1)+1)$ corresponds to Sequence* $\boxed{2}$.

(b) *The function defined by $2(f(x+1)+1)$ corresponds to Sequence* $\boxed{1}$.

(c) *The function defined by $f(-x+2)+2$ corresponds to Sequence* $\boxed{4}$.

(d) *The function defined by $-f(x+2)+2$ corresponds to Sequence* $\boxed{3}$.

FT21.tex

Exercise 14 *Match the sequences of transformations of the function f below to their corresponding formulas.*

Sequence 1		*Sequence 2*	
(1)	*Shift down 2 units*	*(1)*	*Vertical stretch by a factor of 3*
(2)	*Shift left 1 unit*	*(2)*	*Shift left 1 unit*
(3)	*Vertical stretch by a factor of 3*	*(3)*	*Shift down 2 units*
(4)	*Reflection across the y-axis*	*(4)*	*Reflection across the y-axis*

Sequence 3		*Sequence 4*	
(1)	*Shift down 2 units*	*(1)*	*Vertical stretch by a factor of 3*
(2)	*Reflection across the y-axis*	*(2)*	*Reflection across the y-axis*
(3)	*Vertical stretch by a factor of 3*	*(3)*	*Shift down 2 units*
(4)	*Shift left 1 unit*	*(4)*	*Shift left 1 unit*

(a) *The function defined by $3f(-(x+1))-2$ corresponds to Sequence* $\boxed{4}$.

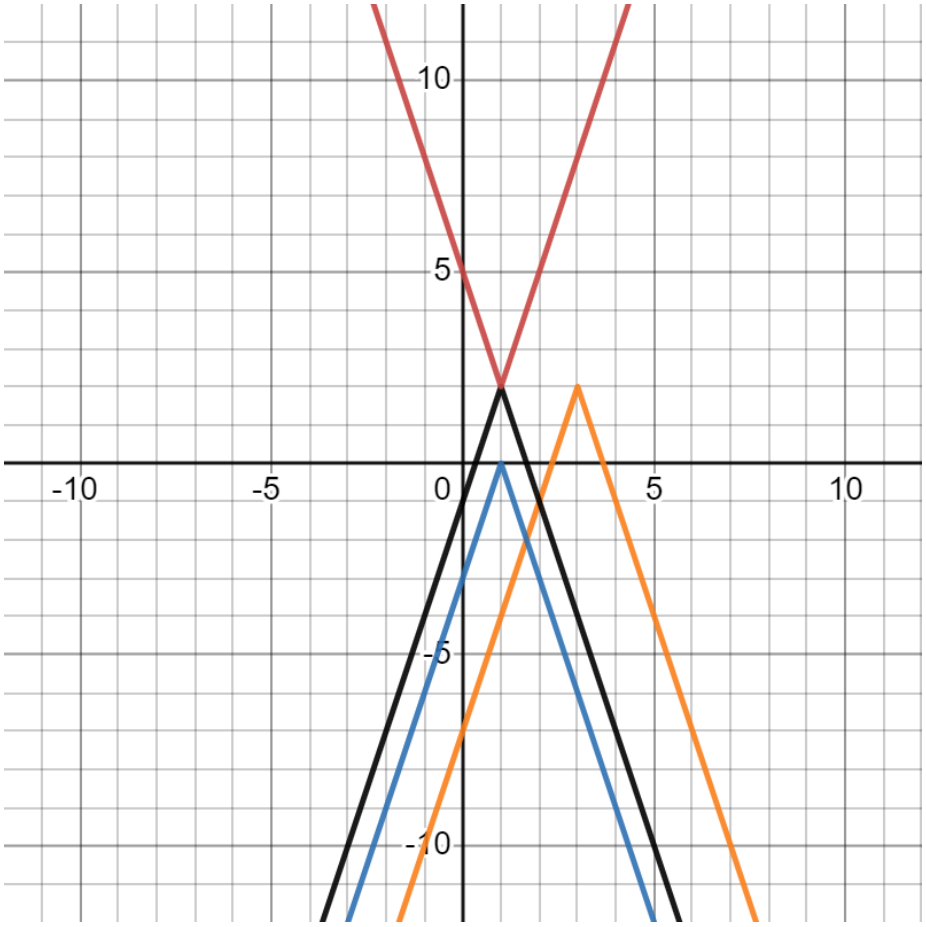
(b) *The function defined by* $3(f(-(x+1))-2)$ *corresponds to Sequence* $\boxed{3}$.

(c) *The function defined by* $3f(-x+1)-2$ *corresponds to Sequence* $\boxed{2}$.

(d) *The function defined by* $3(f(-x+1)-2)$ *corresponds to Sequence* $\boxed{1}$.

FT22.tex

Exercise 15 *Match the graphs below to their corresponding formulas.*



(a) *The graph of* $y=-|3(x-3)|+2$ *is the*

Multiple Choice:

(i) *Red graph*

(ii) *Blue graph*
(iii) *Orange graph* ✓
(iv) *Black graph*

(b) *The graph of* $y = -|3x - 3| + 2$ *is the*

Multiple Choice:

(i) *Red graph*
(ii) *Blue graph*
(iii) *Orange graph*
(iv) *Black graph* ✓

(c) *The graph of* $y = |3x - 3| + 2$ *is the*

Multiple Choice:

(i) *Red graph* ✓
(ii) *Blue graph*
(iii) *Orange graph*
(iv) *Black graph*

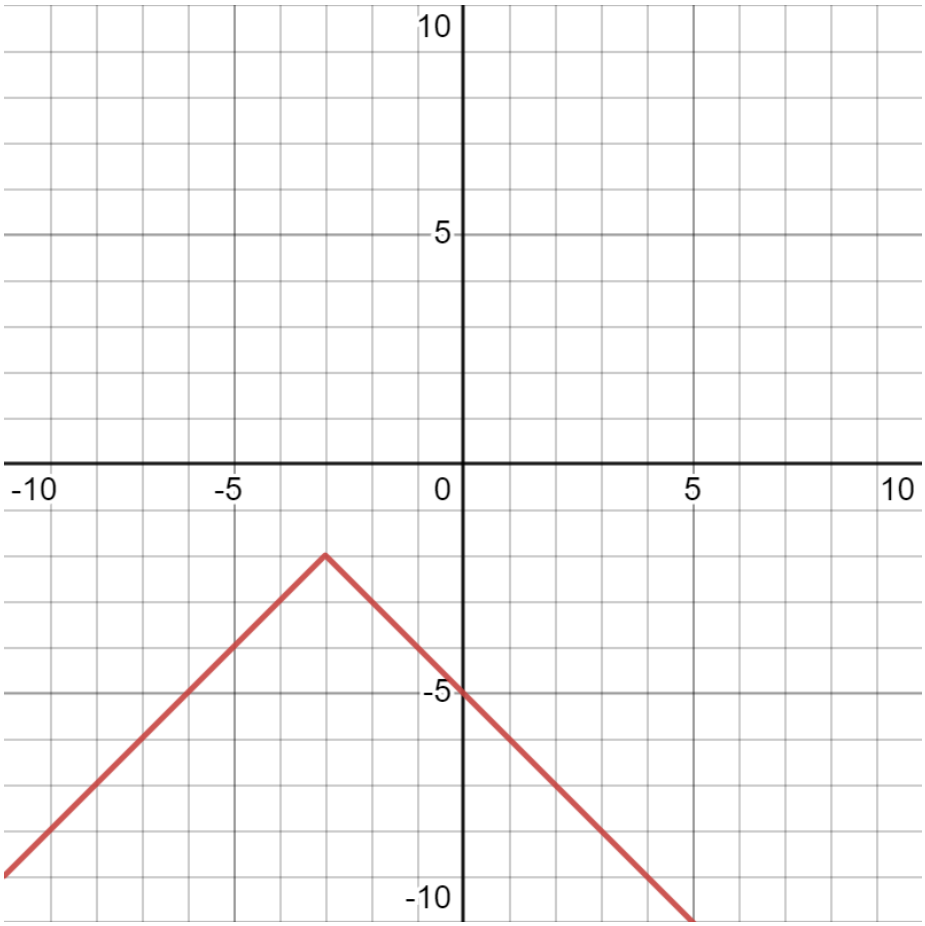
(d) *The graph of* $y = -|3x - 3|$ *is the*

Multiple Choice:

(i) *Red graph*
(ii) *Blue graph* ✓
(iii) *Orange graph*
(iv) *Black graph*

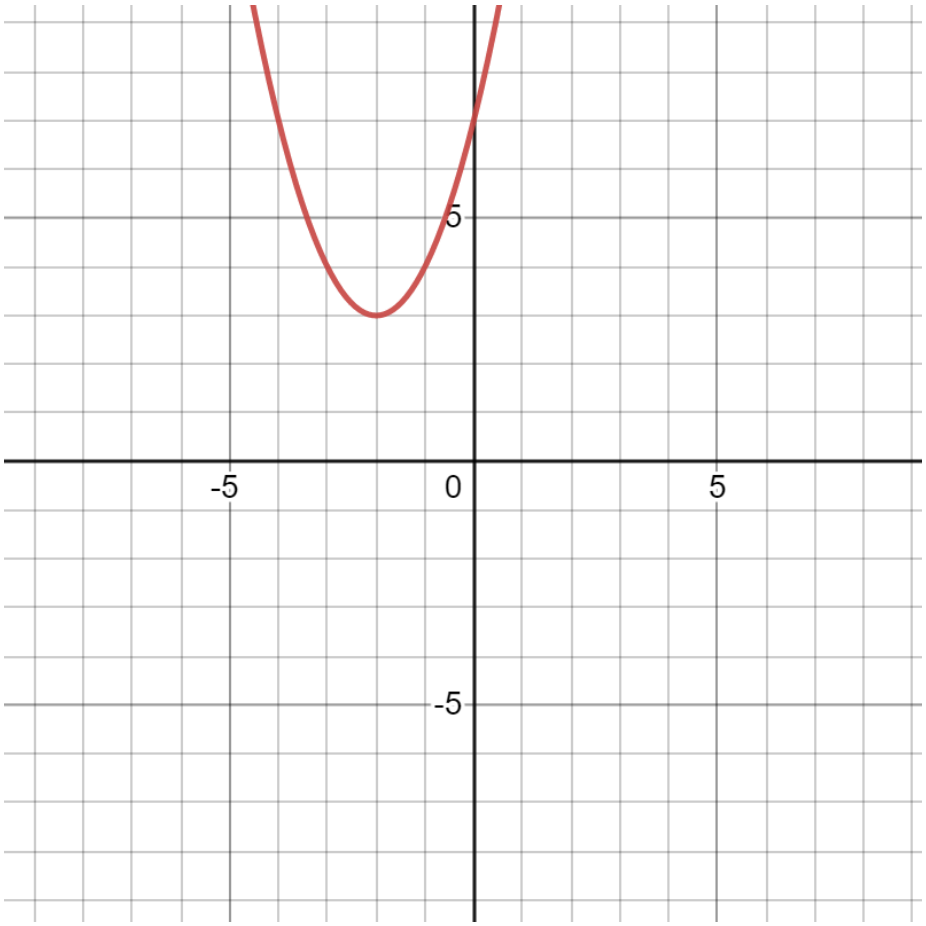
`FT25.tex`

Exercise 16 *Write down the formula for a function* f *that produces the graph below.*



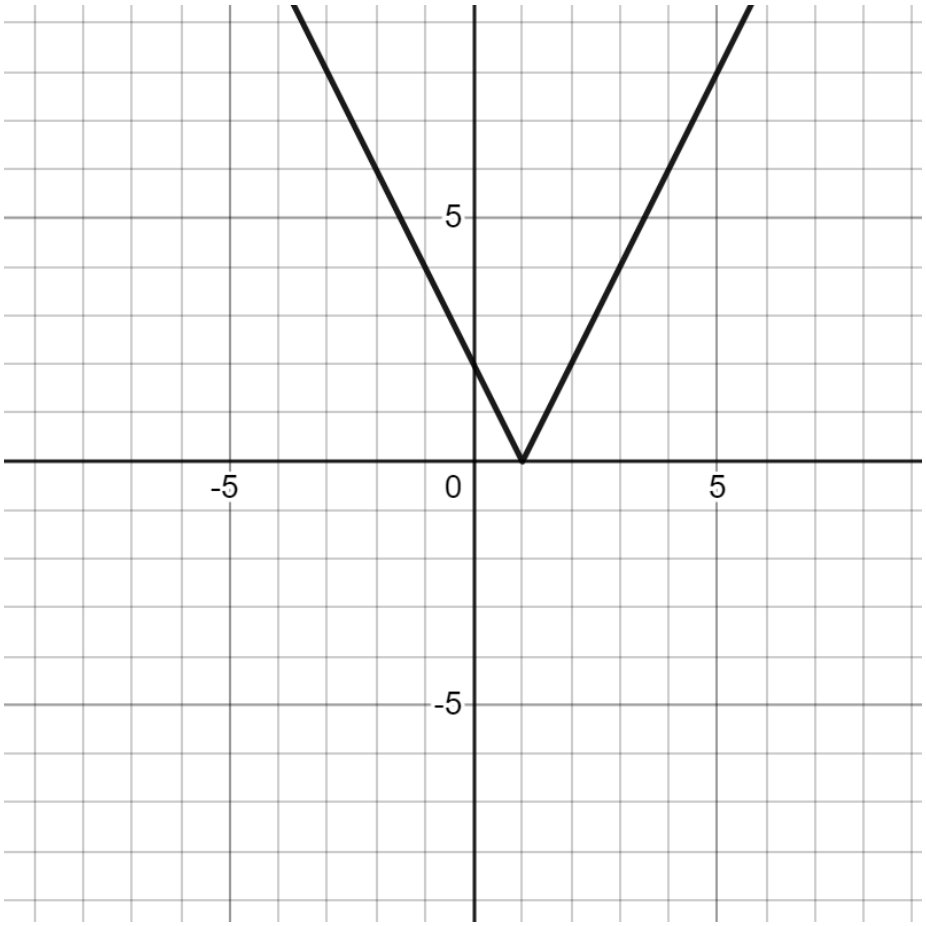
$f(x) = \boxed{-|x+3|-2}$

Exercise 17 *Write down the formula for a function f that produces the graph below.*



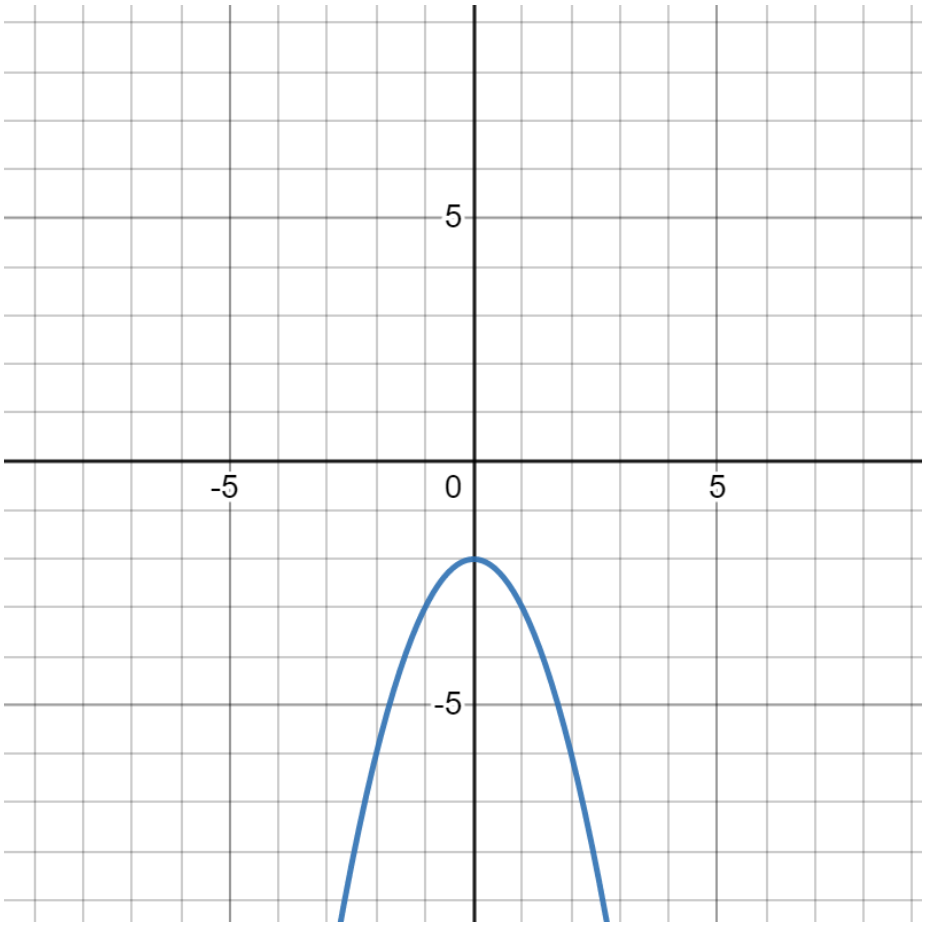
$f(x) = \boxed{(x+2)^2+3}$

Exercise 18 *Write down the formula for a function f that produces the graph below.*



$f(x) = \boxed{|2x - 2|}$

Exercise 19 *Write down the formula for a function f that produces the graph below.*



$f(x) = \boxed{-x^2 - 2}$

