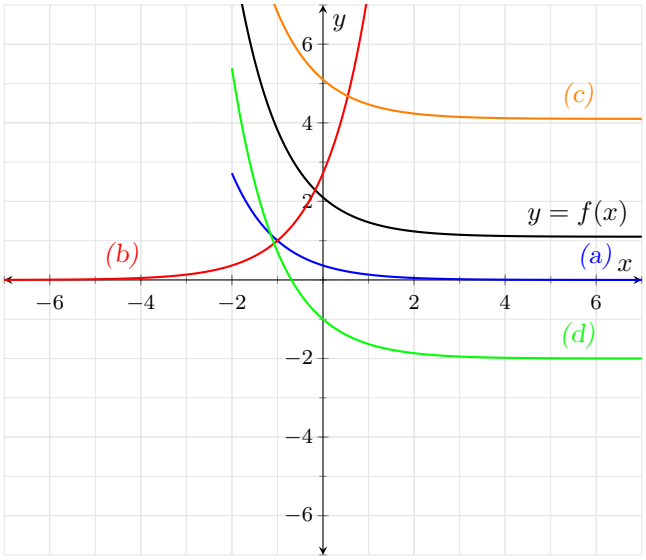
Part 1

Function Transformations

FT1.tex

Exercise 1 *If the graph of $y = f(x)$ is given in black below, which of the following graphs could be the graph of $y = f(x) + 3$?*



Multiple Choice:

(a) *Blue graph*

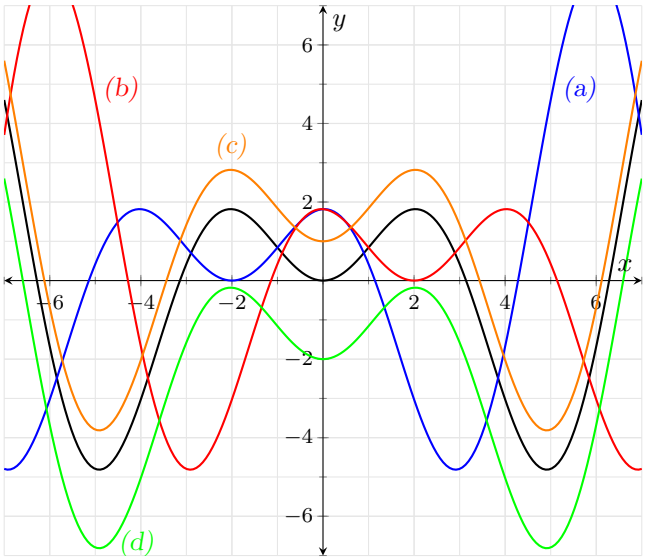
(b) *Red graph*

(c) *Orange graph* ✓

(d) *Green graph*

FT2.tex

Exercise 2 *If the graph of $y = f(x) = x\sin(x)$ is given in black below (namely, the only one below with $f(0) = 0$), which of the following graphs could be the graph of $y = f(x - 2)$?*



Multiple Choice:

(a) *Blue graph*

(b) *Red graph* ✓

(c) *Orange graph*

(d) *Green graph*

`FT3.tex`

Exercise 3 *Write the quadratic function* $f(x) = 4x^2+44x+33$ *in vertex-form, by completing the squares:*

$$f(x) = \boxed{4}\left(x - \boxed{-\frac{11}{2}}\right)^2 + \boxed{-88}.$$

The coordinates of the vertex are:

$$(h,k) = \left(\boxed{-\frac{11}{2}}, \boxed{-88}\right).$$

`FT4.tex`

Exercise 4 *Write the quadratic function $f(x) = -2x^2 + 10x + 13$ in vertex-form, by completing the squares:*

$$f(x) = \boxed{-2}\left(x - \boxed{\frac{5}{2}}\right)^2 + \boxed{\frac{51}{2}}.$$

The coordinates of the vertex are:

$$(h, k) = \left(\boxed{\frac{5}{2}}, \boxed{\frac{51}{2}}\right).$$

`FT5.tex`

Transform the given function by a vertical stretch with a factor of 2.

$$f(x) = x^3 - 4x$$

Exercise 5 *Which of the following represents a vertical stretch by a factor of 2?*

Multiple Choice:

(a) $2f(x)$ ✓

(b) $f(2x)$

Exercise 6 *What is the equation of the fransformed function?*

$$2f(x) = \boxed{2x^3 - 8x}$$

`FT6.tex`

Transform the given function by compressing it horizontally by a factor of 3.

$$f(x) = x^2 + x - 2$$

Exercise 7 *Which of the following represents a horizontal compression by a factor of 3?*

Multiple Choice:

(a) $\frac{1}{3}f(x)$

(b) $f(3x)$ ✓

(c) $f(\frac{x}{3})$

Exercise 8 *What is the equation of the fransformed function?*

$$f(3x) = \boxed{9x^2 + 3x - 2}$$

`FT7.tex`

Transform the given function by a vertical compression with a factor of 3.

$$f(x) = \frac{1}{x+2}$$

Exercise 9 *Which of the following represents a vertical compression by a factor of 3?*

Multiple Choice:

(a) $\frac{1}{3}f(x)$ ✓

(b) $f(3x)$

(c) $f(\frac{x}{3})$

Exercise 10 *What is the equation of the fransformed function?*

$$f(3x) = \boxed{\frac{1}{3x+6}}$$

`FT8.tex`

Transform the given function by a horizontal stretch with a factor of 2.

$$f(x) = x^3 + 2x^2 - 4x + 8$$

Exercise 11 *Which of the following represents a horizontal stretch by a factor of 2?*

Multiple Choice:

(a) $2f(x)$

(b) $f(2x)$

(c) $f(\frac{x}{2})$ ✓

Exercise 12 *What is the equation of the fransformed function?*

$$f(\frac{x}{2}) = \boxed{\frac{1}{8}x^3 + \frac{1}{2}x^2 - 2x + 8}$$

`FT9.tex`

Describe how to transform the graph of f into the graph of g

$f(x) = (x-1)^2$ and $g(x) = -(x+3)^2$

Exercise 13 *What type of shift occurs?*

Multiple Choice:

(a) *horizontal shift to the right*

(b) *horizontal shift to the left* ✓

(c) *vertical shift up*

(d) *vertical shift down*

(e) *no shift*

Exercise 14 *How much of horizontal shift to the left?*
$\boxed{4}$ *units*

Exercise 15 *What type of stretch or shrink occurs?*

Multiple Choice:

(a) *horizontal shrink*

(b) *horizontal stretch*

(c) *vertical stretch*

(d) *vertical shrink*

(e) *no shrink or stretch* ✓

Exercise 16 *What type of reflection occurs?*

Multiple Choice:

(a) *reflection across the x axis* ✓

(b) *reflection across the y axis*

(c) *no reflection*

`FT10.tex`

Describe how to transform the graph of f into the graph of g

$f(x) = (x+2)^3$ and $g(x) = -(x-1)^3$

Exercise 17 *What type of shift occurs?*

Multiple Choice:

(a) *horizontal shift to the right* ✓

(b) *horizontal shift to the left*

(c) *vertical shift up*

(d) *vertical shift down*

(e) *no shift*

Exercise 18 *How much of horizontal shift to the right?*
$\boxed{3}$ *units*

Exercise 19 *What type of stretch or shrink occurs?*

Multiple Choice:

(a) *horizontal shrink*

(b) *horizontal stretch*

(c) *vertical stretch*

(d) *vertical shrink*

(e) *no shrink or stretch* ✓

Exercise 20 *What type of reflection occurs?*

Multiple Choice:

(a) *reflection across the x axis* ✓

(b) *reflection across the y axis*

(c) *no reflection*

`FT11.tex`

Exercise 21 *Find the equation of the reflection of f across the x axis and the y axis.*

$f(x) = x^3 - 5x^2 - 3x + 2$

Reflection across the x-axis: $g(x) = \boxed{-x^3 + 5x^2 + 3x - 2}$

Reflection across the y-axis: $g(x) = \boxed{-x^3 - 5x^2 + 3x + 2}$

`FT12.tex`

Exercise 22 *Find the equation of the reflection of f across the x axis and the y axis.*

$f(x) = 2\sqrt{x+3} - 4$

Reflection across the x-axis: $g(x) = \boxed{-2\sqrt{x+3}+4}$

Reflection across the y-axis: $g(x) = \boxed{2\sqrt{3-x}-4}$

`FT13.tex`

Exercise 23 *Let $A = f(r)$ be the area of a circle of radius r.*

a. *Write a formula for* $f(r) = \boxed{\pi r^2}$.

b. *Which expression represents the area of a circle whose radius is increased by 5%?*

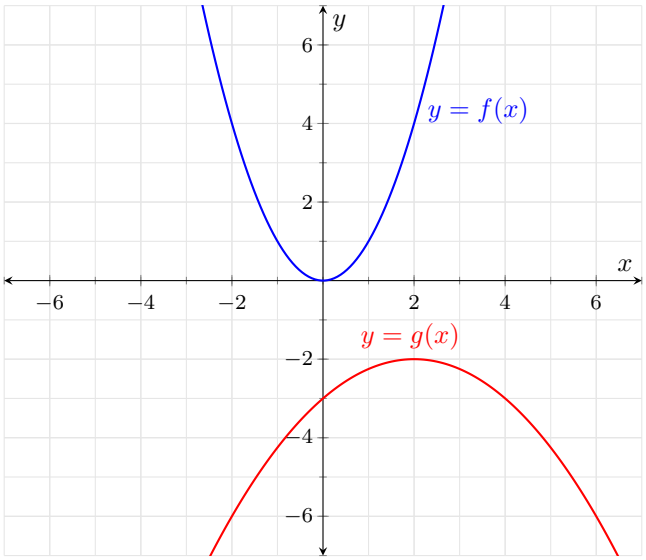
Multiple Choice:

(i) $f(r + 0.05)$
(ii) $0.05f(r)$
(iii) $f(r) + 0.05$
(iv) $f(5 + r)$
(v) $f(1.05r)$ ✓

c. *By what percent does the area increase if the radius is increased by 5%? Round to the nearest 0.01%. Answer:* $\boxed{10.25}$%.

`FT14.tex`

Exercise 24 *Consider the functions $f(x) = x^2$ and $g(x) = -\frac{x^2}{4} + x - 3$. Their graphics are below — $y = f(x)$ in blue and $y = g(x)$ in red.*



To produce the graph of g in terms of the graph of f, in which order should you perform the following steps? Enter the numbers 1, 2, 3, and 4, accordingly.

Hint: *Finding the concrete relation $g(x) = af(bx - c) + d$ might be helpful.*

- *Horizontal shift right 1 unit.* $\boxed{3}$
- *Vertical shift up 2 units.* $\boxed{1}$
- *Reflection across the x-axis.* $\boxed{2}$
- *Horizontal stretching by a factor of 2.* $\boxed{4}$.

`FT15.tex`

Exercise 25 *Given a function f, which of the following represents a vertical translation of 2 units upward, followed by a reflection across the y-axis?*

Multiple Choice:

(a) $y = f(-x) + 2$ ✓

(b) $y = f(2 - x)$

(c) $y = f(x) - 2$

(d) $y = 2 - f(x)$

(e) $y = -f(x-2)$

FT16.tex

Exercise 26 *Let $f(x) = \sqrt{x}$. Find a formula for a function g whose graph is obtained from f from the given sequence of transformations. (1) horizontal compression by a factor of 2; (2) shift right 3 units; (3) shift up 1 unit*

$$g(x) = \boxed{(2(x-3))^{1/2} + 1}$$
